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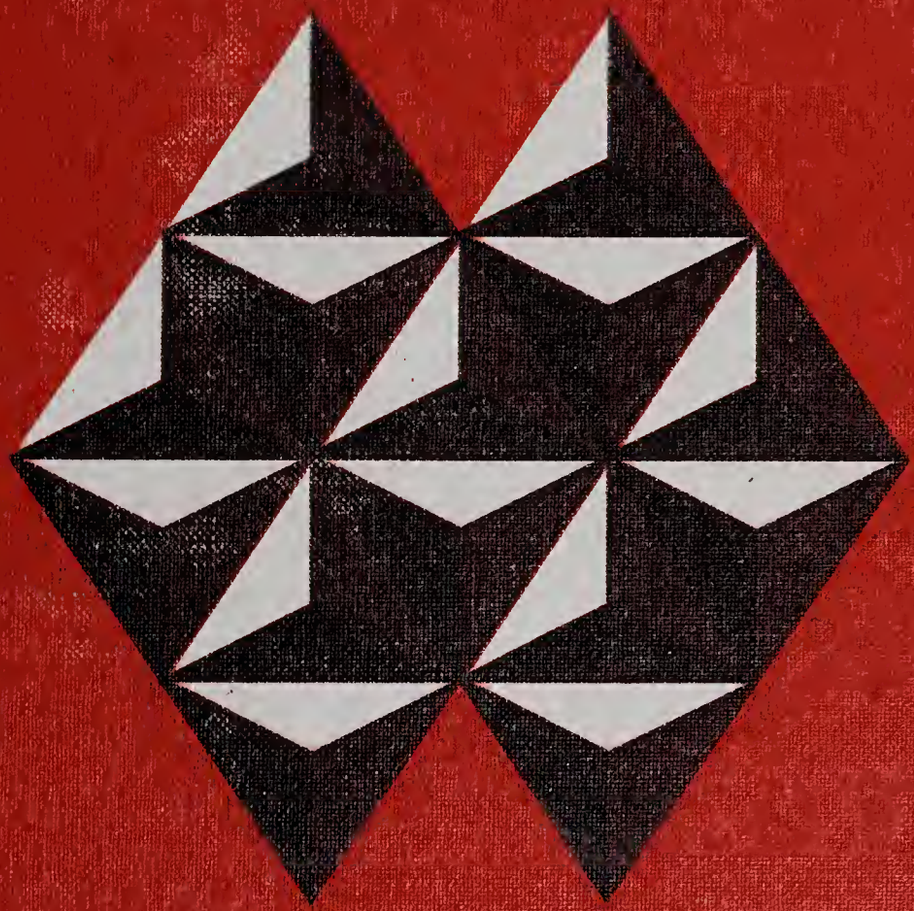

BOOK THREE

- MANUAL
- FACSIMILE OF TEXT
- ANSWERS

CURRICULUM

GINN MODERN MATHEMATICS SERIES

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Teaching Mathematics We Need

GUIDE · FACSIMILE OF TEXT · ANSWERS

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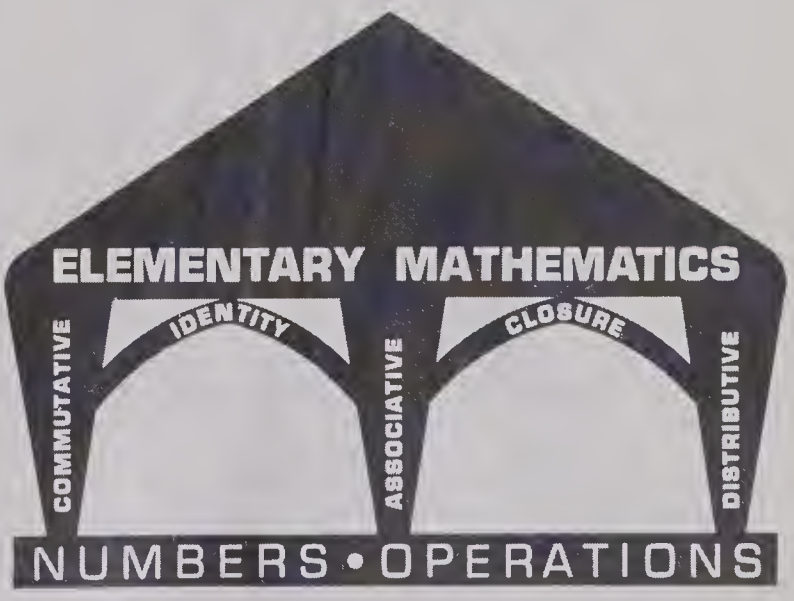

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Throughout the entire period when the School Mathematics Study Group was conducting its writing and experimentation for grades 1–8, several of the *Mathematics We Need* authors, as members of that group, participated in the program and made many contributions to the experimentation. In turn, the *Mathematics We Need* program has benefited from participation by its authors in the SMSG projects.

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Your Teachers' Edition and Modern Mathematics

Teaching Modern Mathematics Confidently

Completeness is the key word which describes a program that may be presented with confidence by a large majority of elementary-school teachers. Achievement of *completeness* is time-consuming in the writing stage; *completeness* is tremendously saving of time for the program in use.

Completeness, as represented in *Mathematics We Need*, implies

- careful blending of mathematics that is new in the curriculum for grades 1–8 and mathematics that is time-tested for those grades;
- adjustment that utilizes the excellent growth that teachers have made in the last three decades;
- well-planned use of the many important principles of teaching and learning of elementary mathematics that have become available in the last thirty years;
- a curriculum organization which provides an adequate variety of presentations at a sensible pace so that thorough understanding is achieved before highly-technical symbols and procedures are introduced;
- textbooks which, instead of merely being excellent mathematical outlines, guide the discovery of mathematical ideas superbly through sufficient and varied verbal questioning and maintain and extend what is learned through strong programs of enrichment, practice, application, and testing;
- teachers' editions so carefully prepared that day by day they provide for every need in building depth of background and in individualizing instruction by including a wide range of suggestions through pre-book, textbook, and post-book lessons;
- supplementary devices and printed materials skillfully integrated to make the total mathematics program a superb instrument for teaching and for learning;
- a program so carefully organized that it can be introduced into a school system with a minimum of confusion and without the sacrifice of time taken from other important subjects of the curriculum which, of course, are also demanding time.

Preparing to Teach a Modern Program

Before we examine more closely with you the details of the *Mathematics We Need* program, we would like to say more about the all-pervasive problem of preparing to teach a modern program.

It is possible to work out programs in elementary mathematics so advanced in character, so skeletonized in form, and geared to so rapid a pace of learning that *nearly all* teachers would encounter great difficulty in teaching them. This, though, is far from the situation you will encounter if you are teaching *Mathematics We Need*. It is the belief of the authors of this series that you have a program which is mathematically powerful, yet is also highly teachable, highly learnable by all children, and highly useful to them.

Some of those who are openly critical regarding the inadequate mathematical background of teachers propose that, if teachers will take a single short, hurried, highly technical in-service course in mathematics, they can be readied to teach modern mathematics. From this hasty survey, teachers are supposed to learn all the complexities and refinements of the subject so as to be able to identify the characteristics of the difficult concepts and to ask the right questions and plan the best activities for pupil discovery. More than that,—it is presumed that if teachers are to teach a program in which the printed material is hardly more than a mathematical outline, they will be sufficiently prepared so they can write the supplementary materials which are not available in the skeleton of prepared materials, and can themselves devise what is needed for practice, maintenance, problem-solving, caring for individual differences, and evaluation.

Those of us who were responsible for the preparation of *Mathematics We Need* recognize that brief courses have value in introducing concepts. Our concern is that such a once-over course, isolated from the classroom, cannot come anywhere near readying teachers for their new duties. This is so, because even those who have strong orientation toward mathematics absorb many misconceptions in the single exposure provided in the course. Even

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one or two regular mathematics courses leave serious gaps. The only solution that we could discern was to take extra years to prepare a program that will provide you each day with a maximum of help. Please note that we say "each day." Our program gives you day-by-day background suggestions to assist you in deepening and refining your mathematical understanding; day-by-day helps in guiding correct development of concepts and skills; and an abundance of day-by-day materials for maintenance, practice, problem-solving, individualizing instruction, and evaluation.

Facsimile of Pupil's Page	Teacher's Page
Title and clue caption to pinpoint content Oral developmental questions to assure asking of right questions for discovery and to clarify concepts for pupils and teachers Variety of written work, including practice to further clarify and to nudge learning in the direction of permanence References to extra activities, extra practice, and extra problems in back of book to help make very complete the program of caring for individual differences	Objectives and Background to clarify and extend the ideas for teachers Pre-Book Lesson that provides the first stage for discovery Notes for Using the Text Page which provide guiding suggestions so that important aspects in the text are emphasized Individualizing Instruction which provides wide range of suggestions as a capstone of the overall program of individualization

Using the Day-by-Day Helps

The highlights of the general organization of a pupil's developmental page and corresponding teacher's page are outlined above. To make more evident to you the value of our suggestions and aids, we discuss with you below the stages that you may want to follow as you plan a lesson.

The items mentioned for the pupil's page are characteristic of developmental pages. Besides developmental pages, there are many pages devoted to resurveying, reteaching, exploring, enrichment, problems, practice, maintenance, and testing.

The facsimiles of the pupil's pages in this teachers' edition have the further help of answers and annotations printed thereon, over and above the content of the pupil's page as such.

• As you start to plan a lesson, you should have clearly in mind the underlying purposes of the authors in writing that text page (hereafter referred to as "pupil's page"). These purposes are identified on the pupil's page by titles and clue captions, and they are further clarified by carefully-stated **Objectives** on the facing page of teaching suggestions (hereafter referred to as "teacher's page"). In addition to orienting you to the underlying purposes, these several helps will enable you to determine whether the pupil's page is to be used to resurvey, reteach, explore, develop new concepts, extend, enrich, improve problem-solving ability, provide practice, maintain learning, or test understanding or skill.

• You will also be concerned about having a thorough understanding of each mathematical concept or idea which is to be taught. As a step in this direction, (a) read through the oral development and the associated reproduced answers on the pupil's page (oral work is designated by [O]; written work by [W]), and then (b) read the **Background** section of the teacher's page. Note that we have not chosen to discuss all mathematical ideas at once in a single place, as in a few pages in the end matter, because this form of treatment, separate from the context in which the ideas are taught, would probably be more confusing than helpful. Instead, in accordance with our day-by-day plan, we cover each idea thoroughly at the time when you teach it, thus enabling you to relate the idea directly to the content prepared for students. Moreover, this plan gives us the opportunity to approach the idea with you in a variety of ways so

that the idea is brought to the work on later pupil pages and, consequently, discussed again in Background discussions on many of those pages.*

• After reading the Background section, read again the oral development on the pupil's page to help you in planning your procedure for guiding discovery on the part of your pupils. Note the order of the statements made and the questions asked. Attempts to guide discovery are successful to the degree that learners are led logically step by step from what they know to the concluding idea or understanding which is sought. Stated differently, clumsy and non-sequential questioning gets nowhere and defeats the purpose of instruction.

We take pride in the developmental lessons in our program. In the writing of each lesson, questions have been worded carefully, only to be reworded and then reworded again in the interest of clarity; and questions have been tried out in one sequence after another in search of the best order attainable. As we have said, we take pride in the final product, but our pride is tempered with awareness that we have done no more than authors of texts should do. Many teachers starting on a new program will not find it possible, on their own, to devise a sequence of questions adequate to guide discovery effectively, and they are entitled to what help can be given them. Teachers who are more sophisticated in mathematics will see in what we offer a means of checking and possibly improving their own patterns of questioning.

The introductory problem and the sequence of questioning in the oral development have these further values: (a) the introductory illustrative problem that is utilized to introduce the development often serves to motivate pupils; (b) the oral responses will help you keep track of your pupils' thinking; and (c) these developmental lessons enable you to help your pupils acquire skill in reading with understanding the closely-knit type of writing that is characteristic of mathematical material. Point (c) needs to be emphasized. Pupils who do not have elementary-school texts that require this kind of reading will almost certainly encounter serious difficulties when they deal with the heavily verbal texts at later grade levels.

• Now with the purposes, a clear understanding of the content to be taught, and the general plan of guided discovery of the pupil's page in mind, read the **Pre-Book Lesson** on the teacher's page. Here, too, we felt that we should take whatever time was necessary to work out with exceeding care model plans of instruction to precede the study of the text page—model plans only in the sense of illustrative patterns to be adapted as desired.

Seek these things from the suggestions in the Pre-Book section: a further opportunity to enhance your own understanding; an excellent technique for inventorying pupil background prior to more systematic development; a first approach to the idea—an approach that offers optimum opportunity for guided discovery, and one that is less abstract and is closer to pupils' experiences than the text development; and an opportunity to strengthen earlier learnings that are needed for the new topic.

• As you turn to the section on the teacher's page entitled **Using the Text Page**, you will be closely relating the suggestions in this section to the [O] and [W] sections of the pupil's page. It is at this point that you crystallize the plan that you will use for the portion of the lesson that pertains to the basic purpose of the page.

• In completing your planning of the lesson, you will need to be concerned regarding the extensions, practice, maintenance, problem-solving, and testing requirements that are so fundamental in caring for individual differences. *Mathematics We Need*, because it is a complete program, provides all of these. First, look at the pupil's page itself. You frequently will find on that page one or more sets of the following: practice items presented in interesting variations; maintenance exercises for concepts developed earlier; a section of enrichment; a set of problems; a brief test. Also, on a pupil's page, you often will find references to one or more of the following: **Reteaching** (except for grades 1–3), **Extra Examples, Extra Problems, Extra Activities.** These are references to the valuable reservoir at the end of the pupil's book. For obvious

* If you are interested in examining a number of the ideas at one time, first turn to **Terms with Illustrations,** page 375 of this Teachers' Edition. Then, for more detailed study, turn to pages xv-xvii to locate references in this Teachers' Edition to pages where further discussion of the ideas may be found.

reasons, reteaching is included in the first chapters in grades 2 and 3 instead of in the reservoir.

• Although we have moved a long way toward completeness in building into the textbook itself a very comprehensive program for *caring for individual differences*, we have not been satisfied to stop there. You will find on many teacher's pages that there are helpful suggestions for *Slower Learners*, *All Pupils*, and *More Capable Children* under the general heading **Individualizing Instruction.** This wealth of materials for individualizing your teaching combined with all that is provided in the pupil's book means that you can help each pupil achieve at his highest level.

In summary, then, as exemplified by almost any pupil's page and its accompanying teacher's page, we have tried to give you a self-contained program that day by day relates the content of each day's work to the total mathematics program; that helps you, the teacher, to have the kind of clear and thorough understanding of the content that is needed for superior teaching; that has a model set of questions for each development to help each pupil, largely through his own efforts, discover concepts and procedures; that gives you guidance in setting up the highly important pre-book lessons; and that provides for you complete programs of practice, maintenance, problem-solving, enrichment, and testing, over and above a most carefully-worked-out sequence of mathematical content.

Notes Regarding the Mathematics We Need Program

Building on Mathematical Structure

In elementary-school mathematics, pupils use and study progressively more sophisticated sets of numbers—counting numbers, whole numbers, fraction numbers (non-negative rational numbers), integers, rational numbers, and real numbers. As new sets of numbers are investigated, operations on these sets are defined and properties of these operations are observed and generalized. In turn, these properties supply the justifications for various algorithms such as those that are used for finding sums or products. This explicit study of various number systems, of operations on their sets of elements, of properties of these operations, and of applications of these properties is frequently called *study of mathematical structure.**

In the *Mathematics We Need* series, stress is placed on a realistic developmental program leading to having pupils eventually see elementary mathematics as a structured subject. The program has a first cycle that introduces during the first four years the elements of the structure in a somewhat intuitive and inductive approach—this builds understandings and skills carefully, unencumbered by extensive symbolism. Then, in a second cycle, the program becomes more explicit and more symbolic so as to provide optimum progress toward a logical, formal, deductive approach to the study of mathematics.

In the early school years, the concept of a set is developed through well-planned study of natural objects, and then by extension to sets of numbers. The sets of counting, whole, and fraction numbers are carefully studied and the operations on these sets are progressively discovered. Similarly, the Commutative and Associative Properties of Addition and Multiplication, the Distributive Property of Multiplication over Addition, zero as the identity element for addition, one as the identity element for multiplication, and additive and multiplicative (mŭl′tĭ·plĭ·kā′tiv) inverses are studied first through observation in many examples. Then after pupils have encountered these concepts repeatedly in a variety of situations and have generalized them at their own level of maturity and understanding, technical names are attached to various sets of numbers and to properties of the operations on these sets.

In the middle grades, the properties of operations are used explicitly to justify the procedures in multiplication algorithms and long-division algorithms. Properties of operations on non-negative rational numbers are explicitly stated and named and are used to justify procedures in adding, subtracting, multiplying, and dividing in this set of numbers.

In grades seven and eight the interrelation of the various sets of numbers studied previously is observed and diagrammed. The negatives of whole numbers are introduced and the set of integers is identified. The properties of the operations on these various sets are summarized and are symbolized in forms such as $a + b = b + a$. In these grades, too, other number systems are introduced, non-negative and negative rational numbers are recognized as making up the set of rational numbers, and numbers that correspond to points on the number line but that cannot be expressed in the form $\frac{a}{b}$, with a and b integers and $b \neq 0$, are named "irrational numbers." Finally, the union of the sets of rational and irrational numbers is found to be the set of real numbers. As these sets of numbers are introduced through applications, their properties are observed and generalized. Throughout this study, the properties of the operations on these sets are seen to justify more complex computations and are used to justify short cuts, mental procedures, and steps in solving open mathematical sentences.

It should be emphasized that the program as a whole involves a gradual, progressively more explicit development starting in the primary grades, extending through grade 8, and then continues into the high-school books in the Ginn Modern Mathematics Series. The pupil is led naturally

* The illustration on the title page of the textbook may be used to indicate to pupils the general goal of the book in the study of structure.

from intuitive reasoning to the threshold of formal study of the number systems. Ideas are introduced when they can be made reasonable to the pupil. Once they are introduced, they are *used*. The pupil is not artificially or unrealistically subjected to formalism, technical terms, and symbols until the ideas are well established.

With such a program, the pupil is helped to understand what he is doing and sees the study of mathematics as a unified, rational whole. At the same time he has a sound basis for a logical deductive study of mathematics in the years that follow.

Building on Mathematical Strands

There are certain basic mathematical concepts, fundamental principles, and computational techniques which permeate and unify the entire mathematics program from grade 1 through grade 8. These we call "mathematical strands." Many strands are introduced in the early grades with provision for consistent reinforcement and extension at each succeeding grade level. At whatever point the first presentation is made, opportunities are given for building on these strands, a number of which are mentioned below:

1. Sets and Operations on Sets—studied as the basis for the development of whole-number concepts, for the understanding of operations on numbers, and for clarification of geometric ideas

2. Number—study of the set of whole numbers extended to the sets of non-negative rational numbers, integers, non-negative and negative rational numbers, and real numbers

3. Numeration Systems—thorough study of our base-10 place-value system; comparison with place-value systems having bases other than 10

4. Operations on Sets of Numbers—study of the meaning of operation, certain defined operations, and the properties of the operations and use of these ideas to make logical the computational techniques which are in general use

5. Estimation and Methods of Mental Computation

6. Mathematical Sentences —emphasis upon symbols for operations ($+, -, \times, \div, \sqrt{\ }$) and symbols which indicate equality, inequality, and order ($=, \neq, <, >, \leq, \geq$); true, false, and open mathematical sentences

7. Problem-Solving—extended from simple one-step situations to complex situations which require the statement and solution of involved mathematical sentences

8. Non-Metric Geometry—starting from the recognition of common geometric figures and progressing gradually to the study of properties of such figures as prisms, pyramids, cylinders, cones, and spheres

9. Measurement—a strand which utilizes many concepts of metric geometry

To illustrate more fully the development of a strand and the manner in which such a sequence contributes to a unified whole, let us examine in some detail a strand that is somewhat closer to applied mathematics—the measurement strand. Illustrations of certain other strands were provided in the section entitled *Building on Mathematical Structure*.

The study of measurement progresses from simple beginnings when pupils intuitively sense a difference in sizes of objects, to the need for comparison through the selection and use of appropriate units of measurement, to the development in the upper grades of ideas of precision. Experiences in the measurement of a variety of physical objects in the environment are provided, but the discussion that follows will be limited to the developmental sequence for the measurement of length, area, and volume.

Basic to the concept of measurement in the elementary grades is the idea of a unit of measurement which is of the same nature as the thing to be measured. The measure of a segment is in terms of a unit that is a line segment; the measure of an angle is in terms of a unit that is an angle; the measure of a region is in terms of a unit that is a region, and so on. However, the unit need not be a standard unit, and the initial phase for the study of each type of measurement is the use of a variety of arbitrary units. Thus, attention is focused upon the process rather than upon the unit.

Pupils see that they may express the length of their desks, of the table, or of the bookcase in such arbitrary units as "finger inch" or "pencil." They may find the length of the room in "paces." Emphasis is placed upon the idea that the measure of a segment is a number which indicates how

many times the unit is applied in the distance from one end point to the other end point of the segment measured. To name a length is to mention both the number that is the measure and the unit of measurement—for example, 5 feet.

As pupils experiment with arbitrary units they find that convenience in measuring requires the use of both large and small units. These and other experiences bring to pupils a realization of the necessity for standard units and the need to develop skill in the use of standard measuring devices scaled in inches, centimeters, feet, yards, and so on.

Since the measure of a segment may not always be a whole number, a logical approach to the study of rational numbers is provided. Throughout grades 4–6, measurement units and the work with rational numbers are closely related and, hence, they reinforce each other.

The part of the measurement strand which deals with the measure of the region builds on the basic concepts developed for the measurement work with segments. Pupils first use unit regions having various shapes—triangular, rectangular, circular, square. When a square region is recognized as the most advantageous unit, then the square inch, the square centimeter, the square foot, and so on are derived from the corresponding standard units for segments. In all phases of the measurement strand, there is emphasis upon the idea that the smaller the unit used, the more precise the measurement.

When measurement of volume is developed in grade 6 and then extended in grades 7 and 8, the basic measurement ideas described for segments and regions are utilized. Further extension of the work in measurement includes the development of such formulas as those for computing perimeter, area, and volume.

Values of Precision in Mathematical Language

In a modern program, more emphasis is placed on the ideas underlying mathematics and more attention is given to abstract patterns of thought. To achieve all this, new symbolism and terminology are needed in the early grades. However, this symbolism and terminology must not be arbitrarily rushed into the program—what is introduced must be precise but, at the same time, it must be introduced gradually. This introduction should occur after sufficient understanding has been achieved so that the use of the symbol or term will promote more complete understanding of the concept under study. Once introduced, such symbolism and terminology has to be maintained throughout the program with frequent reinforcement and extension.

The degree of precision used at a certain level should depend not only on the subject matter of that level but also on how well it will hold up under more advanced treatment. For example, the statement "When numbers are added, the sum is greater than any addend." seems very precise if the only numbers that are known to the pupil are the counting numbers 1, 2, 3, and so on. However, the lack of precision becomes evident when the pupil learns about the set of whole numbers, with its new member 0, and recognizes that for $3 + 0$, for instance, the sum is not greater than the addend 3.

Traditional mathematical areas—algebra, geometry, trigonometry—still form the core of the secondary-mathematics program, but this core is enveloped by modern notation, modern approaches and, to some extent, new concepts. Such a program is easily studied if good foundations have been laid in the elementary program. Thus, precision in an elementary program is necessary to clarify and simplify the material being studied at any particular level and to lay sound foundations for clear and concise definitions for extensive development of work in later grades.

Geometry provides a good illustration of the value of careful use of language. Precision in presenting geometric concepts is needed in order that the pupil understand, early in his training, the difference between metric and non-metric geometry and the difference between abstract sets of points and geometric shapes in the physical universe. Such precision is necessary not only for further study in geometry but also in order to understand better the subject matter that is being presented in many of the modern science programs.

Although the need for precise terminology in enabling pupils to communicate their thoughts with clarity is recognized, it is unfortunate that, even in the so-called modern approach to mathematics, there does not exist universally accepted

terminology. This leads to some confusion as one turns to different sources. However, we are closer to such an ideal situation than ever before. *Mathematics We Need* employs terminology that is up-to-date, generally accepted, and widely used. In cases where variations of terminology for a certain idea exist, *Mathematics We Need* introduces such variations when this does not lead to inconsistency.

Using Mathematics

The expression "using mathematics" means the employing of ideas from mathematics that help solve the many numerical and geometrical problems so common in this scientific age, as well as problems in more advanced phases of mathematics. In other words, mathematics taught in an elementary grade has two major applications. One is external as applied to realistic problems and the other is internal as developmental in instilling an attitude of creative attack on the mathematics of later grades and the mathematics of the future, some of which is now not even known.

We normally think of the external use when we refer to "applied" mathematics. Pupils, of course, do use mathematics in real-life situations outside the classroom, but the immediate practical situations do not constitute a complete curriculum. Pupils also need to study problem-solving situations of a more abstract and, in a sense, less realistic nature. Because of the pressure of many language patterns and combinations of situations, many as yet undreamed of, it is difficult to teach each pattern of words individually in an effort to help pupils interpret independently all the problems they might face.

Providing a powerful foundation for problem-solving is the concept of a mathematical model. A problem normally involves the application of mathematics to some physical situation. But to solve the problem, the pupil must translate the description in words into the language of mathematics. In technical terms, he needs to learn how to set up a mathematical model for the problem. At first, this model may be the basic idea: if the two addends are known, addition is used to find the sum; and if a sum and an addend are known, subtraction is used to find the unknown addend. Similarly, if the two factors are known, multiplication is used to find the product; and if a product and one factor are known, division is used to find the unknown factor. Later, the pupil moves to a more symbolic model—a mathematical sentence for the situation. In many cases, this sentence may be a statement of an addends-sum relationship (for example, $8 + 6 = n$) or a statement of a factors-product relationship (for example, $3 \times n = 24$); or perhaps a statement of a well-known formula such as $F = \frac{9}{5}C + 32$. This means that pupils must learn to express verbal statements as mathematical sentences, and be able to solve these sentences once they have been written. The final phase of problem-solving consists of the pupil's translating into its physical meaning in the problem situation the answer he obtained by working in the model.

Problems chosen to illustrate problem-solving techniques can be very interesting. Fortunately, pupils enjoy a wealth of applied problems which are significant to them. They like to know more about how mathematics is actually used in science, in technology, in sports, in business, in the social sciences, and so on. Even when not immediately practical for pupils, such uses of mathematics help them develop problem-solving skills and enable them to understand how other people use mathematics.

The second major aspect of using mathematics is often neglected when teaching a particular topic. Virtually all of the mathematics developed in the elementary grades is needed later in other mathematics courses and potentially in presently unrecognized uses. It is often difficult to explain in meaningful terms to the pupil that he is studying a topic one year so that he can use it another year or even several years in the future, in connection with other topics in mathematics presently unknown to him.

This study of mathematics for its own sake, which might be called internal applications, takes many forms: learning basic skills for future application, solving problems of a purely numerical nature, solving fun or recreational problems, and others. Indeed, most of the modern topics such as sets, properties, numeration systems, and nonmetric geometry are justified as an important part of the curriculum, because of their usefulness in the

development of insight, in providing an orderly base for mathematical thinking, and in meeting the mathematical needs of the future rather than because of their immediately utilitarian value.

In a complete elementary-mathematics program, then, pupils encounter mathematics which they will use in several ways: actual applications to problems in their own life; applications to problems beyond their relatively limited experiences, techniques and understandings that will help them unlock mathematics in the future; and problems of a puzzle or recreational nature.

There is no sharp distinction between "pure" and "applied" mathematics, and that is as it should be. If mathematics is to be functional and worthwhile for a pupil, he should think of the subject as one which is used so often that it is practical, but also as one which is fascinating in its own right, without regard to application. This "dual but unified" nature of mathematics is excellently reflected in the *Mathematics We Need* series.

Providing for Individual Differences

The extraordinarily great range of differences within and among children at any given age level often is neither well defined nor understood. At the fifth-grade level, for instance, it is normal to find children ranging from early third-grade to late eighth-grade levels in interest, ability, and achievement. Psychologists did not discover individual differences, they merely described their range.

Most of us as teachers and parents are conditioned to judge progress or success in terms of some preconceived notion of a standard which is considered to be average or normal. Our levels of expectation in mathematics instruction and achievement are affected by some such "average." Making adjustments for children who deviate from this normal expectation is a major responsibility if the effectiveness of our instruction is to be at a maximum. The provisions and recommendations for attaining the highest level of interest and achievement by all pupils which are found in the *Mathematics We Need* series stem from the following philosophy of mathematical learning:

a. A sound, carefully prepared, sequential set of mathematical ideas and skills must be made available to all pupils.

b. Regardless of whether or not a school system uses acceleration as one way of providing for individual differences, there still must be a wide variety of activities provided in the basic printed program. This is so because even when pupils are accelerated, there will still be individuals who differ within the accelerated group and who must, therefore, have differentiated assignments.

c. Enrichment activities should be available to all pupils. Slower learners should not be denied the excitement and additional insight often found in enrichment activities. In general, enrichment work extends and supplements the basic mathematics program. It affords an opportunity to take "side trips" into the world of mathematics which may be equally enjoyed by all pupils. *Mathematics We Need* frequently provides enrichment suggestions which can be used with all pupils.

There is a wide range of activities and suggestions for individualizing instruction available in the *Mathematics We Need* series.

- In the pupil's textbook you will find a variety of extra materials. At intervals through the chapters of each book are sections which are clearly indicated as being intended for enrichment. Many other pages offer alternate ways of performing an operation or carrying out new activites for children to try.

- The reservoir at the end of each textbook for grades 3–8 provides three types of extra help. These are found under the following titles: (1) **Extra Problems;** (2) **Extra Examples;** and (3) **Extra Activities.** Further, in grades 4–8, some **Reteaching** helps are provided in the reservoir. Each of these sections can be an excellent and useful source of material for individuals or small groups of children as the occasion demands. References to these extra exercises are provided at the end of appropriate material in the basic pages of the textbook.

- A section entitled **Individualizing Instruction** is available in most of the lesson plans. Suggestions in this section include those for all pupils, for those who are slower learners, and for those who are more capable. Often, several different types of activities are suggested for a given group of children. Thus, a wide variety of experiences in learning mathematical concepts is provided.

• The carefully developed discussions in the Background section of lesson plans provide a source that can be often used for more enrichment.

• Pre-Book Lessons and Using the Text Page in the lesson plans suggest further resources in caring for individual differences.

• At the end of each chapter in the teachers' edition there are further suggestions for supplementing and extending the mathematics program. Some of these activities involve the alternate uses of pages of material in the pupil's text. This feature takes advantage of the fact that many of the activities on the pupil's pages may be modified or adapted slightly to create an exciting and valuable extended-learning or practice experience.

• The workbooks in the series afford many opportunities for individualizing instruction.

• Further extension is possible through using supplementary materials that are available from Ginn and Company. These include *Ginn Arithme-Sticks*, *Related-Facts Cards*, two kits of flannel items entitled *Introduction to Sets and Numbers* and *Operations on Sets and Numbers*, and a number-line model entitled *Modern Number Line*. All of these are designed to present modern mathematics ideas and all have complete guides. Further, for certain situations, we suggest that you write to the publishers, Ginn and Company, to inquire regarding supplementary publications that will provide further enrichment.

The Learning Process in Mathematics

Thirty years ago, when the idea of developing arithmetic through understanding rather than through memorization was being pioneered by Ginn and Company arithmetic authors and editors, a widely recognized expert in the teaching of secondary mathematics asked one of the authors to tell him something of the new movement. At the outset he admitted that he was unsympathetic with the little he had heard about it. As the author proceeded in his explanation of "meaningful arithmetic," this secondary specialist became steadily more impatient, finally breaking out with the statement, "All nonsense! Just see to it that children in the elementary grades learn to add, subtract, multiply, and divide. Then when they come to high school, we'll teach them how to think!" He did not say how and why children who had been learning arithmetic blindly and mechanically for seven or eight years would suddenly be ready, disposed, and able to blossom into the ability to reason mathematically upon becoming high-school freshmen.

The anecdote is worth the telling for two reasons. In the first place, this critic's brief statement epitomized the conception of arithmetic and of the processes of teaching and learning the subject that was dominant thirty years ago: arithmetic was thought to consist primarily of computation skills, to be learned and maintained by incessant practice. In the second place, the conversation reveals the conditions under which "meaningful arithmetic," the precursor of "modern elementary mathematics," made its great contributions in developing the climate for and making excellent progress in the learning of mathematics via understanding—contributions for which it is not too generally given credit.

The key word in any account of the learning process in mathematics is *understanding*. This statement holds true even in the case of attitudes, for how can one be confident in using intellectual skills one does not understand or respect a body of knowledge that possesses little meaning?

Now, understanding is not an all-or-none affair, either present or absent; rather, it is a matter of degree. A child is told that there are five horses in a field. Does he understand "five"? Yes, to some extent, if he realizes that "five" is a correct word to use for the particular set of horses he sees; but No, in a fuller sense of the word, if to him "five" refers to these special animals and is not understood as a characteristic of all sets that are in one-to-one correspondence with that set of horses.

The degrees of understanding just illustrated are rarely adequate in mathematical learning, save as starting points. For the idea of "five," we are dealing with abstract ideas which have no physical existence whatsoever; but, if we understand the idea thoroughly, we can impose this idea on a great variety of external representations. However, note the word "impose." The ideas are not *in* the representations. Instead, we *put* them there.

For every idea there is usually, but not always, a word or other symbol ("five," "5," "3 + 2,"

"8 − 3") which stands for it; and more complex abstractions, such as generalizations which involve a relationship among several ideas are also usually couched in language. But neither an idea nor a generalization can have much meaning if one knows only the corresponding symbol or symbolic expression. And here lies one of the most serious pitfalls in teaching mathematics, or anything else, for we may infer that a learner who can correctly employ the appropriate language in response to a limited number of specific unvarying questions has actually acquired all the understanding we want him to have. It is for this reason that too early introduction of symbols, terms, and definitions defeats sound learning—too early in the sense of an inadequate base of understanding so that memorization is the only recourse.

Do you want an example? If so, read the following: Gom flas mev hukl. What does the sentence mean? Read the series of syllables over and over until you can repeat it without referring to the syllables themselves. Now what does the whole thing mean? Is the sentence correct or incorrect? Does it apply to a game? to economics? to clothing? to a map? Have you not memorized a succession of verbal symbols without acquiring much by way of meaning? And will continued repetition of the series increase its meaning? Finally, do you perceive the applicability of your experiences here to much that passes for learning in the classroom?

As you have seen, repetitive practice does not enhance meaning; but it does do something else: it increases your skill or makes you more efficient in saying the series of syllables, and it may enable you to recall the series for a longer period of time than would a single repetition. And repetitive practice performs these same functions in the learning of mathematics. Needless to say, however, it should be used, not with nonsense material like that above, but with content (generalizations and skills, for example) that are already well understood. Hence, the statement that there is no place for drill in modern mathematics is erroneous. On the other hand, it should be deferred in each instance to a time when the desired degree of meaning has been engendered.

So, we are left with the question, How are we to assist children to develop or to broaden, deepen, sharpen, and enrich understanding? We do so by providing through guided discovery a variety of relevant experiences—not essentially the same experience over and over again as in drill. All these experiences contain the element of meaning to be taught, but in differing contexts.

Thus, in learning "five," children are guided to enumerate in order to find the number of members in sets of objects at home and at school—sets of pencils, marbles, dolls, chairs, and so on; they manipulate movable objects and imaginatively manipulate pictured objects, noting sets which have the number characteristic of "fiveness" and sets that do not have this "fiveness" characteristic; they produce sets which have "fiveness" by making marks, drawing oranges, tapping a bell; they come to recognize at a glance conventionalized groupings like Fig. 1–3, as having the number property of "fiveness." Each time, attention is directed to the

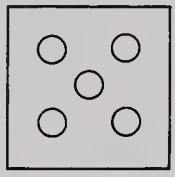

Fig. 1

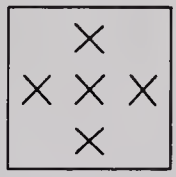

Fig. 2

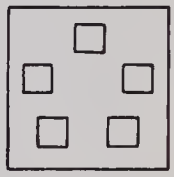

Fig. 3

one idea to be taught, in contrast to other inappropriate ideas. Eventually, children arrive at an abstraction which is symbolized by numerals such as 5, five, V, 7 − 2, and 4 + 1. It should be noted that over and above the necessity of a variety of approaches for refining an idea, such variety is also imperative if those who fail to understand through one approach are to have opportunities to attain understanding through other approaches.

Understanding grows in a manner which, if pictured, would resemble stairs of uneven steps, unlike in height and depth. Children do not need always to traverse all the possible steps, starting invariably with something equivalent to the enumeration of concrete objects. Instead, depending upon their degree of understanding, they may begin with step 3 or step 6 in a hypothetical set of stairs with ten steps. (It would be foolish indeed to teach children the meaning of 307,486 beginning with enumerating objects.) Understanding at any stage rests and builds upon understandings previously acquired and in turn contributes to the next stage. It is, in

fact, this very characteristic which makes it possible to guide pupils in discovery of "new" ideas—"new" because the ideas are actually only extensions of earlier understandings. Much has been said about this earlier in the sections pertaining to building on structure and on mathematical strands.

To conclude, first negatively, then positively: children do not increase understanding of mathematical content by memorizing the language in which it is expressed or by engaging repeatedly in what is practically the same experience; they advance in understanding by undergoing a variety of guided discovery and other experiences, all of them embodying the same content but otherwise different, and all leading by easy stages to progressively more abstract and generalized forms of thought.

In sum, a complete program should be written so that understanding is assured through a careful development that includes an intelligently planned variety of approaches. In essence, such a plan is accomplished if there is true balance in the program. This implies careful selection from the content used in experimental programs; judicious retention of time-tested content; constant use of ideas taught, both in developing other ideas and in application to practical situations; utilization of variety even when repetition is needed for building a skill; and having balance in methodology as represented by guided discovery which blends the efficiency of telling, without the deadliness of telling, and the effectiveness of discovery, without the confusion and time loss of random discovery. The *Mathematics We Need* program is noteworthy for its remarkable balance that assures optimum learning through understanding.

Notes Regarding the Grade-3 Program

Introduction

Since the underlying plan of this program is to be of maximum assistance to teachers day by day, we will not try to give the entire picture of the grade-3 program at this point. Instead, we will present brief summaries in this section and then refer you to teacher's pages throughout this Teachers' Edition where further information is available at those points where it is most useful. You may want to glance through those pages when they are referred to in the paragraphs which follow, so as to get an overall picture of what is done in each of the areas under discussion.

General Plan of the Grade-3 Program

• The books for grades 1 through 4 constitute the first major cycle of the *Mathematics We Need* program. For schools which have kindergartens, the activity program provided by the *Modern Mathematics Kindergarten Kit* enables the school to have five years in this first major cycle. Grade 3 is consistent with the plans for this first cycle in that ideas are first encountered in an intuitive way, the development is gradual, and a variety of presentations is made so that the pupil gets to examine the concept or relationship, or whatever, from a number of angles in order to assure thorough basic learning. Symbolism is introduced slowly at the point where there is sufficient understanding so that the introduction of the symbolism will not just be another addition to the memory load. At all times, maximum provision is made for guided discovery through carefully developed questions in the text, through extensive suggestions for Pre-Book Lessons, and through a number of valuable suggestions in the Individualizing Instruction section of the lesson plans. Elaborate provision is made for developing competence in problem-solving, for the fixing of skills so that they will be available when needed in further developmental learning, for individualizing instruction, for maintenance of earlier-learned ideas, and for evaluation.

Unlike later books in the series, the grade 3 text has extensive reteaching in chapters 1 and 2. It is essential to have this because pupils come into grade 3 from a variety of programs, whereas at later grade levels, it can be assumed that pupils usually have had the earlier book in this series for background. You will want to scrutinize carefully these first two chapters in *Mathematics We Need*, Grade 3, in order to govern wisely the pace of instruction for your particular group. When you recognize that certain content has been thoroughly taught, a quick review may be enough. When content is known to have been only partly taught, the pace of the teaching must be slackened and reteaching will be needed. Finally when content is seen to be new to your group, instruction must proceed slowly and developmentally. A good guiding principle is that it is a waste of time to tarry too long on what is known; and it is just as much a waste of time to hurry instruction unduly on what is not known for, if that portion is not learned, it will cause a greater eventual loss of time.

• A brief overview of the first chapter will be found on the page having the headings "Overview —Chapter 1" and "Teaching Page 1." Overviews for the remaining chapters will be found on Teacher's Pages 52, 94, 140, 178, 214, 256, and 300.

Overview of Mathematical Ideas

Arranged in alphabetic order on pages xvi–xvii, you will find selected headings. Related subtopics are classified under these headings in developmental order. Accompanying each heading and subtopic are references to lesson plans where discussions of terms and topics can be read in direct relationship to the pupil's pages. These references emphasize our plan of day-by-day in-service training.

If you want to have a first look at the ideas in a logical sequence before you start teaching the grade-3 program, read the references for each heading and its related subtopics in this order: Sets; Number; Number Line; Numeration Systems; Inequalities; Properties; Mathematical Sentences; Inverses; Logic; Addition and Subtraction; Multiplication and Division; Arrays; Renaming; Relations; Integers; Geometric Ideas. There will be

some overlapping, but the opportunity to study an idea in relation to the pupil's page more than compensates for the duplications.

When page references are indicated by numerals only, you should read the **Background** on that teacher's page. But, for 8P, read **Pre-Book Lesson** on Teacher's Page 8; for 11U, read **Using the Text Page** on Teacher's Page 11; for 95L, read **Looking Ahead** on Teacher's Page 95; for 304I, read **Individualizing Instruction** on Teacher's Page 304. However, practically all references are to **Background** sections.

• Addition and Subtraction, (see 8P, 12, 15, 18, 32). On the basis of the idea that an operation involves one or more elements and a means of determining a unique element, the idea of *binary operation*, (see 8, 12, 40), is used meaningfully. In essence, the addition operation is taught first and wide use is made of the idea that subtraction derives from addition. For instance, the terms used are sum and addend regardless of whether the form is

$$\underset{\text{(addend)}}{27} + \underset{\text{(addend)}}{32} = \underset{\text{(sum)}}{59} \textit{ or } \underset{\text{(sum)}}{59} - \underset{\text{(addend)}}{32} = \underset{\text{(addend)}}{27}.$$

This works excellently in many situations, including the undoing idea illustrated, as in the above examples, when 32 is added and then the addition is undone by subtracting 32. Out of this addition-subtraction relationship is evolved the extremely useful *addends-sum relationship*, (see 11U, 21P, 62P), which gives us a model for solving mathematical sentences such as $27 + 32 = n$, $n - 32 = 27$, or $59 - n = 27$. The point is that if we identify two addends, we add to find the unknown sum; if we identify a sum and an addend, we subtract to find the unknown addend. This addends-sum relationship also extends into the solving of problems. The ideas of *renaming and carrying* in addition and of *renaming* in subtraction (see 95L, 102, 104, 110, 123, 124, 132) are used very precisely.

• Arrays, (see 206, 206P).

• Geometric Ideas, (see 120). This text develops the elements that are fundamental in geometry through the sets-of-points approach. Such a plan eliminates many ambiguities and fits in ideally with the geometry of later grades when the study of geometry in its logical development is carried out. For the terms taught, see: *angle*, (see 256, 257); *coordinate*, (see 93, 139); *curve*, (see 120); *line*, (see 120); *line segment*, (see 120, 256); *measurement*, (see 64, 65, 116); *path*, (see 120); *point*, (see 120, 121P); *polygon*, (see 120, 200); *ray*, (see 256); *simple closed curve*, (see 120).

• Inequalities, (see 27, 57, 90). The idea, not the symbolism, is most important. In the texts for grades 1 and 2, much is done to have pupils sense the important idea that one number is often not equal to another and to sense which number is greater and which number is less. Developmentally, the pupils first use the expressions *is greater than* and *is less than*. They then used *G* and *L*. Then, in grade 2, they were introduced to the very meaningful *is not equal to* symbol ($\neq$) but still were not introduced to the somewhat confusing *is greater than* and *is less than* symbols ($>$, $<$). It is true that quite a few pupils do remember the direction of pointing of these symbols. However, quite a few younger pupils do not get this matter straightened out and their confusion leads to many errors and poor scores when actually pupils do have clear understanding of the ideas. For this reason, the *Mathematics We Need* program takes much time to build understanding and then, during grade 3, the *is greater than* symbol and *is less than* symbol are brought into the pupil's work (see 90). We must watch constantly that we do not defeat the goals of the modern mathematics movement by tactics such as introducing symbols so early that the pupil's only recourse is to memorize these because the proper foundation of understanding has not been built.

• Integers, (see 214).

• Inverses, (see 15, 280, 282).

• Logic, (see 74).

• Mathematical Sentences, (see 8, 18, 77, 94). The Background discussions referred to will also clarify *number sentences* and *open sentences*.

• Multiplication and Division, (see 202, 205, 207, 246, 248, 258, 264, 278, 288, 304I, 306, 324, 330, 334, 338). *binary operation*, (see 164); *factors-product relationship*, (see 282, 296, 332); *renaming and remembering*, (see 258).

• Number, (see 2, 6). *set of whole numbers*, (see 1, 2); *set of counting numbers*, (see 1, 2); *set of even numbers*, (see 30, 63); *set of odd numbers*, (see 30, 63);

cardinal number, (see 96); *ordinal number*, (see 96); *set of fraction numbers*, (see 140, 230, 310)—these discussions will also clarify the idea of a *fraction* as a numeral and *fraction number* as a number and will also relate the idea of *rational number* to fraction number; *set of rational numbers*, (see 140, 230, 310).

- Number Line, (see 2, 35, 240U, 278).
- Numeration Systems, (see 2–3, 100P, 180).

Further items that will extend understanding of *numeration systems* include: *number names*, (see 229P); *numeral*, (see 2, 178, 342); *expanded notation*, (see 166); *fraction*, (see 140, 230).

- Properties. *Property of Closure*, (see 10, 207); *Commutative Property of Addition*, (see 8, 9P, 12, 54, 150); *Associative Property of Addition*, (see 40, 54, 75, 150, 152); *Identity Property of Addition*, (see 38, 85); *Commutative Property of Multiplication*, (see 206, 206P, 224, 239, 270, 282); *Distributive Property of Multiplication over Addition*, (see 248, 334); *Identity Property of Multiplication*, (see 227).
- Relations, (see 81, 153, 317).
- Renaming, (see 62L, 75, 123, 268, 338).
- Sets, (see 1, 6, 29, 156). For *subset*, (see 6); for *empty set*, (see 261). For introductions to the idea of *equivalent sets*, (see 164).

Summary of Other Features

Just as for mathematical ideas, any discussions of other features in this text will be much clearer if they are read directly in relation to the pupil's text pages. Each reference that follows is to a teacher's page and the accompanying pupil's page. When only the numeral is given, read **Background;** when a numeral is followed by I, read **Individualizing Instruction.**

- Approximations, (see 158, 183, 341).
- Concrete Aids, (see 1I).
- Oral Development. No page references are given because practically any pupil's text page illustrates this plan. For the development of abstract concepts like those that are being brought into elementary mathematics programs today, it is particularly important to have careful oral development provided in the textbooks. Many advantages could be listed for this plan, but suffice it to say at this point that if teaching can be defined as the guiding of learning—and we think this is a good definition—then a prime requisite is that teachers know what "goes on in their pupils' heads" and oral questioning and discussion that has been carefully planned often is the only means of gaining this vital information.
- Practice, (see 98, 99, 114, 122, 201).
- Problem-Solving, (see a selection of these pages: 11, 12, 20, 20I, 22, 28, 34, 54, 67, 72, 109, 118, 143, 149, 182, 183, 219, 220, 228, 237, 274, 285, 300, 314). This list of references is long because of the wide variety of approaches to problem-solving in the grade-3 textbook. This series carefully builds the structure of mathematics, but the authors have also recognized that much must be done to help pupils associate the mathematics with the problem needs that arise as they manage sensibly their quantitative world.

In essence, the *Mathematics We Need* program recognizes that if pupils just learn a single approach —that of restating verbal problems in equation form, they will have a very narrow background for problem-solving. This is so, because they will not sense what actually occurs in the managing of the quantitative situation. Consequently, this program builds problem-solving competence on a developmental basis instead of leaping at once to the highly abstract equation plan. To begin with, pupils identify in terms of the set situation as it is described. They are guided to recognize that if two subsets are described and the problem is to find the number for a total, one adds the numbers for the subsets; if a set and subset are described and the problem is to find the number of the other subset, one subtracts the number of the known subset from the number of the known set. A wide variety of activities is used to help pupils in this identification of sets and subsets as described in the problem.

The developmental plan then moves on to determining more directly the fact that if there are two addends, the operation of addition is to be used; or if there are a sum and an addend, the solution is obtained by subtracting. All of this keeps the pupil closely associated with the imagined action in the problem, and yet it clearly keeps the mathematics separated from the social.

Paralleling these initial approaches to problem-solving is the development of the idea of equations, as such, and the plan of solving these equations through use of the addends-sum relationship.

Gradually, pupils are guided to associate the equations with the problem situation but, even then, the solution is in terms of abstracting the known addends or the sum and known addend from the verbal description and not from the equation as such.

By the end of grade 3, pupils will have had a very thorough development of the sensing of the quantitative situation in terms of sets, the more abstract use of the addends-sum relationship as applied to the sets situation, and a truly thorough build-up of the association of the equation with problems. Besides, they will have acquired competence in the solution of equations. This general development means that pupils will enter grade 4 with an ideal background for problem-solving that is henceforth to be based on the equation plan.

What has been said for problems that require the operations of addition and subtraction can also be said for the plan of building problem-solving competence when the problems require the operations of multiplication and division.

• Psychology of Learning, (see 18, 36, 44, 45, 52, 55, 59, 68, 85, 98, 110–111, 114, 124, 157, 234, 239, 318, 336). Again, we have provided a lengthy listing. There has been too much assuming on the part of many that, if material is logical, it will more or less automatically be learned. Many other facets of learning need to be kept in the picture constantly in the development of a program and in the use of that program. The Background sections referred to will highlight the extent to which this textbook has utilized the many concepts from the field of learning.

• Evaluation, (see 49–51, 90, 115, 122, 129, 144, 173, 204P, 326). We do not agree with those who contend that if mathematics is developed on the basis of understandings, little testing is needed. The opposite is the case, as our wide experience through the years has clearly demonstrated to us. Over and above the many opportunities to evaluate through the numerous oral developments in the pupil's text, we do have a thorough testing program. Often this Guide will suggest that a particular set of exercises which is not specifically designated as a test might be used for evaluation. In these cases, we will indicate the kinds of things to be evaluated. Then, too, each chapter has a most thorough testing program that includes diagnosis, computation, problem-solving, and information and meaning.

Elementary Mathematics Aids

Throughout the Teachers' Edition, included in the lesson plans, there are many references to the items listed below. These references can be located by noting footnotes for many of the teacher's pages. You may want to study this list early in the year in order to arrange well in advance for obtaining the items you will want to use.

The following may be purchased from Ginn and Company (Boston, New York, Chicago, Atlanta, Dallas, Palo Alto, Toronto):

1. Flannel Board Fractional Parts
2. Flan-O-Graph
3. Ginn Arithme-Sticks (with guide)
4. Introduction to Sets and Numbers (flannel items and guide)
5. Modern Mathematics Kindergarten Kit (flannel items and guide)
6. Modern Number Line (with guide)
7. Number Concept Cards
8. Related-Facts Cards (with guide)
   8A Addition and Subtraction
   8B Multiplication and Division
9. Operations on Sets and Numbers (flannel items and guide)

9X. Modern Mental Computation (flannel items and guide)

The following are available from Milton Bradley Company. Comparable items are often available from other sources.

10. Calcuframe
11. Clock Dial
12. Plastic Counting Discs
13. Cubical Counting Blocks
14. Day by Day Calendar
15. Educational Thermometer
16. Educational Toy Money
17. Flannel Board Cut-Outs
18. Flannel Board Numbers
19. Geometric Figures and Solids
20. Hundred Chart
21. Place Value Indicator
22. Primary Number Line
23. Self-Teaching Flash Cards: addition
24. Self-Teaching Flash Cards: subtraction
25. Self-Teaching Flash Cards: multiplication
26. Self-Teaching Flash Cards: division
27. Tick-Tock-Primary Clock
28. New Math Symbols

Contents of Pupil's Text

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2	MAINTAINING AND EXTENDING UNDERSTANDINGS AND SKILLS: OPERATIONS – addition and subtraction, sums as great as 99 • MATHEMATICAL SENTENCES – true and false; equalities and inequalities • PROPERTIES, ADDITION – associative; compensation • MEASUREMENTS – linear • LOGIC – if-then sentences • USE OF SETS, NUMERATION, PROPERTIES, NON-METRIC GEOMETRY ENRICHMENT • PROBLEM-SOLVING • TESTING	52
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4	SETS – numbers: whole, fraction • NUMERATION – expanded notation • OPERATIONS – addition and subtraction, sums as great as 999; readiness for multiplication • MATHEMATICAL SENTENCES – introducing multiplication • MEASUREMENTS – dozen; weight • ESTIMATION MAINTENANCE THROUGH USE: SETS, NUMERATION, MATHEMATICAL SENTENCES, PROPERTIES, MEASUREMENTS, LOGIC, NON-METRIC GEOMETRY ENRICHMENT • PROBLEM-SOLVING • TESTING	140

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— 5 — SETS – equivalent • NUMERATION – Roman; renaming for adding and subtracting • OPERATIONS – addition; subtraction; multiplication • PROPERTIES, MULTIPLICATION – commutative • SEQUENCES • NON-METRIC GEOMETRY – naming polygons; number line **178**

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ENRICHMENT • PROBLEM-SOLVING • TESTING

— 6 — SETS – equivalent; fraction numbers • NUMERATION – expanded notation • OPERATIONS – addition and subtraction; multiplication, products as great as 99 • MATHEMATICAL SENTENCES – using other number names • PROPERTIES, MULTIPLICATION – distributive; identity • SEQUENCES • MEASUREMENTS – temperature; liquids **214**

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— 7 — NUMERATION – renaming for multiplying • OPERATIONS – multiplication, products as great as 999; division • MATHEMATICAL SENTENCES – multiplication; introducing division • PROPERTIES, MULTIPLICATION – inverse • SEQUENCES • ESTIMATION • NON-METRIC GEOMETRY – line segments; lines; rays; angles **256**

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— 8 — SETS – fraction numbers • NUMERATION – expanded notation; 4-place numerals; renaming for multiplying and dividing • OPERATIONS – multiplication; division, products as great as 999 • MATHEMATICAL SENTENCES – multiplication and division (solved by using factors-product relationship) • SEQUENCES • MEASUREMENTS – use of fraction numbers • GRAPHS – picture **300**

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Overview — Chapter 1

• **Reteaching.** The basic plan to have in mind in teaching Chapter 1 is that important mathematical principles are intuitively developed and then used as the means of providing interesting presentations of other instructional materials. Since pupils come to grade 3 with varied backgrounds, this development is done thoroughly. Among the ideas that are utilized for organizing the instructional content are the commutative, associative, closure, and identity properties; the idea of inverses—If a number is added (subtracted), you subtract (add) that number to undo the prior operation; the addends-sum relationship which suggests that if two addends are known, you add to find the sum and if the sum and an addend are known, you subtract to find the unknown addend; number sentences involving use of the addends-sum relationship; the number line; and arranging facts in sets that utilize important relationships. These are used extensively in the review of number, numeration through 2-place numerals, and addition and subtraction involving facts through sums of 10 and higher-decade work.

• **Extension.** More is done with the idea that both addition and subtraction involve a *sum* and *two addends*. The idea of using frames (squares, triangles, and so on) to represent the unknown in number sentences is extended to the use of *n*, and this idea is related to the addends-sum idea. Other extensions include measurements involving the calendar and the clock, utilizing the ideas of even and odd numbers in generalizing about the addition and subtracting of 1 and 2, and further work with the symbol = and the symbol $\neq$.

• **Problem-Solving.** For the problem-solving strand, pupils specify, according to the parts-total idea, why they add in certain problems and why they subtract in others. They also select an equation from those that are associated with a particular problem, describe how addition is used for problems that require finding the number for the sum, and how subtraction is used for problems that require finding the number for the other part. They then do the basic work in solving problems.

• **Enrichment.** In addition to the enrichment suggestions in each lesson plan in this Teachers' Edition and the reservoir of suggestions at the end of the pupil's text, Chapter 1 provides enrichment in relation to the addition and subtraction chart and in examples that involve *n* as a third addend.

• **Testing.** Since needs vary at the beginning of the year, the only pages designated as testing pages are the four chapter tests at the end of the chapter: Problem-Solving Test; Diagnostic Test with suggestions for remedial work where necessary; Test of Information and Meaning; and Computation Test covering the skills reviewed in the chapter. However, several of the exercises throughout the chapter may be used for testing according to the needs of the individual class.

Teaching Page 1

Pupil's Objectives

To review (a) the concept of a set; (b) that by identifying the number of elements in the set we identify the number property of that set; (c) that numbers may describe the order of things.

New Words*: *dens, Scouts*

* The vocabulary of the pupil's text has been controlled so no page contains more than 3 new words and each new word is used at least twice where introduced.

Teacher's Preparation

Be sure that several sets of things found in the classroom are identified during the discussion of the text page.

A *set* is any well-defined collection of things. The term *things* is interpreted broadly to include numbers and other abstractions. Each thing in the set is called an *element* or a *member* of that set. We can use the term *set* almost any time that we use the word *group* and also in situations in which we

wouldn't use *group*, such as for sets with 1 member or no members. The concept and language of sets will be encountered by the children throughout their study of mathematics.

The *cardinal* number of a set indicates the number of elements in the set. An *ordinal* number for a set is a number associated with a member of a set because of a certain rule of order. Such a number often answers the question "Which one?"

The set of *counting* numbers, frequently called *natural* numbers, begins with 1 and continues indefinitely. The set of *whole* numbers includes 0 (zero) and all the counting numbers.

Using the Text Page

In the interest of brevity, the author frequently will say "Ask" or "Do this" or "Write such and such." These comments are intended to be suggestions only.

• Direct the attention of the class to the picture opposite page 1. (From this point on, it should be assumed that a reference of this type means text page —. If a teaching page is involved, it will always be referred to as such). Mention that it is a picture of a big race at a Cub Scout meeting. Talk about the various things pupils may see in the picture, without mentioning how many of each thing there are.

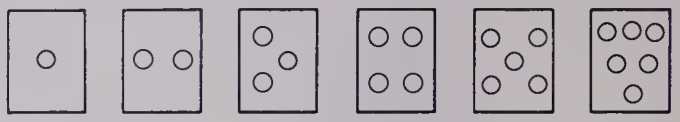
• Proceed with the questions on page 1. As the questions are answered and discussed by the class, emphasize that some numbers tell "how many" in a group, and some numbers tell "which one" of a group.

• Tell the class that *set* is another word we sometimes use for *group*. Ask them to point out sets of boys, tables, children, window panes, and so on, and tell something about the set.

Individualizing Instruction

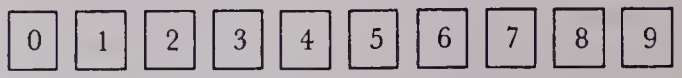
• Start building an arithmetic corner for *all pupils*. Collect objects such as milk-bottle caps, one-inch squares of construction paper, cubes, and so on which will be used in the manipulative stage of number work. Other teaching aids, such as a clockface, flannel board and cutouts and numerals, Ginn Arithme-Sticks, Ginn Number Concept Cards, a hundred chart, and so on will be added from time to time as occasion arises for their use.*

• A set of dot cards similar to those shown below is useful to give practice in immediate recognition of number patterns. The cards should be about 5″ by 8″. Make all dots about 1″ in diameter and of the same color. Ginn Number Cards of this type are available.



Flash a dot card and ask the children to tell the number for the set of dots on the card. Show the card for just an instant to encourage pupils to recognize a pattern instead of counting by ones. Ask such questions as, "How did you know it was 4?"

• A very useful teaching aid for *all pupils* is the set of cards pictured below.



Each child should make a set of 2″ by 3″ cards—one card for each of the numerals 0 through 9. Then, when you flash a dot card, each child will pick up his card with the appropriate numeral and hold it high for you to see. In this way, all children are involved in the activity, and you can check the responses of each. We will call this activity "Show the Answer."

The type of practice afforded by *Show the Answer* cards is extremely valuable for the children as well as the teacher. In working with a relatively large group of children, the teacher has an opportunity to diagnose the difficulties of individual children. As a consequence of identifying a source of misunderstanding, the teacher can immediately direct the questioning in an effort to correct the situation. A great deal of the understanding developed in the arithmetic program is difficult to measure with a paper-pencil test. Teachers must be constantly alert to situations in which mathematical understandings are revealed. Directing the *Show the Answer* practice session is one such opportunity.

* See list on page xix.

* The answer tells how many, so it is a name for a cardinal number.
** Here, and in 7b and c, the idea of ordinal number is stressed.

If We Could Not Use Numbers

Using cardinal and ordinal numbers [O]

As shown in the picture, Cub Scouts from two dens have brought their racing cars together for a big race.

* 1. How many racing cars are there in the group, or set, of red cars? 5

2. How many are there in the other set of cars? 3

3. How many of the Cub Scouts are from Den 2? 5

4. How did numbers help you to get your answers? to tell how many

5. If we could not use numbers, could you tell
   a. how many racing cars there are in each set? No
   b. how many boys are from Den 6? No
   c. how many boys are from the two dens together? No

**6. Why is each racing car numbered? to tell which one

7. If we could not use numbers, could you tell
   a. how many there are in the set of prize buttons? No
   b. which button is red? No blue? No
   c. the place where your home is? No
   d. how old you are? No
   e. at what time the school bus comes? No
   f. how tall you are? how much you weigh? No
   g. how many children are in your room? No

***When we talk about 5 sticks of candy or 3 apples or 16 books, we are using some of the whole numbers.

8. Find sets of things in your room and tell about them by using whole numbers.

*** The set of whole numbers is introduced first since pupils are familiar with this set.

* Take advantage of the opportunity to distinguish between the set of whole numbers and the set of counting numbers.

** **A**

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Numbers through 99

Whole numbers and counting numbers [O]

In box A are names of the whole numbers 0 through 99. Number names are called numerals. So, 5 is a numeral we may write or read when we mean the number five.

As you see, 0 is in box A. It is the numeral for the number that means not any. So, we can write, "0 mice," to mean no mice at all.

1. When you count a set of things by 1's,
   a. which number is first?
   b. would you ever start counting things with 0? No Why?

0 means not any.

* 2. We call the numbers 0, 1, 2, 3, and so on the set of whole numbers. Why do we call the numbers 1, 2, 3, and so on the set of counting numbers? We use these numbers for counting things.

3. In box A, which are the 1-place numerals?

one digit in each numeral

4. In box A, which is the first 2-place numeral?

two digits in each numeral

5. Read the 2-place numerals under 0 in box A. If you need help, the names are shown in box B.

ten, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety

** The numerals in box A name the whole numbers from 0 through 99. Pupils should distinguish between the uses of number—an idea—and numeral—a symbol that names the number.

Teaching Pages 2 and 3

Pupil's Objectives

(a) To review the skills of reading and writing numerals for numbers from 0 through 99; (b) to discover the systematic way in which these numbers are organized; (c) to begin to learn the difference between a number and a numeral; (d) to begin to learn the use of a number line; and (e) to discover that 0 is not a counting number.

New Words

(page 2) *mice*
(page 3) *ends*, *least*, *-ty* words

Background

The chart on page 2 may be different from the number chart with which you are familiar (on which the first numeral is 1). Both counting numbers and whole numbers are included for this chart. Since children's experiences in the earlier grades have been chiefly limited to the counting numbers, charts used there often contain only numerals for the counting numbers. The chart on page 2 begins with 0, a whole number, and is limited to the ten number symbols of the base-ten numeration system —the 1-place numerals. Each succeeding row contains 2-place numerals which are the result of the place-value feature of the base-ten numeration system and repeated use of the ten number symbols, 0–9.

The terms "1-place," "2-place," and "3-place" will be used frequently throughout the pupil's book, so the children should become thoroughly familiar with them. The systematic character of 2-place numerals is often overlooked when the reading and writing of numerals are taught purely as mechanical skills. The material on these pages is intended to begin understanding of the base-ten numeration system.

Number and Numeral. *Number* is an idea—it is abstract. It is the idea commonly associated with all sets of a given size. The *numeral* is the symbol that has evolved over many years and is used to express or indicate the number. Children recognize that the name "Rover," used to identify a pet dog, is not the dog itself but its name. When in doubt, use the word "number." It is less serious to misuse "number" than to misuse "numeral."

Number Line. A number line consists of points going on and on in opposite directions with given points emphasized to indicate a set of numbers. On paper or on the board, we can show only a small portion of a number-line picture. And so, when we speak of working with a number line, we are always referring to only a portion of it. We draw arrowheads toward each end of the picture of the line shown to indicate that it continues on and on. On a number-line picture the points for whole numbers are generally shown equidistant. By inspection, one can see that there are many points between the points for whole numbers. Many of these *in-between* points may be thought of as representing a fraction number, later termed "a rational number."

Teacher's Preparation

A class chart similar to the one on page 2 is very useful. One made of lightweight tagboard or paper is preferable to one written on the board. The former may be easily moved about as the need requires.

Pre-Book Lesson

- Draw pictures of these sets of objects on the chalkboard: 5 triangles; 3 stars; 8 squares. Ask questions about the sets as follows: "How many are in the set of triangles?" "What set has 8 things in it?" (Use the geometric shapes and terms as often as possible—when the use fits in naturally without disrupting the mathematical concept being developed.)
- Write this set of numerals on the chalkboard: 7, 2, 6, 9, 0. Ask, "How many numbers are in the set shown on the chalkboard?" "What is the greatest number in the set?" "What is the least number in this set?" "What whole number comes just after 7? after 2? after 6? after 9? after 0?" "Is there a whole number that comes just before each number in this set?" (Be alert for the child who recognizes that there is no whole number less than 0.)

• Write the numerals 38, 31, and 35 on the board. Have children arrange them to show numbers in order from least to greatest. Repeat with several other sets of numerals.

Using the Text Pages

• As selected children read each question, direct the discussion to cover the material in the Background section.

• The following points about chart A should be emphasized: (1) Only the ten numerals (number symbols) of our numeration system appear in the top row. (2) All of the numerals in the top row appear as digits in the numerals in each row under it. (3) Each row starts at the left with the numeral for the tens of the decade. (4) All the numerals for numbers in each decade such as 30–39 are in the same row.

• The number names for all tens are written in box B. This is a good opportunity to teach the meaning of the suffix "-ty." Explain that "-ty" means "tens," so "twenty" means "two tens," "sixty" means "six tens," and so on. The word "tens" has been abbreviated or changed and "-ty" at the end of a word has taken its place. Page 3 is the only page in the pupil's text which contains more than 3 new words. However, when pupils understand about the -ty words, they will not think of the words as new.

• For Ex. 9 draw a number-line picture on the board. Show points equidistant on it. Put arrowheads near each end to show that the line goes on and on. Select a point for zero and write the numeral. Then, to the right of that point, have pupils write the number names at the appropriate points for the whole numbers through 25. Emphasize that any point in the line may be selected for zero.

• Read the directions aloud for each of Ex. 10–15 to make certain the pupils understand what is expected. You should give directions as to how you want each child to number his paper using Ex. 10 as a model on the chalkboard.

NOTE: Throughout this series, each page has portions marked [O] for oral work or [W] for written work. You may wish to use some [O] material for a written assignment or may wish to make an oral presentation out of [W] designated material. You are encouraged to use your discretion in this regard as the occasion demands.

Individualizing Instruction

• Use the written work for *all pupils* who understand about the sets of numerals given and directions about certain numbers: greatest; least; just before; just after. The *slower learners* may need to do more work on the board before they are ready to work independently.

• Use the class number chart with the *slower learners*. Point to numerals for children to read. Say a number and ask a child to find the numeral for it on the chart. Ask the children to indicate sets of numbers, such as the set shown by numerals with 2 in one's place or the set shown by numerals with 5 in ten's place.

• Have the *more capable children* draw number-line pictures to show and label points for the following sets of whole numbers:

a. from 5 through 15 b. from 25 through 35
c. between 50 and 60 (but not including either)
d. greater than 90 and less than 99

* Some of your pupils might be able to use a number line to explain why there is no greatest whole number. Zero is the least whole number.

6. Tell how each 2-place numeral under 0 in box A starts. 10 starts with 1, 20 (twenty) with ?, and so on. 2, 3, 4, 5, 6, 7, 8, 9

7. Under 0, each 2-place numeral ends in what? 0

8. Read the row in box A starting with 40.

a. How do these numerals start? 4

b. How does each end? 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

B
10 ten
20 twenty
30 thirty
40 forty
50 fifty
60 sixty
70 seventy
80 eighty
90 ninety

The number names shown in box A may be thought of as names for some points on a number line. Points for 0 through 21 are shown on the number-line picture below.

* **9.** On the board, draw a number-line picture and name the points for 0 through 25.

[W]

10. For each set, show the greatest number. To show the greatest number for 10a, write the numeral 69.

a. 47, 52, 69 69 **b.** 40, 37, 29 40 **c.** 38, 46, 51 51

11. For each set, show the number which is least.

a. 36, 33, 35 33 **b.** 53, 60, 49 49 **c.** 62, 75, 95 62

12. Show the whole number which comes just before

a. 50. 49 **b.** 39. 38 **c.** 78. 77 **d.** 63. 62 **e.** 91. 90

13. Show the whole number which comes just after

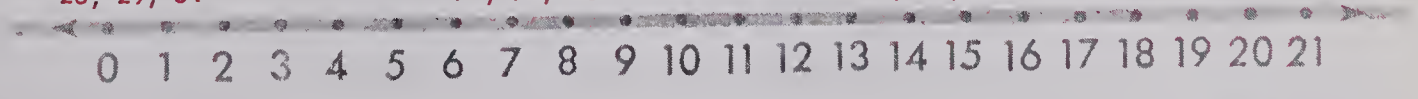
a. 44. 45 **b.** 60. 61 **c.** 29. 30 **d.** 82. 83 **e.** 51. 52

14. Write a numeral for

a. forty-two. 42 **b.** ninety-five. 95 **c.** fifty-six. 56

15. Show the numbers from least to greatest:

a. 31, 26, 29 26, 29, 31 **b.** 73, 78, 71 71, 73, 78 **c.** 76, 55, 64 55, 64, 76



* The symbol ⊕ is an abbreviated form for ⫴, a bundle of 10 sticks—a symbol easily used by young children.

What 2-Place Numerals Show

Tens and ones [O]

Many times we work with bundles of sticks. For a bundle of 10, we can write ⊕. For 1 stick we can write |. * For ⊕ *think*, "1 ten, or 10." For | *think*, "1 one, or 1." When you see ⊕⊕|| || | *think*, "2 tens and 5 ones, or 25."

Tell how to finish examples 1 and 2.

1. **a.** || | 3 ones, or 3 **b.** ⊕⊕ 2 tens, or 20
   c. ⊕⊕⊕⊕|| || | 4 tens and 5 ones, or 45

2. **a.** 32 ⊕⊕⊕||, or 3 tens and 2 ones
   b. 47 ⊕⊕⊕⊕|| || ||, or 4 tens and 7 ones

[W]

3. Use ⊕'s and |'s to show **a.** 43. ⊕⊕⊕⊕ || | **b.** 70. ⊕⊕⊕⊕⊕⊕⊕ **c.** 65. ⊕⊕⊕⊕⊕⊕ || | |

Second place	First place
9	2

In 92, the second-place digit, 9, shows the number of tens. The first-place digit, 2, shows the number of ones. *Think*, "92 = 9 tens and 2 ones." We might say that 92 shows both tens and ones. We mean that 92 shows both the number of tens and the number of ones.

Write and finish each of the following:

	a	b
4.	27 = 2 tens and 7 ones	4 tens and 9 ones = 49
5.	69 = 6 tens and 9 ones	3 tens and 0 ones = 30
6.	80 = 8 tens and 0 ones	9 tens and 5 ones = 95

The digit in first place, on the right in a 2-place numeral, shows the number of ones.

The digit in second place, on the left in a 2-place numeral, shows the number of tens.

Pupil's Objectives

(a) To recognize and use the symbols ⌀ (for a bundle of ten things) and / (for one thing); and (b) to read and to write correctly a 2-place numeral to show a number of tens and a number of ones.

New Words: *bundles, second*

Background

Pupils who used *Mathematics We Need* in the second grade should have had experiences with the positional nature of the decimal numeration system and the relation between tens and ones. They were taught to use the symbols ⌀ (for tens) and // // / (for ones). Note that, when possible, the marks for ones are shown in groups of two. This is to facilitate counting to find the number of ones, and to foster counting by 2's.

Teacher's Preparation

You will need to have available an appropriate place-value teaching aid* to help pupils visualize the 2-place numerals in our base-10 numeration system.

Pre-Book Lesson

• Tell your class the following story:

THE FISHERMAN'S SYSTEM OF KEEPING A RECORD

Many years ago a boy in Japan went fishing with his father each day. The boy kept a record of how many fish they caught by putting on a string one bead for each fish. (If you have some wooden beads, it would be a good idea to show the children how the boy kept a record of the fish caught.)

One day the boy and his father caught many fish in a net. The boy used his fingers (all ten of them) to show how many fish (hold up your hands so the class can see the 10 fingers) and then put 10 beads on the string. But, there were more fish, so he used all his fingers again and put 10 more beads on the string.

Suddenly, a wonderful idea came to the boy. He thought, "Each time I count this many fish (hold up 10 fingers, again), I'll put just one bead on another string." (Say to the children, "What name do we have for this many? The boy did not have a name for this many, but we call it *ten*, don't we?")

But, when the boy showed his father how many fish they caught that day, the father was angry. He did not understand the boy's system and he thought that the boy had not counted correctly. The boy finally explained the system to his father, but he had one big problem. He had two strings—one string to show tens, and one string to show ones. He didn't always know which string was which. With his father's help, he made a frame to keep the strings straight. The string on the right would always be for ones and the string next on the left would always be for tens.

• Direct the children to show a "picture" of 35 with the teaching aid you have selected to use. Through guided questioning and direction in using the aid, foster the understanding that the 3 in its place in the numeral 35 shows *3 tens* and the 5 in its place shows *5 ones*. Have the children picture several more 2-place numerals, each time asking a child to explain what each digit in its place shows.

Using the Text Page

• Bring out that use of the symbols ⌀ (for tens) and / (for ones) is a quick way to draw a picture showing a number.

• Make sure that pupils have clearly in mind that, for a 2-place numeral, (a) the place on the right means ones and is called *first place*, or one's place; and (b) the place at the left of one's place means tens and is called *second place*, or ten's place.

• Have pupils state in their own words the generalization that appears on the text page.

Individualizing Instruction

• For *all pupils* give practice, as you feel it is needed, in work using ⌀'s and /'s and *tens* and *ones* for 2-place numerals.

• For *slower learners*, you may find it necessary to allow for time to be spent in the arithmetic corner to work with the concrete bundles of ten and single sticks. These pupils may need to assemble a bundle of 10 and 4 more sticks with the tens shown at the left and the ones at the right before drawing the pictures ⌀ // // and then writing the numeral 14.

• See if *more capable children* recognize the idea that the digit in ten's place means 10 times what the same digit would mean in one's place.

* See list of materials on page xix.

Pupil's Objectives

(a) To review counting by 2's, 3's, 5's, and 10's; (b) to learn to count by 4's; and (c) to distinguish between numbers that belong to a defined set and numbers that do not belong to that set.

New Words: *circles, triangles*

Background

Pupils who used *Mathematics We Need* in the second grade were introduced to counting by the following sequences: by 2's to 100; by 3's to 30; by 5's to 100; and by 10's to 200. Counting by 4's is new in this lesson.

Children recognize patterns for sets of 2 through 6 readily. Have them use this skill in counting.

Teacher's Preparation

You should have sets of dot cards and numeral cards available for the Pre-Book Lesson. See the directions in Individualizing Instruction for Teaching Page 1. After the pupils have become thoroughly familiar with the dot cards, additional cards should be prepared to show several other patterns for a group, or set.

Pre-Book Lesson

• Have children line up with partners. Count them by 1's. Count them by 2's. Let children tell the advantages of counting by numbers greater than 1.

• If things exist in the room that can be counted by 2's, 3's, and so on use these for practice in counting (for example, window panes, children at tables, books on shelves).

• Use the dot cards for practice in immediate recognition of patterns. Have the children show the number of members in the set of dots by raising the appropriate numeral card (*Show the Answer*).

• On the board, write, "12, 14, _ ? _, _ ? _, 20." Have children write the missing numerals and explain the pattern.

Using the Text Page

• Be sure the children can identify the circles, squares, and triangles. Make certain, also, that pupils realize that the circle, square, or triangle does not include the space within it.

• Make sure the children understand the directions for the written work. For Ex. 5–9, they are to show numbers that belong to the defined set. For Ex. 10–12, they are to show numbers that *do not belong* to the defined set.

Individualizing Instruction

• If difficulties arise in Ex. 5–9, you may use one or more of the following:

a. Arrange beads of the Arithme-Sticks in sets of 2, 3, 4, or 5. Have children count by 1's and then by the number for each set of beads shown.

b. Prepare a grid that shows 4 blocks horizontally and 10 blocks vertically. Have children write in these blocks the numerals from 1 through 40, working across each row. Help pupils understand that the right-hand numerals then represent the counting-by-4's sequence.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
21	22	23	24
25	26	27	28
29	30	31	32
33	34	35	36
37	38	39	40

c. A grid such as the following for counting by 2's is helpful in fostering counting by numbers greater than 1.

2		4		6		8		10
12		14		16		18		20
22		24		26		28		30

d. Several copies of a blank grid can be duplicated on one sheet of paper so that each child can have practice in counting by several different numbers. He may begin with any number.

• Provide *all pupils* opportunities to count objects in the classroom—such as counting books, papers, and so forth to be distributed.

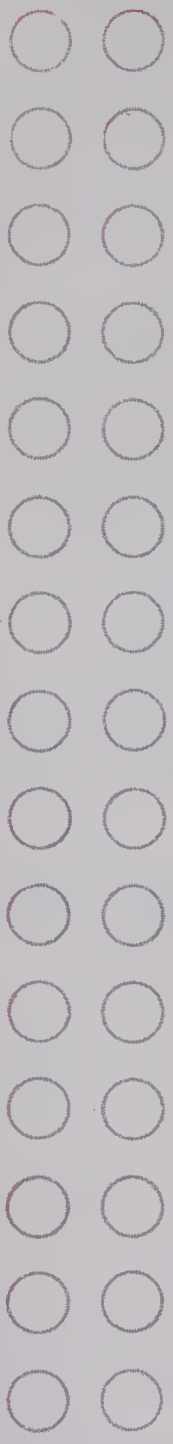
• With the *more capable children*, have them try beginning with 7 and counting by 3's or beginning with 23 and counting by 4's, and so on.

• Have *more capable children* begin with a number, such as 27, and count backward by a number greater than 1.

Counting by sequences is an immature method of finding the number for a total, but an important one. You may be surprised at how many have trouble with 3 and 4.

Faster Ways to Count

Sequences [O]

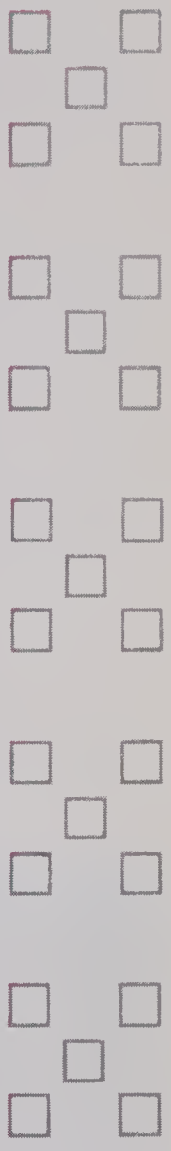


1. Count the circles by 1's. Then count them by sets of 2. Which way is faster to count the circles? by sets of 2

2. Count the squares by 1's, then by sets of 5. Which way is faster to count the squares? by sets of 5

3. Count the triangles by sets of 10. Could you count the triangles by 1's? Yes

4. Why do we like to count by sets of more than 1? Faster and easier



[W]

For example 5 write, "54, 56, 58, 60, 62, 64, 66." In this way finish examples 6 through 9.

5. Count by 2's: 54, 56, 58, 60, 62, 64, 66

6. Count by 3's: 3, 6, 9, 12, 15, 18, 21

7. Count by 4's: 12, 16, 20, 24, 28, 32, 36

8. Count by 5's: 5, 10, 15, 20, 25, 30, 35, 40

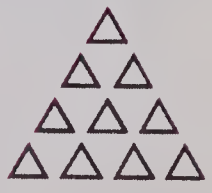
9. Count by 10's: 40, 50, 60, 70, 80, 90

For each of examples 10 through 12, write the numerals for the numbers that are not in that set.

10. Counting by 2's: 8, 9, 10, 12, 13, 14 9, 13

11. Counting by 3's: 6, 8, 12, 15, 17, 21 8, 17

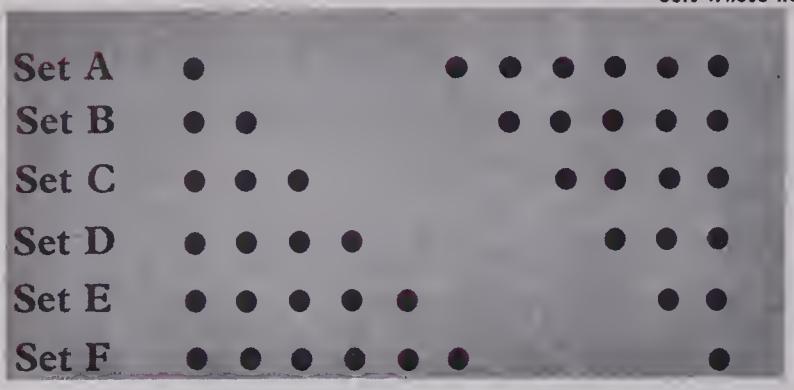
12. Counting by 4's: 8, 12, 17, 21, 24, 28 17, 21



* The notation 7; 2 and 5 provides an easy way to indicate a set of 7 things and two of its parts.

Numbers for Sets and Their Parts

Sets whose number is 7 or 6 or 5 [O]



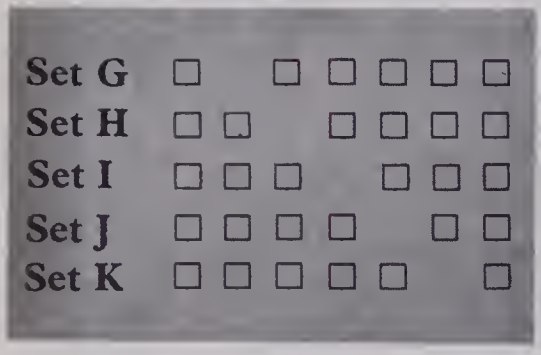
1. What are the numbers for the set and the parts in set A? (7; 1 and 6) set B? (7; 2 and 5) set C? (7; 3 and 4) set D? (7; 4 and 3) set E? (7; 5 and 2) set F? (7; 6 and 1)

* 2. For which two sets do we *think*, "7; 2 and 5"? B and E

3. For which two sets do we *think*, "7; 3 and 4"? C and D

4. Which set has the same two parts as set F? A

5. If the numbers for a set and one of its parts are 7 and 1, what is the number for the other part? 6 Which sets show this? A and F



6. What numbers for a set and its parts are shown in set G? (6; 1 and 5) H? (6; 2 and 4) I? (6; 3 and 3) J? (6; 4 and 2) K? (6; 5 and 1)

7. Which set has the same two parts as set G? K set H? J

8. Why is there no other set with the same two parts as set I? The two parts are the same size.

** 9. For set L, we can say that the numbers for the set and its parts are 5; 4 and 1. Why might we say, "5; 1 and 4"? The numbers for the parts are 1 and 4.

10. For set M, say the numbers for the set and its parts in two ways. 5; 2 and 3 5; 3 and 2

** The idea that two parts of a set may be arranged in different order is easily associated later with the principle that two numbers may be added in different order without changing the sum.

Pupil's Objectives

(a) To review the whole numbers 5, 6, and 7; (b) to discover numbers for parts of a set of 7, of 6, and of 5; and (c) to discover that if you know the number for a set and one of the parts of that set, you can find the number for the other part.

Background

An understanding of the number for a set and the numbers for its parts (subsets) is fundamental to building a knowledge of the addition and subtraction facts. This lesson and several that follow lead children to rediscover and review two important relationships between the number for a set and the numbers for its parts: (1) if the numbers for the parts are known, then the number for the set can be found; and (2) if the numbers for a set and one of its parts are known, then the number for the other part can be found.

Intuitively, the pupil is being introduced to the idea of adding and subtracting numbers. Learning about the numbers for a set and its parts lays the foundation upon which the A. and S. facts and the operations are built.

The use of compensation appears in the work on this page. In the list of numbers for parts of a set of 7, as one number becomes 1 greater, the other number becomes 1 less.

Teacher's Preparation

Have some kind of manipulative materials available for each child. Discs, bottle caps, blocks,* and so on are examples. One-inch squares of construction paper may also be prepared.

Have a flannel board** ready with some geometric designs to use on it (circles, squares, triangles, and so on).

Pre-Book Lesson

Ask the children to count 7 of their squares and to put all the others away. Direct the pupils to arrange the squares to show a *picture* of 7. As you watch, select one child to go to the flannel board and show the picture of 7 he has made. (It is not necessary that all pupils show the same pattern.) Elicit from the child one pair of parts of a set of 7.

Direct the participation of the children and the discussion so that all pairs of parts of a set of 7, from 1 and 6 to 6 and 1, will be *seen*. You may wish to conduct the same type of activity to show parts of a set of 5 or of 6.

Using the Text Page

- As selected children read and answer each question, make sure that all pupils see that, except for 3 and 3, each pair of numbers for parts of a set of 7 or of 6 occurs twice.
- Help the children discover in rows A–F and G–K that, as the number for one part is one greater, the number for the other part is one less. Intuitively the child is introduced to the idea of *compensation*.

Individualizing Instruction

- Give *all pupils* oral practice of this type:

If 5 is the number for one part of a set of 7, what is the number for the other part?

- *Slower learners* may need more practice with manipulative materials in order to *see* the numbers for a set and its parts. Give them more of the type of activity in the Pre-Book Lesson.
- *More capable children* may write the numerals for parts of a set of 7 as shown in rows A–F. They may also write the numerals for the parts of a set of 6 and of 5 in the same way.
- *All pupils* may do the following type of worksheet exercise for written practice:

Ring expressions that show parts of a set of 7.

5 and 2	1 and 6	3 and 4
4 and 1	3 and 2	5 and 1

Ring expressions that show parts of a set of 6.

4 and 2	5 and 1	2 and 4
2 and 2	3 and 3	3 and 2

Ring expressions that show parts of a set of 5.

1 and 3	3 and 2	2 and 3
2 and 4	4 and 1	3 and 4

* See 12 and 13, page xix. ** See 2 and 1, page xix.

Teaching Page 7

Pupil's Objectives

(a) To review the numbers for sets of 6 through 9 and the numbers for their pairs of parts; (b) to make tables of numbers for parts of sets; and (c) to reinforce the parts-total relationship.

New Word

copy

Background

Read again the Background section for Teaching Page 6.

Teacher's Preparation

Write the heading *Parts of a Set of 9* on the chalkboard high enough so that all numbers for parts of a set of 9 elements can be shown under it.

Pre-Book Lesson

- Ask 9 children to stand side by side across the front of the classroom. Ask a seated child to separate the children in front to show two parts of a set of 9 children. Ask him to tell the numbers for the parts and then to show them under the heading on the board. (Be sure the child writes "4 and 5" on the board. He is writing an expression for two parts of a set, and not an expression for the operation of addition.)
- Have another child repeat the activity to show a different pair of parts of the set of 9.
- Continue the activity until all parts of a set of 9 have been demonstrated and the numbers for them shown on the board.
- Use manipulative materials to review parts of a set of 8, of 7, and of 6.

Using the Text Page

- If there are any reading difficulties, go over the directions with the class before beginning the written work. Review the word *triangle*.
- Again lead the children to discover the use of compensation when writing the tables of numbers for the parts of a set.
- Be sure the pupils know what 7; 2 and _ ? _ means. Write Ex. 6 on the board as a model for them to follow.

Individualizing Instruction

- For *slower learners*, you may wish to have them use concrete objects or Ginn Arithme-Sticks to find parts of sets of 9, 8, and 7. Guide the pupils carefully to be sure that they are doing the written work correctly.
- For *more capable children*, duplicate across the top of a page or write on the chalkboard:

If □ is one part of ○, the other part is △.

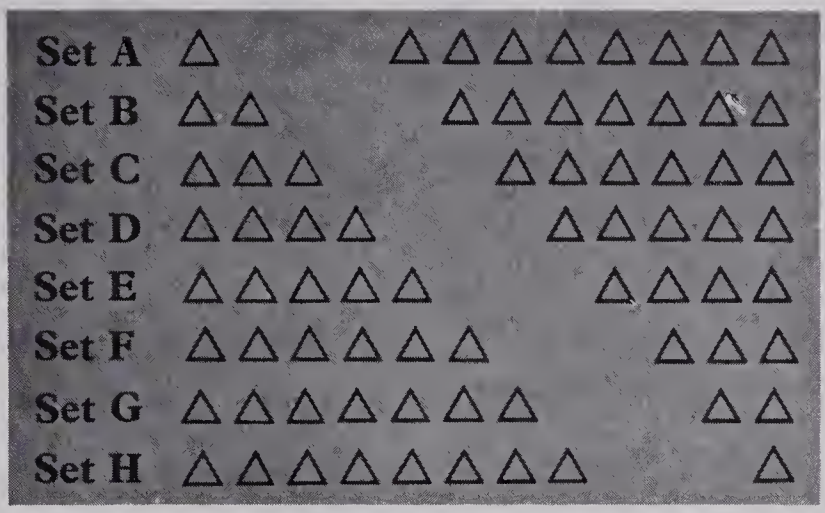
Direct the children to copy the above sentence several times and write a numeral in each geometric shape to make true sentences. They may show numbers for the parts of any set having as many as 9 members. Each shape in the sentence stands for a different number. Watch for the capable thinker who discovers that numbers for a set of 6 elements and its parts 3 and 3 cannot be expressed in this kind of sentence because, in this case, the two parts do not have two different numbers. See if that pupil can write an appropriate sentence such as,

If is one part of , the other part is .

More Sets and Their Parts

Sets whose number is 9 or 8 or 7 [W]

1. The number for each set of triangles is 9. Copy and finish the table of numbers for the parts.

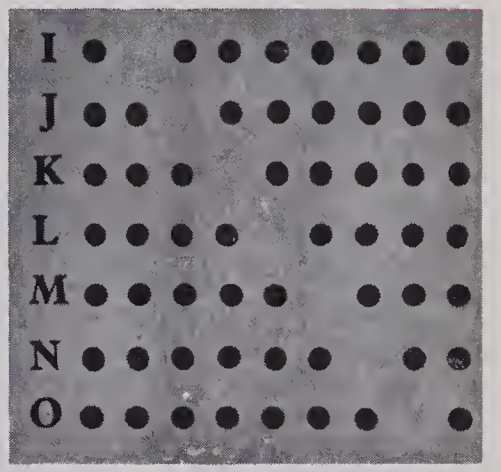


	Ex. 3
1 and 8	1 and 7
2 and 7	2 and 6
3 and ?[6]	3 and 5
4 and ?[5]	4 and 4
?[5] and 4	5 and 3
?[6] and 3	6 and 2
?[7] and ?[2]	7 and 1
8 and 1	

2. What is the number for the other part when the number for a set is 9 and the number for one part is

a. 3?[6] **b.** 8?[1] **c.** 4?[5] **d.** 7?[2] **e.** 6?[3]

3. Make a table of numbers for the parts when the number for a set is 8. Start with 1 and 7. Use the sets of dots for help. (Ans. above)



4. Which set of dots shows two parts that have the same number? L

5. Draw six sets of 7 dots to show the pairs of parts. Make a table of the numbers for the parts.

Copy and write what is missing. Make each example tell about the numbers for a set and the two parts.

6. 7; 2 and _?[5]_

7. 9; 3 and _?[6]_

8. _?[8]_; 4 and 4

9. 7; 3 and _?[4]_

10. _?[9]_; 5 and 4

11. 8; _?[5]_ and 3

12. 6; _?[2]_ and 4

13. 8; 2 and _?[6]_

14. 9; _?[2]_ and 7

With this page, your pupils will have reviewed work with parts which have the numbers 2-9 as foundation for more general work with the operations, addition and subtraction.

Putting Parts Together for the Total

Sums as great as 9; number sentences [O]

1. Putting parts together makes the total set. Tell the numbers for the two parts and the total set in the picture. 7; 2 and 5

* 2. Finish sentences 2a and 2b.

a. 2 boys and 5 boys are 7 boys.

b. 5 boys and 2 boys are 7 boys.

Using the numbers for the parts of the total, you can write a pair of number sentences, or adding examples.

$2 + 5 = ?$ is the number sentence for 2a.

$5 + 2 = ?$ is the number sentence for 2b.

3. We can *think*, "2 plus 5 equals what?" for $2 + 5 = ?$ Tell what to think for the number sentence for 2b. "5 plus 2 equals what?"

4. Will the number sentences for 2a and 2b both have the same answer, or sum? Yes Why? In each example, the numbers for the two parts are 2 and 5.

5. Tell the numbers for the parts and the total set in the dot picture. 9; 6 and 3

a. Tell two sentences about the parts and the total. 6 dots and 3 dots are 9 dots. 3 dots and 6 dots are 9 dots.

b. On the board, show the pair of adding examples with their sums. Using plus and equals, read each of the adding examples with its sum. 6 + 3 = 9 3 + 6 = 9

When you know the numbers for the parts, you can add to find the number for the total.

* The idea developed here is one of the fundamentals of problem-solving as taught in the Mathematics We Need program. Now pupils begin translation from sets language to numbers language, a skill needed constantly in problem-solving.

Teaching Page 8

Pupil's Objectives

To review (a) the idea that parts of a group, or set, may be put together, and that an adding example can show what is being done; (b) that the order in which two numbers are added does not affect the sum; (c) work with pairs of addition examples, or number sentences; and (d) using *plus* and *equals* in reading addition number sentences.

New Words

plus, *sentences*, *total*

Background

On this page the child is introduced to the *Commutative Property of Addition*, though not by name. *For any two addends, we can change the order of adding without affecting the sum.*

$$2 + 5 = 5 + 2$$

Learning about properties (sometimes called principles) is essential in developing full understanding of a number system and its operations.

We say that addition is a *binary operation*. This means that we operate on two numbers to obtain a unique third number.

Children must develop skills in expressing ideas with clarity and precision. The language of mathematics is particularly suited to this purpose. Experiences with *mathematical sentences* should grow naturally out of experiences with arithmetic. A mathematical sentence simply expresses an idea (sometimes true, as $2 + 3 = 5$; sometimes false, as $2 + 2 = 5$; and sometimes neither true nor false, as $2 + 3 = ?$) concerning numbers. In the primary or middle grades, a mathematical sentence is frequently called *an arithmetic sentence* or *a number sentence*. The concept will be used extensively throughout this program.

An arithmetic sentence that asks a question is an *open sentence* and often is called an *example*. $2 + 5 = ?$ is an open sentence, or example; it is a mathematical sentence for which a number is unknown. Use the terminology *number sentence* as frequently as possible.

Pre-Book Lesson

• Ask two girls to stand together at the front of the room. Ask three other girls to stand together about five feet from the two. Ask, "How many girls are in this set?" (2) "How many girls are in this set?" (3)

• Have the three girls join the two girls. Ask, "How many girls are there in all?"

This activity is to let the class see the joining of two disjoint sets (two sets which have no members in common) to form a new set. This activity of joining sets may be associated with addition of numbers.

• Write the following English sentence on the chalkboard and have a member of the class read it:

Two girls and three girls are how many girls?

• Write a number sentence under the English sentence. Explain that to write a number sentence, we use special marks that we have already learned. The transition should look something like this:

Two girls and three girls are how many girls?
2 girls and 3 girls are _?_ girls.
2 + 3 = ?

As you progress from one step to the next in the transition, have children read each sentence. Leave the sentences on the board and refer back to them in the lesson when the words *plus* and *equals* are introduced. Only $2 + 3 = ?$ shows addition.

Using the Text Page

• Use the picture to point out the two parts of the total set. Stress the idea that we can join two parts in either order to get the total, so we can add the numbers for the two parts in either order to get the number for the total set.

• Emphasize the use of the words *plus* and *equals* in reading addition number sentences.

Individualizing Instruction

• *More capable children* may write pairs of number sentences for each of the pairs of parts studied.

• Have *slower learners* turn back to page 7 and do again the written work in Ex. 6–14.

Pupil's Objectives

(a) To learn the difference between an example and a fact; (b) to build an understanding of the Commutative Property of Addition in reviewing pairs of addition facts; (c) to learn that there are some addition facts that do not have reverses; and (d) to learn that examples for addition can be written in either horizontal or vertical form.

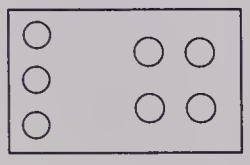
Teacher's Preparation

Have some manipulative materials available for the Pre-Book Lesson. The one-inch squares of construction paper will do very nicely. Each child should have seven of them and a full piece of construction paper, contrasting in color, on which to place the squares.

You should have a flannel board with some felt or flannel geometric shapes to use in this activity.

Pre-Book Lesson

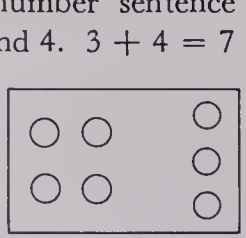
• Have your flannel board in a place where everyone can see it. The child's piece of colored construction paper should be in front of him. Call this colored paper a "number board." Say, "Make a picture of 3 on the left side of your piece of paper. Now, show a picture of 4 on the right side of the paper.



How many are on the left?
How many are on the right?
How many are there in all?"

• On the board, write the number sentence about the set of 7 and its parts 3 and 4. $3 + 4 = 7$

• Direct the children to turn their papers around very carefully so the 4 will be on the left and the 3 on the right. They must be careful not to move any of the shapes while moving the paper.



Have a child say the number sentence about the set of 7 and its parts that the picture now shows. Write the new number sentence on the board under the previous one.

Ask, "In what ways are the two number sentences alike?" The children should respond with three answers: (1) They are both adding; (2) They both show the same numbers; and (3) They both have 7 for an answer.

Ask, "In what way are the two number sentences different?" The answers to this question begin to provide insight into the generalization about the effects of changing the order of two addends and then adding. Emphasize this point and lead children to make a statement about it in their own words.

Using the Text Page

• Emphasize that we can use what we know about the numbers for a set and any two of its parts in order to make two addition facts.

• Make sure pupils understand why they can make only one addition fact about a set of 6 and its parts 3 and 3. Use manipulative material as suggested in the Pre-Book Lesson if necessary.

• Write the words *Horizontal* and *Vertical* on the board and show a pair of A. facts under each.

• Have children state in their own words the generalization about pairs of addition facts.

• Before doing the written work, have pupils tell for each example what is given. For Ex. 8, have them say, "A set of 5 things has the parts 3 and 2."

Individualizing Instruction

• For *slower learners*, provide expressions such as the following: 8; 3 and □ 7; 2 and △

• For the material above, have the *more capable children* write number sentences using the form: _ ? _ plus _ ? _ equals _ ? _.

• For the *slower learners*, have groups of pupils dramatize a fact, such as $5 + 2 = 7$. Have children stand in front of the chalkboard in groups of 5 and 2 with a large *and* written on the board between the two groups. Pupils at their desks can show the addition fact. Then have the groups of pupils change their position to show 2 and 5. Pupils will then show the other fact to make a pair of addition facts.

• Have *all pupils* work with a partner and use their number board to show and say addition facts. One pupil may make the picture of the parts and the other pupil may say the pair of facts.

Discovering Pairs of Addition Facts

Use of Commutative Property; sums as great as 9 [O]

1. When the numbers for a set and its parts are 9; 5 and 4, we can show a pair of adding examples, or addition examples, and their sums. We can show the pair as in a or as in b. Are the sums all the same? Yes

a. $5 + 4 = 9$
$4 + 5 = 9$

or

b.
$$\begin{array}{r} 5 \\ +4 \\ \hline 9 \end{array} \qquad \begin{array}{r} 4 \\ +5 \\ \hline 9 \end{array}$$

2. An addition example and its sum, like $5 + 4 = 9$, is called an addition fact. We read the addition facts shown in 1a and in 1b the same way. Read them.

5 plus 4 equals 9. 4 plus 5 equals 9.

Knowing the numbers for a set and its parts will help in learning addition facts. Finish examples 3 through 5.

3. 7; 4 and 3
$4 + 3 = \overset{7}{?}$
$3 + 4 = \overset{7}{?}$

4. 9; 3 and 6
$3 + \overset{6}{?} = 9$
$6 + \overset{3}{?} = 9$

5. 8; 6 and 2
$6 + \overset{2}{?} = \overset{8}{?}$
$\overset{2}{?} + 6 = \overset{8}{?}$

6. 6; 3 and 3. Why is there only one addition fact when the numbers for a set and its parts are 6; 3 and 3?

The parts are the same size.

7. For which of these is there only one addition fact?

a. 4; 2 and 2 **b.** 9; 1 and 8 **c.** 8; 4 and 4

a and c.

Most addition facts go in pairs.

[W]

Each example below shows numbers for a set and its parts. For each example, show a pair of addition facts as in a in example 1; then as in b in example 1.

8. 5; 3 and 2 ($3+2=5$; $2+3=5$; $+\frac{3}{2}$ $+\frac{2}{3}$)

9. 6; 4 and 2 ($4+2=6$; $2+4=6$; $+\frac{4}{2}$ $+\frac{2}{4}$)

10. 8; 5 and 3 ($5+3=8$; $3+5=8$; $+\frac{5}{3}$ $+\frac{3}{5}$)

11. 9; 7 and 2 ($7+2=9$; $2+7=9$; $+\frac{7}{2}$ $+\frac{2}{7}$)

12. 8; 2 and 6 ($2+6=8$; $6+2=8$; $+\frac{2}{6}$ $+\frac{6}{2}$)

13. 7; 5 and 2 ($5+2=7$; $2+5=7$; $+\frac{5}{2}$ $+\frac{2}{5}$)

* Since for any two whole-number addends, the sum is a whole number, the set of whole numbers has the property of closure under addition.

When We Add

*Terms of addition; *Property of Closure* [O]

2	addend
+6	addend
8	sum

1. What do we call numbers to be added? addends

2. How many numbers can we add at a time? two

3. What do we call the answer? sum

Say the sums for the examples in row 4.

4.	3	2	1	7	4	5	6
	+1	+3	+5	+2	+1	+2	+1
	4	5	6	9	5	7	7

5. In row 4, are the addends and the sums whole numbers? Is any sum less than one of its addends?

When the addends are whole numbers, the sum is a whole number and is never less than any of its addends.

** Try These

Addition facts, sums as great as 9 [W]

Show sums on folded paper. Your teacher will show you how to fold the paper. You may find □ or △ or _?_ in place of ? in examples 3 through 11.

	a	b	c	d	e	f	g	h	i	j	k
1.	1	2	3	1	4	5	2	3	7	1	2
	+8	+7	+4	+7	+2	+4	+5	+6	+1	+5	+3
	9	9	7	8	6	9	7	9	8	6	5
2.	4	1	5	4	5	8	3	3	6	2	6
	+5	+1	+3	+3	+1	+1	+5	+2	+2	+2	+1
	9	2	8	7	6	9	8	5	8	4	7

3. 3 + 3 = ? 6
4. 6 plus 3 = ? 9
5. 1 + 4 equals _?_ 5
6. 2 plus 6 = ? 8
7. 2 + 4 = □ 6
8. 5 plus 2 equals △ 7
9. 3 + 1 = ? 4
10. 1 + 6 = ? 7
11. 4 + 4 equals □ 8

** This quick test will give a picture of your pupils' knowledge of addition facts having sums as great as 9. Notice the different forms used for showing addition examples including the use of frames.

Teaching Page 10

Pupil's Objectives

(a) To review the use of the words *addend* and *sum*; (b) to begin to use the generalization that the sum of two whole numbers is always a whole number and is never less than either of the whole-number addends; and (c) to learn to use folded paper in writing answers.

New Word

folded

Background

The *Property of Closure* is introduced intuitively in the middle grades of this program. We say that *a set of numbers is closed under a particular operation if the result obtained by the operation on those numbers is a number within the same set.* For example, we say that the set of whole numbers is closed with respect to addition because the sum of any two whole numbers is always a whole number. This set of numbers is not closed with respect to subtraction because, under certain circumstances, the answer will not be a whole number (for example, $3 - 5 = {}^{-}2$).

Teacher's Preparation

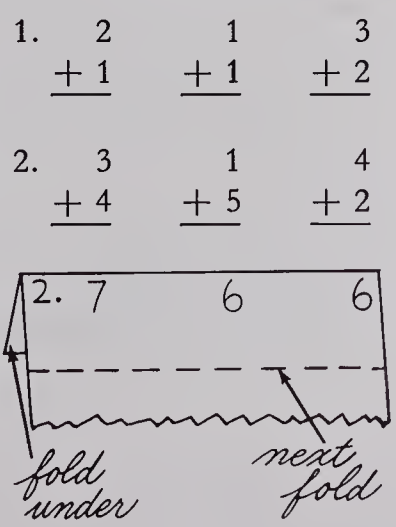
Folded-paper exercises are recommended frequently throughout this book. Most folded-paper exercises will require only one-half sheet of paper. Have paper ready for the folded-paper practice at the bottom of page 10.

The two forms of folded-paper practice are described and illustrated below:

Horizontal

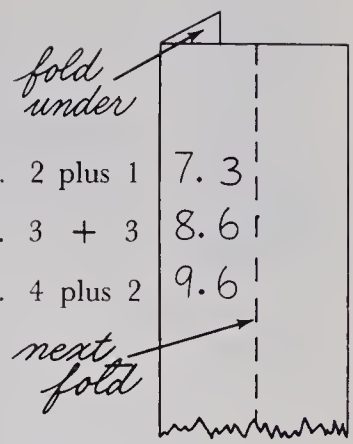
First, lay the top of the paper under the examples in row 1. Show the number of the row and then the answers.

Then, fold this row of answers under. Show the number of the next row and then the answers, and so on (see the diagram).



Vertical

First lay the left edge of the paper so that only the left column of examples can be seen. Show the example numbers and the answers. Then fold this column of answers under (vertically) and proceed in similar manner for the second column and so on (see the diagram).



Pre-Book Lesson

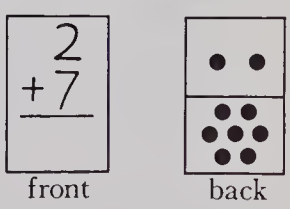
- Review the difference between the set of whole numbers and the set of counting numbers.
- Review adding numbers for parts of a set to find the number for the total set. Lead the children to discover that the number for the total is never less than the number for any of its parts.
- Direct the discussion to see whether the pupils can state the generalization in their own words.

Using the Text Page

- Write the words *addend* and *sum* on the board. Refer to them while discussing the oral exercises.
- Demonstrate and explain the methods of folding paper for use in writing answers only.

Individualizing Instruction

- Use *Show the Answer* practice (see Teaching Page 1) with *all pupils*. Show on flash cards or write on the board such expressions as 7; 2 and □. Have each child select the correct numeral card from his set and hold it up when requested to.
- Have *slower learners* make dot pictures and study cards (about 2″ by 4″ in size) for use with facts they do not know. For practice, have the children work alone or with a partner.



- The *more capable children* may employ classroom situations and write problems for Ex. 3–11 at the bottom of page 10.

Teaching Page 11

Pupil's Objectives

(a) To review *cents* and use of the symbol ¢; (b) to work with addition problems using what is known about parts of a set to find the number for the total set; and (c) to use known addition facts in the solving of the problems.

New Word

pencil

Background

In its social application, a number is usually associated with a set of objects or a set of measurement units. When we perform a mathematical operation it is the numbers upon which we operate and not the objects with which the numbers are associated. Therefore, when writing the work associated with any social problem, we write only the numerals and the operation sign to show that the operation is on numbers. After the answer has been obtained, we rewrite the answer with its appropriate label (oranges, blocks, and so on). An exception is made at the earlier grade levels when a problem shows money numbers since some of the work involves dollars and cents prior to studying operations with decimal-fraction numbers. We write the ¢ sign with the numerals and perform the mathematical operation. The answer does not need to be rewritten with the label, but should show the ¢ sign.

Teacher's Preparation

Provide materials of the type referred to in the problems on page 11. Perhaps you will want to provide some toy or real money to use in connection with the money problems.

Pre-Book Lesson

- Review the values of dime, nickel, and cent.
- Review the symbol ¢.
- Refer to any store situation in which the children will be familiar with the spending of small amounts of money.

Using the Text Page

- As each problem is read, ask questions about the parts of a set mentioned in the problem. Continue to emphasize the parts-total relationship: *When you know the numbers for the parts, you can add those numbers to find the number for the total.* Then ask questions that seek answers to *why* we add in each problem to find the answer.
- Emphasize that a problem can be stated in many different ways. Challenge the children, when making their own problems, to state a problem in as many different ways as possible. Then have them write the work on the chalkboard for each problem in the two ways shown for problem 6. Be sure to use the terms *addend* and *sum* in the discussions.
- The ways of writing the work for problem 6 at the bottom of the page illustrate the point mentioned in the Background section—that only the numerals are written when performing the operation. Then, the answer is rewritten with its label. Problems 1, 2, 7, and 9 on this page are exceptions because the numerals are written with ¢ signs.

Individualizing Instruction

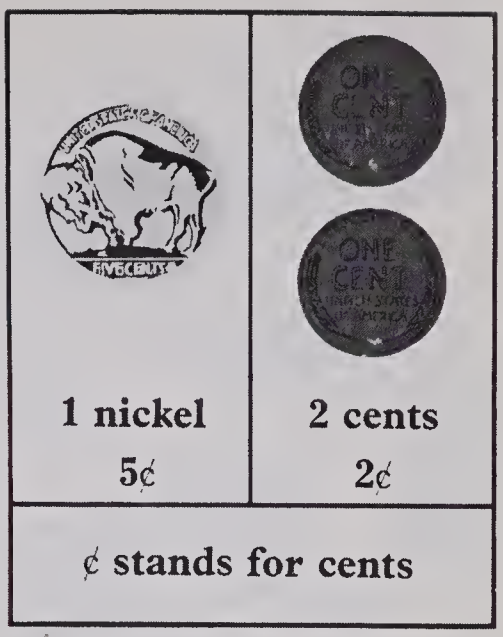
- Give *all pupils* more work in making addition problems.
- The *slower learners* may need more work with materials. Provide each child with some blocks, discs, paper squares, and so on to use while they are working the problems.
- The *more capable children* may write problems about

5 boys and 4 boys	4¢ and 3¢
2 books and 3 books	2 pictures and 6 pictures
3 chairs and 5 chairs	3 toys and 1 toy

* Pupil responses will give you insight into their real understanding of when to use addition.

Addition for Problems

Telling why add; making A. problems [O]



1. Tom has 5¢ in one pocket and 2¢ in another pocket. How much money has he in both pockets? 7¢

a. What are the parts in this problem? 5¢ and 2¢

b. Why would you add 5 and 2? To find the number for the total

* Tell the parts and why you would add in each of problems 2 through 5.

2. One red pencil costs 4¢ and one yellow pencil costs 3¢. How much do the two pencils cost together? 4¢ and 3¢ See 1. b.

3. Ann read 3 stories in one book and 5 stories in another book. How many stories did she read in all? 3 stories 5 stories See 1. b.

4. Six long balloons and 3 round balloons make a group of how many balloons? 6 balloons 3 balloons See 1. b.

5. Jim put 2 blocks on a pile of 5 other blocks. How many blocks were in the pile then? 2 blocks 5 blocks See 1. b.

Make your own addition problems about these things. See how many ways you can say each problem.

6. 3 horses and 4 horses **7.** a 4¢ toy and a 5¢ toy

8. 7 dogs and 2 dogs **9.** a 6¢ candy and a 2¢ candy

We can write the work for problem 6 as in a or as in b at the right.

2. 4¢ + 3¢ = 7¢

$$\begin{array}{r} 4¢ \\ +\ 3¢ \\ \hline 7¢ \end{array}$$

3. 3 + 5 = 8 (8 stories)

$$\begin{array}{r} 3 \\ +\ 5 \\ \hline 8 \end{array}$$

8 stories

On the board write the work for problems 2 through 5 and for problems 7 through 9 in the two ways shown for problem 6.

a. 3 + 4 = 7
** 7 horses

b.
$$\begin{array}{r} 3 \\ +4 \\ \hline 7 \end{array}$$
**7 horses

Work for Problems 4, 5, 7, 8 and 9

4. 6 + 3 = 9
9 balloons

$$\begin{array}{r} 6 \\ +\ 3 \\ \hline 9 \end{array}$$

9 balloons

5. 2 + 5 = 7
7 blocks

$$\begin{array}{r} 2 \\ +5 \\ \hline 7 \end{array}$$

7 blocks

7. 4¢ + 5¢ = 9¢

$$\begin{array}{r} 4¢ \\ +\ 5¢ \\ \hline 9¢ \end{array}$$

8. 7 + 2 = 9
9 dogs

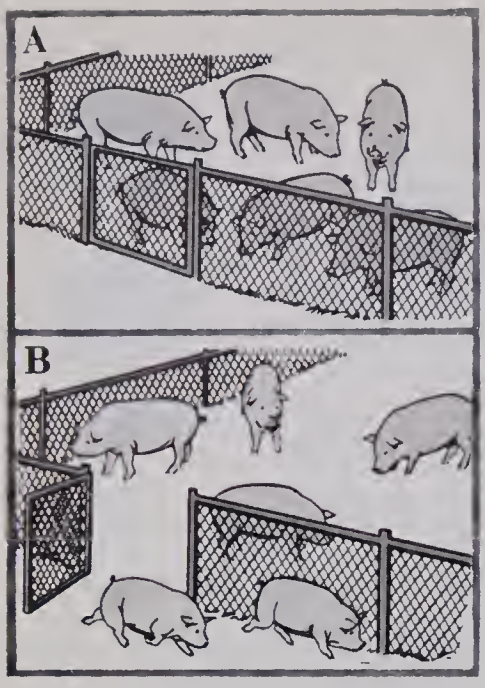
$$\begin{array}{r} 7 \\ +\ 2 \\ \hline 9 \end{array}$$

9 dogs

9. 6¢ + 2¢ = 8¢

$$\begin{array}{r} 6¢ \\ +\ 2¢ \\ \hline 8¢ \end{array}$$

** Have pupils establish the habit of rewriting the answer with the correct label after doing the work with numerals.



Taking Away One Part of a Total

Sums as great as 9 [0]

1. Six pigs were in a pen (box A). Two of them got away (box B). How many pigs were left in the pen? 4

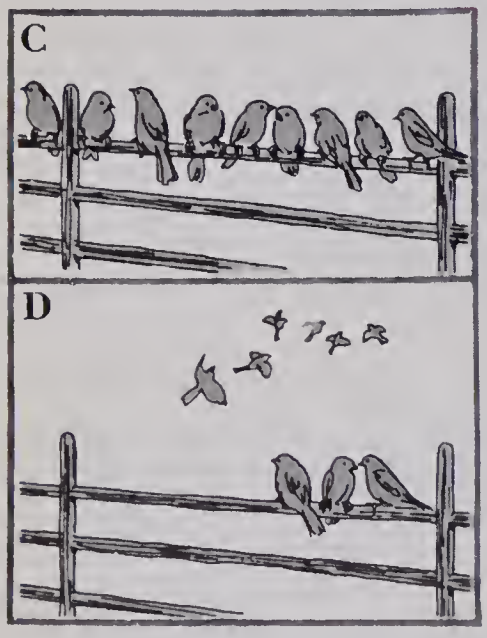
The total set was _?6 pigs. The part you know is _?2 pigs. To find the other part, the part left, you can take away the part you know.

2. Read the sentence about the pigs:
6 pigs take away 2 pigs is _?4 pigs.

3. You can write a subtracting example in the two ways shown at the right. What sign is used to show minus? –

a. $6 - 2 = ?$

b.
$$\begin{array}{r} 6 \\ -2 \\ \hline ? \end{array}$$



4. Read both subtracting examples 3a and 3b the same way and say their answers. Use minus and equals. 6 minus 2 equals 4.

5. Nine birds were on a fence (box C). Some flew away. Three birds are left (box D). How many birds are gone? 6

a. The number for the total is 9?_, and for the part you know is _?3.

b. Subtract to find the number for the other part, the part gone. $9 - 3 = ?$ 6

When you know the numbers for the total and one part, you can subtract to find the number for the other part.

The idea developed here is the second of the fundamentals of problem-solving taught in the Mathematics We Need program. Pupils continue to translate from sets language to numbers language—now for the separation of a subset and the use of subtraction.

Teaching Pages 12 and 13

Pupil's Objectives

(a) To review the idea that a group, or set, of things may be separated and that a subtraction example can show what is being done; (b) to review that a subtraction example can be written in both horizontal and vertical form; (c) to begin to use *minus* and *equals* when reading subtraction examples, or number sentences; and (d) to use what is known about the numbers for the total set and one of its parts in solving subtraction problems.

New Words

(page 12) *minus* (page 13) *between*, *path*

Background

Two uses for subtraction rather than the mastery of subtraction facts are to be emphasized on these pages. The *how-many-left* idea should not be too difficult because it occurs so frequently in the child's daily life. The *number-gone* idea is somewhat more sophisticated. In this use for subtraction, the number of items visible or identified is the part left. The number gone is the part to be found.

The need for the subtraction operation may occur in life in a great variety of language situations. Your pupils must be able to translate these various language patterns into their appropriate mathematical models.

The use of the Commutative Property of Addition was introduced in connection with page 8. We are still dealing with the set of whole numbers, but now we are performing a different operation. It is very important to examine this property in terms of each operation that we use. Pupils will discover that changing the order when writing numerals for a subtraction example will change the answer; whereas, the result did not change in addition. Therefore, they will see that the Commutative Property does not hold for subtraction.

Subtraction is a *binary operation* because we operate on two numbers to obtain a unique third number.

Teacher's Preparation

Have available a variety of materials to use in manipulating and in dramatizing the two ideas presented in this lesson. The suggestions in the Pre-Book Lesson should be a help in deciding what materials to prepare.

Pre-Book Lesson

- Have all children manipulate objects to experience taking away part of a set in order to see what is left.
- To create further interest, dramatize a variety of ways in which a part is left. Do not let the children lose sight of the fact that if they know the numbers for the original set and the part that has been taken away, they subtract to find the number for the part left. Allow a specified number of books to be moved to another place, balls to roll away, pennies to get lost, children to leave a group to get a drink, and so on. The purpose is to discover that the original set of objects has diminished. Only a part of the original total is left. Emphasize that we can subtract to find the number for the part left.
- For the how-many-gone experience, ask 7 children to stand at the front of the room. Make certain that all children count the number of members in the total set. Ask the children at their seats to close their eyes while part of the group of 7 leave the room. Then, ask the class to open their eyes and tell how many are gone. Ask, "How can you find how many are gone, if you know the numbers for the total and the part left?"
- Repeat the how-many-gone activity with a variety of materials. Each time ask the children to close their eyes while part of the original set is removed. Emphasize that we can subtract to find the number for the part gone.
- Emphasize these two subtraction ideas thoroughly before proceeding to the book lesson. Be sure pupils know the terminology "minus sign."
- Have pupils state in their own words the generalization that appears at the bottom of text page 12.

Using the Text Pages

- As exercises 1 and 2 are read and discussed, guide the questioning so that the children know that these are how-many-left situations.

Be sure to emphasize the parts-total relationship: *When we know the numbers for the total and one part, we can subtract to find the number for the other part.*

• After reading Ex. 3, write the two examples. Stress that the number for the total is shown on the left in the horizontal example and at the top in the vertical example. Discuss that there are many words we can use for the sign meaning *subtract* but we should begin to use "minus" now.

• As problem 10 is read and discussed, guide the questioning so that the class realizes that this is a how-many-gone situation. We continue to use what we know about the numbers for the total and one part to solve this type of problem.

• For problems 6–14 make sure that pupils can identify the numbers for the total and one part. For each problem ask, "How can we find the number for the other part of the total?"

• Finally, pupils are to write on the chalkboard in two ways the subtracting example to be used for finding the number for the other part in each problem. Tell pupils that we subtract with numerals having the ¢ sign just as with other numerals.

Individualizing Instruction

• For the *slower learners*, more practice may be necessary with numbers for totals and their parts. They may turn to page 7 and do Ex. 6–14 again.

• Have the *average* and *more capable children* read again each of problems 6–14 and write on paper the kind of subtraction question it asks. They may use HML to mean *how-many-left* and HMG to mean *how-many-gone*.

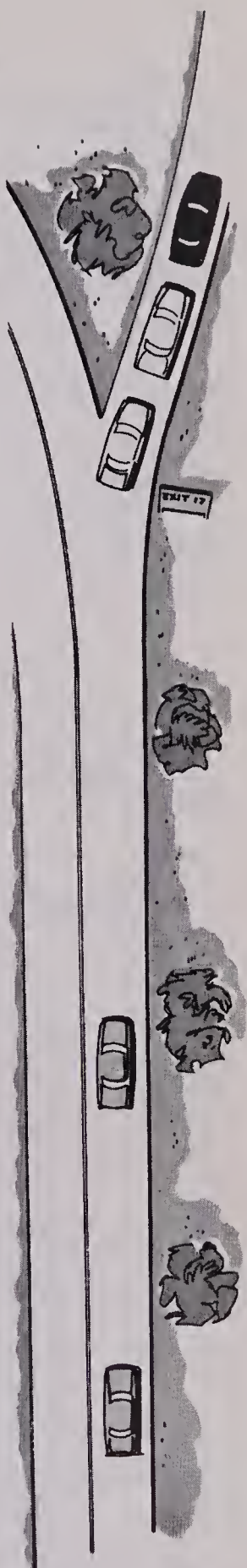
• *All pupils* may dramatize or use pictures to show problems 6–14.

NOTES

* All problems involving finding the other part can be associated with subtraction. On this page, the "other part" is the "number of things left" or the "number of things gone."

Finding how many left or how many gone

For each of problems 6 through 14, tell the numbers for the total and the part you know.



6. There were 5 cars on a road. Three of them turned off the road. How many cars were left on the road? total 5; part 3

7. Tom had 9¢. Now he has only 4¢. How many cents did he spend? total 9; part 4

8. Sam had 7 puppies. He gave away 4 of them. How many of the puppies did he keep? total 7; part 4

9. Mary had 6 doll shoes. She lost 3 of them. How many doll shoes has she now? total 6; part 3

10. Nine children were playing by the water. After some went home, 6 were left. How many went home? total 9; part 6

11. Sue counted 8 dogs on the path between her house and Ann's. When she looked again, there were only 6 dogs on the path between the two houses. How many dogs had gone? total 8; part 6

12. There were 7 pencils on the table. Two of them rolled off. How many pencils were then on the table? total 7; part 2

13. The first day of October, Linda bought a book of 8 bus tickets. At the end of October, she had 3 tickets left. How many bus tickets did she use? total 8; part 3

14. Mike ate 2 of his 9 candy turkeys. Then how many candy turkeys did he have left? total 9; part 2

* For each of problems 6 through 14 write on the board the subtracting example for finding the number for the other part. Write it two ways as in example 3.

6. $5-3=2$ 7. $9¢-4¢=5¢$ 8. $7-4=3$ 9. $6-3=3$ 10. $9-6=3$ 11. $8-6=2$ 12. $7-2=5$ 13. $8-3=5$ 14. $9-2=7$

Discovering Pairs of Subtraction Facts

Sums as great as 9 [O]

1. What is the number for the total set of airplanes? 7

a. Cover the 4-part. What is the number for the other part? $7 - 4 = ?$ 3

b. Cover the 3-part. What is the number for the other part? $7 - 3 = ?$ 4

2. For the dots, the number for the total set is ? 8.

a. Cover the 3-part. $8 - 3 = ?$ 5

b. Cover the 5-part. $8 - 5 = ?$ 3

You have found a pair of subtraction facts.

3. Which of these facts has no other subtraction fact to go with it to make a pair? Why is this so? The parts are the same size.

$6 - 3 = 3$ (6−3=3) $4 - 2 = 2$ (4−2=2) $6 - 4 = 2$ $8 - 4 = 4$ (8−4=4)

Knowing the numbers for a set and its parts will help in learning subtraction facts. Try examples 4 through 6.

4. 6; 2 and 4
   $6 - 2 = ?$ 4
   $6 - 4 = ?$ 2

5. 8; 1 and 7
   $8 - 1 = ?$ 7
   $8 - 7 = ?$ 1

6. 9; 2 and 7

$$\begin{array}{r} 9 \\ -2 \\ \hline ? \\ 7 \end{array} \qquad \begin{array}{r} 9 \\ -7 \\ \hline ? \\ 2 \end{array}$$

Most subtraction facts go in pairs.

$9 - 3 = 6$
$9 - 6 = 3$

$$\begin{array}{r} 9 \\ -3 \\ \hline 6 \end{array} \qquad \begin{array}{r} 9 \\ -6 \\ \hline 3 \end{array}$$

[W]

Show pairs of subtraction facts two ways. For 9; 3 and 6, show the pair of facts as in the box.

7. 9; 3 and 6
8. 8; 6 and 2
9. 9; 4 and 5
10. 7; 4 and 3
11. 7; 6 and 1
12. 7; 2 and 5
13. 9; 7 and 2
14. 6; 4 and 2
15. 6; 5 and 1

Pupil's Objectives

To review (a) the idea that by separating a set of things, you can find a pair of S. facts; (b) the relationship between the two S. facts in a pair (for example, between $6 - 4 = 2$ and $6 - 2 = 4$); (c) showing subtraction facts in number sentences; and (d) the S. facts with sums of 9 or less.

Teacher's Preparation

Have available some manipulative materials for the Pre-Book Lesson. The paper squares and the piece of construction paper ("number board") used in connection with pages 6 and 9 would be very useful. A flannel board should be used for illustrations for the children to follow.

Pre-Book Lesson

• Have the children demonstrate subtraction situations with objects. For example, a child could start with a set of eight books and take away three of them. After the child has demonstrated and stated the fact, write the following sentence on the board: Eight minus three equals five. To strengthen understanding of a number sentence, have the children write the same sentence using numerals and signs of operation.

Eight minus three equals five.
$8 \quad - \quad 3 \quad = \quad 5$

Discuss the difference between the two sentences written on the board. In one, words have been used to write the sentence, while in the other, numerals and signs have been used.

• Have the children manipulate the squares on the construction paper to discover pairs of subtraction facts. Have them place 7 squares on the paper. Direct them to cover or remove 4 of the squares and note the number of squares left; then cover or remove 3 and note the number left. On the board write the subtraction number sentences (facts) to accompany this manipulation and discussion: $7 - 4 = 3 \quad 7 - 3 = 4$

Ask such questions as

How are the two sentences alike?
How are the two sentences different?
In each sentence, where is the number for the total shown?
Where are the numbers for the parts shown?

Answers to these kinds of questions lead to further understanding of the parts-total relationship.

• Use demonstrations and manipulations until the children can express the idea that subtraction facts go in pairs. Allow pupils to put dots or X's, on the board to represent the sets being studied.

Using the Text Page

• Use the oral exercises to reinforce the important concepts developed in the Pre-Book Lesson.

• Before the written work is assigned, show on the board the horizontal and vertical ways of writing subtraction examples.

Individualizing Instruction

• Write the following on the board:

_ ? _ minus _ ? _ equals _ ? _.

Ask the *average* and *more capable children* to write, in this form, sentences using numbers they know for sets and their parts.

• For each of Ex. 7–15, *slower learners* may make dot pictures for the parts and show the pair of S. facts for the picture.

• Have *more capable children* change Ex. 7 so it will be true for teen numbers by making 9 into 19 and 3 into 13 or 6 into 16 this way:

19; 13 and 6 19; 3 and 16

In the same way they may rewrite Ex. 8–15.

• Have *more capable children* show all possible pairs of S. facts for taking away from 6, 7, 8, and 9.

Teaching Page 15

Pupil's Objectives

(a) To devclop fuller understanding of the relationship between addition and subtraction facts; (b) to learn that subtracting a number can undo adding that number; and (c) to give further mastery of the use of *addend* and *sum* in both addition and subtraction facts.

New Words

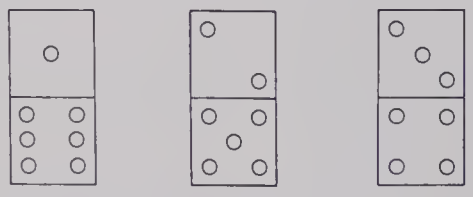
study, undo

Background

The *inverse* relationship between addition and subtraction is an important concept made use of at many different levels in mathematics. At this particular time it is sufficient to know that sums and addends are used in both addition and subtraction. Soon we will begin to use addition to check the result of a subtraction operation. Note, too, that this inverse relationship enables us to confine technical terminology for addition and subtraction to *sum* and *addend* and thus avoid use of the terms *minuend*, *subtrahend*, and *remainder*.

Teacher's Preparation

Materials should be made available for showing the total 7 and its parts. Domino cards as illustrated below would be very useful.



Pre-Book Lesson

- Have a set of 3 children stand together in front of the class. Have another set of 4 children stand together nearby. Have the two sets join together to form a set of 7. In directing 4 children to leave the set, point out to the class the "undoing" of the joining together that occurred at first.
- Use the children to show some more "joining" and "undoing" of parts and their total.
- Use the domino cards to see all of the addition and subtraction relationships for a set of 7 and its parts.

Using the Text Page

- As the facts in box A are examined, continue the discussion about subtracting a number undoing the adding of that number.

After studying box B, go back to box A and identify the addends and sum in each example in rows a-d.

- Use this lesson to build up the use of *addends* and *sum* in connection with both addition and subtraction facts.

Individualizing Instruction

- Have the *more capable children* show another fact to make a pair for each fact shown in rows 7–8. For example, the fact to go with Ex. 7a to make a pair would be $3 + 5 = 8$.
- For each example in row 7 *slower learners* should make a dot picture to show the parts of the total. Then they may show the pair of facts for the dot picture.
- The *more capable children* should show the fact for undoing each of the additions and subtractions in rows 7–8. For example, the fact for undoing Ex. 7a would be $8 - 3 = 5$; for Ex. 7b, the fact would be $4 + 5 = 9$.

* The use of the terms addend and sum will help pupils to understand the important idea of inverses—that adding an addend can be undone by subtracting that addend.

Can a Subtraction Undo an Addition?

Inverses; addends-sum relationship [O]

Did you ever stop to think that subtracting a number can undo adding that number?

A

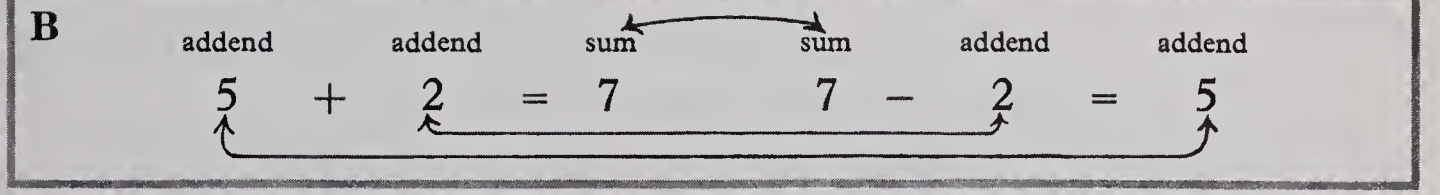
	Addition	Subtraction
a.	1 + 2 = 3	3 − 2 = 1
b.	4 + 2 = 6	6 − 2 = 4
c.	3 + 5 = 8	8 − 5 = 3
d.	5 + 4 = 9	9 − 4 = 5

1. Study the work in box A. Tell how each subtraction can undo the addition shown in the same row.

Now study the work in box B.

*

B



As you answer examples 2 through 6, think that subtracting a number can undo adding that number.

2. When we add two numbers, we get a _?_. sum

3. When we subtract, we can say that the number we subtract from is the _?_. sum

4. The numbers we add are called _?_. addends

5. When we subtract, we can say that the number we subtract is an _?_. addend

6. When we subtract, the answer is an _?_. addend

For each number sentence below, tell the sum.

	a	b	c	d	e
7.	5 + 3 = 8	9 − 5 = 4	6 − 4 = 2	3 + 4 = 7	8 − 2 = 6
8.	2 + 3 = 5	5 − 4 = 1	2 + 7 = 9	9 − 3 = 6	7 − 3 = 4

8 5 6 7 8 5 5 9 9 7

For each number sentence, tell the two addends.

7.	5 and 3	5 and 4	4 and 2	3 and 4	2 and 6
8.	2 and 3	4 and 1	2 and 7	3 and 6	3 and 4

When We Subtract

Addends-sum relationship [O]

8	sum
− 2	addend
6	addend

1. What do we call the number we subtract from? sum

2. What do we call the number we subtract? addend

3. When we subtract, how many numbers do we use? 2

4. What is the answer called in subtraction? addend

What is the other addend for each example below?

5.

7	8	4	5	6	9	7
6	1	2	3	5	1	3
1	7	2	2	1	8	4

6. In row 5, are the answers whole numbers? Yes Is the answer ever greater than the sum? No

When we subtract from a whole-number sum and the answer is a whole number, that answer is never greater than the sum.

* Try These

Subtraction facts, sums as great as 9 [W]

For each example, what is the other addend? Show the other addends on folded paper. Sometimes you will find □ or △ or _?_ in place of ?.

	a	b	c	d	e	f	g	h	i	j	k
1.	7	4	6	7	8	6	8	9	5	9	8
	3	2	4	2	6	2	3	8	2	6	5
	4	2	2	5	2	4	5	1	3	3	3
2.	8	3	4	7	5	9	7	8	9	7	9
	1	2	3	4	1	2	5	4	5	1	4
	7	1	1	3	4	7	2	4	4	6	5

3. 8 less 7 = ? 1

4. 9 − 3 = ? 6

5. 9 − 7 equals _?_. 2

6. 3 subtracted from 6 = □. 3

7. 8 minus 2 = △. 6

* This quick test will give a picture of your pupils' knowledge of subtraction facts having sums as great as 9. Notice the several ways of showing subtraction examples, including frames.

Teaching Page 16

Pupil's Objectives

(a) To review the relationship between a sum and its addends; (b) to review the use of the terms *sum* and *addends*; and (c) to begin to generalize that, in subtracting a whole-number addend from a whole-number sum, the unknown addend will never be greater than the sum.

Background

Mathematically it is incorrect to state that it is impossible to subtract a greater number from a lesser one so this idea is avoided in this program. Instead, in the third grade, we stress that when subtracting with whole numbers, the addend subtracted must not be greater than the sum. If a known addend is greater than the sum, the answer is a negative number and pupils at this point have not studied the set of numbers that includes negatives.

Teacher's Preparation

- Have a class number chart available for the Pre-Book Lesson. If you have not prepared for the class a large number chart like the one on page 2, be prepared to refer to page 2 during the Pre-Book Lesson.
- Have paper available for the folded-paper practice at the bottom of the page.

Pre-Book Lesson

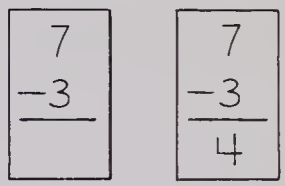
- Have different children count objects in sets of items in the room. Elicit the information from the class that counting numbers were used in counting these objects. Discuss how the sets of counting numbers and whole numbers differ.
- Use the class number chart to identify 0, the numeral for the number that is not a member of the set of counting numbers. Perhaps a child will generalize that all counting numbers are whole numbers but not all whole numbers are counting numbers.
- Use objects in the room to demonstrate subtraction. Have the class identify the number for a set of books. Then have them tell the number for the part that is removed and the number for the part that is left. In repeating this type of activity give practice in using as many different words associated with subtraction as possible—*less*, *minus*, and *subtracted from*.

Using the Text Page

- Guide the children to state in their own words the generalization in the middle of the page.
- Review the directions for folding paper described on Teaching Page 10. Encourage the children to write the answers to each row of examples as quickly as possible. However, do not stress speed at the expense of understanding and accuracy.

Individualizing Instruction

- The *slower learners* may check their answers to the written work by drawing dot pictures for each example.
- The *more capable children* may write a problem for each of examples 3–7 at the foot of the page.
- Let pupils who had difficulty with any example make a study card (about 2″ by 4″) for the fact. He may use the cards to study the facts.



front back

Pupil's Objectives

(a) To review using numbers for the total and one of its parts in problems requiring subtraction; (b) to review the how-many-left and the how-many-gone ideas in problems requiring subtraction; (c) to have practice in making problems requiring subtraction; and (d) to have practice in writing the work for solving problems requiring subtraction.

Pre-Book Lesson

• Ask, "Who can tell us two kinds of problems we have studied that require subtraction?"

• Use objects in the room to demonstrate the HML and HMG ideas. Be sure that the children understand the difference between the two situations and can demonstrate them.

Using the Text Page

• Ask selected children to read each problem. Have other children tell three things about the problem: (1) the number for the total; (2) the number for the part given; and (3) whether a how-many-left or a how-many-gone question is asked. Check to see how many pupils recognize that numbers for two parts are given in Ex. 4.

• Ask the class to use the information given in Ex. 6 to 9 to make subtraction problems. Elicit as many different ways of stating each problem as you can. Each time have pupils identify whether the problem stated is a HML problem or a HMG problem.

• The work for a HMG problem is shown in the text, written in two ways. Use the problem given for Ex. 7 and write the work on the chalkboard in two ways for a HML problem in preparation for the written work required at the bottom of the page.

• Remind the children that, when we have money numbers, we use the ¢ sign with the numerals and do not rewrite the answer. For other numbers we write only the numerals to find the answer. Then, after the answer is found, we rewrite the answer with the proper label. Leave the written work on the board as a guide for the children while they work problems 1–9.

• Each time a problem is worked on the board, ask for: (a) the number for the total; (b) the number for the part given; and (c) the kind of question asked so that we subtract to find the number for the other part.

Individualizing Instruction

• An excellent form of drill is possible with the use of the *Show the Answer* cards suggested under the Individualizing Instruction section of Teaching Page 1. Each child must have his own set of cards. You may wish to provide each child with a tagboard or construction-paper holder in which to put the answer before holding it up for the teacher to see. Such a holder can be made by cutting paper 4″ by 5″, folding up 1″ of the 5-inch side, and stapling it on each end.

The teacher writes an A. or a S. number sentence on the board. (You may want to use Related-Facts Cards instead of the board.*) The child finds on his numeral cards the answer to the sentence on the board and inserts it in his holder. Upon a signal from the teacher, all children show the answer simultaneously. In this way it is possible for the teacher to diagnose the difficulties of the class as well as of each individual in the mastery of the A. and S. facts taught thus far.

• Give the *more capable children* more information like that in Ex. 6–9 so they may write other subtraction problems for *all pupils* to solve. The following may be helpful:

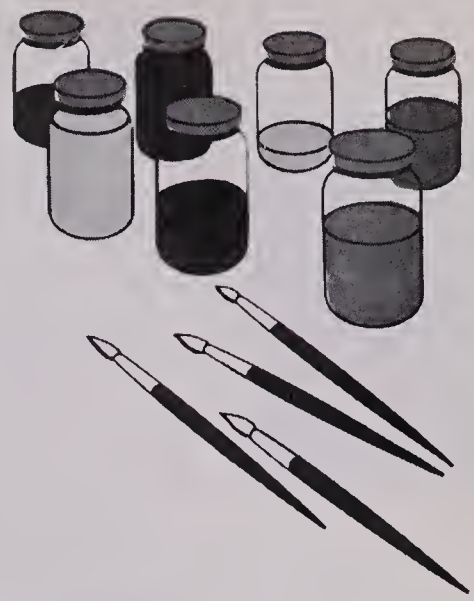
a. 8 flowers; 3 are left.
b. 7 marbles; 2 are lost.
c. 9 crayons; 5 are left.
d. 7 birds; 4 flew away.
e. 8 boys; 6 are left.

• The *slower learner* might draw a picture to show a subtraction situation. He could tell his story to a group and then one child could write the subtraction example on the chalkboard.

* See 8A, page xix.

Subtraction for Problems

Telling what is given; making S. problems [O]



1. There were 7 jars of paint on the table. The children took 5 of them. How many jars of paint were left? 2

a. What are the numbers for the total and for the part you know?
total 7; part 5

b. How can you find the number for the part left? subtract

For each of problems 2 through 5, tell the numbers for the total and for the part you know. Watch, because some problem may not have a total!

2. The children used some of the 7 brushes on the table. 4 brushes were left. How many were used?
total 7; part 4

3. Six boys were at a table. Some of them went away. Two boys stayed at the table. How many went away?
total 6; part 2

4. Mary has 3 red pencils and 5 black pencils. How many pencils does she have? parts 3 and 5

5. Sam had 9¢. He bought an apple for 5¢. How much money did Sam have left? total 9; part 5

For examples 6 through 9, make subtraction problems about finding the part left or the part gone.

6. 9 candies; 6 are left.

7. Joe lost 2¢ of his 9¢.

8. 9 dogs; 5 left now.

9. 8 boys; 4 went swimming.

[W]

Write the work for the subtraction problems in two ways as shown at the right for problem 6.

* **a.** $9 - 6 = 3$

3 candies

* **b.**
$$\begin{array}{r} 9 \\ -\,6 \\ \hline 3 \end{array}$$

3 candies

* Here again, impress on pupils the necessity of rewriting the answer after doing the work with numerals.

Sets of Addition and Subtraction Facts

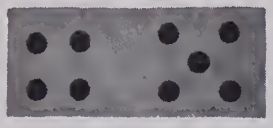
Addends-sum relationship; 4 facts or 2 facts [O]

Many times now we shall use A. instead of the word addition and S. instead of the word subtraction.

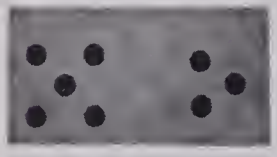
For these dots, we *think*, "The numbers for the total set and its parts are 7; 1 and 6." For the numbers 7; 1 and 6 there are two A. facts and two S. facts. We call these four facts the set of A. and S. facts for 7; 1 and 6.

1. Finish this set of A. and S. facts:

$1 + 6 = \overset{7}{?}$ $\quad 6 + 1 = \overset{7}{?}$ $\quad 7 - 1 = \overset{6}{?}$ $\quad 7 - 6 = \overset{1}{?}$



2. Tell about this total and its parts. 9; 4 and 5 ∧ Then say the set of A. and S. facts. 4+5=9 5+4=9 9−5=4 9−4=5



3. On the board, show the set of A. and S. facts about this total and its parts. 5+3=8 3+5=8 8−5=3 8−3=5

4. Say the set of A. and S. facts about

∧ a. 6; 4 and 2. 4+2=6 2+4=6 6−4=2 6−2=4 b. 8; 2 and 6. ∨ 2+6=8 6+2=8 8−2=6 8−6=2

5. To make the set of A. and S. facts, say the three other facts to go with ∧ a. $4 + 3 = 7$. 3+4=7 7−4=3 7−3=4 b. $9 - 7 = 2$. ∨ 9−2=7 7+2=9 2+7=9

6. Why are there only two facts in the set of A. and S. facts for this total and its parts? The parts are the same size. ∧ Say the set of A. and S. facts. 3+3=6 6−3=3

*** When the two parts of a total are the same size, the set of A. and S. facts has only one A. fact and one S. fact.**

*** When the two parts are not the same size, there are two A. facts and two S. facts in the set.**

* Instead of having to learn 90 separate addition facts and 90 separate subtraction facts, pupils can achieve 4 facts at a time (or sometimes only 2 facts) by using sets of addition and subtraction facts.

Teaching Pages 18 and 19

Pupil's Objectives

(a) To review that there is a set of A. and S. facts for the numbers for every total set and two of its parts; (b) to review that the numbers for most pairs of parts lead to a set of four facts, but that the numbers for some pairs lead to a set of only two facts; (c) to begin to use A. for addition and S. for subtraction; (d) to review all A. and S. facts through sums of 9; and (e) to begin to use the letter *n* as a symbol for an unknown sum in a number sentence.

New Words

(page 18) *instead, word*
(page 19) *exercise, unknown*

Background

Up to now, each fact in addition has been taught along with its corresponding addition faet to make a pair of facts. The same thing has been done for subtraction facts. From now on, the corresponding A. and S. facts will be taught simultaneously to make a set of A. and S. facts using the numbers for any given total and two of its parts. Most sets of A. and S. facts will contain four facts; two A. facts and two S. facts. Exceptions are the sets of facts using numbers such as 6; 3 and 3 for which the set contains but two facts, one A. fact and one S. fact.

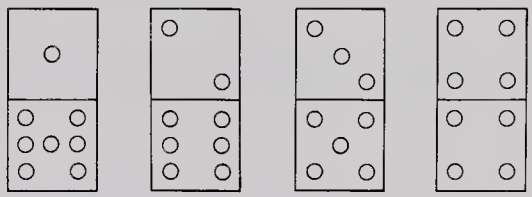
The task of learning the many necessary facts is lightened through utilizing the relationships among the A. and S. faets. The insight gained from examining a sum and its addends in terms of both A. and S. fosters a higher level of understanding than learning A. facts and S. faets separately.

Slower learners may need much practice to develop this understanding and to put it to use. Give them many experiences with objects and pictured situations before expecting them to master the abstract form.

The use of the number sentence has been introduced previously. The letter *n* in the exercises at the bottom of page 19 stands for the unknown sum. There is nothing difficult about this symbol. Pupils have been finding the answer when a question mark or a □ or a △ stands for the unknown number. When reading the sentence, *n* should be read "what number," "a number," "some number," and so on. Since *n* will be used in more complex ways later, be sure that the new symbol is clearly understood. You are cautioned not to be anxious to use *x* to show an unknown number in a number sentence because of the possible confusion at this grade level between *x* and the multiplication symbol.

Teacher's Preparation

- Have manipulative materials available to show a set of 8 and its pairs of parts.
- Have large domino cards made for showing the parts of a set of 8 as follows:



Pre-Book Lesson

- With a set of 8 objects, show the parts 2 and 6. Write on the board *8; 2 and 6*. Elicit the information that we can show a set of four facts when we know that a total set has 8 members and the parts have 2 members and 6 members. Show the two A. facts and two S. faets on the chalkboard: $6 + 2 = 8$, $2 + 6 = 8$, $8 - 2 = 6$, $8 - 6 = 2$.

Ask:

How many A. facts are there?
How many S. facts are there?

- Have the children use objects to derive other sets of four A. and S. facts.
- Use the large domino cards for 8; 7 and 1, and for 8; 5 and 3 and derive the sets of A. and S. facts.
- Use objeets and the domino cards to derive the parts for 8; 4 and 4. Have the children say the set of A. and S. facts about these parts for 8. Ask them to explain why there is only one A. fact and one S. fact in this set. Show the two facts on the chalkboard.

• For the work at the bottom of page 19, write number sentences on the board using ?, □, and △ for the unknown sum: $4 + 5 = ?$ $2 + 3 = \triangle$ $4 + 3 = \square$. Below each, write the same number sentence using n for the unknown sum. $4 + 5 = n$ $2 + 3 = n$ $4 + 3 = n$ Ask,

How are these number sentences alike?
How are they different?
What number does the ? stand for?
What number does the □ stand for?
What number does the △ stand for?
What numbers do the *n's* stand for?

Repeat this activity with several other number sentences before proceeding with the text page.

• Guide the discussion so that the children state in their own words the generalizations at the bottom of page 18 and page 19.

Using the Text Pages

• The material on page 18 is an oral summary of the Pre-Book Lesson dealing with sets of A. and S. facts.

• Be sure the children understand the directions for the written work at the top of page 19. Use Ex. 7 as a model by showing on the chalkboard the four facts in the set for the total 5 and its parts 3 and 2.

• For Ex. 13–18, the children are to copy the example showing the fact given and then show the other A. and S. facts in that set of facts.

• Before assigning the work at the bottom of page 19, discuss the use of n in a number sentence. Identify the addends in Ex. 1–12.

• Explain to the children that *1–18* means *1 through 18*, and that *Ex.* means *exercise* and can be used to designate either a problem or an example.

• Make sure the children understand the directions for Ex. 13.

Individualizing Instruction

• Have *all pupils* work in small groups to practice the A. and S. facts with sums through 9. You may wish to place the class in small groups of 3 or 4 or 5. Have one child in each group write a number sentence with n for an unknown number. The others will show the unknown number using the *Show the Answer* cards. After a child has written 5 sentences, have someone else in the group write the sentences.

• The *slower learners* may need to work with objects or dot pictures before they write their work for Ex. 7–12 at the top of page 19.

• You may wish to draw pictures of the total and its parts for Ex. 7–12 on the board and then have the *slower learners* show the set of A. and S. facts for each picture.

• Have *more capable children* change Ex. 7 at the top of page 19 so it will be true for teen numbers by making 5 into 15 and 3 into 13 or 2 into 12 this way:

15; 13 and 2 15; 3 and 12

In this same way rewrite Ex. 8–12.

NOTES

[W]

Show the set of A. and S. facts about

3+2=5 2+3=5 5-3=2 5-2=3 3+5=8 5+3=8 8-3=5 8-5=3

7. ∧5; 3 and 2. 8. ∨7; 4 and 3. 9. ∧8; 3 and 5.

4+4=8 8-4=4 4+3=7 3+4=7 7-4=3 7-3=4 2+7=9 7+2=9 9-2=7 9-7=2

10. ∧8; 4 and 4. 11. ∨9; 5 and 4. 12. ∧9; 2 and 7.

5+4=9 4+5=9 9-5=4 9-4=5

Copy examples 13 through 18 and show the other fact or facts in the same set of A. and S. facts.

2+4=6 6-4=2 6-2=4 5+4=9 9-5=4 9-4=5

13. ∧ 4 + 2 = 6 14. ∨ 6 − 3 = 3 15. ∧ 4 + 5 = 9

9-3=6 6+3=9 3+6=9 3+3=6 7-4=3 3+4=7 4+3=7

16. ∧ 9 − 6 = 3 17. ∨ 5 + 2 = 7 18. ∧ 7 − 3 = 4

2+5=7 7-5=2 7-2=5

A Different Kind of Example

n, for an unknown sum, in A.; sums as great as 9 [W]

The letter *n* in an example, or number sentence, stands for an unknown number, just as □ or △ or _?_ or ? does. In the examples below, the addends are known, so the unknown number is the sum.

addend addend sum

$5 + 2 = n$

In $5+2=n$, you know that $n=7$ because $5+2=7$.

Copy each example but, in place of *n* or □ or △, write the numeral. For example 1 write, "5 + 3 = 8."

1.	$5 + 3 = n$ (8)	5.	$3 + 4 = n$ (7)	9.	$4 + 2 = n$ (6)
2.	$3 + 2 = n$ (5)	6.	$5 + 2 = \square$ (7)	10.	$3 + 5 = n$ (8)
3.	$5 + 4 = n$ (9)	7.	$3 + 6 = n$ (9)	11.	$6 + 3 = \triangle$ (9)
4.	$2 + 4 = \square$ (6)	8.	$2 + 3 = n$ (5)	12.	$4 + 3 = n$ (7)

If you know two addends, you add to find the sum.

Ex. stands for exercise. An exercise may be a problem or an example. You have just finished Ex. 1–12.

13. Show the other A. and S. facts in the same set as **a.** Ex. 3. **b.** Ex. 5. **c.** Ex. 9. **d.** Ex. 11.

4+5=9 9-5=4 9-4=5 4+3=7 7-3=4 7-4=3 2+4=6 6-4=2 6-2=4 3+6=9 9-6=3 9-3=6

* These two statements provide the key to an intelligent decision as to whether to use addition or to use subtraction.

Should You Add or Should You Subtract?

Differentiating A. and S.; Extra Problems [W]

* If you know the numbers for the parts, you can add to find the number for the total.

* If you know the numbers for the total and one part, you can subtract to find the number for the other part.

For problem 1 below, write "1" on your paper. After the 1 write "A." if you are to add or "S." if you are to subtract. Do this same thing for problems 2–5.

1. Mother had 9 candles. She put 8 of them on the birthday cake. How many candles did she not use? S. 1 candle total 9; part 8

2. Five big girls and 4 little ones came to the party. How many girls came to the party? A. 9 girls parts 5 and 4

3. Eight balloons were tied to the table. Three good ones are left. How many balloons have been broken? S. 5 balloons total 8; part 3

4. Betty blew out 2 of the 8 candles on her birthday cake. How many candles were left burning? S. 6 candles total 8; part 2

5. Betty was given 4 books about other lands and 3 books about Indians. That was how many books in all? A. 7 books parts 4 and 3

For Ex. 1–5, write the work as in Ex. 6, pages 11 and 17.

[o]

Tell why you wrote "A." or "S." for each problem.

For each problem, tell the numbers for the two parts that are given, if you wrote "A." Tell the numbers for the total and a part that are given, if you wrote "S."

Ex. 2 and 5 Add to find the number for the total.
Ex. 1, 3, and 4 Subtract to find the number for a part.

** ◆**Extra Problems.** Your teacher may want you to turn now to page 347 and work Set 1 for extra problems.

** At the foot of this page is the first reference to the reservoir of extra materials at the back of this book—Extra Problems, Extra Examples, and Extra Activities.

Teaching Page 20

Pupil's Objectives

(a) To become proficient in differentiating between the need for addition and the need for subtraction in the solution of problems; (b) to gain additional experience in solving the two kinds of problems requiring subtraction that have already been taught; and (c) to see how well the parts-total relationship is understood.

New Words

extra, pages

Background

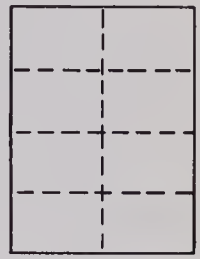
This is the first time the children have had to decide for each problem whether to add or subtract. It is important that they discuss their choices after completing the written assignment. Whether the pupil's response is right or wrong, ask him to defend his answer in order to locate errors in thinking. Often it is necessary for the teacher to accept an incorrect answer and, through guided questioning of the pupil, lead him to see his mistake. An immediate rejection of an incorrect answer tends to place a child on the defensive and causes him to withdraw from class discussion.

Pupils should understand that they use addition when numbers for the parts to be joined are given and the number for a total is to be found. They use subtraction when the numbers for the total and one of its two parts are given and the number for the other part is to be found. These ideas are basic to problem-solving and should be emphasized over and over. In later grades, pupils will learn to write an equation for a problem situation and then solve the equation. However, in Grade 3, they can work more easily by using these generalizations and they will sense better the relationship of mathematics to social situations.

Pre-Book Lesson

Have the children solve some oral problems related to the social situations in the classroom or the school. Include the type requiring addition and two types that require subtraction. Have them tell whether the problem requires addition or subtraction and why. No computation should be required.

Using the Text Page



- Perhaps you will wish to show the children how to make *blocks* on their papers for working problems. First, fold the paper in halves lengthwise. Then, open it and fold it in halves the other way. Fold in halves once more. When the paper is opened, the lines created by the creases will form *blocks* on the paper in which to work 8 problems. Number each block to correspond with the problem being worked.
- First, the children should read each problem and decide between A. and S. Then they write "A." or "S." after the appropriate problem number shown on the paper. Then, they go back and work each problem.
- Remind the pupils that they write only the numerals when working the problem. After the answer has been found, they rewrite the answer with its label. Remind the pupils that for these problems they write their work two ways for each problem—horizontally and vertically.

Individualizing Instruction

- *All pupils* can participate in differentiating between addition and subtraction situations by engaging in one or all of the following activities: (a) reread each problem on page 20 and tell the story in the problem in his own words; (b) make dots to illustrate the problem; (c) dramatize the problem.
- The Extra Problems referred to at the foot of page 20, as well as others on many following pages, provides material for helping pupils who need additional practice in problem-solving. These extra problems throughout the text help to maintain problem-solving skills. Make sure pupils are able to use correct thought procedures before you assign them Extra Problems. Sometimes *more capable children* enjoy the challenge of problem-solving, so these sets of extra problems will provide material for this purpose.

Teaching Page 21

Pupil's Objectives

(a) To develop mastery of the generalization that, when the sum and one of two addends are known, you subtract to find the other addend; and (b) to begin to use *n* to show an unknown addend in a number sentence.

Pre-Book Lesson

• Make use of a problem situation having a total of no more than 9. On the board show the number for the total. Then remove a part of the total set used and on the board show the number for the part removed after the number for the total followed by a minus sign. Finish the subtraction example. Elicit information as to whether this is a *how-many-left* or a *how-many-gone* situation and the numbers for the total and the parts.

• Have a group of children go to the board. Say to one child, "The number for a total is 7, the number for one of its parts is 3; subtract to find the number for the other part." Say to another child, "The number for a part is 4, the number for the total is 9; find the number for the other part."

• Change to using *sum* and *addends*. Say to one child, "The two addends are 3 and 2; find the sum." Say to another child, "One addend is 5, the sum is 8; find the other addend."

• Say, "Now, I am going to read a number sentence with *n* in it. John, you write the sentence on the board." Read, "6 + *n* = 9." "In this sentence does *n* stand for the sum or for an addend?" Have the children show the number that *n* stands for if they can. Ask how they knew the number. Read and discuss both A. and S. number sentences with *n* standing for an addend. Emphasize the terms *sum* and *addend* in directing the board work.

While a child is working at the board, have the remainder of the class write the work on paper. As you circulate among the class, you can find which pupils understand the new work.

Using the Text Page

• The purpose of the oral material is to build a greater understanding of the inverse relationship between addition and subtraction. The important idea developed on this page is that we *can* subtract to find the unknown addend when the sum and one addend are given.

Children often say, "In *n* + 6 = 8, why must I subtract 6 from 8 to find the answer 2 when I can just think that 2 plus 6 equals 8?" You may not want to force children to think differently in working with basic A. and S. facts. However, it is very useful for them to know how to do this work. In more difficult examples, the child will not know the unknown addend and will have to subtract to find it.

• Use Ex. 10 to show the class how to do the written work. For 3 + *n* = 5, when we write "5 − 3 = 2" to find the number for *n*, we are just showing one fact in the set of A. and S. facts for 5, 3, and 2. Even though the children know the number *n* stands for, writing the example to show how to find the number for *n* is good practice.

• Have pupils state in their own words the generalization that appears at the foot of the page.

Individualizing Instruction

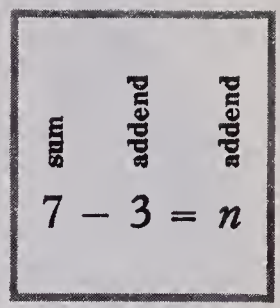
• The *slower learners* may need to do Ex. 10–18 orally using manipulative objects or dot pictures before they are ready to do the written work. Have them tell the sum and the addend. Then say the example to use in finding the unknown addend.

• *More capable children* may write examples for other pupils to work, using *n* for an unknown sum or addend. Use sums through 9.

n in Another Kind of Example

n, for an unknown addend; addends-sum relationship [O]

In these examples, or number sentences, you will need to use what you know about sums and their addends.



1. In the example $3 + n = 7$, 3 is the known addend, n stands for the unknown addend, and 7 is the sum.

To find what n stands for, why can you subtract the known addend from the sum? $7 - 3 = n$, so $n = 4$?

For each of Ex. 2–4, tell the sum and known addend. Then tell what n stands for. For Ex. 2 say, "8 is the sum and 6 is the known addend. $8 - 6 = 2$, so $n = 2$."

2. $n + 6 = 8$ — sum 8; addend 6; 8 − 6 = n, so n = 2

3. $3 + n = 9$ — sum 9; addend 3; 9 − 3 = n, so n = 6

4. $n + 4 = 6$ — sum 6; addend 4; 6 − 4 = n, so n = 2

5. In $6 - 1 = n$ why is 6 the sum? ∨ Why is 1 the known addend? ∨ Why can you subtract to find what n stands for? ∨ $6 - 1 = n$, so $n = 5$?
It is the number we subtract from. It is the number we subtract. Subtract to find an unknown addend.

6. In $9 - n = 5$, what is the sum? 9 the known addend? 5
To find the unknown addend, use $9 - 5 = n$. $n = 4$?

For each of Ex. 7–9, tell the sum and the known addend. Then tell how to find the number n stands for.

7. $7 - n = 6$ — sum 7; addend 6; 7 − 6 = n, so n = 1

8. $5 - n = 2$ — sum 5; addend 2; 5 − 2 = n, so n = 3

9. $6 - 3 = n$ — sum 6; addend 3; 6 − 3 = n, so n = 3

[W]

Write your work to find the unknown number.

10. $3 + n = 5$ — 5 − 3 = 2
11. $7 - 1 = n$ — 6
12. $4 + n = 7$ — 7 − 4 = 3
13. $8 - n = 4$ — 8 − 4 = 4
14. $n + 7 = 9$ — 9 − 7 = 2
15. $9 - n = 2$ — 9 − 2 = 7
16. $7 - n = 2$ — 7 − 2 = 5
17. $1 + 7 = n$ — 8
18. $\square + 6 = 9$ — 9 − 6 = 3

If you know the sum and one addend, you subtract to find the unknown addend.

*n to stand for a number in a number sentence is used in a great many exercises throughout this text. When pupils learn to distinguish between n standing for an addend and n standing for a sum, they should have little difficulty in solving the number sentence.

n-Examples for Problems

Choosing the n-example [O]

1. Ann's brother made her a doll house. It has 4 rooms on the first floor and 3 rooms on the second floor. This is how many rooms in all? 7

* We know the numbers for the two parts. Is a or b or c the correct *n*-example, or number sentence, for finding the number for the total? b

a. $4 - 1 = n$ **b.** $4 + 3 = n$ **c.** $4 - 3 = n$

For each problem, which *n*-example is correct?

2. Sue has 4 blue hair ribbons and 5 yellow hair ribbons. How many hair ribbons does Sue have?

a 9 ribbons

a. $4 + 5 = n$ **b.** $5 - 4 = n$ **c.** $5 - 1 = n$

3. Ann made 7 doll hats. She now has only 4 of them. How many doll hats has Ann lost?

b 3 hats

a. $7 + 4 = n$ **b.** $7 - 4 = n$ **c.** $4 + 7 = n$

4. Sue had 9 dresses for her dolls. She gave Ann 3 dresses. Then Sue had how many dresses?

b 6 dresses

a. $9 + 3 = n$ **b.** $9 - 3 = n$ **c.** $3 + 9 = n$

5. Sue had 8 buttons. She lost 3 buttons. How many buttons does she have left?

b 5 buttons

a. $8 + 3 = n$ **b.** $8 - 3 = n$ **c.** $3 + 8 = n$

6. Ann has 2 white dolls' shoes and 6 others. How many dolls' shoes has she?

c 8 shoes

a. $6 - 2 = n$ **b.** $8 - 2 = n$ **c.** $2 + 6 = n$

* Another step in developing the ability to translate from sets language to numbers language! And ground work is being laid for solving a problem by using an equation.

Pupil's Objectives

(a) To continue to use what we know about numbers for a total and its parts in order to solve A. and S. problems; and (b) to choose from several number sentences, or *n*-examples, the one that is suggested by the information given and the question asked in a problem.

New Words: *ribbons*

Background

An important facet of solving problems involves translation. The child must be able to *translate* the information of the problem into an appropriate mathematical model. Consider the problem, There are 9 marbles in the bag and 5 marbles in the box. How many more marbles does the bag have than the box? Other language patterns which could have been used for this problem are, *Tell the difference between the marbles in the bag and those in the box.* and *How many fewer marbles are there in the box than in the bag?* Though the sentences differ in language they are all alike in that they do not contain "subtract," "minus," or "−." The idea of subtraction is suggested in each, however.

Children must have many experiences in reading problems, identifying the numbers for the total set and the parts, and then finally and gradually translating information into number sentences. This act of translating requires the child to use the information given in such a way as to help him arrive at a reasonable answer.

Pre-Book Lesson

• Use some objects in the room to demonstrate joining a set of 3 with a set of 4. Have the children use the demonstration as the information for stating a problem. Elicit as many different ways as possible of stating the problem in the language of the children.

• Provide on the board three *n*-examples, or number sentences, one of which fits the problem. Ask the pupils to choose the number sentence belonging with the problem. Ask why they chose the particular sentence. This is beginning work with relating equations to problem situations and lays a good foundation for later work.

• Repeat the preceding with other demonstrations of problems, stating them, and choosing the number sentence, or *n*-example, for the problem.

Using the Text Page

• The purpose of the oral exercise is to identify the best *n*-example, or number sentence, for each problem. Be sure to ask children to explain *why* they selected each number sentence. The work on this page is entirely oral but, in connection with work on page 27, pupils will be asked to write the work for these problems.

In this book, for problem-solving, only the direct form of *n*-example is used. This results in the *n* always being used in place of the answer, as in $4 + 5 = n$ or $n = 4 + 5$ or $9 - 4 = n$.

Individualizing Instruction

• Perhaps the *slower learners* should draw dot pictures for problems 2–6 and then find the number sentence for each problem.

• The *more capable children*, and perhaps *all pupils*, may choose the correct *n*-example from the two sentences given below for each of the problems on pages 11 and 17. Have the children tell why they selected a particular number sentence.

(page 11)

1. a. $5¢ + 2¢ = n¢$	b. $5¢ - 2¢ = n¢$
2. a. $4¢ - 3¢ = n¢$	b. $4¢ + 3¢ = n¢$
3. a. $3 + 5 = n$	b. $5 - 3 = n$
4. a. $6 - 3 = n$	b. $6 + 3 = n$
5. a. $2 + 5 = n$	b. $5 - 2 = n$

(page 17)

1. a. $7 + 5 = n$	b. $7 - 5 = n$
2. a. $7 - 4 = n$	b. $4 + 7 = n$
3. a. $6 + 2 = n$	b. $6 - 2 = n$
4. a. $5 - 3 = n$	b. $3 + 5 = n$
5. a. $9¢ + 5¢ = n$	b. $9¢ - 5¢ = n¢$
6. a. $9 + 6 = n$	b. $9 - 6 = n$
7. a. $2¢ + 9¢ = n¢$	b. $9¢ - 2¢ = n¢$
8. a. $9 - 5 = n$	b. $9 + 5 = n$
9. a. $8 + 4 = n$	b. $8 - 4 = n$

Pupil's Objectives

(a) To review uses of the calendar; and (b) to review some of the time concepts associated with the calendar (days in a week, weeks in a month, months in a year).

New Words

calendar, *month*, *week*

Background

You probably have had a calendar in the classroom since the beginning of the school year. Use this page as a formal introduction to the calendar or as a review of the understandings presented thus far in the use of an actual calendar.

Teacher's Preparation

Have available some calendars showing the twelve months.*

Pre-Book Lesson

You may wish to tell the class about some of the ways man has kept a record of time. He made a tally mark on a wall of a cave for each day that passed. He associated important events with the season in which they occurred. Candles with marks on them to indicate hours or days were burned as a means of keeping a record of time. A glass with sand in it recorded hours or days.

Using the Text Page

- If you use this page during some month other than October, use the calendar in the classroom to answer questions like Ex. 4 and 5.
- You will have to give the children some direction in starting a birthday-month calendar. Each will need to know the day on which to start the month of his birthday as well as the number of days in the month. You may wish to divide your class into groups of 4 or 5 and appoint a chairman for each group. The chairman will have a calendar for the year and can show each child in his group the day his birthday month begins and how many days in the month.

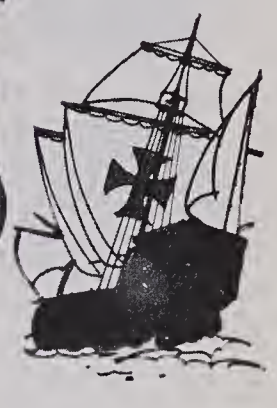
Individualizing Instruction

- *All pupils* should use a large classroom calendar drawn on tagboard or prepared as a bulletin-board display, on which information about the daily weather, important events for the month, holidays, and so on are recorded.
- Individual calendars for the month, season, or year showing holidays may be prepared by *all pupils*. You will have to provide the necessary information appropriate to the type of calendar being prepared.
- *More capable children* may make a class birthday calendar. The necessary information could be obtained from the individual calendars prepared for Ex. 6 or from the teacher's class record.
- *More capable children* may copy in a list the names of the months and show the number of days in each. They may use a calendar showing the twelve months or use the rhyme "30 days hath September." From their list they can tell the longest months and the shortest months. Encourage these pupils to learn the correct spelling for the names of the months.
- *More capable children* may use an encyclopedia to try to find why it was decided that there should be 7 days in a week. They might also find out about plans for a Perpetual World Calendar and a Thirteen-Month Calendar.

* Sec 14, page xix.

OCTOBER

SUN.	MON.	TUES.	WED.	THURS.	FRI.	SAT.
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				



Telling Time with the Calendar

Measures [W]

1. When can we use a calendar to tell time?
To find the day, week, month, and year
2. How many days has the month of October? 31

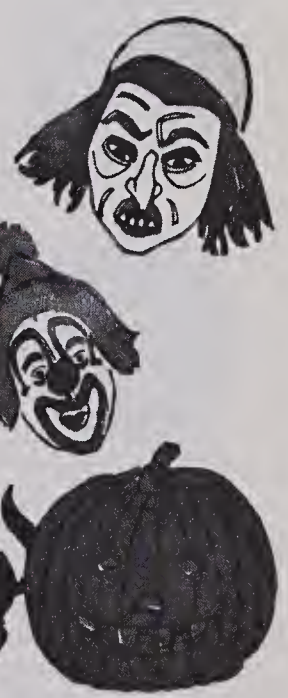
3. How many days are there in a week? 7

4. October has how many whole weeks? 4 After the whole weeks, how many days are there left over? 3

5. For the month of October shown, tell
   a. on what day of the week the month began. Sunday
   b. how many Sundays there were in the month. 5
   c. on what day of the week the last day was. Tuesday
   d. on what day of the month the second Monday was. 9
   e. on what day of the week Columbus Day was. Thursday

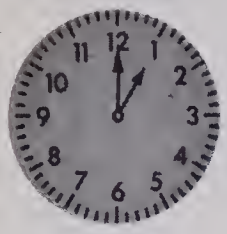
6. Make a calendar for your birthday month.
   a. Write in red the numeral for your birthday.
   b. Put X on the numeral for the first Monday.

7. Most months have about 4 weeks.



7 days (da.) = 1 week (wk.) **1 wk. = 7 da.**

Telling Time on Clocks

A 

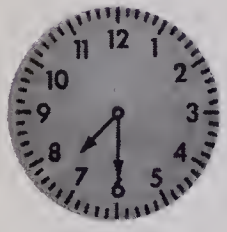
1. Clock A shows that it is 1 o'clock. At 1 o'clock, to what numeral does the short hand point? 1 To what numeral does the long hand point? 12

B

2. Ann's birthday party will start at 2 o'clock. At that time, where will the short hand point? 2 Why will the long hand again point to 12? See clock B. It has again traveled the whole distance.

3. The long hand on the clock is called the minute hand. How far around the clockface does it move in one hour? all the way ∧ Why is the short hand called the hour hand? It shows the hours. ∧

When the minute hand reaches 6, it has moved half around. So we say that the time is half past an hour.

C 

4. Clock C shows half past what hour? 7

D

5. Tell the time shown on clock D. half past 5 ∧ Where is the long hand? 6 The short hand has moved past 5.

E

6. On clock E, the long hand has moved a quarter of the way around. The time is quarter past 8.

7. On clock F, the hour hand is just before 2. This clock shows quarter before 2. Why can we say "quarter before"? The minute hand has only a quarter of the distance to travel.

F

8. On the board draw the face of a clock showing half past 3.

9. On the board draw the face of a clock showing quarter before 5.

10. Draw another clock showing quarter past 1.

Teaching Pages 24 and 25

Pupil's Objectives

(a) To review telling time by the hour and half hour; and (b) to learn telling time by the quarter hour and minute intervals.

New Words

(page 24) *hour*, *minute*
(page 25) *activity*, *spaces*

Background

Because the ability to tell time is so important, it is necessary to identify children who have already developed the skill, those who have some understanding but need more practice, and those who need initial instruction.

Units of measurement are used to approximate the size of things. Although we can think of a thing as having an exact measurement, it is not physically possible to measure anything exactly. However, by using smaller and smaller units of measurement we can increase the preeision, and so decrease the error. The units of measurement of time studied as of this lesson are minutes, hours, days, weeks, months, and years. If you were to measure time using only whole minutes, you would measure any amount of time between $119\frac{1}{2}$ minutes and $120\frac{1}{2}$ minutes as 120 minutes. Notice that by using a minute as the unit of time, the greatest possible error is $\frac{1}{2}$ minute. If you were using the hour as the unit of time, any amount of time between $1\frac{1}{2}$ hours and $2\frac{1}{2}$ hours would measure 2 hours. The greatest possible error in this case is $\frac{1}{2}$ hour. Notice that although the measurements 120 minutes and 2 hours may be thought of as representing the same amount of time, in actual situations, there may be a very great difference in the amount of time being measured. It is because of this difference in the possible error that we say a measurement obtained by using a smaller unit is more precise than one obtained by using a larger unit. For example, if each of the following are measurements from actual situations, then we would say the measurement in row A is more precise than the measurement below it in row B.

A	60 seconds	60 minutes	24 hours	7 days
B	1 minute	1 hour	1 day	1 week

On page 25 of the pupil's text you will find reference to the first of a series of sets of exercises entitled *Extra Activities*. These are enrichment activities which have been prepared for use by your *more capable children*, as definite extension of the work provided for *all pupils* on the text pages.

Teacher's Preparation

If possible, have some discarded clocks, the hands of which may be turned. Have a large commercial or handmade clock dial available for class demonstration purposes.* If possible have an individual clockface with movable hands available for each pupil.

Draw a clockface on the board that shows the time for dismissal for recess. Also, write on the board: "this week," "today," "the hour from 10 to 11."

Pre-Book Lesson

- Discuss the time that the class is dismissed for recess. Point out the different ways of telling time listed on the board. Ask which is the most accurate way of telling you the time of today's recess.
- Use the class clock to identify other known times of the day that are important to this class. In the discussion, bring out the idea that the minute is the most precise unit in these illustrations because it is the smallest unit of time used.

Using the Text Pages

During discussion of the examples on pages 24 and 25, use the real or the class clocks in conjunction with the pictures and questions on the pages. Be sure to emphasize the preciseness of the various units of time in relation to each other.

Individualizing Instruction

- If any doubts remain as to which children need further practice, a quick written exercise may be used wherein children write the time as shown on a clockface manipulated by the teacher.

* See 11, page xix.

• *Slower learners* may profit from making individual cardboard clocks with movable hands. Under the teacher's guidance they can manipulate the hands to show various times. You may wish to duplicate some clockfaces on which the children draw in the hands to record activities they habitually engage in at certain times each day.

• *More capable children* may use the form *4:30* as well as the form *half past 4* when dealing with time on the half hour.

• *More capable children* may do exercises such as:

a. It is now 1 o'clock. What time will it be 3 hours from now?

b. It is now 9 o'clock. What time was it 4 hours ago?

c. It is now 2:30. What time will it be 6 hours from now?

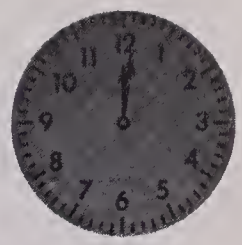
d. It is now 6:30. What time was it 5 hours ago?

• *More capable children* may do Extra Activities Sets 83 and 84 on page 371.

NOTES

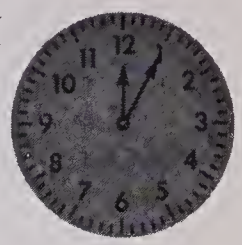
In one hour, the minute hand moves all around the face of the clock.

G



11. On clock G, the minute hand has moved from 12 to the first little mark. How long did it take to move this far? 1 minute Clock G shows 1 minute past 12.

H

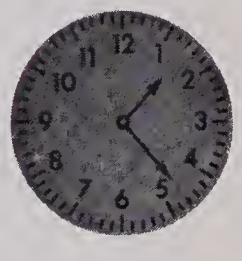


12. Count the spaces on clock H to find how many minutes it took the minute hand to move from 12 to 1. 5 The time shown on clock H is 5 minutes past 12.

13. What time does clock I show? 23 minutes past 1

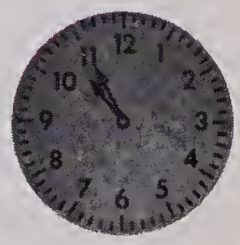
On Clock I, count the spaces by 5's to find how many minutes it took the minute hand to move from 12 to 4. 20 From 4 count by 1's to the mark where the minute hand shows the number of minutes past 1.

I



14. Count by 5's as you move your finger from one numeral to the next all around clock H. How many minutes is it from 12 all around the clock back to 12? 60 Then how many minutes are there in an hour? 60

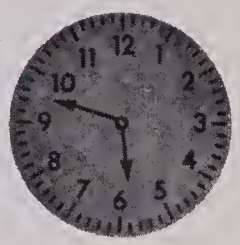
J



60 minutes (min.) = 1 hour (hr.) **1 hr. = 60 min.**

We tell time as minutes past the hour until the minute hand reaches 6. When it is on the way to 12 again, we tell time as minutes before the next hour.

K



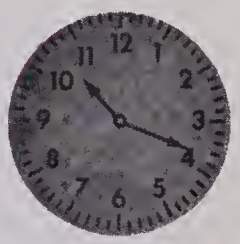
Count to tell the time shown on

15. clock J. 5 minutes before 11

16. clock K. 12 minutes before 6

17. clock L. 18 minutes after 10

L



* ● **Extra Activity.** From time to time, your teacher may want you to have more activities after you finish your work on a page. For extra activities now, work Sets 83 and 84, page 371.

* Your more capable children may well finish their work on an assignment before others. The reference here to Extra Activity refers to sets of extra activities listed in the reservoir at the back of this book.

What Do You Know about Addition?

Addends-sum relationship [W]

Write answers for Ex. 1–9.

1. Show an addition fact.

2. What number does n stand for in $6 + 3 = n$? 9

3. When you add whole numbers, is the sum ever less than any of the addends? No

4. $3 + 5 = 8$ Show the other A. fact in the pair. $5+3=8$

5. If you know the numbers for the parts, how do you find the number for the total set? Add

6. What are the two A. facts about 9; 7 and 2? $7+2=9$ $2+7=9$

7. Show the A. fact $5 + 2 = 7$ another way. $2+5=7$

8. Sometimes the numbers for a set and its parts make only one A. fact. Show a fact of this kind. $4+4=8$, or any fact with the same number for both parts.

9. $8 - 2 = 6$ What are the two A. facts in the same set of A. and S. facts? $2+6=8$ $6+2=8$

Turn to page 10. On folded paper show sums for the examples in rows 1–2.

What Do You Know about Subtraction?

Addends-sum relationship [W]

Write answers for Ex. 1–8.

1. What are the two S. facts about 7; 4 and 3? $7-4=3$ $7-3=4$

2. Do you add or subtract to find the unknown addend? Subtract

3. The numbers for a set and one part are 9 and 6. What is the number for the other part of the set? 3

Teaching Pages 26 and 27

Pupil's Objectives

(a) To find the extent to which certain understandings and generalizations have been absorbed; and (b) to review the use of the symbols = and ≠.

New Word

true

Background

If your pupils have studied *Mathematics We Need*, Book Two, they will be familiar with the use of the sign, ≠, meaning *is not equal to*. Of course, some pupils may be seeing the symbol for the first time on page 27 of the text. In either case, the first treatment in Grade 3 of equalities and inequalities is very simple and should cause little difficulty. By this time, your pupils can sense whether two numbers are equal or not equal. What may be a little more difficult is to sense that both $6 = 6$ and $7 \neq 5$ are true sentences. The negative idea is a bit hard to grasp.

Pre-Book Lesson

• You may want to use the *Show the Answer* activity to help your pupils review the A. and S. facts with sums as great as 9. The teacher may hold up a flash card which shows an A. or S. example having a sum not greater than 9.* The children then select the answer from their numeral cards and show it upon a signal from the teacher.

• After showing a flash card for which the children have shown the correct answer, ask such questions as:

Who can tell us the other facts in the same set of A. and S. facts? Who can tell the two S. facts that belong in this set of A. and S. facts?

Using the Text Pages

• The exercises at the top of page 26 were designed to give the teacher a great deal of information as to each child's skill, knowledge, and understanding of what has been taught about addition. Ex. 4 reviews use of the Commutative Property of Addition.

• The examples at the bottom of page 26 and top of page 27 should reveal each child's skill, knowledge, and understanding of what has been taught about subtraction.

• The oral work in connection with equalities and inequalities at the foot of page 27 should lead gracefully into the written work which follows. Before pupils begin the written work, make sure that they understand clearly that a number sentence containing ≠ can be a true sentence.

• The direction at the foot of page 27 for turning back to page 22 to solve problems is a type of assignment which will occur from time to time throughout the text. The authors have planned that all problems will be solved at some time, even though the first work does not involve solving the problems.

Individualizing Instruction

• The *more capable children* probably can proceed with the written work on these pages independently. Perhaps it would be desirable to have them go ahead with the written work while you use with the rest of the class the *Show the Answer* practice suggested in the Pre-Book Lesson.

• The *more capable children*, and perhaps *all pupils*, may choose the correct *n*-example from the two sentences given below for each of the problems on page 20. Have the children tell why they selected a particular number sentence.

(page 20)

1. a. $9 + 8 = n$	b. $9 - 8 = n$
2. a. $5 + 4 = n$	b. $5 - 4 = n$
3. a. $8 + 3 = n$	b. $8 - 3 = n$
4. a. $2 + 8 = n$	b. $8 - 2 = n$
5. a. $4 + 3 = n$	b. $4 - 3 = n$

• The arithmetic corner should be a constant source of individualizing arithmetic instruction. There are many children's books in which interesting stories and pictures depict the historical development of mathematics. Make these books available in the arithmetic corner for free-time use.

* See 23 and 24, page xix.

The *more capable children* will find much information which they can share with the class from time to time.

• If you find that the *slower learners* need more practice with addition and subtraction facts having sums as great as 9, you may wish to follow up the practice with this Progress Test:

Write the answers on folded paper. Watch the signs!

	a	b	c	d	e	f	g
1.	3	9	2	7	9	8	5
	+ 5	− 2	+ 3	+ 2	− 5	− 4	+ 2
2.	7	4	6	9	2	8	2
	− 4	+ 3	− 4	− 6	+ 4	− 5	+ 7
3.	3	8	6	8	9	2	8
	+ 4	+ 1	+ 2	− 3	− 4	+ 5	− 1
4.	7	8	9	4	6	6	4
	− 3	− 2	− 7	+ 4	+ 3	− 2	+ 5
5.	2	5	7	6	4	3	7
	+ 6	− 3	− 6	− 3	+ 2	+ 6	− 5
6.	9	5	8	5	9	8	3
	− 8	+ 4	− 6	+ 3	− 3	− 7	+ 3

NOTES

4. What are the S. facts in the set of A. and S. facts about 9; 2 and 7? 9−2=7 9−7=2

5. When you subtract with whole numbers and get a whole-number answer, is the answer greater than the sum? No

6. Show a S. fact that has no S. fact to make a pair. 6−3=3
The numbers for the parts of the set are the same.

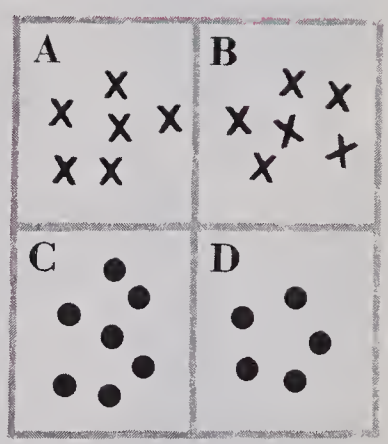
7. What number does n stand for in $9 - n = 4$? 5

8. Write an example using the minus sign. Answers will vary.

Turn to page 16. On folded paper write the answers for the examples in rows 1–2.

* = or ≠

Equalities and inequalities [O]



1. Is the number for set A equal to the number for set B? Yes ∧ To show that 6 is equal to 6, we may write, "6 = 6."

2. Is the number for set C equal to the number for set D? No ∧ To show that 7 is not equal to 5, we may write, "7 ≠ 5."

Tell why each of these sentences is true:
Each sentence shows a true idea.

3. $7 \neq 8$ 4. $5 + 3 = 8$ 5. $7 - 4 \neq 2$ 6. $9 - 2 \neq 8$

[W]

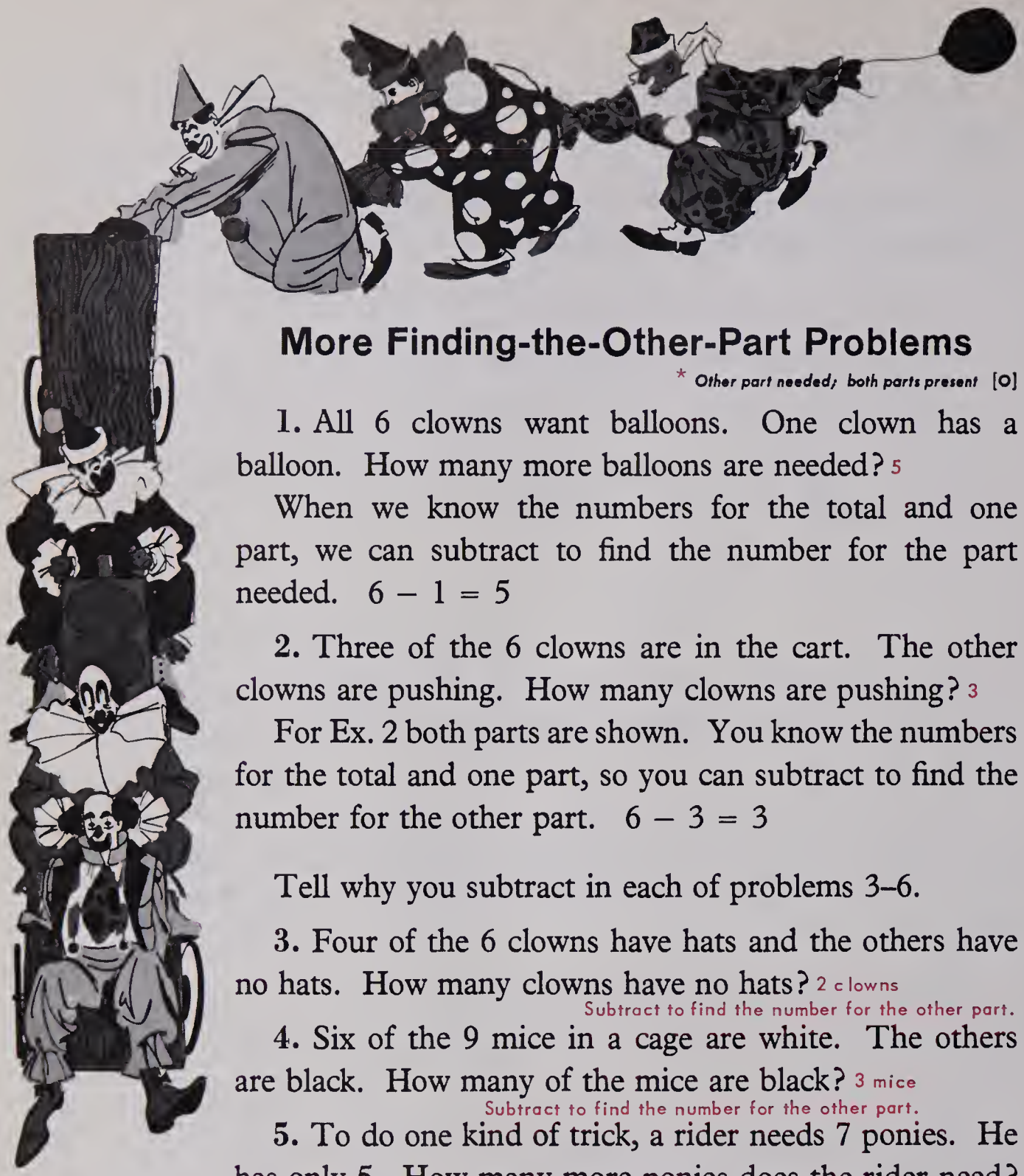
Copy and finish each example below. Write = or ≠ where you see _?_ to make each sentence true.

7. $8 \;\underset{=}{?}\; 8$ 8. $4 + 5 \;\underset{\neq}{?}\; 8$ 9. $2 + 4 \;\underset{\neq}{?}\; 5$ 10. $7 - 5 \;\underset{=}{?}\; 2$

11. $6 + 3 \;\underset{\neq}{?}\; 8$ 12. $5 - 3 \;\underset{=}{?}\; 2$ 13. $6 - 3 \;\underset{=}{?}\; 3$ 14. $3 + 4 \;\underset{\neq}{?}\; 6$

Turn to page 22. Write the work for problems 1–6.

* All of your pupils are probably familiar with the equals sign (=) but some may not know the symbol ≠ for is not equal to. The ideas of equality and inequality are both fundamental in mathematics.



More Finding-the-Other-Part Problems

* *Other part needed; both parts present* [O]

1. All 6 clowns want balloons. One clown has a balloon. How many more balloons are needed? 5

When we know the numbers for the total and one part, we can subtract to find the number for the part needed. $6 - 1 = 5$

2. Three of the 6 clowns are in the cart. The other clowns are pushing. How many clowns are pushing? 3

For Ex. 2 both parts are shown. You know the numbers for the total and one part, so you can subtract to find the number for the other part. $6 - 3 = 3$

Tell why you subtract in each of problems 3–6.

3. Four of the 6 clowns have hats and the others have no hats. How many clowns have no hats? 2 clowns
Subtract to find the number for the other part.

4. Six of the 9 mice in a cage are white. The others are black. How many of the mice are black? 3 mice
Subtract to find the number for the other part.

5. To do one kind of trick, a rider needs 7 ponies. He has only 5. How many more ponies does the rider need?
2 ponies. Subtract to find the number for the part needed.

6. A clown uses 9 marbles for a game he plays. He has 4 marbles. How many more marbles does he need?
5 marbles. Subtract to find the number for the part needed.

[W]

Write the work for Ex. 1–6 in two ways, as on page 17.

* Socially, situations have differences. However, in these situations, just as in earlier how many left and how many gone situations, they all involve the general idea of finding the number for the other part.

Pupil's Objectives

(a) To learn that subtraction can be used for problems where both parts of a set are shown, but where the numbers for the total set and only one part are known; and (b) to have more experience in solving problems and labeling the answer.

New Words

pushing, *rider*

Background

Problem situations in which subtraction is required are much more diverse than those in which addition is required. The how-many-left and how-many-gone situations have been presented thus far. In this new presentation, both parts of a total are present in the problem setting, but the number for only one part is known. Subtraction can be used to find the number for the other part. Of course, in a simple situation a child can very simply count the objects in the other part. After pupils have subtracted, let them count to verify the solution.

The how-many-more-are-needed idea in problems is one frequently encountered. In this situation, the child knows the numbers for the total and one part.

While variations in situations requiring subtraction do exist, fundamentally, all of these situations have the characteristic that the numbers for the total set and one of the parts is known. When a pupil's analysis of a problem leads to the conclusion that the numbers for the total set and one of its parts are known, he has a sound basis for selecting subtraction as the appropriate operation.

Teacher's Preparation

Have a flannel board* available for the Pre-Book Lesson. You will need circle shapes (or other geometric forms) of two different colors (or sizes).

Pre-Book Lesson

• Use children, books, or other objects in the room to dramatize the how-many-left and how-many-gone ideas, by way of review.

* See 2, page xix.

• Have 8 children (3 boys, 5 girls) stand in line at the front of the room. Ask, "How many children are there in all? How many boys are there? Without counting, how can you find out how many girls there are?"

• Use the flannel board and the manipulative materials to demonstrate other examples of this new subtraction situation.

• Place 6 objects on the flannel board. Ask, "How many more objects are needed to make a set of 8?" In this situation the children can see the six objects. They know 8 are needed in all. Therefore, they must subtract to find the number for the part needed to make a set of 8. Of course, they could count up to 8 but it is more efficient to subtract.

Using the Text Page

• Be sure to stress the generalization that we subtract when we know the numbers for a total and one of its parts. Have the children state this idea in their own words.

• In problems 1, 5, and 6 the how-many-more-are-needed situation is used. This is another use of the parts-total relationship in subtraction.

• Before assigning the written activity at the bottom of the page, remind the class that we find the answer to a problem by using numbers shown in the problem and apply the appropriate operation. Then, we rewrite the answer with its label.

Individualizing Instruction

• *Slower learners* may play a game for practice. Show a box and tell pupils that it contains 8 objects. Remove some of the objects in such a way that the pupils cannot see how many have been removed. Let them see the part of the set remaining in the box and then tell the number for the part removed. Use only totals through 9. Let different children remove parts of the total set from the box.

• Have the *more capable children* write, for others to solve, problems of three different kinds, all of which require subtraction.

Pupil's Objectives

(a) To review that sets of things do not need to be alike or identical in all ways in order to join them or to separate them; and (b) to learn to use the correct label for an answer to a problem.

New Words

act, label

Background

The children have been counting, joining, and separating *sets* of things. In earlier grades, they have learned to identify the number property (how many members) of a set and to produce a set which has a specified number property when the number is a whole number. They have learned to use addition to find the number for the total when joining sets and subtraction to find the number for a part when separating sets. When joining sets, it is not necessary that the elements of sets be alike in all ways in order for the sets to be joined. We can join a set of 4 elephants and a set of 2 lions and call them *animals*. Or, we can join a set that contains a garden hose and a knife with a set containing a stove, a lily, and a hammer. The resulting set will contain all of these elements, which are alike in that they are all *things* and the number property of the new set will be 5. Within the child's experience there is usually some easily discerned likeness among sets being joined or separated, so he can think of a common name to use in labeling the answer to a problem.

Teacher's Preparation

In advance of the lesson, prepare a small box with the following items: 2 paper clips, 1 crayon, 1 pencil, 2 rubber bands, and 1 pair of scissors.

Pre-Book Lesson

• With the following statement, help pupils to find the number for the set as well as an appropriate label. "There is a set of objects in this box. Let's see how many objects there are in the set."

• Perhaps this same type of illustration can be accomplished by asking a boy to identify and count the items in his pocket; or a girl to do the same with her pencil case. It may be possible to identify a set of objects in a child's desk.

• Using items that belong to the children, show a set of 2 sweaters (or coats) and another set of 4 caps (or gloves). Ask, "When we join a set of sweaters with a set of caps, in what way are all the members of the new set alike? What can we call the things in the new set?" (pieces of clothing)

Illustrate the idea in thinking about tables and chairs. Pupils may suggest other sets of objects for which to find a label.

• In order to show separation, start with the 6 *pieces of clothing* arrived at in the work above. Remove the 4 caps. Ask what we should call the remaining items in the set. Pupils may want to call the remaining items *sweaters*, but point out that the set from which some items were removed was named *pieces of clothing*; so 2 *pieces of clothing* are left.

This idea is a little difficult, so repeat the activity until pupils seem to understand that the other part of a set has the same name as the set from which part is removed.

Using the Text Page

• Have selected children read each problem and explain why they chose the particular label for the answer. Have different children explain how to find the answer to each problem. Do not emphasize the answer; but, how to find the answer.

• Before assigning the written work (to be shown both horizontally and vertically), review how to work problems,—we write only the numerals and the operation sign and find the answer. Then we rewrite the answer with its correct label.

Individualizing Instruction

The *more capable children* should find it interesting to write different combinations of names for unlike sets and identify the label when the sets are joined. For example:

hammers and saws are tools;
robins and crows are birds;
boys and girls are children;
lions and horses are animals;
nickels and dimes are coins.

Labeling the Answers for Problems

Social labels for answers [O]

1. These three boys and 2 girls are at the circus. When we find the number for the total, will the label for the answer be boys or girls or circus or children? children

* Read Ex. 2–7. Do not write the work. Just tell what is missing. This will help you to label your answer.

2. The children watched 4 horses and 4 dogs in a circus act. How many animals ? were in the act? 8 animals

3. The ring near Joe has 9 animals in it. Two of them are monkeys. How many other animals ? are there? 7 animals

4. Sam watched 5 of the 9 elephants leave their ring. Then how many elephants ? were in that ring? 4 elephants

5. Sue gave away some of the 8 nuts she had. Two peanuts are left. How many nuts ? did Sue give away? 6 nuts

6. Four balls and 3 tops make how many toys ? in all? 7 toys

7. How many cents ? are 3 cents and a nickel? 8 cents

[W]

Write the work for each problem two ways. Then write the answer again with its label, as on page 17.

* Pupils may remember that they should rewrite an answer and label it, but sometimes they don't know what the label should be. This page provides work in deciding what the label should be.

A Set of Even Numbers

2	18	50
84	16	32
70	28	4

A Set of Odd Numbers

45	3	41
89	37	5
1	65	9

*Even Numbers and Odd Numbers

Adding or subtracting 1 [O]

1. Say the one's digits of the numerals for this set of even numbers; of odd numbers. 0, 2, 4, 6, 8 / 1, 3, 5, 7, 9

2. What numbers for a–d are not correct?

a. Odd: 6, 6, 40, 43, 35, 59, 40, 71
b. Even: 8, 15, 15, 47, 20, 16, 47, 64
c. Odd: 27, 12, 88, 9, 12, 88, 3, 11
d. Even: 19, 19, 13, 22, 18, 13, 34, 12

[W]

For each _?_, write "odd" or "even."

Set A

6	8	4
+1	+1	+1
?	?	?

3. Show sums only for set A.

a. The sums are _odd_ numbers.

b. If you add 1 to an even number, the sum is an _odd_ number.

Set B

5	3	7
+1	+1	+1
?	?	?

4. Show sums only for set B.

a. The sums are _even_ numbers.

b. If you add 1 to an odd number, the sum is an _even_ number.

Set C

4	6	8
−1	−1	−1
?	?	?

5. Show answers only for set C.

a. The answers are _odd_ numbers.

b. If you subtract 1 from an even number, the answer is an _odd_ number.

Set D

9	5	7
−1	−1	−1
?	?	?

6. Show answers only for set D.

a. The answers are _even_ numbers.

b. If you subtract 1 from an odd number, the answer is an _even_ number.

* Even numbers are zero and other multiples of 2. All other whole numbers belong to the set of odd numbers.

Teaching Pages 30 and 31

Pupil's Objectives

(a) To discover the relationship between odd and even numbers; (b) to learn the generalization that the result is *odd* when 1 is added to or subtracted from an even number; and (c) to learn the generalization that the result is *even* when 1 is added to or subtracted from an odd number.

Background

Even numbers are defined as the set of all numbers which are products of 2 and another whole number. Since multiplication has not yet been developed, we shall attempt to build an understanding of even numbers as they are related to 2 without using multiplication as such.

Odd numbers may be simply defined as numbers that are 1 greater than an even number.

Eventually children should come to know that (1) the sum of two even numbers is even; (2) the sum of two odd numbers is even; (3) the sum of an odd and an even number is odd; and that the same generalizations are true for the unknown addend when the operation is subtraction and the two numbers we start with are even; are odd; one is even and the other is odd.

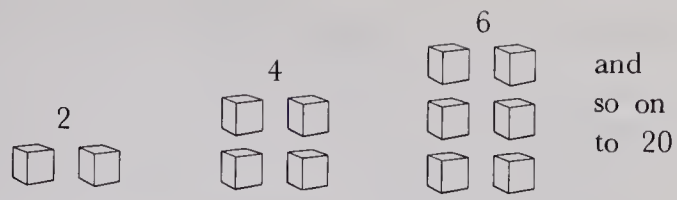
Teacher's Preparation

Have a set of at least 20 blocks or cubes* available. If they are not readily available in your room, kindergarten rooms are usually equipped with sets of multicolored 1″ cubes.

Pre-Book Lesson

• Tell the children that we call some numbers *even numbers*, and that we call some *odd numbers*.

• Use blocks or cubes as follows to demonstrate *even* numbers: "I am going to build up these blocks by 2's. If we put on blocks 1 at a time, we couldn't keep the *two* stacks *even*. Each time we will write the numeral for the number of blocks we have used." As you build the stacks it should progress something like this:



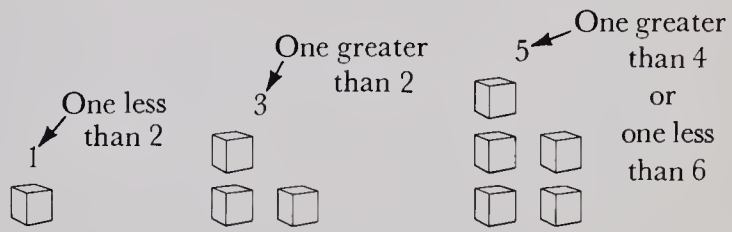
Ask such questions as:

How many blocks must we use each time to keep the stacks even? Let's count the sets of 2 in this stack.

Guide the discussion to lead to the idea that *even* numbers are related to twoness.

As a result of reading the numerals for even numbers, identified with the blocks, most children will recognize that we say the even numbers when we count by two. Numerals for even numbers end in 0, 2, 4, 6, 8.

• After demonstrating and discussing the even numbers, ask if anyone can tell what an odd number is. A frequent response is, "Any number that is not even." Lead the class to "see" that an odd number is one greater or one less than an even number. Use the stacks of blocks to show this relationship.



Using the Text Pages

• Ask the pupils to explain *why* to each of their answers for Ex. 1 and 2 on page 30.

• The *more capable children* may proceed independently with the written work. You may wish to work Ex. 3–4 orally with the *slower learners* before assigning the page as written work.

• Before proceeding to the work on page 31, be sure to discuss the generalizations arrived at in the written work on page 30. Extension of these important ideas is developed on page 31. The pupil must understand one level before proceeding to the next.

• The *more capable children* should proceed with the written work on page 31 independently. It may be more profitable for the *slower learners* if you

* See 13, page xix.

direct exercises 1–8 orally. Then Ex. 9–14 can be assigned as written work for them.

Individualizing Instruction

More capable children may do the following activity for written work:

- Write the numerals for counting by 3's. Do those numerals stand for odd numbers or for even numbers or for some of each? Why is this so?
- Write the numerals for counting by 4's. Do those numerals stand for odd numbers or for even numbers? Why is this so?
- Turn back to the chart on page 2. Do the numerals show even numbers or odd numbers in
  a. the column under 1?
  b. the column under 2?
  c. the diagonal from 0 to 99?
  d. the row started by 10? by 50?

NOTES

More about Even and Odd Numbers

Adding or subtracting 2 [W]

1. Copy the examples in Set A and find the sums.

2. Do the same for set B.

3. In which set are all sums

a. odd numbers? B **b.** even numbers? A

4. Copy the examples in set C and write the answers.

5. Do the same for set D.

6. In which set are all answers

a. odd numbers? D **b.** even numbers? C

7. Will the sum be an odd or an even number

a. if you add 2 to an odd number? odd

b. if you add 2 to an even number? even

8. Will the answer be an odd or an even number

a. if you subtract 2 from an even number? even

b. if you subtract 2 from an odd number? odd

Set A

4	2	6
+2	+2	+2
? 6	? 4	? 8

Set B

7	5	3
+2	+2	+2
? 9	? 7	? 5

Set C

8	4	6
−2	−2	−2
? 6	? 2	? 4

Set D

9	5	7
−2	−2	−2
? 7	? 3	? 5

9. Which are even numbers? 34 41 59 99 8 34, 8

10. Which are odd numbers? 55 62 97 84 43 55, 97, 43

11. Show the set of even numbers from 2 through 24.
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24

12. Show the set of odd numbers from 7 through 29.
7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29

13. What even number comes next after

a. 48? 50 **b.** 90? 92 **c.** 82? 84 **d.** 75? 76 **e.** 26? 28 **f.** 51? 52

14. What odd number comes next after

a. 17? 19 **b.** 44? 45 **c.** 71? 73 **d.** 30? 31 **e.** 13? 15 **f.** 62? 63

Pupils should now understand that adding one to an even number gives an odd number (and vice versa); that adding two to an even number gives an even number; and that adding two to an odd number gives an odd number.

Now that your pupils think about both A. and S. facts in terms of addends and sum, they should find work with the chart on page 33 very interesting.

A New Way to Look at Things You Know

Chart for A. and S. facts; sums as great as 9 [O]

1. Find 5 in the chart on page 33.

a. In the blue column, find the 2 that is in the same row as 5. This shows that 2 is one addend of 5.

b. In the red row, find 3 in the same column as 5. When 2 is one addend of 5, is 3 the other addend? Yes

2. Use a row and a column to find the addends shown for a. 9 (4 and 5). b. 6 (3 and 3). c. 8 (5 and 3).

If you know a pair of addends of a sum, you can give the set of A. and S. facts for that sum and its pair of addends.

3. Find 7. What are the addends? (3 and 4) Say the set of A. and S. facts. $3 + 4 = 7$ $4 + 3 = 7$ $7 - 3 = 4$ $7 - 4 = 3$

4. Say the set of A. and S. facts for

a. 8 (See Ex. 8). $3 + 2 = 5$ $1 + 4 = 5$ b. 5. $2 + 3 = 5$ $4 + 1 = 5$ c. 7 (See Ex. 9). $5 - 3 = 2$ $5 - 1 = 4$ d. 5. $5 - 2 = 3$ $5 - 4 = 1$ e. 9 (See Ex. 7).

5. Why does each black numeral in the chart show a sum for both the A. facts and the S. facts? It shows the number for the total.

6. Find 6 and the addends shown for 6. Then find the other black 6's and the pairs of addends shown for each. Say all the sets of A. and S. facts with the sum 6.

$5 + 1 = 6$ $4 + 2 = 6$ $1 + 5 = 6$ $2 + 4 = 6$ $3 + 3 = 6$ $6 - 5 = 1$ $6 - 4 = 2$ $6 - 3 = 3$ $6 - 1 = 5$ $6 - 2 = 4$

[W]

7. Start with 9 and the addends shown for 9. Show all the sets of A. and S. facts in which 9 is the sum.

$8 + 1 = 9$ $1 + 8 = 9$ $9 - 8 = 1$ $9 - 1 = 8$ $7 + 2 = 9$ $2 + 7 = 9$ $9 - 7 = 2$ $9 - 2 = 7$
$6 + 3 = 9$ $3 + 6 = 9$ $9 - 6 = 3$ $9 - 3 = 6$ $5 + 4 = 9$ $4 + 5 = 9$ $9 - 5 = 4$ $9 - 4 = 5$

8. Start with 8 and the addends shown for 8. Show all the sets of A. and S. facts in which 8 is the sum.

$7 + 1 = 8$ $1 + 7 = 8$ $8 - 7 = 1$ $8 - 1 = 7$ $6 + 2 = 8$ $2 + 6 = 8$ $8 - 6 = 2$ $8 - 2 = 6$
$5 + 3 = 8$ $3 + 5 = 8$ $8 - 5 = 3$ $8 - 3 = 5$ $4 + 4 = 8$ $8 - 4 = 4$

9. Start with 7 and the addends shown for 7. Show all the sets of A. and S. facts in which 7 is the sum.

$6 + 1 = 7$ $1 + 6 = 7$ $7 - 6 = 1$ $7 - 1 = 6$
$5 + 2 = 7$ $2 + 5 = 7$ $7 - 5 = 2$ $7 - 2 = 5$
$4 + 3 = 7$ $3 + 4 = 7$ $7 - 4 = 3$ $7 - 3 = 4$

Teaching Pages 32 and 33

Pupil's Objectives

(a) To begin to use an addition-subtraction number chart; and (b) to review the addends of all sums 2 through 9.

New Word

(page 32) *column*

Background

The addition-subtraction number chart is an excellent device for showing the addends and sums for all A. and S. facts. In the chart, one can readily find the numerals for a given sum and all of its related pairs of addends. And, vice versa, one can see the numerals for any particular addend and its relationship to all possible sums.

In this chart, the numerals for one set of addends appear in red across the top of the chart. The numerals for the other set of addends appear in blue down the left side of the chart. Many teachers prefer to show the operation symbol inside the upper left square (which is otherwise empty). Later, when children are introduced to the multiplication-division chart, it is useful to know by a glance at the upper left square the operation embodied in the chart.

The numeral for a particular sum is found in the chart by starting with the addend shown in the blue column and the addend shown in the red row. By following across from the addend shown in the side column and down from the addend shown in the top row, the two paths ultimately meet. The numeral inside the square at the intersection of the two paths represents the sum of the two addends chosen. The inverse of this operation is to start in the chart with the numeral for a known sum across from or under the numeral for a known addend. Moving upward or to the left from the sum numeral reveals the numeral for the unknown addend.

Teacher's Preparation

• Make a large class chart like the one at the top of text page 33. A chart prepared on lightweight tagboard or newsprint would be more functional for future use than one made on the chalkboard. In either case, you will find it practical to use one color (red) for the *row* of addends shown and another color (blue) for the *column* of addends shown. Numerals for the sums should appear in a third color (black). The chart would be about 30″ square.

• Prepare a master for duplicating a small chart. Do not fill in any of the squares of the chart. It is to be used by the pupils.

Pre-Book Lesson

• Develop the use of the large class chart somewhat as follows:

> Look at the 4 in the blue column. Follow the row across from the 4 to the 6. The 6 shows a sum and 4 is one of its addends. Can you find the 2 in the red row above the 6? It shows the other addend of 6 when one addend is 4.
>
> The numeral for one addend I am thinking of is the 2 in the red row. The sum is 7. Who can point to the numeral for the other addend?

• The discussion and questioning should lead the class to the discovery that, except when the addends are equal, the numerals for two addends of a sum appear on the chart in two places. Continue to ask questions about the class chart until you are sure that the children are thoroughly familiar with how to read and interpret it.

Using the Text Pages

• Refer to the geometric forms *square*, *circle*, and *triangle* as appropriate in these exercises.

• Be sure that the children understand that there are four sets of A. and S. facts for each of sums 9 and 8 and its pairs of addends; and that, for the sum 7 and its pairs of addends, there are 3 sets of A. and S. facts.

For 9:

$1 + 8 = 9,\ 8 + 1 = 9,\ 9 - 1 = 8,\ 9 - 8 = 1$
$2 + 7 = 9,\ 7 + 2 = 9,\ 9 - 2 = 7,\ 9 - 7 = 2$
$3 + 6 = 9,\ 6 + 3 = 9,\ 9 - 3 = 6,\ 9 - 6 = 3$
$4 + 5 = 9,\ 5 + 4 = 9,\ 9 - 4 = 5,\ 9 - 5 = 4$

For 8:

$1 + 7 = 8,\ 7 + 1 = 8,\ 8 - 1 = 7,\ 8 - 7 = 1$
$2 + 6 = 8,\ 6 + 2 = 8,\ 8 - 2 = 6,\ 8 - 6 = 2$
$3 + 5 = 8,\ 5 + 3 = 8,\ 8 - 3 = 5,\ 8 - 5 = 3$
$4 + 4 = 8,\ 8 - 4 = 4$

For 7:

$1 + 6 = 7,\ 6 + 1 = 7,\ 7 - 1 = 6,\ 7 - 6 = 1$
$2 + 5 = 7,\ 5 + 2 = 7,\ 7 - 2 = 5,\ 7 - 5 = 2$
$3 + 4 = 7,\ 4 + 3 = 7,\ 7 - 3 = 4,\ 7 - 4 = 3$

• The enrichment material on page 33 may be appropriate for use by *more capable children* only. The children must understand how to read and interpret the chart before this section is assigned.

You may wish to assign both of these pages as enrichment material for your *more capable children* only and come back to them at a later time for the *average* and *slower learners*. Sections labeled *Enrichment* in a clue caption and titled *Do You Like to Try New Things?* appear in each chapter throughout the text as material for use of *more capable children* who finish regular assignments ahead of the other members of the class. Such sections may be used as they appear in the text or may be saved for later assignments when the teacher sees a need. It is just possible that you will want to use some of the enrichment sections as work for *all pupils*.

Individualizing Instruction

Give *all pupils* the blank addition-subtraction charts you have duplicated. The following are suggestions for use of the charts:

a. Have pupils write the numerals for as many of the addends and sums as they can. Permit the *slower learners* to make dot pictures to help them see the parts and total from which they can derive pairs of addends and their sums.

b. Write number sentences or show flash cards and have the children fill in the corresponding squares on the chart. For consistency, have pupils use the numeral in the left column as the first numeral in an addition number sentence.

c. Have the children write on their charts the row numerals and the column numerals showing addends. Then designate selected squares within the chart to write letters for sums. Have the children write a number sentence for each of these sums and its pair of addends. You may want to alter this somewhat by putting letters in place of some of the numerals for addends.

NOTES

+	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	_?_
3	4	5	6	7	8	9	_?_	_?_
4	5	6	7	8	9	Z	_?_	_?_
5	6	7	8	9	Y	_?_	_?_	_?_
6	7	8	9	_?_	_?_	_?_	Q	_?_
7	8	9	W	_?_	N	_?_	P	_?_
8	9	X	M	_?_	_?_	_?_	S	R

Do You Like to Try New Things?

Enrichment [W]

1. In the chart find X.

a. What sum does X stand for? 10

b. What are the two addends of the sum X stands for? 8 and 2

c. Show the set of A. and S. facts.
8+2=10 2+8=10 10−8=2 10−2=8

2. In the chart find W.

a. What sum does W stand for? 10

b. What are the two addends of that sum? 7 and 3

c. Show the set of A. and S. facts.
7+3=10 3+7=10 10−7=3 10−3=7

3. Find Y. Show the set of A. and S. facts for that sum and its two addends. 5+5=10 10−5=5

4. Find Z. Show the set of A. and S. facts.
4+6=10 6+4=10 10−4=6 10−6=4

5. To show all the pairs of addends of 10, we need another row and another column in the chart. What numeral in the blue column would start the row? 9 What numeral in the red row would start the column? 9

6. Try to show the sum the letter stands for.

a. M 11 b. N 12 c. P 14 d. Q 13 e. R 16 f. S 15

Another facet in developing problem-solving ability!

Telling What Is Given

A. and S. problems; addends-sum relationship [O]

For each problem say, "The numbers for the parts are given" or say, "The numbers for a total and one part are given."

1. Seven large boats and 2 small boats took groups on fishing trips. "The numbers for the parts ore given."

2. Three of the 7 large boats went to one place to fish. The other large boats went to another place.
"The numbers for a total and one part ore given."

3. On one boat there were 2 women and 7 men.
"The numbers for the parts ore given."

4. The wind blew away the hats of 2 of the 7 men.
"The numbers for a total and one part are given."

5. One of the women caught 2 fish. The other one caught 6 fish. "The numbers for the ports are given."

6. Three of the 9 boats were black.
"The numbers for a total and one part are given."

7. A girl caught 4 fish of one kind and 5 fish of another kind. "The numbers for the parts are given."

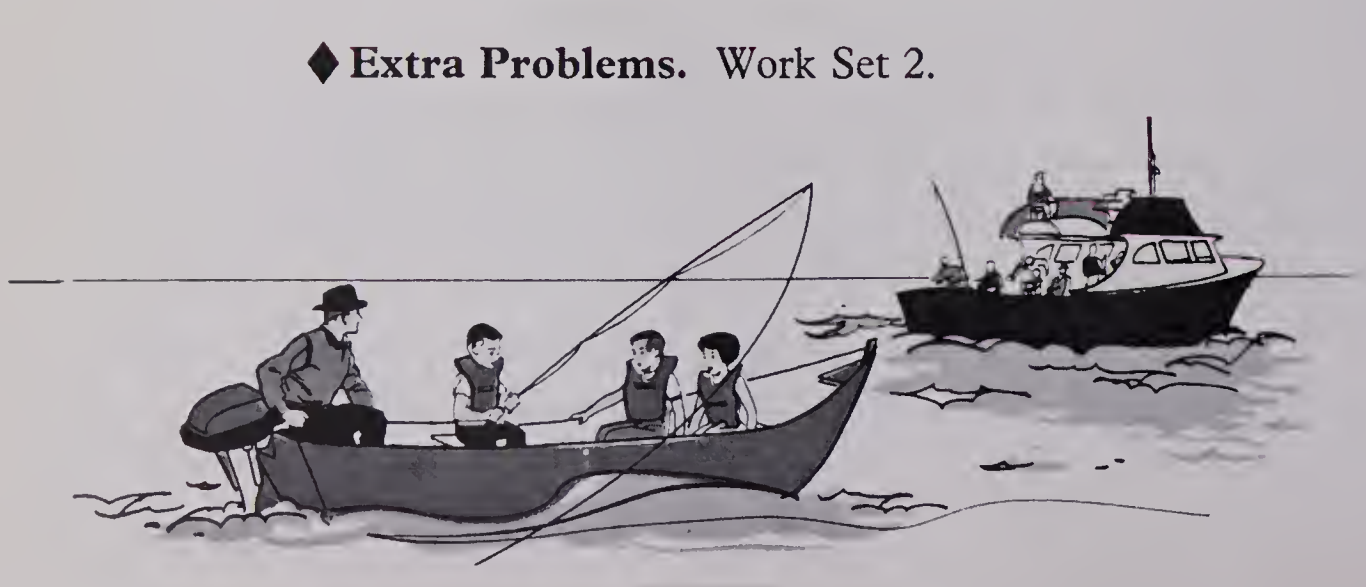
8. Two of the 8 fish one man caught were so small that he had to throw them back into the water.
"The numbers for a totol and one port are given."

9. One man lost some of his 6 fish. He had 4 left.
"The numbers for a total ond one part are given."

[W]

For each of Ex. 1–9, write a problem.

♦ **Extra Problems.** Work Set 2.



Teaching Page 34

Pupil's Objectives

(a) To continue learning to distinguish between problems requiring addition and problems requiring subtraction in terms of the information given; (b) to distinguish between the several types of situations requiring subtraction as a result of the information given in a problem setting; (c) to be able to determine whether the known information involves numbers for two parts, thus requiring addition or the number for a total and one part, thus requiring subtraction; and (d) to use known information as a basis for asking a problem question.

New Words

large, *women*

Background

The problem setting in which we find ourselves determines the application(s) we make of number. As a consequence of knowing certain information, we should be able to predict fairly accurately what the question will be. In addition and subtraction situations the given information must be numbers either for the two parts of a total set or for a total and one of its parts. After pupils have had ample experience in analyzing the given information to determine whether the given data pertain to a total and a part or to two parts, they can then use this as the basis for developing a mathematical model for the problem. They then use the model (number sentence) as a guide in finding the solution to the problem. If number sentences are used as the basic approach to problem-solving without the prior period of development of the ability to really analyze the situation, the problem-solving procedure becomes too mechanical and often misleading.

By going through the stages of the carefully developed problem-solving program of *Mathematics We Need*, pupils are guided to development of the ability to solve problems. The program provides for proceeding through the following stages to arrive at stage 4 in the fourth grade:

1. Analyze the situation.
2. Associate the correct mathematical sentence with the analysis.
3. At the same time, study the method for solving the problem.
4. Eventually bring together stages 1, 2, and 3 by writing the mathematical sentence and then using it to solve the problem.

Teacher's Preparation

Have some manipulative materials ready.

Pre-Book Lesson

• Use the objects you have available to demonstrate the following subtraction situations:

a. How many left
b. How many gone
c. Both parts shown, but the number for only one part known
d. How many more are needed

• As each of the subtraction situations is demonstrated and discussed, have selected children explain what operation to use for solving, and tell why.

Using the Text Page

• Tell the class that each exercise is part of a problem. They are to read the information given and then decide whether what is shown are numbers for a total and one of its parts or numbers for two parts of a total.

• After the children have decided what is given, have them write an addition or a subtraction question to complete each problem.

Individualizing Instruction

• Write the following on the chalkboard. Remind pupils that each shape represents a different number.

If ○ is one addend of △, the other addend is □. Have *all pupils* write 8 sentences in this form, using sums through 9.

• Write on the board:

8 = 5 and 3 7 = 1 and 6 9 = 7 and 2

Have the *slower learners* show a set of A. and S. facts for the numbers for each total and the pair of parts indicated.

• Extra Problems, Set 2, may be assigned.

Teaching Page 35

Pupil's Objectives

(a) To use the ruler to find all the pairs of numbers for parts of a set of 10; (b) to study and master the A. and S. facts with sum 10; (c) to find the number that n stands for in addition and subtraction number sentences showing sums as great as 10; and (d) to differentiate between tables of numbers for parts of a set and tables of facts.

Background

The ruler, which really shows a portion of the number line, helps pupils to understand the geometric idea of number.

A table of numbers for pairs of parts of a set shows all combinations of the whole numbers 1 through 9. A table of A. facts with a given sum shows, with the signs of operation, all possible combinations of the addends 1 through 9 and their sum. A table of S. facts with a given sum shows, with the signs of operation, the sum, the subtracting of each addend 1 through 9, and the resulting addends.

Teacher's Preparation

Be certain that each pupil has a ruler.

Pre-Book Lesson

• Direct the class to use the ruler to discover the parts of 3 inches. By counting, they will discover that when 1 inch is one part of 3 inches, 2 inches is the other part; when 2 inches is one part of 3 inches, 1 inch is the other part. As pupils discover the parts, make a table on the board to show the numbers for the parts.

• Have pupils continue this activity, finding parts of 4 inches, 5 inches, 8 inches, and 9 inches.

• Have pupils use a set of 10 objects and tell the set of A. and S. facts about 10; 6 and 4.

• Ask whether a set of 10 has any pair of parts for which there are only 2 facts in the set of A. and S. facts.

• As preparation for work in the text, have selected pupils go to the chalkboard and write a table of A. facts with sum 6 (beginning with $1 + 5 = 6$ and ending with $5 + 1 = 6$); with sum 7; with sum 8; with sum 9.

Have other pupils write at the board a table of S. facts with sum 6 (beginning with $6 - 1 = 5$ and ending with $6 - 5 = 1$); with sum 7; with sum 8; with sum 9.

Using the Text Page

• Have the pupils use their rulers to find numbers for the parts of 10 and copy and finish the table of numbers for the parts.

• You may wish to use part of this lesson for oral work with children working at the board. Lead pupils to generalize about what happens to the number for the other part if the number for one part increases by 1.

• The work at the foot of the page in finding the unknown number is not a new type. However, it does extend the sums now to include 10.

Individualizing Instruction

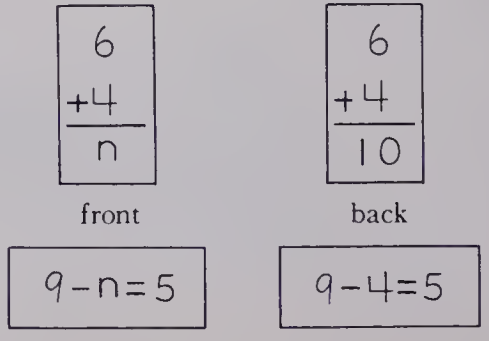
• You may want to use *Show the Answer* practice with the *slower learners* before assigning the written work at the bottom of the page.

a. Use flash cards (see Teaching Pages 26 and 27).

b. Write on the board number sentences containing n or $\square$ or $\bigcirc$.

c. Ask questions such as, If 6 is the number for one of the parts of a set of 10, what is the number for the other part?

• *All pupils* can make study cards (about 2″ by 4″) for the facts they do not know. Cards can be made of tagboard or construction paper. The front and back of two different types of study cards are shown below.



Facts with the Sum 10

A. and S. [W]

1. Use your ruler to find the numbers for the parts of the set of 10 inches. Copy and finish this table of numbers for the parts.

1 and 9
2 and ? 8
3 and ? 7
4 and ? 6
and so on to
9 and 1

5 and 5
6 and 4
7 and 3
8 and 2

2. Show the set of A. and S. facts about each of these:

a. 10; 2 and 8 **c.** 10; 7 and 3
b. 10; 9 and 1 **d.** 10; 4 and 6
e. 10; 5 and 5

3. Copy Ex. a-f and finish each set of A. and S. facts.

a. $10 - 6 = 4$ (6+4=10, 4+6=10; 10−4=6)
c. $10 - 1 = 9$ (1+9=10, 9+1=10; 10−9=1)
e. $5 + 5 = 10$ (10−5=5)
b. $8 + 2 = 10$ (10−8=2, 10−2=8; 2+8=10)
d. $3 + 7 = 10$ (10−3=7, 10−7=3; 7+3=10)
f. $10 - 4 = 6$ (4+6=10, 6+4=10; 10−6=4)

4. Copy and finish the table at the right, A. Facts with Sum 10.

A. Facts with sum 10
1 + 9 = 10
2 + ?8= 10
and so on to
9 + 1 = 10

3+7=10
4+6=10
5+5=10
6+4=10
7+3=10
8+2=10

5. Make the table S. Facts with Sum 10. Start with $10 - 1 = 9$, and end with $10 - 9 = 1$.

10−1=9 10−3=7 10−5=5 10−7=3 10−9=1
10−2=8 10−4=6 10−6=4 10−8=2

Copy and finish Ex. 6–23, writing the numeral for the unknown number.

6. $2 + 7 = n$ (9)
7. $1 + 9 = n$ (10)
8. $10 - 8 = n$ (2)
9. $9 - 6 = n$ (3)
10. $6 + 4 = \square$ (10)
11. $3 + 5 = n$ (8)
12. $10 - 6 = n$ (4)
13. $10 - 3 = n$ (7)
14. $2 + 8 = n$ (10)
15. $5 + 5 = n$ (10)
16. $3 + 7 = n$ (10)
17. $10 - 4 = n$ (6)
18. $8 + 2 = n$ (10)
19. $10 - 9 = n$ (1)
20. $5 + 4 = n$ (9)
21. $7 + 2 = n$ (9)
22. $9 - 5 = \bigcirc$ (4)
23. $4 + 6 = n$ (10)

2. a 2+8=10, 8+2=10, 10−8=2, 10−2=8
2. b 9+1=10, 1+9=10, 10−9=1, 10−1=9
2. c 7+3=10, 3+7=10, 10−7=3, 10−3=7
2. d 4+6=10, 6+4=10, 10−4=6, 10−6=4
2. e 5+5=10, 10−5=5

Helpers for Addition and Subtraction Facts

Mental procedures; Extra Examples [O]

You can use these helpers for A. and S. facts. Finish the examples given in Ex. 1–4.

1. A total and its parts. Knowing that the numbers for a total set and its parts are 9; 6 and 3, helps you find

$9 - 6 = ?$ $3 + 6 = ?$ $9 - 3 = ?$ $6 + 3 = ?$

2. Equal addends. Finish Ex. a–c. Then, for each of Ex. d–f, tell a helping fact with equal addends.

a. $3 + 3 = 6$,
so $4 + 3 = ?$ 7
and $3 + 4 = ?$ 7

b. $8 - 4 = 4$,
so $10 - 4 = ?$ 6
and $10 - ? = 4$ 6

c. $4 + 4 = 8$,
so $3 + 4 = ?$ 7
and $4 + 3 = ?$ 7

d. $5 + 4 = ?$
4+4=8 or 5+5=10

e. $7 - 3 = ?$
6−3=3

f. $9 - 4 = ?$
8−4=4

3. Sets of A. and S. facts. Knowing one fact in a set of A. and S. facts can help you find other facts in that set.

a. $3 + 4 = 7$,
so $7 - 3 = ?$ 4

b. $8 - 5 = 3$,
so $8 - ? = 5$ 3

c. $4 + 6 = 10$,
so $10 - 4 = ?$ 6

4. Facts near each other. Finish Ex. a–c. Then tell a fact that can help with each of Ex. d–f.

a. $4 + 2 = 6$,
so $5 + 2 = ?$ 7
and $4 + 3 = ?$ 7

b. $9 - 7 = 2$,
so $8 - 7 = ?$ 1
and $10 - 7 = ?$ 3

c. $8 + 2 = 10$,
so $8 + 1 = ?$ 9
and $7 + 2 = ?$ 9

d. $10 - 4 = ?$
10−5=5

e. $2 + 7 = ?$
1+7=8

f. $5 + 3 = ?$
5+2=7

Answers for d – f will vary.

[W]

* ◆ **Extra Examples.** Turn to Sets 17 and 18 on page 357 for practice on A. facts; Sets 19 and 20 for practice on S. facts; and Set 21 for practice on A. and S. facts.

* This is the first of many references to sets of extra examples provided in the reservoir at the back of this book. These sets of examples are carefully developed so that they may be used at the time they are recommended or later.

Teaching Page 36

Pupil's Objectives

To find different ways to think about numbers for parts and totals to help in finding the answers to A. and S. facts not yet mastered.

New Word

practice

Background

Pupils who used *Mathematics We Need* in the second grade were introduced to *helpers* for verifying the answer to an A. or S. fact. These *helpers* stem from generalizations about number relationships that foster understanding. In order to forestall blind memorization, we want children to have a flexible means of self-help in finding answers. As the child approaches mastery of a fact the generalization will remain with him, but the use of "helpers" is considered to be second-best as a means of obtaining answers—something to fall back on if memory fails. Such devices will be discarded as pupils are guided toward use of more efficient methods.

The rule of *Compensation* permits us to change the numbers on which we are operating in order to facilitate the solution. For example, in addition, the change we make in one addend must *compensate* for the change we make in the other addend. In the example $6 + 4$, we can subtract 1 from 6 and *compensate* for this change by adding 1 to 4, thereby keeping the total relationship the same. The generalization for Compensation in Addition is: *For addition of two numbers, if you increase one addend and decrease the other addend by the same number, the sum remains the same.*

In subtraction, compensation permits us to add the same number to both the sum and the known addend without changing the answer. In the example $52 - 27$, we may find it easier to add 3 to both numbers and find the answer to $55 - 30$. In both cases the answer is 25. The generalization for Compensation in Subtraction is: *For subtraction, if you increase or decrease both sum and known addend by the same number, the unknown addend remains the same.*

Pre-Book Lesson

- Review sets of A. and S. facts when numbers for two parts of a set or for a total and one part are given.
- Use objects in the classroom to demonstrate the relationship between adding $4 + 2$ and $3 + 3$. Place 4 books in one pile and 2 books in another pile. Let the class see that *1 more* on the 2-pile and *1 fewer* on the 4-pile will not change the total number of books. This is a demonstration of the use of compensation. Demonstrate this idea with several other groups.

Using the Text Page

Be sure to emphasize each of the *helpers* presented in this material.

Individualizing Instruction

- The *helpers* may be particularly useful in aiding the *slower learners*. It will be well worth the time spent to be sure that the children understand the idea behind each *helper* and that they know how to use it.
- Be sure that *all pupils* can tell sets of A. and S. facts, using the number for a total and the numbers for its pairs of parts. Ask them to tell other facts in the set when one fact is given.
- Give extra practice in learning the facts with equal addends ($2 + 2 = 4$, $4 - 2 = 2$, $3 + 3 = 6$, $6 - 3 = 3$, $4 + 4 = 8$, $8 - 4 = 4$, and so on). These facts use numbers for parts of the same size. See if pupils can tell the set of two A. and S. facts about such a set and parts of the same size.
- Extra Examples, Sets 17–21, may be assigned as needed.

The Extra Examples Sets which are referred to on page 36 in the pupil's book, and on many pages to follow, provide a wealth of material for helping individuals who need additional practice. Be sure pupils are using correct thought procedures before you assign them Extra Examples; i.e., they should be getting answers for the facts by direct recall, not by counting or other immature procedures. However, do not press this too hard.

Teaching Page 37

Pupil's Objectives

(a) To extend the use of n in number sentences; and (b) to gain experience in finding the unknown sum in a subtraction number sentence using n.

New Word

explain

Background

The letter n, which previously stood for a sum in an addition number sentence and for an addend in both A. and S. number sentences, now stands for the sum in a subtraction number sentence.

$$n - 4 = 2$$

Teacher's Preparation

- Have n written on a 4″ by 6″ card for use in the Pre-Book Lesson.
- Have the following set of A. and S. facts shown on the chalkboard:

$6 + 4 = 10$	$4 + 6 = 10$
$10 - 4 = 6$	$10 - 6 = 4$

Pre-Book Lesson

- With the n-card, cover different numerals on the board in the number sentences showing facts. Each time a numeral is covered, have a child read the number sentence. In this activity, place the emphasis on the reading of number sentences and not upon finding the unknown number which n stands for.
- Next ask the children to close their eyes. With the n-card, cover a numeral in a fact shown. Ask a pupil to open his eyes and tell the number shown by the covered numeral.
- Write other number sentences on the board until the children understand that n may be used to stand for the sum in both addition and subtraction sentences. Also use Related-Facts Cards.*

Using the Text Page

- Elicit responses from the children to reveal understanding of the addends-sum relationship.
- Be sure the pupils understand that □ or △ or ⬡ may be used to stand for an unknown number just as n does.
- Caution pupils to work carefully in rows 12–17 because the unknown number may sometimes be an addend and sometimes a sum.

Individualizing Instruction

- Have the *slower learners* make a domino picture for each number sentence in the written work. See the domino pictures on Teaching Pages 18 and 19. You may want to make the written assignment for this group somewhat shorter.
- The *more capable children* may write S. number sentences using n to stand for an unknown sum. They may work with partners and solve each other's number sentences.

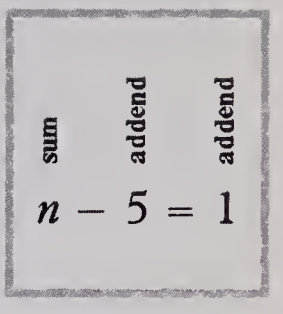
* See 8A, page xix.

NOTES

n in Another Kind of Example

n, for an unknown sum, in S. [O]

Think about the numbers for a total and its parts.



1. For the example, or number sentence, $n - 5 = 1$,

a. the number for the part taken away is 5. The number for the other part is 1. You know that these numbers are both addends. Then is their sum the number for the unknown total? Yes

b. Do we add 5 and 1 to find what *n* stands for? Yes $n = ?$ 6

2. In the example $n - 2 = 8$, explain why $n = 10$.

It is the sum for 2 and 8.

Explain why $n = 7$ in each of Ex. 3–5.

3. $n - 5 = 2$ 4. $n - 6 = 1$ 5. $n - 4 = 3$

Seven is the sum for the two addends shown.

If you know two addends, you add to find the sum.

What number does *n* or △ or ⬭ stand for?

6. 9 $n - 1 = 8$ 7. 10 $n - 5 = 5$ 8. 10 ⬭ $- 8 = 2$

9. 8 △ $- 4 = 4$ 10. 9 $n - 2 = 7$ 11. 8 $n - 5 = 3$

[W]

Copy and write the numeral for the unknown number.

a	b	c	d
12. 7 $n - 1 = 6$	10 $n - 3 = 7$	$6 + 3 =$ 9 n	7 $n - 2 = 5$
13. 10 $n - 9 = 1$	9 $n - 7 = 2$	$8 - 6 =$ 2 n	3 $n + 6 = 9$
14. 8 $n - 3 = 5$	9 $n - 4 = 5$	$6 -$ 5 $n = 1$	$9 - 7 =$ 2 △
15. $9 + 1 =$ 10 n	10 $n - 6 = 4$	$4 +$ 5 ⬭ $= 9$	10 $n - 4 = 6$
16. $4 + 3 =$ 7 n	$5 + 1 =$ 6 □	8 $n - 1 = 7$	$3 + 6 =$ 9 n
17. 8 $n - 2 = 6$	10 $n - 7 = 3$	10 $n - 8 = 2$	9 $n - 5 = 4$

* The number sentences in the box provide a summary of the different ways in which n may be used to stand for either an addend or a sum.

$5 + 3 = \overset{\text{sum}}{n}$	A.
$8 - 3 = \overset{\text{addend}}{n}$	S.
$\overset{\text{addend}}{n} + 5 = 8$	S.
$3 + 5 = \overset{\text{sum}}{n}$	A.
$8 - 5 = \overset{\text{addend}}{n}$	S.
$\overset{\text{sum}}{n} - 5 = 3$	A.
$3 + \overset{\text{addend}}{n} = 8$	S.
$8 - \overset{\text{addend}}{n} = 3$	S.
$\overset{\text{sum}}{n} - 3 = 5$	A.
$8 - \overset{\text{addend}}{n} = 5$	S.

n Is Here! *n* Is There!

n in any position in A. or S. [O]

In each example in the box, *n* stands for the unknown sum or for an unknown addend.

1. For each example, tell if *n* stands for the sum or for an addend. Then tell how to find the number for *n*. [W]

For Ex. 2–7, show what is given and what *n* stands for. For Ex. 2 write, "Sum 9 and addend 4. $n = 5$." For Ex. 3 write, "Addends 4 and 6. $n = 10$."

2. $9 - n = 4$

3. $4 + 6 = n$ "Addend 2 and sum 8. n=6."

4. $10 - 3 = n$ "Sum 10 and addend 3. n=7."

5. $2 + n = 8$

6. $n + 6 = 10$ "Addend 6 and sum 10. n=4."

7. $n - 3 = 7$ "Addends 3 and 7. n=10."

For each of Ex. 8–16, show only what *n* stands for.

8. $8 - 6 = \overset{2}{n}$

9. $\overset{3}{n} + 7 = 10$

10. $3 + \overset{6}{n} = 9$

11. $8 - \overset{4}{n} = 4$

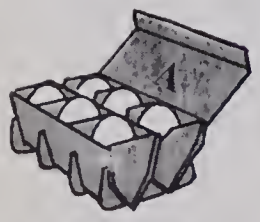
12. $\overset{9}{n} - 5 = 4$

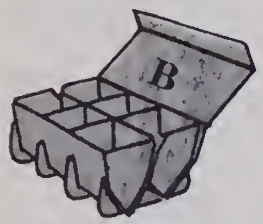
13. $8 + \overset{2}{n} = 10$

14. $\overset{9}{n} - 7 = 2$

15. $9 - \overset{2}{n} = 7$

16. $6 + 4 = \overset{10}{n}$





Zero in Addition

Identity element for A. [O]

Box A shows 6 eggs. Box B shows 0 eggs. How many eggs are in the two boxes?

$6 + 0 = ?_6$ $\qquad$ $0 + 6 = ?_6$

****When one of two numbers is 0, their sum equals the other number.**

Say the sums for these:

1. $0 + \overset{8}{8}$
2. $9 + \overset{9}{0}$
3. $1 + \overset{1}{0}$
4. $0 + \overset{7}{7}$
5. $0 + \overset{5}{5}$
6. $3 + \overset{3}{0}$
7. $\begin{array}{r} 4 \\ \underline{0} \\ 4 \end{array}$
8. $\begin{array}{r} 0 \\ \underline{9} \\ 9 \end{array}$
9. $\begin{array}{r} 7 \\ \underline{0} \\ 7 \end{array}$

** Make sure that pupils realize that they do not have to learn the A. facts having 0 as an addend, if they understand the principle that when one of two addends is zero, the sum is equal to the other addend.

Pupil's Objectives

(a) To summarize the use of n in number sentences; (b) to be able to identify whether n stands for a sum or for an addend; and (c) to understand the use of zero as an addend.

Background

From experience, children know that zero describes the number for an empty set and that when one of two addends is zero, the sum equals the other addend. But they do not always apply this knowledge when confronted with 0 in written examples. Therefore, do not assume that all children will comprehend the generalization immediately. Provide sufficient help so that they will see sense behind the statement and will be able to apply the principle rather than just try to memorize facts.

The *identity element* is another important property which increases in significance at each new level of mathematical sophistication. For the set of whole numbers, there is one and only one number which can be added to any number in the set without changing that number. Zero is the *Identity Element for Addition* in the set of whole numbers. There is no identity element for addition for the set of counting numbers.

This property is not named at the third-grade level, but it is introduced intuitively and used throughout the program.

Teacher's Preparation

Have available two small boxes and some objects to place in them.

Pre-Book Lesson

- Show the two boxes to the class—one with 4 objects, the other empty. Ask, "How many objects are there in all?" Write on the board the number sentence $4 + 0 = 4$.
- Repeat the demonstration with a different number of objects in one box.
- Ask children to tell what is always true about the sum when 0 is added to a number.

Using the Text Page

- Emphasize the reading of the number sentences in the box and have pupils tell for each whether n stands for the sum or for an addend.
- The *more capable children* should be able to proceed with this work independently. The *slower learners* may need individual or some group direction.
- Have pupils state in their own words the generalization at the bottom of the page about zero as an addend.
- Have pupils use their understanding of the Commutative Property of Addition by saying the other A. example to go with each of Ex. 1–9 at the bottom of the page.

Individualizing Instruction

- *All pupils* should be permitted and encouraged to draw domino pictures and make study cards when it seems appropriate. Using objects and drawing pictures help to build confidence and foster understanding, both important ingredients necessary in complete mastery of a concept.
- Matching domino pictures with study cards is a good type of practice for most children.
- Occasionally it may be useful to have some of the *more capable children* help the *slower learners*.
- *More capable children* may see if they can change each of Ex. 2–16 so it will show n to stand for the answer. For instance, Ex. 2 would be written as "$9 - 4 = n$."

Pupil's Objectives

(a) To become more familiar with a number-line picture as a way of showing members of the set of whole numbers; and (b) to learn to add and subtract using a number line.

Background

Understanding the number line is important to the understanding of numbers and number relationships. It will be used to build further understanding of the operations, properties of numbers, and the language of mathematics in general. The following conventions should be developed in detail for this grade for most efficient use of the number line.

- We can show only a portion of the number line on paper or on the chalkboard, so arrowheads are shown near each end of the picture to indicate that the line goes on and on.
- Labeled curved arrows above and below the number line indicate from what point to what point a move is made in adding or subtracting.
- In this grade a labeled arrow pointing to the right suggests adding.
- In this grade a labeled arrow pointing to the left suggests subtracting.

Teacher's Preparation

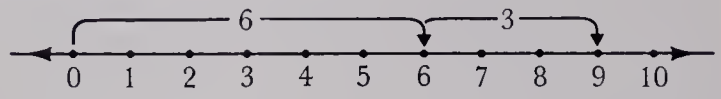
- Make a picture of a number line on the chalkboard with points for whole numbers through 15.
- You may find it useful to make a permanent number-line model. It should be made on a roll of paper about 4″ wide and long enough to show the points for whole numbers 0 through 50 with the numerals about 2″ apart.
- Duplicate a page of pictures of number lines (about 12 on the page). There should be about 15 points shown on each line one-half inch apart.

Pre-Book Lesson

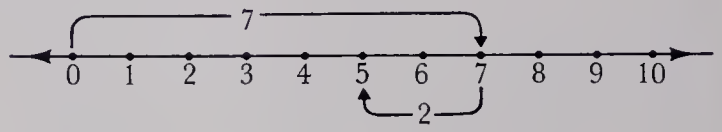
- Explain the important features of the number line, as discussed in the Background section.
- Draw a curved arrow from 0 to 6 (⌐6→) above the number-line picture on the chalkboard. Explain that this arrow shows that we started at the point for 0 and moved 6 points to the right. Ask, "Where does this arrow stop?"

Then draw a curved arrow above the number-line picture, starting at the point for 6 and going to the point for 9 (⌐3→). Ask a child to explain what this arrow shows.

- Have different children draw labeled arrows above the number-line picture to show other additions. Have children explain each drawing.



- To illustrate subtracting, draw a curved arrow above the picture of the number line from the point for 0 to the point for 7 (⌐7→). Then, draw an arrow below the line, starting at the point for 7 and going to the left 2 points to the point for 5 (←2⌐).



Have pupils explain the subtraction being shown.

- Have pupils show other subtractions using the number line. Check their understanding.

Using the Text Page

Oral discussion of Ex. 1–14 will show how well your pupils understand the Pre-Book Lesson.

Individualizing Instruction

- Have *all pupils* use the duplicated sheets of number-line pictures prepared earlier and show the adding and subtracting for the examples in row 4 on page 48. It will be necessary for pupils to label with a numeral each whole-number point on each line. Make sure they begin with 0.
- Have *more capable children* make pictures of number lines with points labeled for whole numbers from 0 through 25. In working each of Ex. 7–14, have them add 10 to the first addend in A. examples or to the sum in S. examples.
- *More capable children* probably would enjoy manipulations with a modern number line.*

* See 6, page xix.

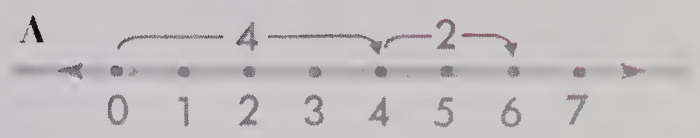
The idea of a number line for ordering numbers is a facet of a thorough mathematics program. Pupils enjoy using a number-line picture for adding and subtracting.

Using a Number Line to Add and Subtract

Using points for whole numbers [O]

Picture A shows adding 4 and 2, using a number-line picture.

A



1. Look at ⌐4¬ to see how you move for addend 4.

a. How many points to the right from 0 do you move? 4

b. To what point do you move? 4

2. Look at ⌐2¬ to see how you move for addend 2.

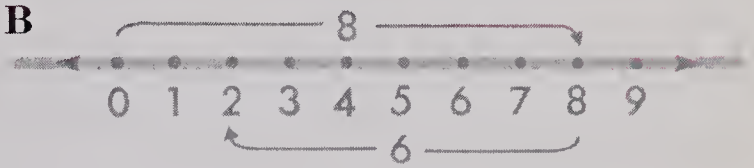
a. How many points from point 4 do you move? 2

b. Do you move right or left? right c. To what point? 6

3. From 0, you moved 4 points and then you moved 2 points. $4 + 2 = ?$ 6

Picture B shows subtracting 6 from 8, using a number-line picture.

B



4. You begin with the sum 8, so look at ⌐8¬.

a. How many points from 0 do you move? 8

b. Do you move right or left? right c. To what point? 8

5. For subtracting 6, look at ⌙6⌡.

a. Where do you begin? 8 b. Do you move right or left? left

c. How many points do you count back? 6

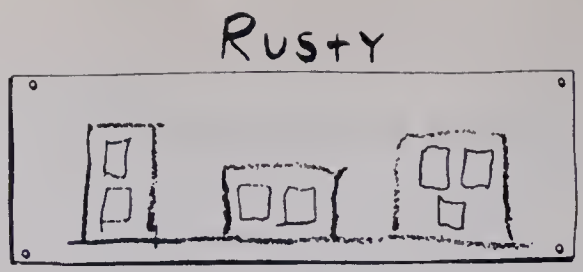
d. To what point do you count? 2

6. You moved 8 points from 0 and back 6. $8 - 6 = ?$ 2

On the board, use a number-line picture to find

7. $6 - 2$. 4 8. $5 + 4$. 9 9. $15 - 7$. 8 10. $6 + 7$. 13

11. $4 + 7$. 11 12. $10 - 8$. 2 13. $6 + 5$. 11 14. $11 - 7$. 4



Adding Three Numbers

* *A. a binary operation* [O]

1. To find how many windows in all are in Nan's houses, we can think of 2, 1, and 4 as three addends. How do we find their sum? add

a. $2 + 1 + 4 = n$

b.
$$\begin{array}{r} 2 \\ 1 \\ \underline{4} \\ n \end{array}$$

We can write the example two ways, as in the box. In Ex. a, add two numbers at a time:

$2 + 1 = 3$, then $3 + 4 = ?^{7}$ $n = ?^{7}$

In Ex. b, add downward two numbers at a time:

$2 + 1 = 3$, then $3 + 4 = ?^{7}$ $n = ?^{7}$

2. Use the number-line picture to find the number of windows in Nan's houses.

3. How many windows in all are in Rusty's houses? 7

a. What are the addends this time? 2, 2, and 3

b. On the board write the example in two ways. $\begin{array}{r} 2 \\ 2 \\ +\,3 \\ \hline 7 \end{array}$ $2+2+3=7$

c. Add in the two examples. In Ex. b add downward.

d. Now use the number-line picture to find the sum.

On the board write an addition example for each of Ex. 4–13. Write each example in two ways and add. See if your answer is correct. Use a number-line picture.

4. 1, 3, 3 ^{7}	5. 6, 0, 4 ^{10}	6. 3, 5, 1 ^{9}	7. 4, 2, 2 ^{8}	8. 1, 7, 1 ^{9}
9. 1, 2, 5 ^{8}	10. 6, 2, 2 ^{10}	11. 3, 2, 3 ^{8}	12. 2, 7, 1 ^{10}	13. 6, 1, 3 ^{10}

* It may be a new idea to the pupils that, when there are more than two addends, only two numbers can be added at a time.

Teaching Pages 40 and 41

Pupil's Objectives

(a) To learn about the addition of three addends using examples written either horizontally or vertically; (b) to practice the horizontal and column addition skills learned; and (c) to learn that the way we group addends will not affect the sum.

New Words

(page 40) *downward*
(page 41) *order*

Background

Each of the operations addition, subtraction, multiplication, and division is said to be a *binary* operation. By this, it is meant that we operate on two and only *two* numbers in the set of numbers at one time. The binary nature of addition becomes more obvious in adding three addends. In the example, $2 + 3 + 4$, we may add $2 + 3$ and get the number 5. Then, we add 5 and 4 to arrive at 9. We might have added $3 + 4$ first to obtain the number 7 and then added 7 to 2 to arrive at 9. At no time did we add more than *two* members of the set in performing the operation.

The Property of Association helps us in selecting a pair of numbers to use for the binary operation. *The Associative Property of Addition* is stated: *For three addends in a certain order, the first may be paired with the second or the second with the third without changing the sum.* This property is sometimes referred to as the *Property of Grouping* because it is concerned with the way in which we *group* addends. In the most abstract form, the Associative Property of Addition is stated: $(a + b) + c = a + (b + c)$. In this program, the understanding is very carefully developed prior to presenting the technical terminology.

If, with three addends, the association is made between the first addend and the third, of course some use is made of the Commutative Property as well as the Associative Property. Since, with third-grade children, we do not use the terminology in dealing with the operation of addition, pupils will intuitively use both properties as they pair any two addends for the first addition.

Teacher's Preparation

- Have objects available that can be manipulated by the children during the Pre-Book Lesson.
- If you have a permanent number-line model (see Teaching Page 39), fasten it on the chalkboard with masking tape. Otherwise, draw a number-line picture on the chalkboard.

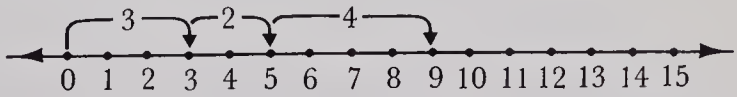
Pre-Book Lesson

- Use 3 sets of objects which make a total of no more than 10. Move two of the sets together. Then move the third set to form a total set. Discuss the joining of three sets to make one total set.
- Start with the original three sets. Move two other sets together, first. Then move the remaining set to form the total set. Elicit the following information:

a. We want to form a total from three separate sets;

b. It makes no difference whether the first two sets or the last two sets are joined first—the total is unchanged.

- Write on the chalkboard the number sentence for joining groups of 3, 2, and 4. $3 + 2 + 4 = n$. Have selected children read the sentence. Then, help them find the sum for this addition example.
- Use the number-line picture for adding in this example.



Elicit information in answer to these questions:
What does each arrow show?
How do you read the sum of the three addends?

- Illustrate other situations with 3 sets of objects. Then have number sentences written, and then have pupils add, using the number line. Be sure to emphasize the understanding implicit in the Associative Property.

Using the Text Pages

- For each of Ex. 4–13, have selected children go to the chalkboard and first write two examples (horizontal and vertical). Then have others draw

a number-line picture and show the adding by using curved arrows.

- You may wish to make Ex. 4–13 paper work for *all pupils* or for only the *more capable children.*
- Have *all pupils* write a number sentence and find the sum for each of Ex. 14–28. They should check each answer by using the number-line picture on page 40.
- Guide the discussion so that children state in their own words the generalization on page 41.
- For the assignment at the foot of page 41, explain why it is necessary to write four different examples using the addends for each of Ex. 14–28. Have pupils write these examples in vertical form.

Individualizing Instruction

- *All pupils* may draw pictures showing three sets and a total. If possible, use situations that apply to content under study in other areas of the curriculum.
- *More capable children* may make number-line pictures and draw arrows to illustrate the adding for Ex. 14 through 28.
- Use Set 22 of Extra Examples as needed.

NOTES

[W]

For Ex. 14–28, write addition examples and add. For Ex. 14 write, "2 + 4 + 3 = 9."

14. 2, 4, 3	17. 0, 8, 1	20. 1, 4, 3	23. 3, 3, 1	26. 2¢, 5¢, 2¢
15. 5, 1, 2	18. 3, 5, 0	21. 3, 1, 3	24. 1, 7, 2	27. 3¢, 6¢, 1¢
16. 7, 0, 3	19. 3, 4, 1	22. 2, 1, 7	25. 4, 1, 5	28. 4¢, 5¢, 0

Are your answers correct? Use a number-line picture.

◆ **Extra Examples.** Work Set 22.

Changing the Order of the Addends

Sums as great as 10; use of parentheses [O]

1. In the box, what two addends are to be added first in a? ^ in b? ^ in c? ^ in d? ^ The ( )'s show the addends to be added first.

For a *think*, "5 + 2 = 7; 7 + 1 = ?"

For b *think*, "2 + 1 = 3; 3 + 5 = ?"

For c *think*, "1 + 5 = 6; 6 + 2 = ?"

For d *think*, "1 + 2 = 3; 3 + 5 = ?"

*

a. $(5 + 2) + 1 = n$
b. $(2 + 1) + 5 = n$
c. $(1 + 5) + 2 = n$
d. $(1 + 2) + 5 = n$

Did changing the order of addends change the sum? No

2. Add downward for Ex. e-h. Which two addends are to be added first in e? ^ in f? ^ in g? ^ in h? ^ What can you say about the sums? The sums are all the same.

e	f	g	h
3	2	5	2
2	5	3	3
5	3	2	5
n	*n*	*n*	*n*

Changing the order of addends and then adding does not change the sum.

[W]

For each of Ex. 14–28 at the top of this page, write four A. examples in columns. Show the addends in a different order each time. Then add. (Ans. may vary.)

For Ex. 14

4	2	3	3
3	3	4	2
+2	+4	+2	+4

* In Mathematics We Need, Book Two, by the use of a tint block, pupils were shown which two of three addends to add first. Now, pupils are introduced to the use of parentheses which, again, indicate the two numbers to be added first.

* Note the reminders to help in deciding whether to add or subtract in solving a problem.

When Do We Add? When Do We Subtract?

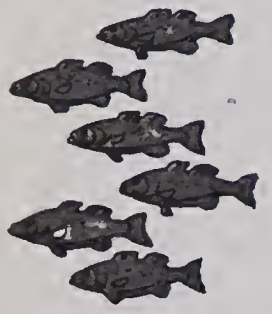
Addends-sum relationship; choosing the n-example [W]

Choose and copy the *n*-example for each of problems 1–6. * To choose, *think* "The numbers for the parts are given, so we add" or "The numbers for the total and one part are given, so we subtract."

1. Tom told his friends that he has 7 rabbits but he wants 10 rabbits. How many more rabbits does he need?

b. 3 rabbits

a. $7 + 10 = n$ **b.** $10 - 7 = n$ **c.** $10 + 7 = n$



2. Bob gave Tom some of his 9 fish. He still has 6. How many fish did Bob give Tom?

a. 3 fish

a. $9 - 6 = n$ **b.** $9 + 6 = n$ **c.** $6 + 9 = n$

3. Don has 3 white mice, 2 gray mice, and 4 brown mice. How many mice has he in all?

a. $4 - 2 = n$ **b.** $4 - 3 = n$ **c.** $3 + 2 + 4 = n$

c. 9 mice

4. Don spent 8¢ of his 10¢ for food for his mice. How much money did he have then?

b. 2¢

a. $10¢ + 8¢ = n¢$ **b.** $10¢ - 8¢ = n¢$ **c.** $8¢ + 10¢ = n¢$

5. Dick's father has 8 dogs. Six of them are puppies. How many older dogs has he?

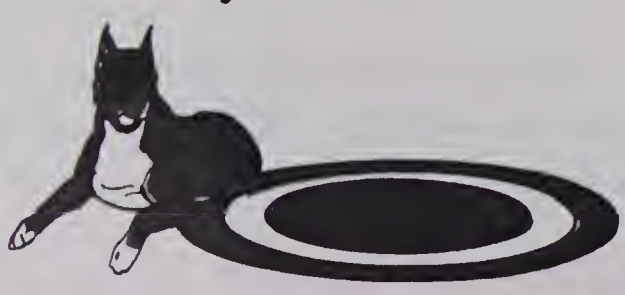
a. 2 dogs

a. $8 - 6 = n$ **b.** $8 + 6 = n$ **c.** $6 + 8 = n$

6. Dick's father is going to buy 2 more dogs to go with the 8 he has. How many dogs will he have then?

b. 10 dogs

a. $8 - 2 = n$ **b.** $8 + 2 = n$ **c.** $8 + 2 + 2 = n$



●**Extra Activity.** Work Set 85.

Teaching Page 42

Pupil's Objectives

(a) To select from three given *n*-examples the one that is associated with that problem; (b) to distinguish between the need for addition and the need for subtraction in solving problems; and (c) to be able to determine whether the numbers given in a problem are for parts of a total or for a total and one part.

New Word

choose

Pre-Book Lesson

• Review quickly the meanings of addition and of subtraction (see Pre-Book Lesson for page 34). Use objects to demonstrate different A. and S. situations.

• Use problem situations in the classroom and help the children decide whether the numbers for the parts are given or the numbers for a total and one part are given. Help them use this information to decide when to add and when to subtract.

Using the Text Page

• Review making *blocks* for working problems (see Teaching Page 20).

• The *more capable children* should be able to proceed with this work independently. You may wish to cover the exercises orally with the *slower learners* before assigning them as written work.

Solving of the problems is not required at this time since certain skills are being stressed. On page 43, pupils will find directions for going back to page 42 and writing the work.

Individualizing Instruction

• If the *slower learners* are not yet able to decide whether to add or subtract for a problem, they should have more concrete experiences. For example, it would be appropriate to use representative materials (geometric forms on the flannel board and at each child's desk) to illustrate each problem. Use 7 circles to represent the 7 rabbits in Ex. 1 so these pupils can see what is involved in the problem.

• Have *more capable children* plan dramatizations for each problem on page 42 to demonstrate that it involves finding the number for a total or for an unknown part.

• Assign Extra Activities Set 85 on page 371 to the *more capable children*.

NOTES

Teaching Page 43

Pupil's Objectives

(a) To make problems for given situations requiring addition and subtraction; and (b) to work with three addends horizontally and vertically, where sums are no greater than 10.

Background

The *translation* of information from the problem setting to the number sentence is one skill involved in efficient problem-solving. All too often we do not emphasize the mere act of *translating*.

On the other hand, to enhance problem-solving skills, it is useful to the learner to have practice in making problems from information about a problem setting. The activity at the top of page 43 calls for experiences in this kind of work. Numbers are provided in social settings. The pupils are to use this information to formulate problems. The emphasis is not upon the solving of problems, but the stating of them with the proper kind of question.

Pre-Book Lesson

- Review the meaning of addition and subtraction (see the Pre-Book Lesson for page 34).
- Use objects in the room to demonstrate $4 + 2 = 6$. Have different children make a problem based upon the demonstration. Write one of the problems on the chalkboard. Use this time to review the important parts of a complete sentence.
- Repeat the activity above with a problem requiring subtraction.

Using the Text Page

- Emphasize the use of complete sentences in stating problems.
- When the children turn back to page 42 to solve problems 1–6, do not emphasize the writing of number sentences. Let them work each problem in any way they wish. They will recall the models on page 11 for working A. problems and on page 17 for working S. problems.
- Remind the class that we add money numbers in the same way that we add any other numbers.
- For the examples at the foot of the page, the children may be challenged to see how many examples they can get correct in a stated time. The results (number attempted and number correct) may indicate to you the pupils who are fast and accurate, fast and inaccurate, slow and accurate, or slow and inaccurate. With this information, proper remedial measures may be applied. In the remedial work, constantly check an individual pupil's thought processes to discover if he is developing efficient ways of working with numbers.

Individualizing Instruction

- Use *Show the Answer* practice in adding three numbers with *all pupils*. After each "show," ask a child to explain how he got the answer.
- Assign Extra Examples Sets 23 and 24 as needed.

NOTES

* If pupils can make A. and S. problems according to direction, they understand how to differentiate between the operations needed for solving these problems.

Making A. and S. Problems

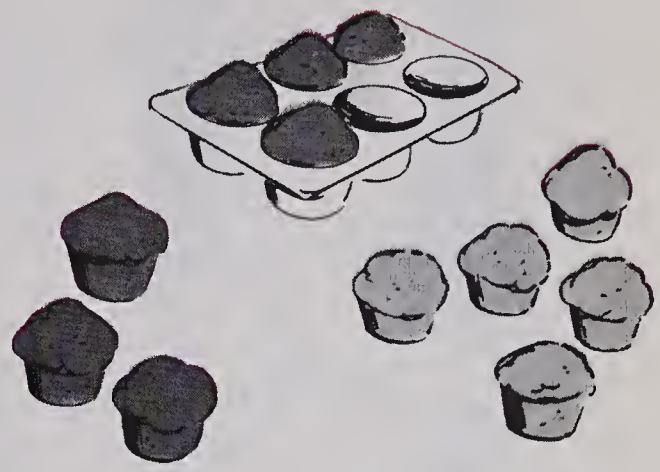
[O]

* Make addition problems about

1. the cupcakes not in the pan. 8 cupcakes
2. the 2 groups of dark cupcakes. 7 cupcakes ∧
3. 3 cents, 2 cents, and 5 cents. 10 cents

* Make subtraction problems about

4. the cupcake pan; the cakes in it. 2 cupcakes ∧
5. 10 eggs; 2 used for cupcakes. 8 eggs ∧
6. 10¢; a nickel of it lost. 5 cents



[W]

Now write your work for problems 1–6 on page 42.

Practice in Adding Three Numbers

Sums as great as 10 [W]

For rows 1 and 2 use folded paper. Show sums only. We add for columns i-k just as for columns a-h.

	a	b	c	d	e	f	g	h	i	j	k
1.	5	2	1	2	0	6	2	5	3¢	8¢	0
	4	4	2	3	3	2	5	1	1¢	0	7¢
	0	3	5	5	7	2	2	4	3¢	2¢	3¢
	9	9	8	10	10	10	9	10	7¢	10¢	10¢
2.	2	3	1	2	4	4	1	2	5¢	2¢	5¢
	0	4	8	2	5	1	3	6	2¢	0	3¢
	7	3	1	6	0	3	4	1	1¢	8¢	1¢
	9	10	10	10	9	8	8	9	8¢	10¢	9¢

Copy in rows, and add. For Ex. 3 write, "0 + 5 + 5 = 10."

3. 0, 5, 5 10 4. 3, 2, 0 5 5. 3, 3, 4 10 6. 3, 6, 0 9 7. 6¢, 0, 3¢ 9¢

8. 4, 5, 1 10 9. 1, 6, 2 9 10. 4, 2, 2 8 11. 1, 5, 4 10 12. 4¢, 1¢, 4¢ 9¢

◆ **Extra Examples.** Work Sets 23 and 24.

Adding with a 2-Place Numeral

[o]

Ann has sold 24 boxes of cookies for the Scouts. If she sells 5 more boxes, how many boxes of cookies will she have sold in all? $24 + 5 = n$

1. Explain how to find the sum by using
Count 10's and 1's. Add tens and ones.
a. Ex. A. **b.** Ex. B.

2. For Ex. C explain the sum 2 tens and 9 ones. 4 + 5 = 9 Write 9 in one's place. Write 2 in ten's place.

A

ϕ ϕ | | | |
| | | |

B

2 tens and 4 ones
\+ 5 ones
_?_ and _?_,
or _?_

C

Tens	Ones
2	4
+	5
2	9

Changing the order of addends does not change the sum.

3. $24 + 5 = 29$. $5 + 24 = 29$, too. Why?

4–6. Tell how to find sums for Ex. D–F. On the board write each example as in box C. Find the sums.

D

ϕ ϕ ϕ | | | |
|
38

E

8 tens and 2 ones
\+ 7 ones
8 tens and 9 ones, or 89

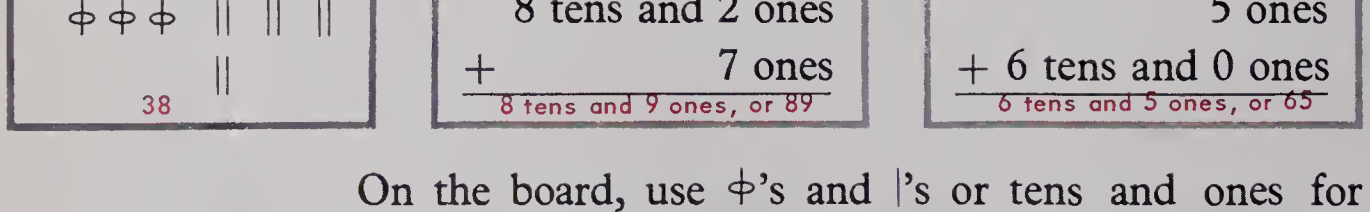
F

5 ones
\+ 6 tens and 0 ones
6 tens and 5 ones, or 65

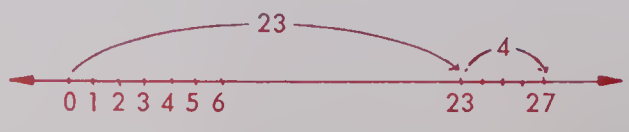
On the board, use ϕ's and |'s or tens and ones for Ex. 7–9. For Ex. 10, use the number-line picture below.

ΦΦ | | | ΦΦΦ | | | | ΦΦ | |

7. $6 + 22$ 28 8. $35 + 3$ 38 9. $4 + 20$ 24 10. $23 + 4$ 27



Ex. 10



Pupil's Objectives

(a) To begin to build an understanding of higher-decade addition; (b) to review the positional nature of our numeration system and use this understanding in higher-decade addition; and (c) to review using the Commutative Property through applying it to addition of whole numbers in the higher decades.

New Word

cookies

Background

This page and a few which follow deal with higher-decade addition in which no carrying is involved. Understanding, and not a permanent thought pattern, is the immediate purpose of this material. At this point, pupils should acquire insight and be able to explain the operation.

Teacher's Preparation

Have some kind of place-value teaching aid available. Wall place-value pockets*, metal or wooden containers with positional values marked on the outsides, and the Ginn Arithme-Sticks**, are examples of materials that would be useful for this purpose. Have available a permanent number-line model (see Teaching Page 39) or a picture of one drawn on the chalkboard.

Pre-Book Lesson

• Show 2 tens and 3 ones in the place-value teaching device. Use these materials to review *2-place numerals*.

Have selected children show some tens and ones in the place-value device and others tell what the number is, write the numeral on the chalkboard, and tell what it shows.

• Review other ways of showing the meaning of 2-place numerals; i.e., ⌀ ⌀ /// , 2 tens and 3 ones.

• After reviewing 2-place numerals and ways of writing them, once again place 2 tens and 3 ones in the place-value device. Ask children to put in 5 more ones. Guide them to suggest putting the 5 *ones* with the 3 *ones*.

Then, picture the operation on the chalkboard as follows:

⌀ ⌀ // /	2 tens and 3 ones	23
// // /	+ 5 ones	+ 5
	2 tens and 8 ones	28

• Repeat the above activity with other 2-place numerals. Make sure at this stage that no carrying is involved.

• Use the place-value aid to review in this new setting the Commutative Property of Addition. Place 2 objects in the one's place. Then have a child place 4 more objects in the one's place. Change the order of placing the objects in the one's place. Lead the children to state in their own words the generalization about what happens when we change the order of adding two numbers.

Review use of the number line to show adding.

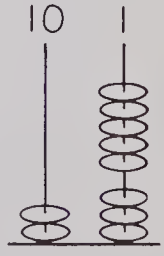
Using the Text Page

• In connection with Ex. 3, lead the children to state the Commutative Property in their own words.

• Encourage the use of the Commutative Property in Ex. 7 and 9.

Individualizing Instruction

• *Slower learners* should have much practice in using objects to foster greater understanding of what 2-place numerals show and the adding with 2-place numerals. Peg boards and pegs from the kindergarten room may be useful for this purpose. The peg board allows for 10 pegs in a row, thus making it possible for a child to see rows of ten for the number shown by a 2-place numeral.



• Have the *more capable children* draw line-abacus pictures to show adding with a 2-place numeral. For example, a picture for 23 + 5 would be drawn as at the right.

* See 21, page xix. ** See 3, page xix.

Pupil's Objectives

To begin to use the knowledge and understanding of the basic A. facts in organizing higher-decade addition examples into sets.

New Word

belong

Background

Not too many years ago, pupils were required to commit to memory the higher-decade facts. *Mathematics We Need* achieves the same end by helping pupils think the sum at once by going through a carefully planned series of developmental steps. You have just taught the principle governing higher-decade addition. The next step is to *organize* learning around relationships. The sums in examples, such as 52 + 7, 92 + 7, 42 + 7, can be found by first identifying the addition set using 2 + 7 = 9.

Pre-Book Lesson

• Using a place-value teaching aid,* ask children to add 22 + 4, then 42 + 4, and then 52 + 4. They should note that each time 2 and 4 are added for the ones. Help pupils to discover and state that each of the given examples belongs to the addition set using 2 + 4 = 6.

• Let pupils find the sums for examples in other addition sets, such as 31 + 6, 51 + 6, 71 + 6. Have pupils tell more examples that belong to this addition set.

• Use a model of a number line** showing points for 1 through 99. Have pupils note that 3 more than 25 is 28, 3 more than 35 is 38, and so on.

* See 21, page xix.
** See 6, page xix.

Using the Text Page

• After obtaining satisfactory responses for the oral work on the page, have pupils do some of the examples in row 17 orally before proceeding with the written work.

• Review the technique of folding paper (see Teaching Page 10) for writing answers to examples.

Individualizing Instruction

• Help the *slower learners build* addition sets using concrete objects such as blocks*, discs**, peg boards, and so on.

Have these children use ϕ's and /'s to find sums for examples in an addition set. For example:

	// // //	4 + 2 = 6
ϕ ϕ	// // //	24 + 2 = 26
ϕ ϕ ϕ ϕ ϕ	// // //	54 + 2 = 56

Call to the pupils' attention that there are the same number of ones to be added in each example.

• Have the *more capable children* try to write all of the examples using 2-place numerals that make up a set using a given A. fact. For example, the set using the A. fact 4 + 5 = 9 would include:

14 + 5	44 + 5	74 + 5
24 + 5	54 + 5	84 + 5
34 + 5	64 + 5	94 + 5

• Have *more capable children* write problems that require the use of higher-decade addition.

* See 13, page xix.
** See 12, page xix.

Pupils find work in higher-decade addition much easier when they realize that they may use A. facts they already know.

Sets Using Addition Facts

[O]

A

a. 13 + 5
b. 23 + 5
c. 63 + 5
d. 83 + 5

1. All the examples in box A belong to the A. set using 3 + 5 = 8. Explain why we can say this. The addition fact 3 + 5 = 8 is used in each example.

2. For Ex. a *think*, "3 + 5 = 8, so 13 + 5 = 18." Why is this so? We use the A. fact and show 8 in one's place. Since one addend is 10 greater, the sum is 10 greater.

3. For Ex. b *think*, "3 + 5 = 8, so 23 + 5 = ?" 28

4. Tell what to think for Ex. c and Ex. d. "3+5=8, so 63+5=68" "3+5=8, so 83+5=88"

B

e. 7 + 12
f. 7 + 22
g. 7 + 82
h. 7 + 42

5. The examples in box B belong to what A. set? 7+2=9

6. Say three examples in the A. set using 7 + 2 = 9. Answers will vary. 32+7 52+7 62+7

7. For Ex. e, finish 7 + 2 = 9, so 7 + 12 = ? 19

8. Tell what to think for Ex. f, Ex. g, and Ex. h. "7+2=9, so 7+22=29" "7+2=9, so 7+82=89" "7+2=9, so 7+42=49"

9. Why are 37 + 2 and 57 + 2 in the A. set using 7 + 2 = 9? The addition fact 7+2=9 is used in each example.

Each of Ex. 10–13 belongs to what A. set?

10. 53 + 4 (3+4=7) **11.** 2 + 20 (2+0=2) **12.** 3 + 43 (3+3=6) **13.** 25 + 4 (5+4=9)

Write on the board three other examples in the A. set using Answers will vary. See samples below.

14. 0 + 7 = 7. (20+7) **15.** 6 + 3 = 9. (26+3) **16.** 8 + 1 = 9. (48+1)

[W]

For rows 17–19, show the sums on folded paper. For Ex. 17a *think*, "0 + 4 = 4, so 30 + 4 = 34."

	a	b	c	d	e	f
17.	30 + 4 (34)	75 + 2 (77)	91 + 8 (99)	52 + 3 (55)	86¢ + 2¢ (88¢)	4¢ + 43¢ (47¢)
18.	3 + 60 (63)	95 + 4 (99)	2 + 76 (78)	62 + 4 (66)	40¢ + 6¢ (46¢)	52¢ + 7¢ (59¢)
19.	85 + 1 (86)	53 + 6 (59)	2 + 75 (77)	4 + 54 (58)	64¢ + 2¢ (66¢)	3¢ + 82¢ (85¢)

A Quicker Way to Add

[O]

1. Tom found 5 apples on the ground. He put them in the basket with 32 other apples. Then how many apples were in the basket? $32 + 5 = n$

a. You can *think*,

$2 + 5 = 7$, so $32 + 5 = 37$.

b. It is quicker just to look at 32 and 5 and *think*, "37."

Say sums this quick way for Ex. 2–7.

2.	3.	4.	5.	6.	7.
5	22	40	5	4	72¢
30	5	6	94	83	3¢
35	27	46	99	87	75¢

[W]

Add the quick way. Show the sums on folded paper.

	a	b	c	d	e	f	g	h	i
8.	40	25	3	61	13	64	26	54¢	6¢
	5	3	35	4	4	3	2	5¢	73¢
	45	28	38	65	17	67	28	59¢	79¢
9.	6	53	3	82	72	8	6	28¢	2¢
	31	6	13	4	7	70	43	1¢	32¢
	37	59	16	86	79	78	49	29¢	34¢
10.	53	1	2	17	50	1	34	5¢	24¢
	2	76	83	2	2	67	5	42¢	4¢
	55	77	85	19	52	68	39	47¢	28¢
11.	14	2	74	83	22	5	5	17¢	51¢
	4	30	2	6	6	44	72	2¢	8¢
	18	32	76	89	28	49	77	19¢	59¢
12.	20	2	33	5	6	7	25	4¢	82¢
	8	46	5	21	30	61	3	41¢	7¢
	28	48	38	26	36	68	28	45¢	89¢

◆**Extra Examples.** Work Sets 25 and 26.

Teaching Page 46

Pupil's Objective

To learn to shorten the thinking of sums for higher-decade addition.

Background

Pupils constantly encounter the need for higher-decade addition as they perform column addition and certain multiplications. It is imperative that they be able to know sums immediately. To be proficient in higher-decade addition, the pupils must know the basic addition facts very well.

Pre-Book Lesson

Use this problem: There are 25 children present today. Three children are absent. How many children do we have in our class? $25 + 3 = ?$

Although the answer may be found by adding the ones and bringing down the tens, guide pupils to understand that it is quicker just to consider the numbers involved and say the answer at once without adding the ones and tens separately.

Using the Text Page

- Encourage the children to use their knowledge of the basic facts and say the answer to each of Ex 2–7 without actually adding ones and tens.
- Check their understanding of the Commutative Property of Addition by saying the other addition example that goes with each of Ex. 2–7.
- Review the technique of folding paper (see Teaching Page 10) before having pupils write answers to rows 8–12.
- As children are writing, walk among them and ask individuals to think an example aloud for you. Be sure they are working on the principle that it is quicker just to consider the numbers and say the answer. Encourage this kind of thinking even with those children who have not arrived at the full level of mastery of the A. facts.

Individualizing Instruction

- Continue to use objects with the *slower learners* to help them master the basic addition facts as well as use the facts in higher-decade addition.
- The *more capable children* may enjoy writing number sentences of the following types in which they use higher-decade addition examples with no carrying.

$$n + 42 = 49$$
$$4 + \square = 75$$

- Use Extra Examples Sets 25 and 26 on page 358, as they are needed.

NOTES

Pupil's Objective

To begin to use higher-decade addition examples with three addends of the type $23 + 2 + 3$.

Background

Heretofore, pupils have not worked with a 2-place numeral for an addend in 3-addend addition. However, the thought process is the same as that for two addends. Some youngsters have difficulty with the unseen addend when adding three numbers. For example, in adding $42 + 4 + 2$, the child may first think that $42 + 4 = 46$. Then, he must think that $46 + 2 = 48$. The 46 is an unseen addend in this example. If you have a child who has difficulty with this particular step in adding three numbers, he will need special attention.

The material in the enrichment section (for the *more capable children*) involves two steps to arrive at a solution for each number sentence. When n stands for one of three addends in an example, the child must first determine the sum of the two known addends. This first step reduces the sentence to an addends-sum situation where the sum and one of its addends are known. Then, the child subtracts to determine the unknown addend. In the example $3 + n + 2 = 9$, the child must think, "$3 + 2 = 5$; $5 + n = 9$; $9 - 5 = 4$; so $n = 4$."

Of course, children are not always conscious of each specific step they use in arriving at a solution of this type. A useful technique to diagnose a child's understanding in this instance is to ask him to "think" the problem aloud.

Pre-Book Lesson

• Write on the chalkboard the number sentence $3 + 5 + 1 = n$. Have a child illustrate the example using a number line as he explains what the sentence asks and means.

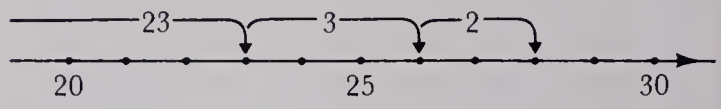
Take this opportunity to review the use of curved arrows in showing adding when using the number line and the use of the Associative Property when adding.

• Use a classroom situation similar to the following as the basis for number sentences requiring higher-decade addition:

There are 23 children present today. There are 3 boys and 2 girls absent. How many children belong in this class?

After writing the sentence $23 + 3 + 2 = n$ on the chalkboard, review the thinking process involved in higher-decade addition with three addends. Start with 23. Be sure pupils use the skill developed in the previous lesson to add 23 and 3 and then use it again to add 26 and 2.

• Make a picture of a number line on the chalkboard and show the adding for the number sentence above (a portion shown below).



Using the Text Page

• For Ex. 2–7, children may work as partners, first one and then the other telling how sums were found. Try to arrange partners so that two *slower learners* do not work together. As you listen, you may decide which pupils you wish to help before you allow them to proceed with written work.

• You may wish to assign only part of the examples in rows 8–11 for written work.

Individualizing Instruction

• Children who show a lack of understanding should be grouped for special instruction. Use objects to help them arrive at answers the short (fast) way when using higher-decade addition.

• The pupils who understand and complete the written work with no difficulty may try the enrichment activity at the bottom of the page. Be sure to encourage the 2-step thinking required in arriving at the solution of each number sentence.

• Use Extra Examples Sets 27 and 28 as needed. Encourage use of the short way.

Using Addition Sets

[O]

1. Joe's father has 23 brown cows, 2 white cows, and 4 black cows. How many cows has he? $23 + 2 + 4 = n$.

Look at 23 and 2. Use $3 + 2 = 5$. *Think*, "25." Look at 4. Use $5 + 4 = 9$. *Think*, "29."

23
2
4
?
29

Tell how to find the sums in Ex. 2–7. Each time tell the A. sets you use and what you think.

2. 22	**3.** 30	**4.** 63	**5.** 12	**6.** 11	**7.** 22¢
Use 2+2 2	Use 0+4 4	Use 3+1 1	Use 2+5 5	Use 1+0 0	Use 2+4 4¢
Use 4+5 5	Use 4+3 3	Use 4+5 5	Use 7+1 1	Use 1+5 5	Use 6+3 3¢
29	37	69	18	16	29¢

[W]

Each time, copy the numerals in a column, and add.

	a	b	c	d	e
8.	19 13, 2, 4	78 73, 3, 2	16 12, 2, 2	88 83, 4, 1	29¢ 26¢, 2¢, 1¢
9.	37 34, 1, 2	38 32, 6, 0	47 44, 0, 3	58 52, 4, 2	59¢ 53¢, 5¢, 1¢
10.	89 81, 1, 7	48 43, 1, 4	69 63, 5, 1	29 20, 6, 3	89¢ 82¢, 4¢, 3¢
11.	47 42, 3, 2	58 53, 4, 1	79 71, 3, 5	79 73, 3, 3	49¢ 41¢, 3¢, 5¢

◆ **Extra Examples.** Work Sets 27 and 28.

Do You Like to Try New Things?

Enrichment [W]

$5 + 4 + n = 10$. Look at $5 + 4$; *think*, $9 + n = 10$. If $9 + n = 10$, then $10 - 9 = n$, so $n = 1$.

Copy each example, writing a numeral in place of n.

1. $4 + 2 + \overset{2}{n} = 8$

2. $3 + \overset{4}{n} + 3 = 10$

3. $\overset{1}{n} + 2 + 6 = 9$

4. $0 + 3 + \overset{5}{n} = 8$

5. $\overset{1}{n} + 1 + 8 = 10$

6. $6 + 1 + \overset{0}{n} = 7$

* Another possible test—this time involving mixed A. and S. facts having sums as great as 10!

* Finding Unknown Sums and Addends

Sums as great as 10 [W]

Write the answers on folded paper. Study the facts you do not know. Make study cards for help.

	a	b	c	d	e	f	g	h	i	j	k
1.	10	4	9	10	2	4	7	9	2¢	3¢	8¢
	−7	+0	−5	−1	+7	+6	−3	−7	+8¢	+3¢	−2¢
	3	4	4	9	9	10	4	2	10¢	6¢	6¢
2.	9	8	3	9	5	10	8	4	10¢	2¢	2¢
	+1	−4	+7	−1	+5	−4	+2	+5	−3¢	+5¢	+3¢
	10	4	10	8	10	6	10	9	7¢	7¢	5¢
3.	3	10	9	2	5	6	9	6	9¢	8¢	7¢
	+6	−2	−8	+6	+3	+2	−3	+3	−2¢	−7¢	−2¢
	9	8	1	8	8	8	6	9	7¢	1¢	5¢
4.	8	9	3	10	8	7	7	10	9¢	2¢	5¢
	−5	+0	+4	−5	−6	+3	−4	−8	−4¢	+4¢	−3¢
	3	9	7	5	2	10	3	2	5¢	6¢	2¢
5.	10	7	6	4	8	0	3	9	5¢	10¢	7¢
	−6	+2	+4	+3	−3	+6	+5	−6	+4¢	−9¢	−5¢
	4	9	10	7	5	6	8	3	9¢	1¢	2¢

To Keep in Practice

[W]

Copy in columns, and find the sums.

1. 20 + 7 27	2. 33 + 4 37	3. 6 + 51 57	4. 61¢ + 3¢ 64¢
5. 8 + 31 39	6. 80 + 9 89	7. 5 + 14 19	8. 33¢ + 6¢ 39¢
9. 0 + 62 62	10. 3 + 45 48	11. 64 + 4 68	12. 7¢ + 12¢ 19¢
13. 3 + 3 + 4 10	14. 2¢ + 6¢ + 2¢ 10¢	15. 7, 0, 3 10	
16. 42 + 0 + 7 49	17. 34¢ + 3¢ + 1¢ 38¢	18. 64, 2, 3 69	
19. 0 + 4 + 5 9	20. 73¢ + 2¢ + 2¢ 77¢	21. 51, 4, 3 58	
22. 22 + 5 + 2 29	23. 51¢ + 5¢ + 2¢ 58¢	24. 30, 3, 6 39	

Teaching Page 48

Pupil's Objectives

(a) To review addition and subtraction facts with sums no greater than 10; and (b) to show addends in columns and find sums.

Pre-Book Lesson

• Write A. and S. number sentences on the board in both horizontal and vertical forms. Review the following ideas that have been developed to date:

a. Reading the number sentences
b. The meanings of the various symbols
c. The Commutative Property of Addition
d. The Associative Property of Addition

• On a picture of a number line, illustrate both adding and subtracting using a number line.

Using the Text Page

• Review the use of folded paper if necessary (see Teaching Page 10). This assignment may be considered a progress test.

• Before assigning the written work at the bottom of the page, emphasize the importance of copying carefully and neatly.

Individualizing Instruction

• Have the *slower learners* make dot pictures for facts they do not know.

• Have the *more capable children* use number-line pictures to illustrate the examples in row 5 at the top of the page and Ex. 19–24 at the bottom of the page.

NOTES

Teaching Page 49

Pupil's Objective

To work the end-of-chapter test on problem-solving.

New Word

miles

Background

Beginning on page 49 and extending through pages 50 and 51 is a simple, well-rounded testing program for chapter 1. The proper appraisal of the child's total mathematical ability must be based on the results of all of these tests. Furthermore, an effort must be made to observe what desirable arithmetic attitudes, appreciations, and values the child has come to possess.

Pre-Book Lesson

• Set the stage for the evaluation program by discussing with the children what they learned and what they think they should remember from studying chapter 1. Make a note of what they emphasize and omit in order to help evaluate your teaching program. Try to promote a healthy attitude toward tests.

• You may wish to review some of the situations requiring subtraction and how we can use what we know about a total and its parts to find the answers to addition and subtraction problems.

Using the Text Page

• After pupils have worked the problems and the papers have been collected, discuss the answers so that each child knows which problems he did not work correctly. Review page 8 for the meaning of addition; pages 12, 16, and 28 for the meaning of subtraction; and page 29 for choosing the correct label for the answer to a problem.

• Further work with the problems on this page is suggested on Teacher's Page 55.

Individualizing Instruction

• Stress the meanings of A. and S. in problems with the *slower learners* by having A. and S. problems dramatized in the classroom.

• Encourage the *more capable children* to

a. draw pictures for 2- and 3-place numerals on a line-abacus.

b. draw number-line pictures showing the adding or subtracting for each problem on page 49.

• Assign sets of Extra Problems or Examples, as seems necessary.

• For suggestions about scoring the pupils' papers, see Teaching Pages 50 and 51.

NOTES

A table for converting each pupil's score to a per-cent score is found on Teacher's Page 50.

Can You Solve Problems?

Problem Test 1

Write your work for each problem.

1. Mr. Fry found that he had used 7 of his 9 bus tickets. How many bus tickets did he have left? 2 tickets
$9-7=2$

2. One day on the bus Jimmy Fry counted 3 girls, 2 boys, and 4 men. That was how many people? 9 people
$3+2+4=9$

3. Another day there were 9 boys on the bus. After some got off, 3 boys were left. How many boys got off the bus? 6 boys
$9-3=6$

4. To go to school, Bob rides 10 blocks. He rides 3 blocks on one street and the other blocks on a second street. How many blocks does Bob ride on the second street? 7 blocks
$10-3=7$

5. Ten girls are going to the school play. They have 6 tickets. How many more tickets do they need? 4 tickets
$10-6=4$

6. A bus had 42 seats with people in them and 6 seats with no people. How many seats does that bus have? 48 seats
$42+6=48$

7. Joe's ride on the bus is 9 miles. After he has gone 5 miles, how many more miles does he have to ride on the bus? 4 miles
$9-5=4$

8. One week Sue spent 14¢, 3¢, and 2¢. How much in all did Sue spend that week? 19¢
$14¢+3¢+2¢=19¢$

9. Ann wants to buy a 10¢ box of popcorn. She has 7¢. How much more money does she need? 3¢
$10¢-7¢=3¢$

10. Joe has 5 pencils, Jimmy has 2, and Bob has 3. How many pencils in all do the boys have? 10 pencils
$5+2+3=10$

Do You Make Mistakes?

Diagnostic Test 1

Tests like this help you to find where you make mistakes, and why you make them.

Write the answers on folded paper.

	a	b	c	d	e	Study Pages *	Practice Sets *
1.	10 − 6 = 4	8 − 6 = 2	2 + 7 = 9	10 − 8 = 2	3¢ + 5¢ = 8¢	10, 16, 35	17–21
2.	7 − 4 = 3	4 + 6 = 10	10 − 3 = 7	2 + 5 = 7	8¢ − 5¢ = 3¢		
3.	5 3 1 = 9	5 1 4 = 10	1 5 2 = 8	3 1 3 = 7	2¢ 6¢ 2¢ = 10¢	40–41	22–24
4.	50 + 3 = 53	86 + 3 = 89	4 + 95 = 99	73 + 4 = 77	7¢ + 72¢ = 79¢	44–47	25–28
5.	42 2 1 = 45	91 4 2 = 97	80 4 4 = 88	31 0 8 = 39	61¢ 2¢ 6¢ = 69¢		

For mistakes in rows 1 and 2, study pages 10, 16, and 35. For extra practice, use some of Sets 17–21.

For mistakes in row 3, study pages 40–41. For extra practice, use some of Sets 22–24. For rows 4 and 5 study pages 44–47. Then work some of Sets 25–28.

After you have done this, take the test again.

* Both study pages and practice sets are referred to because a sound remedial program involves restudying of ideas prior to additional practice.

Teaching Pages 50 and 51

Pupil's Objectives

(a) To diagnose difficulties with facts and skills reviewed in the chapter and to find help in developmental and practice materials on pages listed; (b) to show knowledge of certain concepts, generalizations, and items of information; and (c) to reveal computational ability on some of the skills taught or retaught, involving sums no greater than 10.

New Words

(page 50) *mistakes*, *tests*

Teacher's Preparation

Provide for the type of paper that you wish pupils to use for each of the tests on these two pages. It will also be helpful when you correct papers if you give pupils a plan for displaying their answers.

Using the Text Pages

- For Diagnostic Test 1, make certain that each child knows how to fold the paper for writing answers only (see Teaching Page 10).
- After pupils have written their work for the Test of Information and Meaning 1 on page 51, you will be able to determine which pupils need review.
- Have pupils write answers for Computation Test 1 at a suitable time.

Individualizing Instruction

The three tests on pages 50 and 51, along with the Problem Test on page 49, comprise the standard program for diagnosing and measuring achievement which is provided at the close of each chapter in each book in this series. Scores for work on the Problem Test (page 49) and on the Computation Test (page 51) may be converted to per-cent scores and individual records may be kept for each pupil on 3″ by 5″ cards like the model shown below.

Ralph Thomas		Grade 3
Chapter	*Per Cent Right: Problems*	*Per Cent Right: Computation*
1		
2		
3		
4		
5		
6		
7		
8		

The tables of per cents shown below will enable you to determine quickly the per cent for each pupil's score. For instance, Ralph's Problem-Test score for chapter 1 was 5, so, from the table, his per cent correct is found to be 50. His Computation-Test score was 23, so his per cent correct is 85.

Table of Per Cents for Chapter 1 Scores

Problem Test		Computation Test					
Score	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*
1	10	1	4	10	37	19	70
2	20	2	7	11	41	20	74
3	30	3	11	12	44	21	78
4	40	4	15	13	48	22	81
5	50	5	19	14	52	23	85
6	60	6	22	15	56	24	89
7	70	7	26	16	59	25	93
8	80	8	30	17	63	26	96
9	90	9	33	18	67	27	100
10	100						

Suggestions for Material to Accompany End-of-Chapter 1 Tests

After administering the tests on pages 49, 50, and 51, you will find that some pupils need additional help or review before proceeding to the work of the next chapter. Meanwhile, you may find one or more of the following suggestions valuable for use with those children who do not need additional review.

Alternate Uses of Pages in Chapter 1

Duplicate the following material or ask one of the *more capable children* to read it to the others:

Page 2. In the chart, what do you notice about the position of the numerals when you

a. start with 0 and keep adding 10?
b. start with 0 and keep adding 11?
c. start with 1 and keep adding 11?
d. start with 10 and keep adding 11?
e. start with 1 and add the numbers shown by the digits of each numeral under it?
f. start with 10 and add the numbers shown by the digits of each numeral in the same row?

Find other interesting things about the numerals in the columns and the rows.

Page 5. Write the numerals for counting by

a. 6's to 96.
b. 7's to 98.
c. 8's to 96.
d. 9's to 99.

Write the numerals for counting to or beyond 100 by

a. 2, then 3, then 2, then 3 again (2, 5, 7, 10, · · ·).

b. 3, then 4, then 3, then 4 again.

Page 9. Rewrite Ex. 8–13 showing a tens number for each sum. Make the sentence true by showing a tens number for one of the two addends. Ex. 8 may be rewritten:

$$25 = 3 + 22 \quad \text{or} \quad 25 = 23 + 2$$

Page 35. For each of Ex. 6–23, write a tens digit beside one of the numerals in each sentence. Then write the numeral to show the number that n stands for.

Page 50. For each example in rows 1–5, write three n-sentences showing n in a different position each time. For Ex. 1a, you would write, $10 - 6 = n$, $10 - n = 4$, $n - 6 = 4$.

Supplementary Activities

1. Start with 2 in the first box and write the numerals for counting by 2's. Work by rows.

_?_	_?_	_?_	_?_	_?_	_?_
_?_	_?_	_?_	_?_	_?_	_?_

How many numerals did you write? _____
How many even numbers did you show? _____
How many odd numbers did you show? _____

2. Start with 5 in the first box and write the numerals for counting by 3's. Work by rows.

_?_	_?_	_?_	_?_	_?_
_?_	_?_	_?_	_?_	_?_

How many numbers are shown in the chart? _____
How many even numbers are shown in the chart? _____
How many odd numbers are shown in the chart? _____

3. Start with 7 in the first box and write the numerals for counting by 4's. Work by rows.

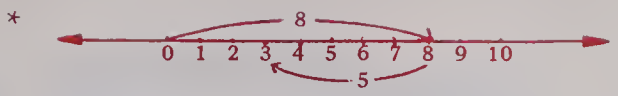
_?_	_?_	_?_	_?_
_?_	_?_	_?_	_?_

4. Start with 30 in the first box and write the numerals for counting backwards by 3's. Work by rows.

_?_	_?_	_?_	_?_
_?_	_?_	_?_	_?_

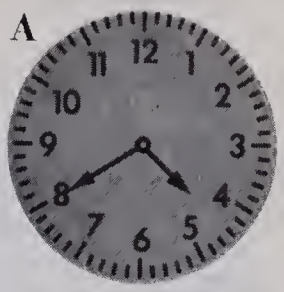
5. Start with 250 in the first box and write the numerals for counting by 2's. Work by rows.

_?_	_?_	_?_	_?_	_?_	_?_	_?_
_?_	_?_	_?_	_?_	_?_	_?_	_?_



Do You Understand?

Test of Information and Meaning 1

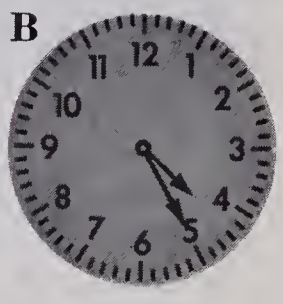


For Ex. 1–10 write the answers.

1. What time does each of clocks A–C show?
A. 20 minutes before 5. B. 25 minutes after 4. C. 9 o'clock.

2. 85 = ?(8) tens and ?(5) ones

3. Draw a number-line picture and show $8 - 5 = 3$.

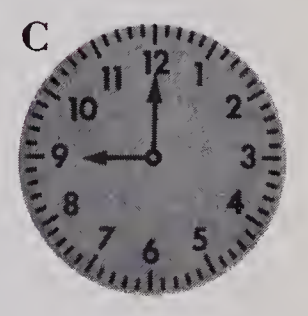


4. In counting by 4's, the number next after 24 is ?(28).
7+3=10 3+7=10 10−7=3 10−3=7

5. 10; 7 and 3. Show the set of A. and S. facts.

6. Do you add or subtract to find the number for the other part? (subtract) Find what n stands for in $10 - n = 4$. (6)

7. Which are odd numbers? 17 20 41 13 56 9 (17, 41, 13, 9)



8. Show the S. fact that goes with $9 - 3 = 6$. 9−6=3

9. For 5; 2 and 3, show two A. facts. 2+3=5 3+2=5

10. Copy and finish: $10 - 2$?(≠) 6 Use = or ≠.

How Well Can You Compute?

Computation Test 1

Write the answers on folded paper.

	a	b	c	d	e	f	g	h	i
1.	4	6	7	2	10	4	7	10¢	9¢
	+5	+4	−4	+7	−7	+6	−5	−3¢	−6¢
	9	10	3	9	3	10	2	7¢	3¢
2.	21	3	9	10	8	45	7	10¢	4¢
	+8	+14	−3	−9	−3	+4	+3	−8¢	+64¢
	29	17	6	1	5	49	10	2¢	68¢
3.	4	5	63	3	2	92	3	33¢	6¢
	0	3	3	4	5	3	6	0	2¢
	2	1	3	1	2	2	1	5¢	2¢
	6	9	69	8	9	97	10	38¢	10¢

2

Adding Tens

Meaning [O]

1. When Sam was in camp, 20 days were sunny and 10 were not sunny. How many days was that? $20 + 10 = n$.

a. Does Ex. A show another way to write the example? Yes

A	B	C	D
20 +10 *n*	ϕ ϕ ϕ	2 tens +1 ten ?, or _?_	20 +10 ?

b. Tell how to find the sum by using Ex. B; Ex. C.

c. For Ex. D, *think*, "Twenty plus ten equals _?_." 30

You add tens in the same way that you add ones.

Say the sums for Ex. 2–6.

2. 2 tens + 6 tens = 8 tens
3. 4 tens + 2 tens = 6 tens
4. 10 + 60 = 70
5. 30 + 60 = 90
6. 50 + 30 = 80

On the board use a number-line picture to add

7. $20 + 30$. 50
8. $60 + 20$. 80
9. $40 + 50$. 90

Overview — Chapter 2

• **Reteaching.** Chapter 2 extends the review of basic addition and subtraction facts to include sums as great as 18 and the use of the subtraction facts in higher-decade subtraction. Again, the work is organized around the ideas of using base ten, number-line pictures, the identity idea, and properties of addition.

• **Extension.** The work involving adding and subtracting uses two 2-place numerals, is extended to the more mature form. No renaming is brought in at this time. Continuous use is made of properties and other mathematical ideas. Work with inequalities carefully distinguishes between *sets* language and *number* language. For use with the number language, the symbols $>$ for *is greater than* and $<$ for *is less than* are now introduced. Informal logic is brought in through use of *if-then* relationships.

• **Problem-Solving.** Pupils write *n*-sentences to show how to work the problem. Other problem-solving techniques include the use of data from a table, the writing of addition and subtraction problems, providing questions for problems with emphasis on understanding where addition is used or where subtraction is used.

• **Enrichment.** As usual, the number of enrichment activities is great in the lesson plans for this chapter and in the suggestions at the end of the text, but in the text pages there is discovery of generalizations about even and odd numbers when these are added and when these are subtracted and there are more difficult situations involved in finding the number for *n* that involve parentheses.

• **Testing.** The chapter is closed with the four basic chapter tests that provide you with a very good picture of each pupil's readiness for moving ahead in grade-3 work. It is suggested that occasionally the sets of exercises within the chapter be handled as testing material.

Teaching Page 52

Pupil's Objective

To use knowledge and understanding of the basic A. facts to acquire skill in adding tens shown by 2-place numerals.

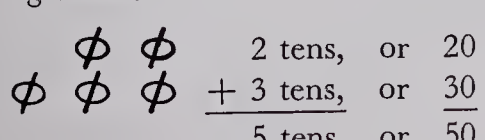
New Words: *camp*, *sunny*

Background

This page is important because the pupil now begins to generalize and use the addition facts which were learned earlier. After generalizing for adding tens, he will later generalize for adding hundreds, thousands, fourths, tenths, and so on.

Pre-Book Lesson

• Have pupils make sets of ten, using bundles of objects (tongue depressors, dowels, strips of cardboard or paper, and so on). The adding and recording should be done as follows:



2 tens,	or	20
+ 3 tens,	or	30
5 tens,	or	50

As the demonstration continues and the model is written on the chalkboard, children may write on their own papers. Make it clear to the class that, although the 10-bundle shows 10 separate sticks, they may use the symbol ⌀ to represent a bundle of 10.

• Guide the pupils to discover and state in their own words the fundamental idea that tens are added like ones.

Using the Text Page

• Have pupils make sets of ten with objects to show the answers for Ex. 1–6.

• Use the Modern Number Line* or draw a picture of a number line on the board showing and labeling points for tens only. Use it in connection with Ex. 7–9.

• Illustrate other examples of adding tens using the number line.

* See 6, page xix.

Pupil's Objective

To begin to use knowledge and skills acquired in higher-decade addition in adding now with two 2-place numerals (no renaming).

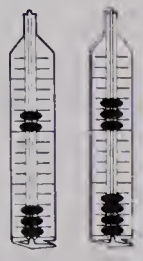
Background

This page is a logical follow-up of the previous lesson. In chapter 1, pupils learned to add in higher-decade addition where only one of the numbers was shown by a 2-place numeral. On page 52, the children were exposed to adding tens only, with two 2-place numerals having 0 in one's place. This new lesson combines the two previously learned skills, adding now with two 2-place numerals showing both ones and tens to be added.

Children are encouraged to add the ones first. In examples of the type presented in this lesson (no renaming), many children will observe that the numbers either for tens or for ones may be added first. In fact, these same youngsters will not be fully convinced of the necessity for adding ones first until they are confronted with an example which involves renaming and carrying to ten's place.

Pre-Book Lesson

• Use a place-value teaching device to show the following example: $34 + 23 = n$. On two Arithme-Sticks,* for instance, show buttons to represent 4 ones and 3 ones on the right-hand stick and 3 tens and 2 tens on the left-hand stick.



• Encourage the children to join the sets of buttons representing ones (to suggest adding the ones first) and then join the sets of buttons representing tens (to suggest adding the tens next).

• Visualize the adding for the same example with each of the following:

3 tens and 4 ones	34
+ 2 tens and 3 ones	+ 23
5 tens and 7 ones	57

• Have the *slower learners* build an understanding of this operation by demonstrating with bundles of objects, while the *more capable children* demonstrate on the board with ⌀'s and /'s.

Using the Text Page

• Have some pupils explain Ex. A–D in the boxes.

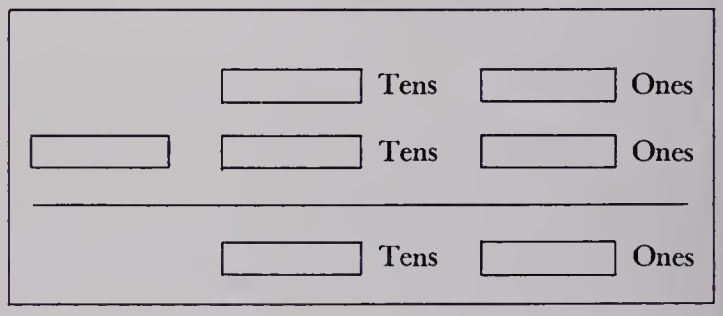
• While other pupils are working Ex. 2–6 on the board have the remainder of the class work on paper at their seats. As you circulate among the class, you can determine which pupils need further help.

• Ex. 7–11 call for oral discussion and will help to reveal lack of understanding where there is any.

Individualizing Instruction

• Use *Show the Answer* practice with *all pupils*. The children can use the set of numeral cards for 0 through 9 that they have been using in this type of practice (see Teaching Page 1). The child can pick up one card that tells how many ones in the example and another card that tells how many tens. Be sure that you do not give an example that will result in the same numeral for both tens and ones in the sum, or an example that requires renaming and carrying.

• Perhaps you will want to make a pocket chart like the following to use for individual or small-group work with all types of addition and subtraction examples.



Each pocket can be stapled in place. The pocket for the operation sign makes it possible to use the chart for both addition and subtraction. Pupils may use their numeral cards in the pockets.

• *All pupils* may do addition with 2-place numerals as follows:

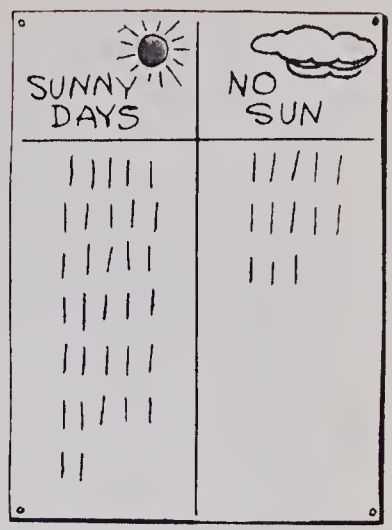
$$\begin{array}{r} 34 = 30 + 4 \\ +\ 23 = 20 + 3 \\ \hline 50 + 7 = 57 \end{array}$$

* See 3, page xix.

* Remind pupils that we add numbers. The numerals, as well as the bundles and the number words, are only means by which we find the unique third number, the sum.

* Adding with 2-Place Numerals

[o]



1. While Ruth was in camp, there were 32 sunny days and 13 days with no sun. How many days was Ruth in camp? $32 + 13 = n$

** First add ones and then add tens as you explain how to find the sum by using

a. Ex. A. **b.** Ex. B.

c. For Ex. C, explain the addition.

A	B	C	D
⏀ ⏀ ⏀|| ⏀|||	3 tens and 2 ones +1 ten and 3 ones _?_ tens and _?_ ones, or 45	Tens | Ones 3 | 2 +1 | 3 4 | 5	32 13 45

d. Ex. D shows the short way to add 32 and 13. First add ones: $2 + 3 = 5$. Write "5" in one's place. Then add tens: $3 + 1 = 4$. Write "4" in ten's place.

Work on the board. For Ex. 2–4 use ⏀'s and |'s.

2.	3.	4.	5.	6.
24 +32 56	10 +33 43	65 +30 95	32 +26 58	53 +44 97

For Ex. 5–6, use tens and ones as in box B.

Now work Ex. 2–6 on the board. Add as in box D.

Add the short way in Ex. 7–11 to find which sums are not correct.

7.	8.	9.	10.	11.
24 21 225 45	50 23 73	22 57 69 79	54 14 68	43 35 87 78

** Be certain pupils recognize that when they are adding 3 and 1, they are 3 tens and 1 ten

Try These

Adding with 2-place numerals [W]

Add the short way. Show the sums on folded paper.

	a	b	c	d	e	f	g	h	i
1.	35	73	34	60	21	64	79	32¢	28¢
	32	26	41	32	48	13	10	52¢	31¢
	67	99	75	92	69	77	89	84¢	59¢
2.	15	61	41	10	74	53	20	36¢	70¢
	74	25	43	60	12	31	58	20¢	6¢
	89	86	84	70	86	84	78	56¢	76¢

Copy in columns. Add the short way.

3. 32 + 25 57
4. 20 + 6 26
5. 7 + 50 57
6. 36 + 2 38
7. 86 + 10 96
8. 74 + 25 99
9. 51 + 27 78
10. 25 + 71 96
11. 41 + 40 81
12. 6¢ + 43¢ 49¢
13. 30¢ + 44¢ 74¢
14. 6¢ + 81¢ 87¢

◆ **Extra Examples.** Work Sets 29 and 30.

Checking Your Sums

Use of Commutative Property [O]

A	B
2 +8 n	42 +15 57

1. Ex. A. Add downward: $2 + 8 = n$. 10

a. Check by adding upward. $8 + 2 = n$. 10

b. Are the sums the same both ways? Yes

c. Did changing the order of adding the two numbers change the sum? No

2. Ex. B. Adding downward, the sum is 57.

a. Now add upward $5 + 2$ and $1 + 4$.

b. Are the sums the same both ways? Yes Why? $2+5=5+2$, $4+1=1+4$

Changing the order of adding does not change the sum. After you add downward, you can check by adding upward.

Teaching Pages 54 and 55

Pupil's Objectives

(a) To continue the use of higher-decade addition skills in adding with two 2-place numerals; (b) to gain additional understanding of the Commutative and Associative Properties of Addition in connection with checking the results of adding two or more numbers; and (c) to learn to write number sentences for A. and S. problems.

New Words

(page 54) *check*, *upward*

Background

The children were first introduced to the use of the Commutative Property of Addition while reviewing addition facts on page 8. At the bottom of page 54 the use of this property is extended to the checking of addition with 2-place numerals.

An extension of the use of the Associative Property of Addition, first developed in connection with the work on pages 40–41, appears with the work at the top of page 55. The property is now used for the purpose of checking the result of adding three and four addends. Of course, with four addends some use of the Commutative Property of Addition is made also (see Teaching Pages 40 and 41).

With more than 2 addends we still add only two numbers at a time because A. is a binary operation.

In previous lessons, where *n*-examples (number sentences) were considered in connection with problem-solving pages, pupils were given three *n*-examples from which they were to choose the correct one for the problem. Now the writing of *n*-examples is an extension of the work and provides for the pupil's converting information presented in a problem situation into a mathematical model. At this time, place the emphasis on the writing of appropriate *n*-examples for the problems and not upon solving the problems. When children write *n*-examples involving cent signs, they should use the ¢ sign with *n*, as *n*¢.

Using the Text Page (page 54, top)

• Provide paper and review the folding procedure for adding in rows 1–2.

• Encourage pupils to be neat in copying examples 3–14 in the vertical form.

Pre-Book Lesson (pages 54–55)

• Use objects in the room to illustrate the adding of 4 + 5. Guide the discussion to emphasize

a. the sum of 4 and 5.

b. the effect of changing the order in adding the two addends, 4 and 5. 4 + 5 = 9 and 5 + 4 = 9

c. ways in which we can check our work in adding 4 and 5. In using materials you can anticipate the statement that counting is the surest means of confirming the sum. However, changing the order of the addends and then adding is a more mature way of checking a sum.

• Raise the question with the class as to why it is necessary to check our work. Have them describe situations in which accuracy is essential. Help them experience the satisfaction that comes from checking and finding that the sum is the same, and so probably correct.

Using the Text Pages (page 54–55)

• Parentheses are used in number sentences to show just part of the sentence. The use of ( ) will be developed more fully in later lessons. However, in Ex. 3 it would be well to tell the pupils that the parentheses are used to show the part of the sentence that is 2 + 3. In fact, the pupils will soon learn that the use of ( ) around the 2 + 3 is a way of renaming 5. The example then becomes 5 + 4 = 9.

• As the children are doing the written work, you might find it valuable to circulate among them and ask individuals to tell you what they think as they work a given example. In this way, as one means of measuring understanding, you can diagnose the thought processes pupils are using.

Pre-Book Lesson (page 55, bottom)

• Use children, books, blocks, and so on to dramatize the putting together of three sets of objects. The number for the total of the three sets should be less than 10. Ask a child to write an *n*-example on the board that goes with the addition

situation dramatized. Have him use n to stand for the unknown sum.

• Use objects and materials in the classroom to illustrate and review the different subtraction situations that have been taught. Then have children write n-examples for the subtraction problems.

For both A. and S. problems, guide pupils to use at this time only the direct form for the n-examples; that is, n appears always in place of the answer (see Teaching Page 22).

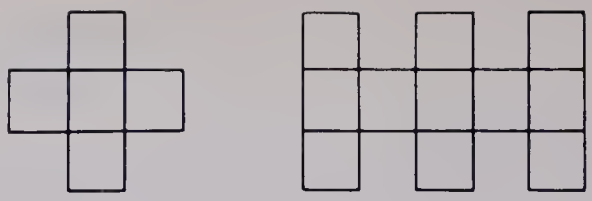
Using the Text Page (page 55, bottom)

• In reading and discussing each of Ex. 1–4, place the emphasis on the making or choosing of appropriate n-examples for the problems rather than on the solving of the problems.

• Some pupils may need to use concrete objects in order to dramatize the problems. For this purpose use the problems on page 49.

Individualizing Instruction

• On the chalkboard draw some diagrams similar to those at the top of the next column.



Have *all pupils* select any diagram they wish and write a 1-place numeral in each small square so that the sum of any three numbers shown, taken horizontally or vertically or diagonally, will be 9. The activity can be modified so that the sum will be 10 or less.

Encourage the children to check each sum of three numbers by adding in one direction and then in the other.

• Sometimes, instead of having children check addition by adding upward, ask them to provide evidence that their answer is correct. That is, after adding the short way, have the pupil provide concrete evidence of the correctness of his answer by using bundles of sticks, cardboard strips, or other such materials.

• Assign Extra Examples Sets 29 and 30 as needed.

NOTES

* Use of the commutative and associative properties permits reversing the addition.
2+3+4=3+2+4=3+4+2=(4+3)+2

3. Ex. C. Add downward: $2 + 3 = 5$, $5 + 4 = 9$. We can show what we do this way: $(2 + 3) + 4 = 9$.

* Add upward: $(4 + 3) + 2$. Are both sums the same? Yes

C	D
2	4
3	1
4	2
9	3
	10

4. You add 4 numbers just as you add 3 numbers. Explain the work for Ex. D. Add downward. 4+1=5 5+2=7 7+3=10

Add upward. Is the sum the same both ways? Yes ^Why? v Changing the order of addends does not change the sum.

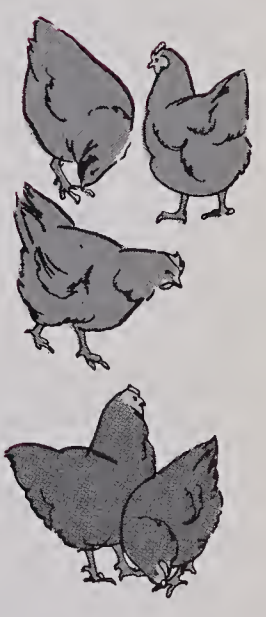
[W]

Copy in columns. Add. Check your work.

5. 63 + 21 84
6. 7 + 80 87
7. 42 + 57 99
8. 40¢ + 30¢ 70¢
9. 80¢ + 9¢ 89¢
10. 9¢ + 60¢ 69¢
11. 50¢ + 3¢ + 3¢ + 3¢ 59¢
12. 2¢ + 2¢ + 0 + 1¢ 5¢
13. 13¢ + 2¢ + 0 + 4¢ 19¢

Making *n*-Examples for Problems

A. and S.; addends-sum relationship [O]



1. Three hens, 2 hens, and 4 hens are how many hens? 9 hens
Why is the *n*-example $3 + 2 + 4 = n$? Add the numbers for the parts to find the number for the total.

2. You have 3¢. You earn 5¢. How much have you? 8¢

You know the numbers for the parts, so how do you find the number for the total? Add ^Say the *n*-example, or number sentence, for this problem. 3¢ + 5¢ = n¢

3. Tom lost 2 of his 8 pencils. How many were left? 6 pencils

Do you know the numbers for two parts or for the total and one of its parts? the total and one of its parts ^Is a or b or c the *n*-example? b

a. $2 + 8 = n$ b. $8 - 2 = n$ c. $8 + 2 = n$

4. I want 9¢. I have 5¢. How much more do I need? 4¢
Say the *n*-example. 9¢ − 5¢ = n¢

[W]

Now write the *n*-example for each problem on page 49.

1. 9−7=n
2. 3+2+4=n
3. 9−3=n
4. 10−3=n
5. 10−6=n
6. 42+6=n
7. 9−5=n
8. 14¢+3¢+2¢=n¢
9. 10¢−7¢=n¢
10. 5+2+3=n

More *n*-Examples

A. and S. problems; making n-examples [O]

Sometimes a problem is given with the question first. For this kind of problem, you may like to write the *n*-example, or number sentence, with the *n* first, as $n = 2 + 3$.

1. How many cents has Ann in her bank? 10¢
She had 8 cents in her bank. She put in 2 more cents. The numbers for two parts are given, so add to find the number for the total.
Is a or b or c the *n*-example for problem 1? b

a. $n¢ = 8¢ - 2¢$ **b.** $n¢ = 8¢ + 2¢$ **c.** $n¢ = 10¢ - 2¢$

2. How much money does Tom have left in his bank? 3¢
He had 10¢. He took out 7¢ for the Red Cross. The numbers for one part and the total are given, so subtract to find the number for the other part.
Which is a way to write the *n*-example, a or b or c? c

a. $n¢ = 10¢ + 7¢$ **b.** $n¢ = 7¢ + 10¢$ **c.** $n¢ = 10¢ - 7¢$

Now write on the board the *n*-example for each of problems 3–6. Put the *n* first in each *n*-example.

3. How much money did Jean have at first? 9¢ ∧She paid 4¢ for candy and had 5¢ left for her bank. $n¢ = 4¢ + 5¢$
The numbers for two parts are given, so add to find the number for the total.

4. How much more money does Jane need? ∨She wants to buy a 10¢ ruler. She has 6¢. 4¢ $n¢ = 10¢ - 6¢$
The numbers for one part and the total are given, so subtract to find the number for the other part.

5. How much money does Joe have left? ∨He had 9¢. He paid 5¢ for a pencil. 4¢ $n¢ = 9¢ - 5¢$
The numbers for one part and the total are given, so subtract to find the number for the other part.

6. How much money does Roy have? ∨He has a nickel and 3 cents. 8¢ $n¢ = 5¢ + 3¢$
The numbers for two parts are given, so add to find the number for the total.

Now say the answer for each question in problems 1–6. Tell why you added or subtracted for each problem.

Teaching Page 56

Pupil's Objectives

(a) To show an understanding of the meanings of addition and subtraction in connection with problem-solving; and (b) to say and write *n*-examples for addition and subtraction problems.

New Words

bank, question

Background

Fostering a feeling for mathematics as a language, a means of communication, is continued in this lesson. The act of translating from the written and spoken problem language situation to the number sentence is a valuable tool in solving mathematical problems. The emphasis in these lessons must be upon the translation and not upon the solving of the problem.

Pre-Book Lesson

• Write on the board the number sentence, $4 + 5 = n$. Ask the children to make up a problem to go with this number sentence. After a short time, ask different children to state their problems. Ask each child to explain why his problem is correct for this sentence. Some of the problems stated by the children may be dramatized to bring out the relationship between the problem and the number sentence. Then reverse the procedure, having pupils make an *n*-example for a given problem.

• Review the pairs of parts of a set of 8 and the addition facts with 8 as a sum that can be made. Show the addition facts on the chalkboard, beginning each time with the sum.

Ask a child to tell about a set of 8 and two of its parts and the two A. facts he can say for those parts. Show the facts on the board, showing the sum first each time, as at the right.

$8 = 1 + 7$
$8 = 7 + 1$
$8 = 2 + 6$
$8 = 6 + 2$
$8 = 3 + 5$
$8 = 5 + 3$
$8 = 4 + 4$

Ask other children to tell about different pairs of parts.

The purpose of this activity is to familiarize the pupils with writing number sentences having the sum shown at the beginning instead of at the end of the sentence. Sentences of this type will be used in this lesson.

Using the Text Page

• Use real or toy coins to dramatize Ex. 1 and 2 before having pupils choose the correct *n*-example.

• Perhaps you will want to ask the *more capable children* to write the *n*-examples for problems 3–6 on paper while the remainder of the class works with you at the chalkboard. Point out that *n* to stand for the sum may be written first just as well as last in an example.

• For the work at the bottom of the page have pupils say the answer for each problem. Each time have them tell why they added or subtracted.

Individualizing Instruction

• *Slower learners* may now write on paper the *n*-examples for problems 3–6 which were developed as board work.

• Provide duplicated pictures of number lines for the *more capable children* to use. Have these children draw and label arrows to illustrate the adding and subtracting for each problem on page 56.

Pupil's Objectives

(a) To recognize the order of inequality between two numbers; (b) to begin to use "is greater than" and "is less than" when reading number sentences; (c) to learn that numbers shown to the right on a number line are always greater than numbers shown to the left, and vice versa; and (d) to extend the use of parentheses in number sentences to indicate a name for a number.

Background

Most of the child's mathematical learnings have emphasized *equality* in number sentences. Often, the relationship between two numbers is one of *inequality* and is expressed in terms of "is greater than" or "is less than" instead of merely "is not equal to." Thus, order also becomes important.

Children should begin now to learn the relationship of points in a number line for whole numbers, in terms of *greater than* and *less than*. Unless otherwise indicated, we assume that the number line is thought of as being horizontal. The number for any point A is greater than the number for any point to the left of A, and less than the number for any point to the right of A. This becomes especially significant to the child later when he discovers that 0 is greater than $^{-}12$. However, in Grade 3, the applications of the number-line idea are limited to the set of whole numbers.

The number sentences on this page contain _ ? _ to indicate that something is missing. In these sentences the missing symbol is one to denote relationship. Pupils already know the symbol of equality (=). On this page the pupils will be introduced to terms of inequality but not the symbols. While many children can learn that $<$ means *is less than* and $>$ means *is greater than*, others confuse these. Consequently, it is desirable to work on the basic idea for a considerable time before introducing the symbols. If you find that your pupils are ready for these symbols before they are introduced on page 90, you should feel free to have them use the symbols. The parentheses are also used to denote or identify a particular part of the sentence. Teach the youngsters to complete the work inside the ( ) before doing anything else called for in the sentence.

To review the use and meaning of the symbol $\neq$, which was introduced on page 27, Ex. 22 asks the general question about substituting for the words *is greater than* and *is less than*.

Teacher's Preparation

Mount your permanent number-line model on the chalkboard or draw a number-line picture.

Pre-Book Lesson

Draw on the chalkboard pictures of 5 circles and beside it pictures of 2 circles. Identify each set with a numeral. Make certain that the pupils are thinking about the number ideas of fiveness and twoness and not the physical sizes of the numerals 5 and 2. Then ask them to explain why the number for a set of 5 is greater than the number for a set of 2. See if anyone actually matches 2 items from the set of 5 with 2 items in the set of 2, to compare the sets involved.

Refer to the number-line model to determine where points for 2 and 5 are located. Discuss the ideas of rightness and leftness on a number line. Then, to illustrate the idea that 5 is greater than 2, elicit from the class that the point for 5 is to the right of the point for 2.

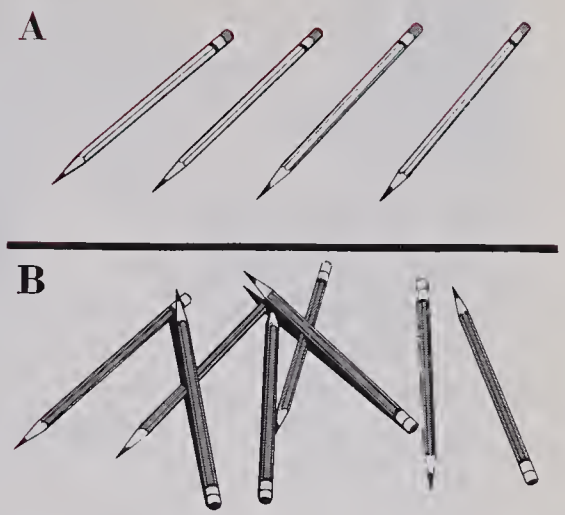
Using the Text Page

- For Ex. 5–10, refer to the number-line picture and reinforce the idea of numbers shown to the right being greater.
- Teach the use of the ( ) bringing out that the answer for the operation shown within the ( ) is a name for a number. Consequently, the operation inside the ( ) should be performed first. In these sentences we then compare the two numbers shown and read the sentence using the words "is greater than" or "is less than" to make the sentence true.
- Write Ex. 13 on the chalkboard if necessary to show how to write Ex. 14–21. Circulate among the pupils, giving help where it is needed. Now and then ask a child to tell you how he thinks the work.

* Now a refinement of inequality is suggested and pupils, knowing that two numbers are unequal, tell and record whether the first is greater than or less than the second.

* Greater Than or Less Than?

Comparisons; inequalities [O]



1. Is set A larger than or smaller than set B? smaller

2. Is the number for set A greater than or less than the number for set B? less

3. On a number-line picture, are the numbers shown to the right of 4 greater than or less than 4? greater than

4. Are the numbers shown on a number-line picture to the left of 15 greater than or less than 15? less than

** In each of Ex. 5–12, use the words is greater than or is less than. If you need help, use a number-line picture.

5. 6 _?_ 8 — is less than
6. 23 _?_ 19 — is greater than
7. 4 _?_ 10 — is less than
8. 12 _?_ 10 — is greater than
9. 16 _?_ 13 — is greater than
10. 24 _?_ 20 — is greater than
11. $(4 + 2)$ _?_ 8. $4 + 2 = 6$; so $(4 + 2)$ is less than 8.
12. $(8 - 2)$ _?_ 11. $8 - 2 = 6$; so $(8 - 2)$ is less than 11.

[W]

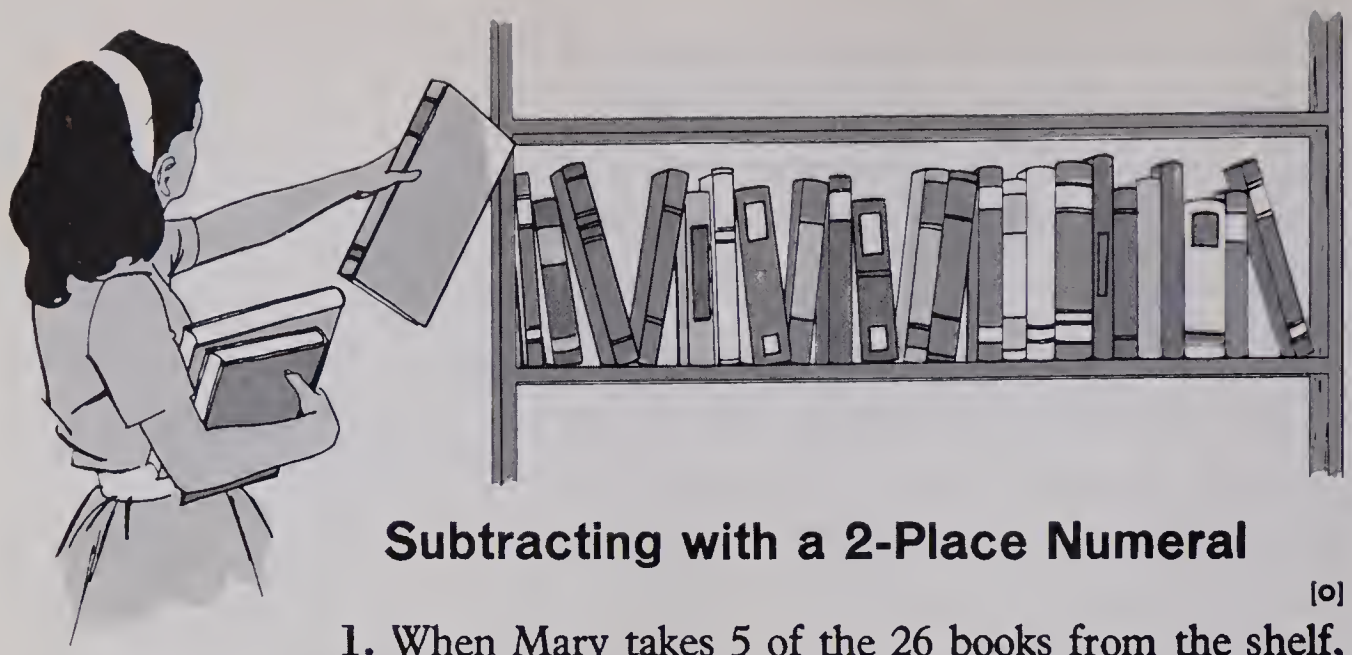
As for Ex. 11 and 12, first do the work in the ( )'s. Then copy the example writing "is greater than" or "is less than" where you see _?_.

13. $(10 + 8)$ _?_ 20 — is less than
14. $(4 + 6)$ _?_ 15 — is less than
15. 11 _?_ $(5 + 4)$ — is greater than
16. $(40 + 40)$ _?_ 60 — is greater than
17. 56 _?_ $(4 + 55)$ — is less than
18. $(43 + 30)$ _?_ 70 — is greater than
19. 40 _?_ $(6 + 52)$ — is less than
20. $(22 + 25)$ _?_ 46 — is greater than
21. $(8 + 30)$ _?_ 50 — is less than

22. For each _?_ in Ex. 13–21, could you write $\neq$? Yes

● **Extra Activity.** Work Set 86.

** Instead of using shorthand symbols at this point, basic understanding is fostered by having pupils actually say and write the complete expressions.



Subtracting with a 2-Place Numeral

[o]

1. When Mary takes 5 of the 26 books from the shelf, how many will be left? $26 - 5 = n$

Explain how to find the answer by using

a. Ex. A. **b.** Ex. B. **c.** Ex. C.

A	B	C	D
⊕ ⊕ |||||	2 tens and 6 ones − 5 ones _?_ tens and _?_ one, or 21	Tens | Ones 2 | 6 − | 5 ? | ?	26 5 21

d. Ex. D shows the short way to subtract. First subtract ones: $6 - 5 = 1$. Explain why the answer shows 1 one; why it shows 2 tens. There are 5 ones to subtract from the 6 ones. There are no tens to subtract.

Copy Ex. 2–7 on the board, and subtract. For Ex. 2 and 3 use ⊕'s and |'s as in box A.

2.	3.	4.	5.	6.	7.
28	34	99	47	84	56
7	3	3	4	2	5
21	31	96	43	82	51

For Ex. 4–7 use tens and ones as in box B.

Now work Ex. 2–7 on the board. Subtract as in box D.

Pupil's Objective

To build an understanding of higher-decade subtraction (no renaming).

Background

Pages 58–62, dealing with higher-decade subtraction, parallel almost exactly the corresponding material for higher-decade addition recently taught. Pages 58 and 59 are designed to teach the rationale of the process without encouraging children to formulate and practice a thought process for obtaining a result immediately. The subsequent pages develop the latter skill and afford adequate practice.

Teacher's Preparation

A place-value teaching aid* should be available for the Pre-Book Lesson.

Pre-Book Lesson

• Use a social problem situation similar to the following:

Twenty-eight children in a class were cutting paper. Three children have not finished cutting their paper. How many children have finished?

• Guide children to discover how to subtract 3 from 28 using a place-value teaching aid. (Caution: Do not try to show both the total and the part known on the place-value aid. Show only the total. When the known part is removed, the remaining part on the place-value aid is the unknown part that represents what is to be found for the subtraction problem.)

• Keep written records on the chalkboard while each child makes a written record on paper at his desk as follows:

a. Use ϕ's and /'s and cross off three ones.

ϕ ϕ // // /+++

b. Use words tens and ones as below.

2 tens and 8 ones
− 3 ones
2 tens and 5 ones, or 25

c. Use numerals as below.

Tens	Ones	
2	8	28
−	3	− 3
2	5	25

Emphasize the fact that ones should be subtracted first. Reiterate it at appropriate intervals.

• Suggest other situations, such as the following, and work as in a, b, and c above. Be sure no renaming is required in the examples presented at this time.

a. 36 scissors, 5 in the box. Find the number gone from the box.

b. 24 crayons, 3 crayons in use. Find the number left in the drawer.

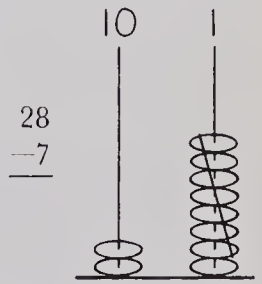
c. 48 pencils, 4 in a box. Find the number gone.

Using the Text Page

• If children have difficulty explaining Ex. A, Ex. B, or Ex. C, do more work as in the Pre-Book Lesson.

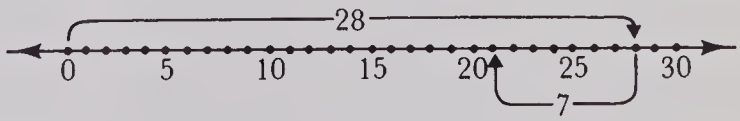
• As each of Ex. 2–7 is written on the chalkboard, be sure to ask children to explain their work.

Individualizing Instruction



• *Slower learners* may draw on their papers abacus pictures for Ex. 2–7. Possibly have one of the *more capable children* demonstrate at the board how to do this. Later, allow these *slower learners* to demonstrate to the class some examples on the Arithme-Sticks or draw some line-abacus pictures on the chalkboard.

• Ask *more capable children* to illustrate the subtracting for Ex. 2–7, using a number line.



28 − 7 = 21

• *All pupils* may work Ex. 2–7 as follows:

28 = 20 + 8
− 7 = 7
20 + 1 = 21

* See 21, page xix.

Pupil's Objective

To learn to organize higher-decade subtraction examples into sets that are based on the simple S. facts.

Background

In this lesson, as in the parallel addition lesson, the children will organize learning around relationships. Answers for many examples, such as 27 − 4, 47 − 4, 57 − 4, are to be found, for a time, by first identifying the subtraction set using the fact 7 − 4 = 3. Eventually, the children will develop the ability to think the answers, and only the answers, in higher-decade subtraction.

Pre-Book Lesson

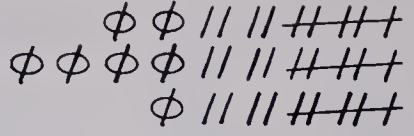
• Use a pocket chart (see Teaching Page 53), bundles of sticks, a flannel-board abacus picture, or the Ginn Arithme-Sticks to demonstrate finding answers for the following:

7	17	47	57
− 4	− 4	− 4	− 4

Ask, "How are all of these examples alike?" Though several answers are possible, make certain that all children see that the basic S. fact 7 − 4 = 3 is involved in the completion of each example. Illustrate for other sets using a subtraction fact.

• Ask each child to make a written record in several ways of solving one set using a subtraction fact (such as 29 − 5, 49 − 5, 19 − 5), first by using ⌀'s and /'s, then with tens and ones, and then using numerals.

a. Using ⌀'s and /'s:



b. Using tens and ones:

2 tens and 9 ones
− 5 ones
2 tens and 4 ones, or 24

c. Using numerals:

Tens	Ones	
2	9	29
−	5	− 5
2	4	24

Using the Text Page

• Review the folded-paper technique for writing only the answers for rows 16–18 (see Teaching Page 10).

• *Slower learners* may need more help with the sets using S. facts before proceeding with the written work. You may want to do more with Ex. 9–15 with the *slower learners* at this time.

Individualizing Instruction

• Write on the board some number sentences of the type 27 − 6 = *n*. Have *all pupils* tell or write a problem for each sentence.

• *More capable children* may draw a line-abacus picture or demonstrate on an abacus or the Ginn Arithme-Sticks some sets using S. facts.

• *Slower learners* should demonstrate the same sets using S. facts with more concrete visual aids.

Sets Using Subtraction Facts

[o]

A

a. 17 − 4
b. 27 − 4
c. 37 − 4
d. 47 − 4

B

e. 19 − 3
f. 29 − 3
g. 59 − 3
h. 89 − 3

1. All the examples in box A belong to the S. set using 7 − 4 = 3. Explain why we can say this. Each example uses the S. fact 7−4=3.

* **2.** Ex. a. To subtract 4 from 17 *think*, "7 − 4 = 3, so 17 − 4 = 13." Why is this so? There is also 1 ten.

3. For Ex. b, *think*, "7 − 4 = 3, so 27 − 4 = 23?" 7−4=3, so 37−4=33; 7−4=3, so 47−4=43

4. Tell what to think for Ex. c and Ex. d. ∧

5. The examples in box B belong to what S. set? 9−3=6 ∧

6. Tell other examples in the S. set using 9 − 3 = 6. Answers will vary. 39−3, 49−3, 69−3 ∧

7. For Ex. e, finish: 9 − 3 = 6, so 19 − 3 = 16? 9−3=6, so 29−3=26; 9−3=6, so 59−3=56; 9−3=6, so 89−3=86

8. Tell what to think for each of Ex. f-h. ∧

Each of these belongs to what S. set?

9. 65 − 2 (5−2=3) **10.** 19 − 4 (9−4=5) **11.** 37 − 5 (7−5=2) **12.** 48 − 6 (8−6=2)

Write on the board three examples in the S. set using

13. 8 − 3 = 5. (18−3, 28−3, 38−3) **14.** 9 − 8 = 1. (19−8, 29−8, 49−8) **15.** 7 − 6 = 1. (17−6, 37−6, 57−6)

∨ Answers will vary.

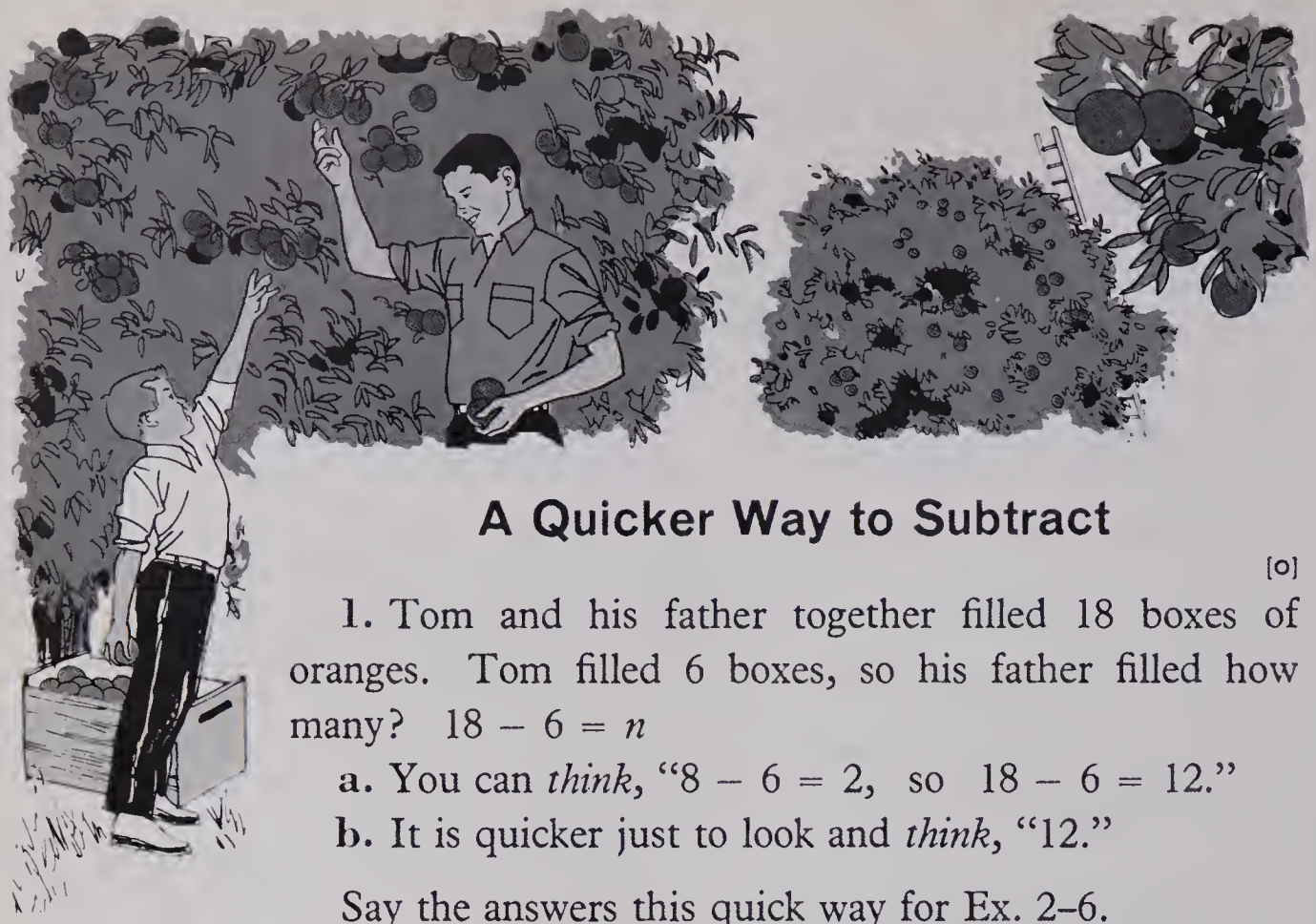
[w]

For rows 16–18, write answers on folded paper. For Ex. 16a *think*, "7 − 3 = 4, so 37 − 3 = 34." Write "34."

	a	b	c	d	e	f
16.	37 − 3 (34)	58 − 4 (54)	66 − 4 (62)	87 − 2 (85)	19 − 2 (17)	24¢ − 1¢ (23¢)
17.	28 − 1 (27)	85 − 2 (83)	57 − 6 (51)	38 − 2 (36)	79 − 6 (73)	87¢ − 4¢ (83¢)
18.	49 − 7 (42)	42 − 1 (41)	84 − 2 (82)	97 − 5 (92)	69 − 8 (61)	38¢ − 6¢ (32¢)

19. Tom spent 3¢. Find how much is left if at first he had **a.** 74¢. 71¢ **b.** 66¢. 63¢ **c.** 35¢. 32¢ **d.** 49¢. 46¢ **e.** 58¢. 55¢

* Pupils find work in higher-decade subtraction much easier when they realize they may use S. facts they already know.



A Quicker Way to Subtract

[O]

1. Tom and his father together filled 18 boxes of oranges. Tom filled 6 boxes, so his father filled how many? $18 - 6 = n$

a. You can *think*, "$8 - 6 = 2$, so $18 - 6 = 12$."

b. It is quicker just to look and *think*, "12."

Say the answers this quick way for Ex. 2–6.

2.	3.	4.	5.	6.
29	46	74	85¢	37¢
-2	-3	-2	$-4¢$	$-5¢$
27	43	72	81¢	32¢

[W]

Write the answers on folded paper. As you work, subtract the quick way.

	a	b	c	d	e	f	g	h	i
7.	98	69	75	56	37	46	17	19¢	26¢
	3	6	2	4	3	5	2	5¢	3¢
	95	63	73	52	34	41	15	14¢	23¢
8.	45	76	24	57	63	39	58	39¢	47¢
	3	2	3	6	2	4	4	3¢	5¢
	42	74	21	51	61	35	54	36¢	42¢
9.	29	68	49	18	35	99	64	58¢	78¢
	7	5	8	7	1	2	1	2¢	6¢
	22	63	41	11	34	97	63	56¢	72¢

Extra Examples. Work Sets 31 and 32.

Teaching Page 60

Pupil's Objective

To learn to use a faster way of thinking higher-decade subtraction answers.

New Word

filled

Background

For higher-decade subtraction examples with no renaming, (e.g., 58 − 4), the skillful pupil will note immediately that the answer for the basic S. fact used, 8 − 4 = 4, does not involve renaming. Consequently, he then knows that the answer will be in the same decade as the sum and he will think the tens right away and then the sum for the ones. To be proficient in higher-decade subtraction, the pupils must know the basic subtraction facts which the higher-decade examples use.

Teacher's Preparation

Have paper ready for the folded-paper practice in the written part of the lesson.

Pre-Book Lesson

Consider a classroom situation similar to the following:

There are 34 children in our class. Three children are absent today. How many pupils are present?

Write the example on the board. Although the answer may be found by subtracting the ones, then thinking the tens, guide the pupils to understand that it is quicker just to consider the numbers and say the answer without first subtracting in each position.

Using the Text Page

- During oral discussion of Ex. 1–6, be sure the children understand why they can think the answers immediately.
- *More capable children* may proceed with written work independently.
- *Slower learners* may profit from some more oral work. If necessary, do rows 7 and 8 orally. Encourage each child to give the answers immediately, but allow him to refer to S. facts if he encounters difficulty.

Individualizing Instruction

- Once pupils have adopted the new direct thought pattern for thinking answers for higher-decade subtraction examples involving no renaming, they may use the skill in a more realistic way. For instance, you could inaugurate for *all pupils* a sale in a classroom store. The sale price could be established as 4¢ less than the regular price. If the teacher sees to it that the items on sale have a price such that the numeral will have a digit from 5 to 9 in one's place, several higher-decade subtraction examples become available. The items "on sale" and the established discount may be changed from time to time in order to provide as many examples as possible. This project can be an oral or a written activity at the teacher's discretion.
- Extra Examples Sets 31 and 32 may be assigned as necessary. Perhaps you will find it useful to impose a time limit on one or more of the sets to ascertain how many correct examples each child can work in a specified amount of time. The results may help to indicate which pupils are arriving (and which are not) at an immediate response. Allowance should be made for any factors that may affect the time needed by an individual to work the examples.

Pupil's Objectives

(a) To build an understanding of the principles for S. facts involving 0; (b) to deal with 0 in subtraction examples of the types that have been taught; and (c) to generalize about the use of 0 in the addends-sum relationship.

Background

The *identity element for addition* was discussed on Teaching Page 38. The *inverse* relationship between addition and subtraction was discussed on Teaching Page 15. The use of zero has been developed by principle thus far in addition. In this lesson the sum-addends relationship when 0 is an addend is generalized as follows:

a. When the sum is known, and the known addend is zero, the other addend is the same as the sum.

b. When the sum is known, and the known addend is the same as the sum, the other addend is zero.

Teacher's Preparation

Have a small box available and nine objects that will fit in it. Have paper ready for the folded-paper written practice.

Pre-Book Lesson

• Place 7 objects in the box. Be sure that all pupils know that there are 7 things in the box. Ask a child to take some of the objects out of the box. Have the child show the objects he removed from the box. Have the class determine how many things are still in the box as a result of seeing how many were removed. Ask them,

What was the total in the box at first?
What part of the total did Betty take out?
What part of the total was left in the box?

• Repeat this activity several times with different totals and parts.

• With 6 objects in the box, the teacher should pretend to remove some from the box. Open your hand so the class can see that no objects were removed. Repeat the questioning about the total, part removed, and part remaining.

• Repeat the activity above, removing no objects and then removing all of the objects. Write the corresponding number sentences on the chalkboard.

Using the Text Page

• Be sure to emphasize the addends-sum relationship as the various exercises are being considered. Have children state the generalizations in their own words.

• Have pupils use a number line to show the subtracting for Ex. 7–12.

• *Slower learners* may need additional oral work before proceeding with the written part of the lesson.

Individualizing Instruction

• Children are often motivated by participating in races. Examples of the types studied to this point may be written on the board in two or more sets, and races may be conducted between individuals or between teams. In team competition, record the number of errors made by each team. The team with the lowest score wins.

• *Slower learners* may demonstrate the two zero principles with the Arithme-Sticks, with $\not 0$'s and /'s, or with the pocket chart (see Teaching Page 53).

• Assign to *all pupils* Extra Examples Set 33, as deemed necessary.

Zero in Subtraction

[O]

1. Sam went into a store with 5¢. He spent nothing. How much money had he then? $5¢ - 0 =$ _?_¢ 5

2. Say the answers. **a.** $2 - 0$ 2 **b.** $6 - 0$ 6 **c.** $4 - 0$ 4

3. Does a number minus 0 equal that number? Yes

4. In another store Sam spent all of his 5¢. How much money did he have left? $5¢ - 5¢ = ?$ 0

5. Say the answers. **a.** $3 - 3$ 0 **b.** $6 - 6$ 0 **c.** $9 - 9$ 0

6. When a number is subtracted from the same number, what is the answer? 0

Subtract. Say the answers for Ex. 7–12.

7.	8.	9.	10.	11.	12.
1	7	8	2	0	8
0	7	0	2	0	8
1	0	8	0	0	0

* **Any number minus 0 equals that number.**

* **Any number minus the same number equals 0.**

[W]

Subtract. Write answers on folded paper. In examples like $29 - 0$, the answer equals the sum.

In examples like $44 - 4$, you subtract all the ones, so the answer must have 0 in one's place. $44 - 4 = 40$.

	a	b	c	d	e	f	g	h	i
13.	17	68	69	23	84	61	95	88¢	56¢
	0	8	9	0	4	0	5	0	3¢
	17	60	60	23	80	61	90	88¢	53¢
14.	29	47	16	89	52	37	73	56¢	34¢
	7	7	6	0	2	6	3	0	2¢
	22	40	10	89	50	31	70	56¢	32¢

◆ **Extra Examples.** Work Set 33.

* Pupils should realize that they do not have to memorize the S. facts having 0 as an addend if they understand the principles about subtracting zero from a number or subtracting an addend equal to the sum.

To Keep in Practice

A. and S. [W]

In rows 1–3 write answers on folded paper.

	a	b	c	d	e	f	g	h	i
1.	33	79	45	17	18	20	58¢	89¢	34¢
	+25	−6	+30	−3	−8	+30	−6¢	−0	+5¢
	58	73	75	14	10	50	52¢	89¢	39¢
2.	84	4	28	45	50	75	45¢	38¢	26¢
	−4	+34	−5	−0	+39	−3	+41¢	−3¢	−5¢
	80	38	23	45	89	72	86¢	35¢	21¢
3.	16	3	17	46	13	40	2¢	4¢	2¢
	−0	+26	−5	−6	0	4	2¢	11¢	10¢
	16	29	12	40	+4	+2	+6¢	+2¢	+6¢
					17	46	10¢	17¢	18¢

For Ex. 4–11 copy the numerals in columns and work.

4. 17 − 4 13
5. 32 + 37 69
6. 4 + 80 84
7. 10 + 72 82
8. 69¢ − 4¢ 65¢
9. 15¢ − 5¢ 10¢
10. 10¢, 1¢, 3¢, 5¢ 19¢
11. 1¢, 4¢, 2¢, 3¢ 10¢

◆ **Extra Examples.** Work Sets 34 and 35.

* No Pencils, Please!

n in any position in A. and S. [O]

In each example find the sum or addend that n stands for. Then read the example using the number for n. Work first by rows, then by columns.

	a	b	c
1.	9 n less 9 = 0	10 n − 8 = 2	2 plus 8 n = 10
2.	5 + 4 n = 9	10 − 4 = 6 n	9 = 7 + 2 n
3.	10 − 2 n = 8	0 n + 7 = 7	10 n less 2 = 8
4.	2 + 7 = 9 n	5 n = 5 − 0	8 n minus 8 = 0
5.	0 n plus 4 = 4	10 minus 4 n = 6	8 n less 2 = 6

* This is a good review of open A. and S. sentences containing n.

Teaching Page 62

Pupil's Objectives

(a) To write answers to addition and subtraction examples of the types studied to this point; and (b) to give orally the unknown number which n stands for in a number sentence.

Pre-Book Lesson

• Use objects in the room, children, or items on the flannel board* to illustrate and review sums through 10 and their addends. For each illustration, emphasize the addends-sum relationship: If two addends are known, then we find the sum by adding. If the sum and one of its addends are known, then we find the other addend by subtracting.

• Emphasize the joining and separating of sets of things and the associated adding or subtracting of numbers. Each time, ask pupils to explain what is known, what is not known, and how to find it.

Using the Text Page

• The written work at the top of the page reviews A. and S. examples when no renaming is necessary.

• Perhaps you will want pupils to do the oral work at the bottom of page 62 before assigning the written work at the top of the page.

Individualizing Instruction

• Assign Extra Examples Sets 34 and 35 as they are needed. Perhaps you will wish to impose a time limit for some children.

* See 2, page xix.

• Have the *more capable children* write sentences of *inequality* using *is greater than* and *is less than* (e.g., 24 is greater than 17).

• Have *slower learners* tell if n stands for a sum or an addend in the examples at the foot of page 62.

• Have *more capable children* rewrite each of the examples in rows 1–5 at the foot of page 62 to show n in three different positions. For Ex. 1a, they would write, "$n - 9 = 0, 9 - n = 0, 9 - 0 = n$."

Looking Ahead

Many times in their arithmetic work, pupils will need to talk about *renaming a number*. In the next chapter (page 101) a more formal development of the concept will be presented. However, the following are suggestions for laying the groundwork now.

• Have the *slower learners* write as many different names as they can think of for the number 7. Write on the board, "Other Names for 7." To get them started, explain that $3 + 4$ is another name for 7. Write "$3 + 4$" under the heading. Point out that $9 - 2$ is another name for 7. Write this under the heading. Have the children write other names for 7 that they know.

Perhaps you will want to do the same for other numbers.

• The writing of other names for a number is a good activity to stimulate creativity. Perhaps you will want to encourage the *more capable children* to try to write names they can think of that involve two 2-place numerals.

Pupil's Objective

To give orally and also to show the unknown sum or addend which n stands for in higher-decade addition and subtraction examples.

Background

Earlier, the pupils were introduced to odd and even numbers. An understanding of some of the relationships between odd and even numbers is useful. *The sum of two even numbers is an even number. The sum of two odd numbers is an even number. The sum of an odd and an even number is an odd number.* The same kinds of statements hold when the operation is subtraction.

In this enrichment section, the treatment is limited to the addition and subtraction of even numbers. In an earlier lesson for *all pupils*, pages 30 and 31, the result of adding 1 and of adding 2 to an even or to an odd number was explored.

Pre-Book Lesson

Review the thought processes involved in finding the answers in higher-decade addition and subtraction. Ask several children to tell how they think the answer.

Using the Text Page

- Ask pupils to tell whether n stands for an addend or a sum in each of the sentences in Ex. 1 and 2, and explain why.
- You may wish to have your class tell whether n is an addend or a sum in the number sentences in rows 3–5 before proceeding with the written work.

Individualizing Instruction

- Perhaps you will want to develop the enrichment section on page 63 orally with the *more capable children* only. Then, ask them to write the answers to the questions. Or, you may want to develop the material with *all pupils*, and then assign it as written work for the *more capable children* only.
- To emphasize further that often the elements of two sets are to be thought of in terms of a different label in order to be joined in a new set, give to *all pupils* exercises such as the following:

Set 1	*joined with*	*Set 2*	*would be*	*New Set*
2 carrots		1 onion		3 vegetables
3 tuna		2 trout		
4 collies		1 spaniel		
5 saucers		3 plates		
2 sparrows		6 robins		

- Have *more capable children* rewrite each of the examples in rows 3–5 so that it shows n in three different positions. For Ex. 3a, they would write $7 + n = 59$, $n + 52 = 59$, $7 + 52 = n$.

NOTES

More *n*-Examples

Finding the unknown addend or sum [O]

1. Find the number n stands for. Why must you add? n stands for an unknown sum.

a. $n = 60 + 9$ (69) **b.** $n - 6 = 32$ (38) **c.** $50 + 30 = n$ (80)

2. Find the number for n. Why must you subtract? n stands for an unknown addend.

a. $n + 4 = 68$ (64) **b.** $n = 75 - 3$ (72) **c.** $42 - n = 2$ (40)

[W]

Write the work to find the number for n.

	a	b	c	d
3.	$7 + n = 59$ (52)	$63 - n = 3$ (60)	$31 + 56 = n$ (87)	$6 + n = 76$ (70)
4.	$78 - n = 5$ (73)	$n + 0 = 33$ (33)	$48 - n = 3$ (45)	$n - 30 = 59$ (89)
5.	$n - 40 = 30$ (70)	$n = 84 - 4$ (80)	$n + 7 = 59$ (52)	$5 + n = 58$ (53)

Do You Like to Try New Things?

Enrichment [W]

1. In set A are the addends even or odd numbers? even

2. Are the sums for set A even or odd numbers? even

* 3. Copy and finish: When two even numbers are added, the sum is an _?_ (even) number.

4. Test your answer for Ex. 3. Make three examples like $44 + 2$ and three like $4 + 14$ and add.

Set A

2	6	8
+6	+4	+2
?	?	?

Set B

6	4	8
−2	−2	−6
?	?	?

5. In set B are even or odd numbers used? even

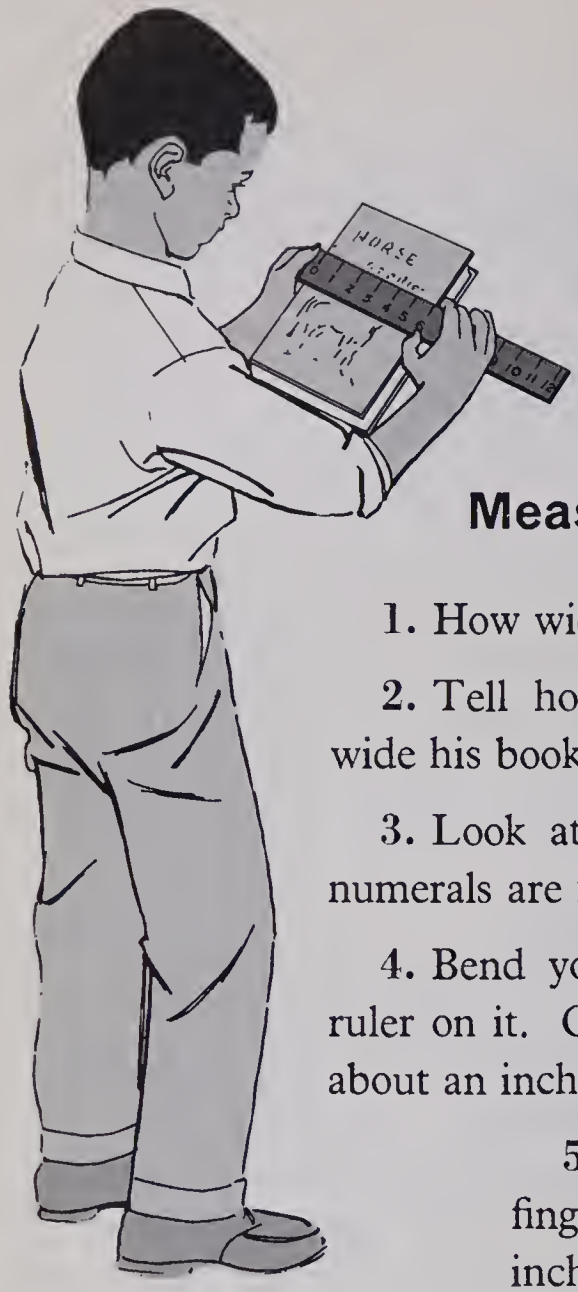
6. Are the answers for set B even or odd numbers? even

* 7. Copy and finish: When an even number is subtracted from an even number, the answer is an _?_ (even) number.

8. Test Ex. 7. Show three examples with answers.

Answers will vary.

* Nudge pupils to discover that since two even-number addends are associated with an even-number sum, it logically follows that an even-number sum and an even-number addend are associated with an even-number addend.



Measuring in Inches and in Feet

Discovering reference measures; mental procedures [O]

1. How wide is Jack's book? 6" ∧ How can you tell? The ruler shows 6. ∧

2. Tell how Jack placed his ruler to measure how wide his book is. With 0 at the left edge of the book

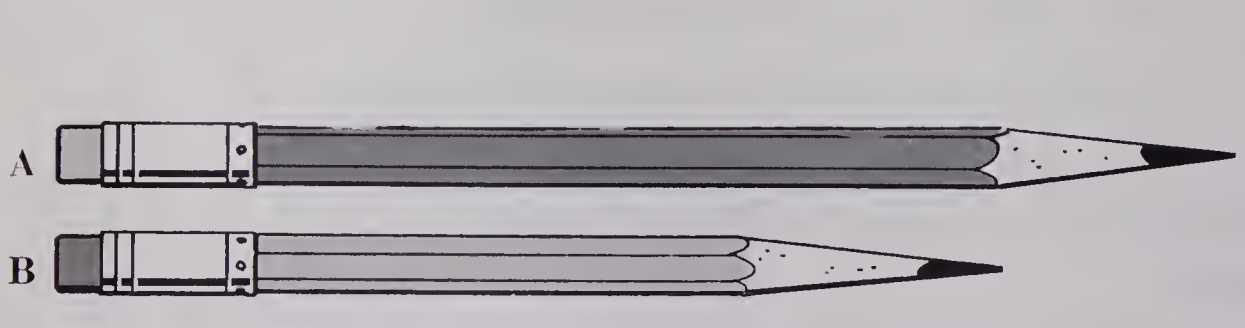
3. Look at your ruler. The longest marks and the numerals are for inches. How many inches equal 1 foot? 12

4. Bend your finger as in the picture, and lay your ruler on it. Check its length. Is this part of your finger about an inch long? Yes

5. Bend and measure the parts of other fingers. Can you find a finger part about an inch long? If so, call it your finger inch.

6. Measure pencils A and B with your finger inch, then with your ruler. Do you get the same length with

5" 4"



Teaching Pages 64 and 65

Pupil's Objectives

(a) To learn the use of the ruler as an instrument for finding the measure in inches and in feet; (b) to acquire a functional concept of *inch* and *foot*; and (c) to establish a *finger inch* and an *arm foot* to use as *reference measures*.

New Words

(page 64) *bend*, *wide*
(page 65) *arm*, *desk*, *feet*

Background

Our common units of measurement, such as the inch and the foot, do not always mean much to children. To most third-grade children *foot* means little more than *twelve inches* and *inch* means very little indeed. To enhance the meaning of these two words, the two *reference measures* are suggested on these two pages.

If the units *inch* and *foot* are to have meaning, pupils must have some handy concrete way of thinking of them. Hence, the ideas of the *finger inch* and the *arm foot* are developed. Such reference units also help pupils develop a sense of quantity.

The measure is the number of times the unit is applied to the particular item to be measured. The measure may be 3 and, to make clear the unit we are using, we may add "in inches" or "in feet," and so on. We usually combine the unit and the measure and tell the measurement, such as when we say, "3 inches" or "3 feet."

Measurements are always approximations. The smaller the unit of measurement we select, the more accurate our approximation becomes. However, it is impossible to be entirely exact in measuring, no matter how small the unit of measurement we select to use. On one occasion the mile is as close an approximation as is needed, while on another occasion the thousandth of an inch is required to describe our measurement. However, in neither instance is the measurement exact.

Teacher's Preparation

Have a 12-inch ruler available for each child. Have available small articles whose length or width can be measured in inches. Suggested are labels, filing cards, tickets, ribbons, papers, and so on.

If possible, cut strips of tagboard or construction paper about 1 inch wide. Have the lengths of the strips in multiples of 1 inch so that the strips may be labeled "1 inch," "2 inches," "3 inches," and so on. If possible, all pupils should have a set of these strips.

Pre-Book Lesson

- Have the pupils estimate the length or the width of different articles in the room; then measure each article with a ruler.
- To develop the idea that measurements are approximations, encourage pupils to say, "The measure in inches is a little more than 3" or "The measure in feet is a little less than 4," and so on.

Using the Text Pages

- As you follow the suggestions given in the pupil's book, be sure to allow for many experiences in (a) estimating distances in both inches and feet; (b) checking the estimate by using a "body" (reference) measure; (c) checking the estimate with the ruler (a more precise instrument for finding the measure).
- Call attention to the fact that the ruler in the picture shows 0 near the left end, whereas the children's rulers may not show it at all. Be sure they know how to place their rulers to measure.
- To find his arm foot, a pupil may need to measure the underside of his arm from the elbow, instead of the inside as shown in the picture.
- Note that, in Ex. 9 and 15, pupils are lead to realize that the size of the object being measured helps determine the unit of measurement (in this instance, inch or foot) to be used.

Individualizing Instruction

- Suggest that *all pupils* measure at home to find some articles whose length may be described as "about 3 in inches" or "about 2 in feet," and so on.
- Have *all pupils* label the tagboard strips that have been made. The length of each strip should

be labeled to the nearest inch. Each child should have strips labeled "1 inch," "2 inches," and so on. The strips may be used as their standards for measuring and checking estimates. Place one or more sets of these strips in the arithmetic corner.

These strips are also excellent devices for verifying A. and S. facts. For example, ask the children to find two strips that together will be equal in length to the 7-strip. The strips can be used somewhat like the number-line picture. Place the 5-strip on top of the 8-strip. How many units of the 8-strip are still showing?

NOTES

Note the introduction of the unique finger-inch and arm-foot reference measurements!

7. On the board, draw a mark that you think is
a. 2 inches long. **b.** 10 inches long. **c.** 7 inches long.

8. Now, measure the marks you have drawn.
a. Use your finger inch. **b.** Use your ruler.

9. Which is the best way to measure small things, so that you really know their size: Use a ruler.
a. Guess? **b.** Use a ruler? **c.** Use your finger inch?

10. In the picture, Mary is finding her arm foot. Explain what she is doing.

11. Find your own arm foot.

12. Guess how wide these things are in feet. Write your guesses on the board.
a. the teacher's desk **c.** a window
b. the room door **d.** your desk

13. Now measure the things in Ex. 12 with your arm foot, then with a ruler. Write the new answers after the guesses in Ex. 12. How good were your guesses as measures?

14. When might you use your arm foot instead of a ruler to find how many feet long or tall something is?
When you do not need to know the exact measure.

15. To tell about how long these things are, would you measure with your finger inch or with your arm foot? Why? For large things, use arm foot and for small, finger inch.
a. a horse (arm foot) **b.** a cat (arm foot) **c.** a big table (arm foot) **d.** a crayon (finger inch)

12 inches (in.) = 1 foot (ft.) **1 ft. = 12 in.**

Problems Using Addition or Subtraction

Telling why A. or S.; making n-examples [W]

Write your work for problems 1–8.

1. The bag on the scales has 20 pounds of potatoes. The bag in Mrs. Gray's hands has 10 pounds. If she buys all these potatoes, how many pounds will she have? 30 pounds. The numbers for the parts are given. Add to find the sum. $20+10=n$ $n=20+10$

2. How much more money does Betty Gray need to buy a measuring cup? The cup costs 15¢. Betty has 5¢. 10¢ The numbers for the total and one part are given. Subtract to find the number for the other part. $15¢-5¢=n¢$ $n¢=15¢-5¢$

3. Mrs. Gray bought apples for 23¢ and oranges for 45¢. How much did these things cost? 68¢ The numbers for the parts are given. Add to find the sum. $23¢+45¢=n¢$ $n¢=23¢+45¢$

4. Betty took down 3 of the 18 boxes of crackers on a shelf. Then how many boxes of crackers were on the shelf? 15 boxes. The numbers for the total and one part are given. Subtract to find the number for the other part. $18-3=n$ $n=18-3$

5. Mrs. Gray wants to buy 18 oranges. She has picked out 8 of them. How many must she still pick out? 10 oranges The numbers for the total and one part are given. Subtract to find the number for the other part. $18-8=n$ $n=18-8$

6. How much did Mrs. Gray pay for a funny hat for Betty? The hat once cost 69¢, but she bought it for 9¢ less. 60¢ The numbers for the total and one part are given. Subtract to find the number for the other part. $69¢-9¢=n¢$ $n¢=69¢-9¢$

Teaching Pages 66 and 67

Pupil's Objectives

(a) To solve addition and subtraction problems; (b) to determine whether to add or to subtract through using the basic ideas that adding is finding the number for the total and that subtracting is finding the number for how many left, how many gone, how many more needed, or how many in the other part shown; (c) to think and write for given problems appropriate number sentences using *n*; and (d) to begin to acquire skill in subtracting tens.

New Words

(page 66) *crackers*, *potatoes*
(page 67) *loaves*

Background

Translation of information in a problem setting to a mathematical equation is an important phase of the problem-solving act. The problems on page 66 and at the top of page 67 should be assigned for the pupils to work in any way they can. Then, there should be the oral lesson in which children tell the kind of *n*-example appropriate for each problem. Conformity in translation and the writing of the equations should not be enforced. Rather, you should strive for creativity and encourage and accept any correct number sentence for the problem. However, at this grade level, steer the work so that pupils write *n* to stand for the answer.

The material at the bottom of page 67 is restricted to the subtraction of numbers for the even decades only. Following pages will extend the skill to cover work with other numbers shown with 2-place numerals.

Pre-Book Lesson (pages 66–67)

Use objects in the classroom to review addition situations and the subtraction situations studied to this point (see (b) under Pupil's Objectives above).

Using the Text Pages (pages 66–67)

- Remind the pupils to work with numerals only and then rewrite the answer with the correct label.
- As the children do the oral work, constantly help them to see that the important thing to decide is whether what is wanted is the number for the total or the number for a part of a total. If a total is to be separated, they may have to find the number for what is left or what is gone or how many more are needed or simply the other part shown.
- In writing the *n*-examples for the problems, the emphasis should be upon translation and then writing an appropriate number sentence. In order to maintain in this grade the use of the direct form only, suggest that pupils write *n* in place of the answer each time.

Individualizing Instruction

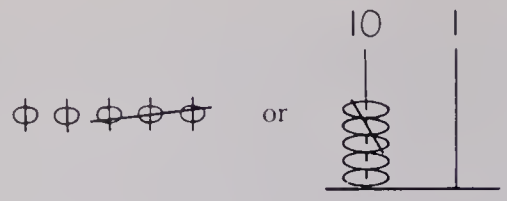
- If *slower learners* are not yet able to decide whether to add or to subtract for a problem, they should have more concrete experiences (e.g., use actual objects involved in the problem or use representative materials so they can see the sets of things). Then they are better able to decide whether the problem asks them to join sets or separate a set.
- *All pupils* need extensive experience in *translation*. An activity that should prove fruitful is the translation from the *n*-example situation to the problem. Provide some *n*-examples of several different A. and S. types (*n* always standing in place of the answer) and ask the children to write problems to go with the *n*-examples.
- Have *more capable children* try rewriting each of problems 1–8 to make an A. problem if it is now a S. problem and a S. problem if it is now an A. problem.

Pre-Book Lesson (page 67, bottom)

The following steps may be used in a demonstration with bundles of sticks or strips of paper. *More capable children* may assist in keeping records on the chalkboard while others write their own records at their desks.

Step 1. Demonstrate with bundles of sticks. Show 5 bundles of ten and remove 3 bundles.

Step 2. Draw on the chalkboard ⌀'s or an abacus picture. Show 5 tens. Cross out 3 tens.



Step 3. Write numerals using digits and words to show 5 tens minus 3 tens.

$$\begin{array}{r} 5 \text{ tens} \\ -3 \text{ tens} \\ \hline 2 \text{ tens} \end{array}$$

Step 4. Write numerals using digits only to show 50 minus 30.

$$\begin{array}{r} 50 \\ -30 \\ \hline 20 \end{array}$$

Use the same steps for other even-decade examples.

Using the Text Page (page 67, bottom)

- After discussing Ex. 1, lead the children to state the generalization in their own words.
- If you have a permanent number-line model with points for whole numbers through 100, use it to demonstrate Ex. 7–9 (see Teaching Page 39). Otherwise, draw on the board a picture of a number line for this part of the lesson.

Individualizing Instruction

Use the TENS and ONES chart with pockets (see Teaching Page 53) for the *slower learners* in order that they may gain additional practice on the positional nature of the numeration system as a basis for understanding subtracting ones and subtracting tens.

NOTES

* In the gradual development toward successful use of operations for problem-solving, pupils now begin to write the equations under the teacher's guidance.

7. On a shelf are 38 loaves of bread. Eight of them are brown bread. How many loaves are of other kinds?

30 loaves. The numbers for the total and one part are given. Subtract to find the number for the other part. $38 - 8 = n$ $n = 38 - 8$

8. In a box there are 3 brown cakes, 2 pink cakes, and 5 white cakes. How many cakes are there in the box?

10 cakes. The numbers for the parts are given. Add to find the sum.

$3 + 2 + 5 = n$ $n = 3 + 2 + 5$ [O]

* Tell why you added or subtracted in each problem.

* On the board write the *n*-example two ways for each problem. For Ex. 1 write,

$$20 + 10 = n \quad \text{and} \quad n = 20 + 10.$$

Subtracting Tens

Meaning [O]

1. If Mrs. Gray buys 10 of the 50 boxes of cards on a store shelf, how many boxes will be left? $50 - 10 = n$

a. Does Ex. A show another way to write the example? Yes

b. Tell how to find the answer by using Ex. B; Ex. C.

A	B	C	D
50 −10 *n*	⊕ ⊕ ⊕ ⊕ ⊘	5 tens − 1 ten ?, or _?_	50 −10 ?

c. For Ex. D *think*, "50 − 10 = 40 ?"

You subtract tens in the same way you subtract ones.

Subtract. Say the answers for Ex. 2–6.

2.	3.	4.	5.	6.
9 tens	7 tens	80	60	90
2 tens	5 tens	50	30	40
7 tens	2 tens	30	30	50

On the board use a number-line picture to subtract

7. 50 − 20. 30 8. 90 − 30. 60 9. 70 − 40. 30

Subtracting with 2-Place Numerals

[o]

1. Sue's mother wants to make 36 cookies. She has made 24. How many more must she make? $36 - 24 = n$

Explain how to find the answer by using

a. Ex. A. **b.** Ex. B. **c.** Ex. C.

A	B	C	D
⧵⧵⧵⧵ | | ⧵⧵⧵⧵	3 tens and 6 ones −2 tens and 4 ones _?_ and _?_, or _?_	Tens | Ones 3 | 6 −2 | 4 ? | ?	36 24 12

d. Ex. D shows the short way to subtract.

First, subtract the ones: $6 - 4$. *Think,* "2." Why is the 2 written in one's place in the answer? to show 2 ones

Then subtract the tens: $3 - 2$. *Think,* "1." Where is the 1 written in the answer? in ten's column Why? to show 1 ten

Copy Ex. 2–7 on the board and subtract. Use tens and ones as in box B. Then subtract as in box D.

2.	3.	4.	5.	6.	7.
86	78	57	69	86¢	97¢
43	56	30	59	34¢	77¢
43	22	27	10	52¢	20¢

Teaching Pages 68 and 69

Pupil's Objective

To acquire skill in subtracting with 2-place numerals (no renaming).

New Word

(page 68) *written*

Background (pages 68–69)

The detailed language of rational explanations, such as use of the words "tens" and "ones," is used to foster meaning and understanding. However, the thought pattern that the child will want to use habitually is described as the *short way* in Ex. 1d on page 68 in the pupil's book.

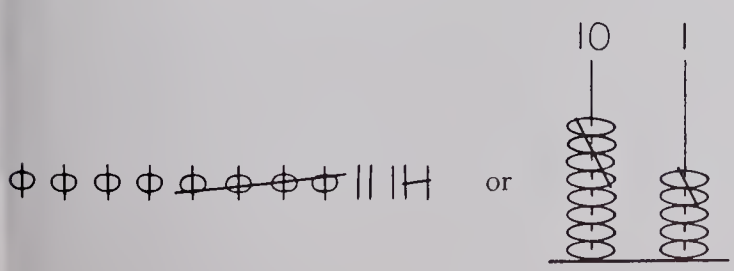
The enrichment material with odd numbers at the bottom of page 69 is an extension of the ideas with even numbers introduced as enrichment on page 63. In the previous lesson the generalizations were concerned with the adding and subtracting of *even* numbers. In this lesson the same generalizations are developed for the adding and subtracting of *odd* numbers.

Pre-Book Lesson (pages 68–69)

Follow a plan similar to that suggested for Teaching Page 67, but use both tens and ones. Stress the importance of subtracting the ones first and always working from positions of lesser value to positions of greater value (from right to left).

Step 1. Demonstrate with bundles of 10 sticks and other loose ones.
Show 8 bundles of ten and 5 ones. Take away 4 bundles of ten and 2 ones.

Step 2. Draw on the chalkboard ϕ's and /'s or an abacus picture. Show 8 tens and 5 ones. Cross out 4 tens and 2 ones.



Step 3. Write numerals using digits and words to show 8 tens and 5 ones minus 4 tens and 2 ones.

$$\begin{array}{r} 8 \text{ tens and } 5 \text{ ones} \\ -4 \text{ tens and } 2 \text{ ones} \\ \hline 4 \text{ tens and } 3 \text{ ones} \end{array}$$

Step 4. Write numerals using digits only to show 85 minus 42.

$$\begin{array}{r} 85 \\ -42 \\ \hline 43 \end{array}$$

Using the Text Pages (pages 68–69)

- While some pupils are working Ex. 2–7 at the board, have the remainder of the class work on paper at their seats.
- Be sure to emphasize the importance of first subtracting ones and then subtracting tens.
- Circulate among the children and check on their work as they complete this assignment. Ask questions of individual children to determine the nature of each child's understanding.

Individualizing Instruction

- *All pupils* may do Ex. 2–7 in this way:

$$\begin{array}{r} 86 = 80 + 6 \\ -43 = 40 + 3 \\ \hline 40 + 3 = 43 \end{array}$$

- Have *team* races using Extra Examples Set 36.
- Some of the *slower learners* may need additional work using the TENS and ONES pocket chart described on Teaching Page 53.

Using the Text Page (page 69, bottom)

- Perhaps you will want to use this material for all children orally. If so, before beginning the work on this page, review the generalizations presented on page 63 relative to adding two even numbers and subtracting an even number from an even number. See if those children who studied the earlier material can recall and state in their own words the result of adding two even numbers. Elicit the same kind of statement about subtracting an even number from an even number.

• Use a number-line picture on the chalkboard to illustrate the generalizations about even numbers and again to help in the oral presentation of the ideas and questions about odd numbers.

• Summarize all of the generalizations that have been presented thus far:

(a) adding 1 to an even number

(b) adding 1 to an odd number

(c) adding two even numbers

(d) subtracting an even number from an even number

(e) adding two odd numbers

(f) subtracting an odd number from an odd number

NOTES

[W]

First subtract ones, then subtract tens. Write on folded paper the answers for rows 8–9.

	a	b	c	d	e	f	g	h	i
8.	36	69	87	97	50	98	34	30¢	46¢
	32	33	64	72	30	50	13	20¢	25¢
	4	36	23	25	20	48	21	10¢	21¢
9.	79	89	80	56	48	98	56	84¢	46¢
	51	28	50	21	43	47	5	12¢	10¢
	28	61	30	35	5	51	51	72¢	36¢

Copy in columns, and subtract.

10. 63 − 61 2
11. 36 − 2 34
12. 27 − 4 23
13. 98 − 78 20
14. 90 − 40 50
15. 78 − 8 70
16. 59¢ − 16¢ 43¢
17. 58¢ − 54¢ 4¢

◆**Extra Examples.** Work Set 36.

Do You Like to Try New Things?

Enrichment [W]

1. In set A are the addends even or odd numbers? odd

2. Are the sums for set A even or odd numbers? even

3. Copy and finish: When two odd numbers are added, the sum is an _even_ number.

4. Test your answer for Ex. 3. Make three examples like 53 + 3 and three like 3 + 64 and add. Answers will vary.

5. In set B are even or odd numbers used? odd

6. Are the answers for set B even or odd numbers? even

7. Copy and finish: When an odd number is subtracted from an odd number, the answer is an _even_ number.

8. Test Ex. 7. Show three examples with answers. Answers will vary.

Set A

5	3	9
+3	+7	+1
?	?	?

Set B

7	9	5
−3	−7	−1
?	?	?

Checking Subtraction Answers

Inverses [O]

A

$$\begin{array}{r} 10 \\ -\,8 \\ \hline 2 \end{array}$$ ↑ Add to check

B

$$\begin{array}{r} 48 \\ -\,13 \\ \hline 35 \end{array}$$ ↑ Add to check

1. In Ex. A, the sum is 10, the known addend is 8, and the addend found is 2.

Adding a number can undo subtracting that same number, so we can check $10 - 8 = 2$ by adding $2 + 8$.

Is the sum of the addends the same as the sum you subtracted from? Yes

2. In Ex. B, explain how to check the subtraction.

Check Ex. 3–7 to find answers that are not correct.

3.	4.	5.	6.	7.
70	68	89	37	56
−30	−3	−46	−6	−23
30 (40)	65	42 (43)	33 (31)	32 (33)

* **In any subtraction example, the sum minus one addend equals the other addend.**

* **So, in checking subtraction, one addend plus the other addend equals the sum.**

[W]

Subtract. Write answers on folded paper. Check.

	a	b	c	d	e	f	g	h	i
8.	19	58	98	60	10	59	77	46¢	79¢
	2	24	6	40	7	7	35	23¢	64¢
	17	34	92	20	3	52	42	23¢	15¢
9.	64	87	39	27	10	68	90	58¢	85¢
	50	22	8	4	6	20	50	48¢	31¢
	14	65	31	23	4	48	40	10¢	54¢

Copy Ex. 10–17 in columns. Subtract and check.

10.	38 − 7 31	12.	95 − 63 32	14.	93¢ − 92¢ 1¢	16.	49¢ − 33¢ 16¢
11.	10 − 9 1	13.	35 − 5 30	15.	84¢ − 54¢ 30¢	17.	27¢ − 20¢ 7¢

◆ **Extra Examples.** Work Set 37 and check.

*Here is another opportunity to emphasize the very useful addends-sum relationship.

Teaching Page 70

Pupil's Objective

To learn how to check the answer in subtraction.

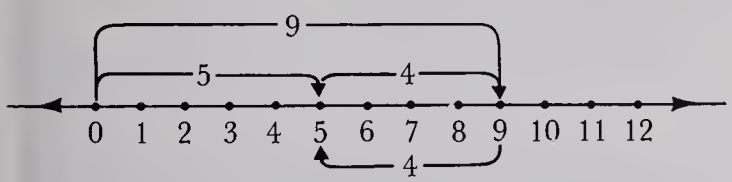
Background

Checking subtraction answers should be taught as a rational operation, not simply as a mechanical trick that works. This operation provides more opportunity to build complete understanding of the addends-sum relationship and also the inverse relationship between addition and subtraction of the same number.

Pre-Book Lesson

• Demonstrate the separation of 9 objects into sets of 5 objects and 4 objects. Then, join the two sets together to show that this joining reconstructs the total group of 9 objects.

• Use a picture of a number line or the Modern Number Line* to show that adding the number subtracted will *undo* the subtraction.



Show on the number-line picture the arrow for the sum 9. Show a 4-arrow back from the point for 9 to the point for 5, to show subtracting 4 from 9. Ask children to tell what has been shown on the number-line picture. Then, above the number-line picture show a 4-arrow from the point for 5 to the point for 9. Ask other children to explain what the 4-arrow above the number-line picture shows. Be sure that they stress the idea that the addition arrow on the number-line picture is "undoing" the subtraction arrow. In other words, it takes you back to the point where you were before you subtracted.

* See 6, page xix.

• On the board write the record as follows:

$$\begin{array}{rl} 9 & \text{the sum} \\ -4 & \text{one addend} \\ \hline 5 & \text{other addend} \end{array} \qquad \left.\begin{array}{r} 5 \\ +4 \end{array}\right\} \text{the addends} \quad \begin{array}{l} \\ \\ \hline 9 \quad \text{the sum} \end{array}$$

Emphasize the addends-sum relationship. Be sure to elicit the important idea that adding a number can undo subtracting that number.

• Use higher-decade S. examples to extend the ideas presented above.

Using the Text Page

• Guide the class to state the generalizations in their own words.

• As the class writes answers on folded paper for the examples in rows 8 and 9, circulate among them and have some pupils think their work aloud.

• Be sure pupils check their work for Ex. 10–17.

Individualizing Instruction

• Discuss reasons for accuracy in subtraction, bringing out the importance of

a. knowing whether the amount of money left after a purchase is enough to make another purchase.

b. being able to plan the use of materials to allow for future needs after present use (as in sewing or construction work).

c. knowing the other part of a set, as in the number right and the number wrong in a spelling or an arithmetic test.

• Discuss the personal satisfaction obtained from self-checking of any work.

• There are, in all probability, some activities you have used that the children enjoy more than others. It is not necessary to have a new type of activity each day. The use of enjoyable activities is to be encouraged. Repetition in the use of these activities provides a motivational as well as a mathematical function.

• Assign Extra Examples Set 37 as needed.

Pupil's Objectives

(a) To write answers for addition and subtraction examples; (b) to check answers for addition and subtraction examples; and (c) to find the unknown sum or addend in *n*-sentences.

Pre-Book Lesson

• Use objects and the number line to demonstrate and illustrate the addends-sum relationship and to review the inverse relationship between the addition and subtraction of the same number. Be sure to stress that subtracting a number can undo adding that number, and vice versa. This idea is essential to understanding the rationale for checking the results of each operation.

• Write some *n*-sentences on the chalkboard. Have children tell whether *n* in each sentence stands for a sum or an addend. Use basic A. and S. facts, as well as higher-decade addition and subtraction examples, in this review.

Using the Text Page

• Perhaps you will find it useful to impose time limits row by row for the practice material at the top of the page. This can be accomplished in two ways: (1) Give pupils a certain amount of time to complete each row and count the number of examples that each child has attempted in that time; or (2) use a code for identifying a particular time interval (e.g., a—30 seconds, b—60 seconds, c—90 seconds, d—120 seconds, and so on) and write the code on the board at the end of each time interval that has elapsed. As each child finishes the row of examples, he is to write on his paper the last letter written on the board. Thus, the teacher can tell, within the limits of the predetermined time interval represented by each letter on the board, the length of time it takes each child to do a row of these examples. Speed should not be emphasized to the detriment of accuracy or understanding. You may wish to consider this assignment as a progress test.

• The *more capable children* should be able to proceed independently with the work at the bottom of the page. Perhaps you will want to concentrate on helping the *slower learners* orally. As these *slower learners* gain understanding, allow them to write a row or two independently as evidence of progress. Be sure each child is secure in his learning before you allow him to proceed independently.

Individualizing Instruction

• Select three examples from the bottom of the page. Have *all pupils* write problems to go with the *n*-examples you have selected.

• Have *more capable children* write each of Ex. 1–15 so as to show *n* in three different positions.

NOTES

To Keep in Practice

[W]

Write answers on folded paper, and check your work.

	a	b	c	d	e	f	g	h	i
1.	29	3	68	29	54	70	35	12¢	41¢
	−20	+75	−47	+40	−32	−30	−5	+63¢	+25¢
	9	78	21	69	22	40	30	75¢	66¢
2.	89	12	40	37	3	48	76	75¢	76¢
	−37	+77	+37	−4	+56	−45	+3	−12¢	−35¢
	52	89	77	33	59	3	79	63¢	41¢
3.	70	29	38	62	65	19	23	33¢	93¢
	+20	−3	−12	+5	−24	−9	+63	+24¢	−23¢
	90	26	26	67	41	10	86	57¢	70¢
4.	58	5	68	87	46	46	17	45¢	62¢
	+20	+43	−6	−80	+52	−6	−7	+41¢	−31¢
	78	48	62	7	98	40	10	86¢	31¢
5.	79	29	51	48	60	87	89	81¢	94¢
	−54	−5	+48	−4	+10	−6	−16	+14¢	−13¢
	25	24	99	44	70	81	73	95¢	81¢

◆**Extra Examples.** Work Set 38.

Finding the Unknown Sum or Addend

Addends-sum relationship [W]

Find the unknown sum or addend. Then write each example with a numeral in place of n or □ or ⬡.

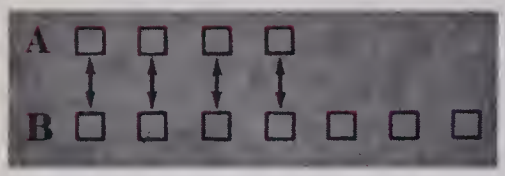
1. $\overset{66}{n} - 40 = 26$
2. $\overset{68}{n} = 50 + 18$
3. $68 - \overset{38}{n} = 30$
4. $\overset{1}{\square} + 27 = 28$
5. $79 - 72 = \overset{7}{n}$
6. $43 - \overset{43}{n} = 0$
7. $32 + 43 = \overset{75}{n}$
8. $\overset{20}{n} - 20 = 0$
9. $97 - \overset{82}{n} = 15$
10. $\overset{57}{n} - 51 = 6$
11. $86 - \overset{36}{n} = 50$
12. $\overset{85}{\bigcirc} - 40 = 45$
13. $37 - 2 = \overset{35}{n}$
14. $61 + 5 = \overset{66}{n}$
15. $\overset{34}{n} + 50 = 84$

◆**Extra Problems.** Work Set 3.

Here is another situation requiring subtraction—the how-many-more and how-many-fewer questions!

Using Subtraction to Compare Numbers

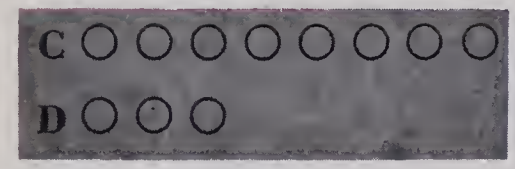
Finding the difference [O]



1. There are 4 squares in set A and 7 in set B. How many more squares are there in set B than in set A? 3

*a. Match squares in set A with squares in set B. You match the 4 squares of set A with one part, or 4 squares, of set B. What is the other part of set B? 3 squares

b. Instead of matching, can you use the numbers for the sets and find the difference by subtracting? $7 - 4 = ?$ 3 The difference between 7 and 4 is 3. So 7 is 3 greater than 4. You have compared the numbers 7 and 4.



2. There are 8 circles in set C and 3 in set D. How many fewer circles are there in set D than in set C? 5

a. If you match circles in set D with circles in set C, what part of 8 circles is matched? 3 circles ∧ not matched? 5 circles

b. Subtract to find the difference. $8 - 3 = ?$ 5 So 3 is 5 less than 8. What numbers have you compared? ∨ 8 and 3

You can compare two numbers by subtracting.

For each of Ex. 3 and 4, make a problem using the word more or the word fewer.

3. A book of 8 pages; a book of 18 pages.

4. A paper with 9 examples; one with 29 examples.

[W]

Copy Ex. 5–8, putting in what is missing.

5. 10 is 2 greater than 8.
6. 23 is 20 greater than 3.
7. 3 is 6 less than 9.
8. 7 is 42 less than 49.

* There is one-to-one correspondence between set A and a part of set B. So, the number of set B can be used as a sum and the number of the matched set can be used as an addend to find the number for the other part.

Teaching Pages 72 and 73

Pupil's Objectives

(a) To learn another type of situation for which subtraction can be used—the idea that subtraction is an operation to use for comparing two numbers and finding the difference between them; (b) to show an understanding of when to use addition and when to use subtraction in problem-solving; and (c) to write *n*-examples for A. and S. problems.

New Words

(page 72) *compare*, *difference*

Background

The use of subtraction to find the difference does not seem, on the face of it, to be like subtracting to find how many (or much) left or how many (or much) gone or how many (or much) more needed or the other part of the number when it is shown. For each of these types, a single known total set may be separated into two parts; but, in finding the difference, it would seem that there are two known total sets at the start. However, when, in order to find the difference, the members of two sets are matched on a one-to-one basis, part of the larger set is necessarily left over. So, for the larger set, the members-matched provide one part and those left over can be thought of as the other part. Because matching illustrates so clearly the need for subtraction when comparing two numbers, this technique of using pictures is employed in the pupil's book (Ex. 1 and 2).

Ex. 3–11 require the comparison of abstract numbers in different ways. The children must become accustomed to the notion that the use of subtraction for comparison of numbers, like subtraction for the other situations, may be expressed in several different language forms.

Teacher's Preparation

To enable children to build an understanding of the difference idea, have available several different sets of objects that may be compared readily—chairs, tables, stacks of books, stacks of crayons, blocks, pencils, scissors, and so on.

Pre-Book Lesson (pages 72–73)

• Have two stacks of books placed side by side so that the children may easily see that one set contains more than the other. Then elicit from the children ways in which they can tell how many more the larger set contains than the smaller. Finally, lead children to understand that, although counting may be acceptable under certain conditions, subtraction is usually the quicker and more satisfactory way.

ILLUSTRATION: If we have a set of 35 pencils and a set of 23 pencils, what is the quickest way to find the difference between the numbers for the two sets?

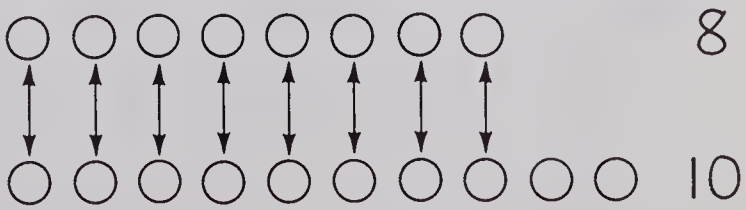
• While using different sets of objects so that the difference may be observed, illustrate the same situation on the board with geometric designs. Phrase questions so that the comparison idea is expressed in various ways: "How many more?" "How many fewer?" "Which is larger?" "How much larger?" "How much smaller?" "Compare." "Find the difference." For some of the sets, have variation among the members.

As each of these different language forms is developed, be sure to continue to emphasize the addends-sum relationship. You will want to make the distinction and have children learn to do so, so that, when we compare two sets, we may use the words "larger," "smaller," "more," "fewer" but, when we compare two numbers, we use the words "greater" and "less."

Using the Text Pages (pages 72–73)

• After discussing Ex. 1 and 2, pupils should be able to state the generalization in their own words.

• Have the children make circle pictures for Ex. 5–8. The circle picture for Ex. 5 is shown below to illustrate the type of pictures pupils should draw.



10 is 2 greater than 8.

• The written work in rows 9–11 is an application of the generalization on page 72 and should cause no trouble.

Using the Text Page (page 73)

• The *more capable children* should be able to work the problems at the bottom of page 73 independently.

• Go over the written problems orally with the *slower learners*. Ask different children to read each problem. Have the children tell about the information given in terms of parts and total. Have them explain what kind of subtraction situation is involved in each of the S. problems. Then have this group of children write their work for the problems.

• Remind pupils again that, when solving problems, they work with numerals only and then rewrite the answer with its label.

• The oral exercise should be very profitable. Telling why A. or S. was chosen as the operation used reveals whether there is an understanding of meanings.

• Writing each *n*-example two ways re-enforces the idea that the *n*-example for a problem may be given in more than one way. In this grade, however, make sure that pupils show *n* in place of the answer only, giving it last or first in the two ways asked for. By using this form of *n*-example, the pupil is, in effect, writing the example he uses for solving the problem.

Individualizing Instruction

• Prepare an arithmetic scrapbook with appropriate titles on separate pages in the book for the different situations requiring subtraction. As pupils encounter one or another of the needs for subtraction in problem situations, they may write the problem on the appropriate page in the book and illustrate it. Keep the scrapbook in an easily accessible place in the room for frequent use and review. *Our Subtraction Book* may be an appropriate name for the cover of the scrapbook. The scrapbook can be made very easily by folding butcher paper or laundry paper to any size and stapling along one edge.

• Have *more capable children* rewrite each of problems 1–6 to make it an A. problem if it is now a S. problem and a S. problem if it is now an A. problem.

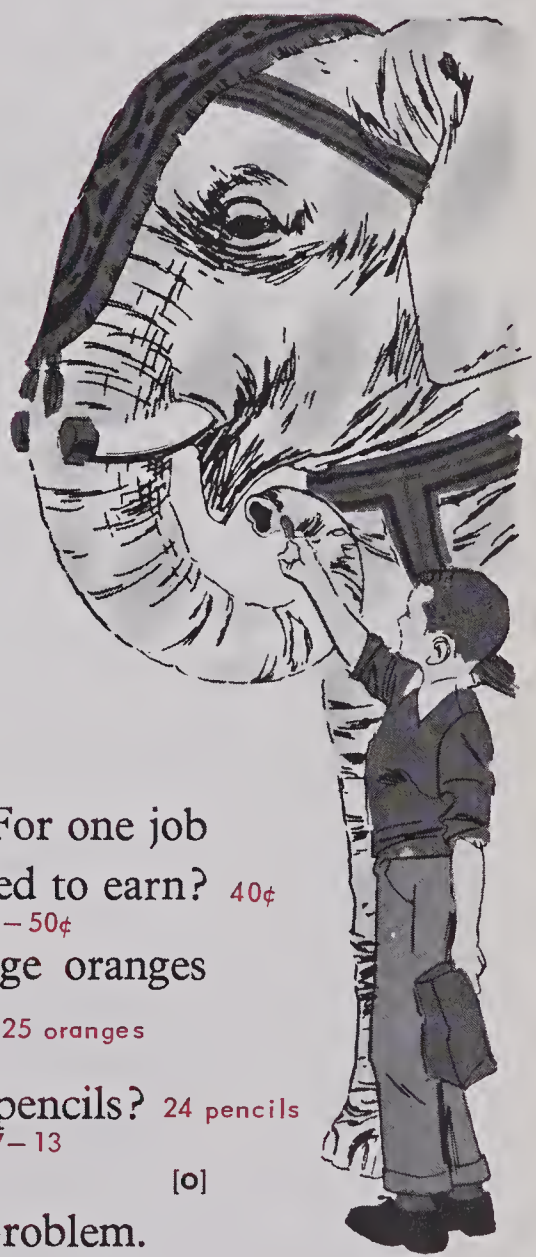
NOTES

Write your work to compare the numbers in each pair.

	a	b	c	d	e
9.	46 and 6 40	87 and 10 77	18 and 78 60	52 and 42 10	14 and 98 84
10.	50 and 30 20	98 and 96 2	84 and 20 64	6 and 79 73	96 and 53 43
11.	38 and 7 31	80 and 30 50	32 and 37 5	47 and 7 40	75 and 65 10

A. or S., Which?

Telling why A. or S.; making n-examples [W]



Write your work for Ex. 1–6.

1. Sam had 25 peanuts. He gave 12 of them to an elephant. How many peanuts did he have left? 13 peanuts
25 − 12 = n n = 25 − 12

2. One night all but 5 of Joe's 38 cows went into the barn. How many cows went into the barn? 33 cows
38 − 5 = n n = 38 − 5

3. Sue has worked 5 examples. She has 20 more examples to work. How many examples are there in all? 25 examples
5 + 20 = n n = 5 + 20

4. Tom wants to earn 90¢ for a new pen. For one job he was paid 50¢. How much more does he need to earn? 40¢
90¢ − 50¢ = n¢ n¢ = 90¢ − 50¢

5. There are 10 small oranges and 15 large oranges in a box. How many oranges are in the box? 25 oranges
10 + 15 = n n = 10 + 15

6. How many fewer are 13 pencils than 37 pencils? 24 pencils
37 − 13 = n n = 37 − 13

[O]

Tell why you added or subtracted in each problem.

On the board, write the *n*-example two ways for each problem.

In examples 1, 2, 4, and 6, numbers for one part and the total are given, so subtract find the number for the other part.
In examples 3 and 5, numbers for the parts are given, so add to find the number for the total.

If . . . , Then . . .

Logic (if-then sentences) [O]

In Ex. 1, tell how the answers were found. Then say the answers for Ex. 2 and 3.

1. If $n = 6$, then
   a. $n + 10 = 16$ (6 + 10 = 16)
   b. $19 - n = 13$ (19 − 6 = 13)
   c. $12 + n = 18$ (12 + 6 = 18)

2. If $n = 7$, then
   a. $n + 12 = ?$ 19
   b. $30 + n = ?$ 37
   c. $27 - n = ?$ 20

3. If $n = 2$, then
   a. $49 - n = ?$ 47
   b. $n + 26 = ?$ 28
   c. $68 - n = ?$ 66

[W]

Copy each of Ex. 4–9, writing the numeral in place of each n. Work each example to find the answer.

4. If $n = 4$, then
   a. $n + n = ?$ 8
   b. $n - n = ?$ 0
   c. $n + 53 = ?$ 57

5. If $n = 5$, then
   a. $25 - _?_ = n$ 20
   b. $38 - n = ?$ 33
   c. $n - 0 = ?$ 5

6. If $n = 3$, then
   a. $40 + n = ?$ 43
   b. $63 - n = ?$ 60
   c. $n + n = ?$ 6

7. If $n = 40$, then
   a. $60 - n = ?$ 20
   b. $n + 31 = ?$ 71
   c. $20 + _?_ = n$ 20

8. If $n = 39$, then
   a. $n - 6 = ?$ 33
   b. $40 + n = ?$ 79
   c. $89 - n = ?$ 50

9. If $n = 17$, then
   a. $40 + n = ?$ 57
   b. $10 + _?_ = n$ (7)
   c. $47 - _?_ = n$ (30)

Do You Like to Try New Things?

Enrichment [W]

Find the number n stands for in $32 + 3 + n = 39$.
Think, "$32 + 3 = 35$; $35 + n = 39$. $39 - 35 = 4$, so $n = 4$."

Copy each example, writing the numeral in place of n.

1. $61 + 4 + n = 67$ (2)
2. $20 + n + 6 = 28$ (2)
3. $84 + 1 + n = 87$ (2)
4. $n + 0 + 8 = 29$ (21)
5. $50 + n + 3 = 56$ (3)
6. $13 + 4 + n = 19$ (2)
7. $n + 3 + 4 = 37$ (30)
8. $12 + n + 5 = 19$ (2)
9. $31 + 6 + n = 39$ (2)

Pupil's Objectives

(a) To begin to use *if-then* thinking as a form of mathematical reasoning; and (b) to have practice in finding the answers in "If-then" sentences, when n stands for a given number.

Background

At the elementary-school level, we are concerned with building a strong background of readiness experiences in *logical thinking* to facilitate the acquisition of patterns of precise mathematical reasoning. Logic provides a way of organizing mathematical ideas and of clarifying their meaning. We do not propose a formal study of logic at this level, but an intuitive development of some of the important features of logical thinking is a good beginning.

"If-then" stories, relationships, or sentences afford the opportunity to begin developing a pattern of precise mathematical reasoning. "If" certain information is proved or accepted as true, "then" what statement or statements can be made that are also true? In each exercise on this page the child is given an "if" statement of truth that n equals some specific value; then the child is to complete some "then" statements involving n. Ultimately, he will be making "then" statements based exclusively on the "if" information without any additional help or knowledge.

Another facet of logic which is being intuitively developed in this program involves giving reasons for certain mathematical ideas to clarify their meaning. In this kind of logic, there is an agreed-upon principle or definition which can be given as an immediate justification for a particular case [e.g., $(4 + 2) = (1 + 5)$ is justified by the agreed-upon principle of using other names for numbers. $(4 + 2)$ and $(1 + 5)$ are ways of renaming the number 6.]. Justifying a procedure is developed extensively in the next lesson (on page 75) of the pupil's book. There the renaming and associative ideas are used quite extensively. The concept of logic involved in these examples is that *all instances of accepted or proved generalizations are true.*

Pre-Book Lesson

• Write n on the chalkboard. Ask pupils to tell how we have been using n in number sentences. What does n stand for? Emphasize that we can use n to stand for *any* number in the set of numbers we are using.

• Write, "$n = 3$" on the chalkboard. Say, "*If* n equals 3, *then* what is the answer to this question?" Write on the chalkboard, "$n + 5 = ?$" Repeat this kind of questioning, using different values for n and using subtraction as well as addition.

• Emphasize that *if* we know something is *true* in arithmetic, *then* we can usually make some other statements that are also true.

Using the Text Page

• Follow through Ex. 1–3 orally with the class.

• You may wish to write Ex. 4a on the board as a model to show how to do the written work.

Individualizing Instruction

The enrichment material at the bottom of page 74 uses n to represent one of three addends. This may be thought of as an introduction to 2-step problem-solving and, for that reason, may be a good lesson for *all pupils*. However, the steps necessary in this work may seem to be too difficult for some pupils and this exercise should be retained for *more capable children* only. First, the child must find the sum of the two given addends. Then, he must subtract that sum from the given sum to find the unknown addend which n stands for. In many of these examples some of the *more capable children* will be able to determine the unknown addend by comparing the sum of the two addends and the sum of the three addends without consciously subtracting. It is not necessary to force pupils to perform the subtraction operation unless they need to do so.

Perhaps you will want to vary the assignment by having the children copy Ex. 1–5 and write the answers and then write answers only for examples 6–9.

Pupil's Objective

To learn a quick way to find answers for basic A. facts when the sum is more than 10.

Background

It is the experience of most teachers that A. and S. facts with sums 11 through 18 are comparatively hard for children. One reason is that, in daily activities, children are seldom required to deal in an exact way with these greater numbers. The aim of this lesson is to provide pupils an intelligent and useful way of understanding these numbers. The method taught is admittedly somewhat roundabout, but it makes sense mathematically. After some use of it, pupils should be encouraged to abandon this method and memorize the facts. This fundamental procedure, however, will serve as something to fall back on when memory fails and, for a time, can serve as a check on higher-decade addition results.

Two important mathematical ideas are fundamental to the procedure developed in this lesson. First is the concept of *renaming* numbers. This was introduced in Looking Ahead on Teaching Page 62.

The second is the *Associative Property of Addition.* Its use was first introduced on page 40 and then extended on page 47 of the pupil's book.

These two ideas function as follows in the example $8 + 7 = n$:

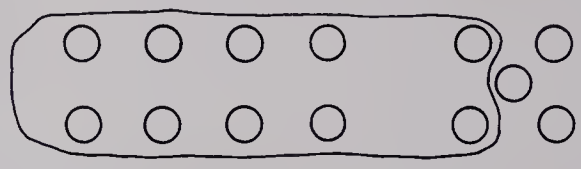
• First the lesser of the two addends, 7, is "renamed" so that one addend of 7 becomes available to be associated with the other addend, 8, in order to make 10. In this instance the 7 is renamed $(2 + 5)$. Thus, the example $8 + 7 = n$ becomes $8 + (2 + 5) = n$. Remember, ( ) are used in number sentences to show part of the sentence, or another name for a number.

• Now, we have the sentence $8 + (2 + 5) = n$. When we have three addends, the Associative Property of Addition allows us to add first either the first two addends or the last two, without changing the sum. For many children, it is easier to add 5 to 10 than to add 7 to 8. Consequently, in this sentence the child learns to add 8 and 2 and then to the sum 10 he adds 5. $(8 + 2 + 5 = n)$

Pre-Book Lesson

• Use geometric designs or some other objects on the flannel board. Show 8 objects in one set and 5 in another set. Review that 10 (ones) = 1 (ten). Guide children to state that they can make a set of 10 by taking 2 objects from the set of 5 and placing them with the 8 objects.

• Make pictures, and write the number sentences as follows:



$$8 + 5 = n$$
$$8 + (2 + 3) = n$$
$$(8 + 2) + 3 = n$$
$$10 + 3 = 13$$
$$\text{so, } 8 + 5 = 13$$

• Give practice with other combinations which permit a group of 10 to be arranged and pictured. Include combinations with the greater addend shown at the right, such as $4 + 9$, in which enough ones are added to the greater addend to make 10.

Using the Text Page

• You may use the Ginn Arithme-Sticks to show making a 10-group during oral discussion of Ex. 1–3.

• The *more capable children* should be able to proceed with the written work independently.

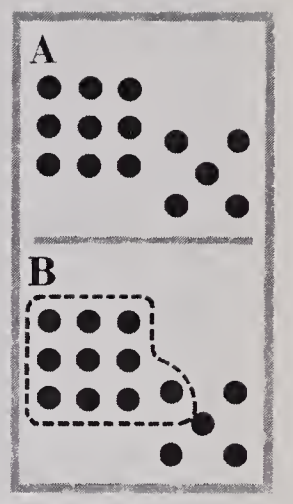
• *Slower learners* may need more oral work before working on the written assignment. For these pupils, write each of Ex. 4–18 on the chalkboard one at a time. Ask the children to tell which number should be renamed; how it should be renamed; and to explain why.

Individualizing Instruction

For *more capable children*, discuss the Associative Property of Addition as explanation for the procedure being used. Then have pupils work some of Ex. 4–18 at the board and simultaneously discuss how the associative property is being applied.

Finding Sums 11 through 18

Using Associative Property and compensation [O]



1. How many children are 9 boys and 5 girls?

The dots in box A stand for the parts of a set, 9 boys and 5 girls. Box B shows a way to find the number for the total set by first making a set of 10 dots. Why is the ring around the 9 dots of the larger part and 1 dot from the smaller part? To show a set of ten dots.

Using numbers, we *think*,

$9 + 5 = (9 + 1) + 4 \quad 10 + 4 = 14$, so $9 + 5 = 14$.

2. $7 + 4 = ?$ On the board draw dots to show the parts, 7 and 4. Ring the larger part, 7 dots, and 3 of the dots in the other part to make a set of 10 dots.

$7 + 4 = (7 + 3) + 1 \quad 10 + 1 = \underline{?}$, so $7 + 4 = ?$ 11

3. $7 + 8 = ?$ You can think of 7 as $5 + 2$. Then add 2 to the greater addend, 8, to make 10.

a. $7 + 8 = 5 + (2 + 8) \quad 5 + 10 = 15$, so $7 + 8 = ?$ 15

b. Use the number-line picture to check the sum.

[W]

By thinking and making 10 first, find the sums in Ex. 4–18. It is better to add to the greater addend to make 10. To check sums, use the number-line picture.

For Ex. 4 write, "$7 + 9 = 6 + (1 + 9) \quad 7 + 9 = 16$."

4. $7 + 9$ $6+(1+9)=16$	7. $9 + 3$ $(9+1)+2=12$	10. $9 + 6$ $(9+1)+5=15$	13. $5 + 7$ $2+(3+7)=12$	16. $6 + 9$ $5+(1+9)=15$
5. $9 + 8$ $(9+1)+7=17$	8. $6 + 7$ $3+(3+7)=13$	11. $9 + 9$ $(9+1)+8=18$	14. $5 + 8$ $3+(2+8)=13$	17. $7 + 5$ $(7+3)+2=12$
6. $6 + 5$ $(6+4)+1=11$	9. $7 + 8$ $5+(2+8)=15$	12. $9 + 4$ $(9+1)+3=13$	15. $3 + 8$ $1+(2+8)=11$	18. $8 + 9$ $7+(1+9)=17$

This technique of first finding a sum 10 is a great help to pupils who find it difficult to remember A. facts having sums greater than 10.

Facts with the Sum 11

Addends-sum relationship [O]

1. Instead of using a ruler, how can you use the number-line picture to help you find numbers for the parts when the number for a set is 11?

Use spaces between points 0 and 11.

2. What is the number for the other part, when the number for a set is 11 and the number for one part is

a. 2? 9 **b.** 5? 6 **c.** 8? 3 **d.** 4? 7

[W]

3. Make a table of numbers for the parts when the number for a set is 11. Go from 2 and 9 to 9 and 2.

2 and 9; 3 and 8; 4 and 7; 5 and 6; 6 and 5; 7 and 4; 8 and 3; 9 and 2

4. Show the set of A. and S. facts about each of these:

a. 11; 2 and 9 **c.** 11; 8 and 3

b. 11; 6 and 5 **d.** 11; 4 and 7

See bottom margin for help.

5. Copy each of these. After each, show the three other facts in the same set of A. and S. facts.

a. $11 - 8 = 3$ **c.** $11 - 2 = 9$

b. $5 + 6 = 11$ **d.** $7 + 4 = 11$

See bottom margin for help.

6. Make the table A. Facts with Sum 11. Start with $2 + 9 = 11$ and end with $9 + 2 = 11$.

See bottom margin for table.

7. Make the table S. Facts with Sum 11. Start with $11 - 2 = 9$ and end with $11 - 9 = 2$.

See bottom margin for table.

Copy and finish each example in rows 8–10, showing in place of n the unknown sum or addend it stands for.

	a	b	c
8.	$3 + n = 11$ (8)	$11 - n = 6$ (5)	11 minus 5 = n (6)
9.	$11 - n = 9$ (2)	$4 + 7 = n$ (11)	7 plus n = 11 (4)
10.	$5 + n = 11$ (6)	$n - 8 = 3$ (11)	n plus 4 = 11 (7)

6. $2+9=11$ $6+5=11$ $3+8=11$ $7+4=11$ $4+7=11$ $8+3=11$ $5+6=11$ $9+2=11$

7. $11-2=9$ $11-6=5$ $11-3=8$ $11-7=4$ $11-4=7$ $11-8=3$ $11-5=6$ $11-9=2$

Teaching Page 76

Pupil's Objectives

(a) To identify and learn to know the numbers for the pairs of parts of a set of 11; (b) to identify and learn the A. and S. facts with sum 11; and (c) to find unknown sums and addends in *n*-sentences.

Background

Up to this time, the *table of parts* of a set and the *table of facts* for the numbers have shown the same numbers. However, the A. facts are limited to the addition of two numbers that can be shown by single-digit numerals. Parts, on the other hand, are any subdivisions of a total set. For example, when a set has 13 members, there are, of course, the parts 12 and 1, 11 and 2, 10 and 3. These numbers are not included in the table of A. facts with sum 13. The remaining numbers for parts of a set of 13; 9 and 4, 8 and 5, 7 and 6 provide for the A. facts because they are numbers which can be expressed by single-digit numerals.

In order to avoid confusion at this time about the numbers for parts and the numbers for facts, the Tables of Parts of the larger sets will only show those numbers that are shown in the table of basic A. facts.

Pre-Book Lesson

Since most of the text page is suggested as a written activity, the Pre-Book Lesson is especially important for reviewing and extending these key ideas:

(a) the use and discovery of the parts of a set of 11

(b) the use of the addends-sum relationship to state and write tables of A. and S. facts

(c) the systematic organization of the eight A. facts and the corresponding S. facts with sum 11

(d) the use of *n* to stand for an unknown number in a number sentence

- Use objects in the classroom to have pupils show the different pairs of parts of 11.
- Compare the ruler and the picture of a number line in the text. Use pictures of a number line and a ruler on the chalkboard. Have pupils tell how they are alike.
- Make a game out of the following activity: Say, "The sum I am thinking of is 11. One of its addends is 3. What is the other addend?" If a child answers correctly, he is allowed to tell a sum and a known addend and ask for the unknown addend; or he can tell two known addends and ask for the sum. He, in turn, selects a child to answer a question.
- Write 11 = 4 and 7 on the board. Ask, "How many addition facts can be written about this set and this pair of parts?" "What are they?" Ask the same questions about subtraction facts. Then ask, "What is the set of A. and S. facts for the set of 11 and its parts 4 and 7?" Elicit from the children that there are 4 A. and S. facts for any given set and two of its parts, with the exception of the pair of parts of the same size.
- Write on the chalkboard *n*-sentences with sums no greater than 11 and have pupils tell the number that belongs in place of the *n*.

Using the Text Page

- Use Ex. 1 and 2 for oral work.
- *All pupils* then should be able to proceed with the written work independently.

Individualizing Instruction

- Have *slower learners* use objects when necessary to assist them in learning the pairs of parts and the A. and S. facts for a set of 11. Have them study independently or with Student Helpers.
- Have *more capable children* make a set of number-fact cards using *n* as shown below for all the A. and S. facts with sum 11.

7+4=n	11-4=n	11-n=7
n+4=11	11-7=n	n-4=7
7+n=11	11-n=4	n-7=4

All pupils may make cards to extend their sets of numeral cards through 11. When the cards have been prepared, use the *Show the Answer* activity (see Teaching Page 10).

Pupil's Objectives

(a) To review some of the basic understandings that have been taught; (b) to learn more about the use of parentheses in number sentences; and (c) to learn to differentiate between true and false number sentences.

Background

A number sentence may be *true* or *false* or *neither true nor false*. Eventually children will learn that any number sentence containing = is called an *equation*. At this time, attention will be given to number sentences that are either *true* or *false*.

Pupils have been introduced to parentheses as a means of associating numerals to indicate another name for a number. For this reason, we complete the operation indicated inside the ( ) before doing anything else with numbers shown in the number sentence. If the sentence contains two or more sets of parentheses, then the work indicated within each pair of parentheses must be completed before doing anything else with the number sentence.

In this lesson, pupils are to compare two numbers (each shown in a pair of parentheses) in a number sentence. If the two numbers are equal, then the sentence is *true*. If the two numbers are not equal, then the sentence is *false*.

Pre-Book Lesson

• Write some English sentences on the chalkboard that are obviously true or false similar to the following:

a. Larry is 9 feet tall.

b. Alice is 8 years old.

c. There are 78 children in our class.

Discuss each sentence with pupils deciding whether a sentence is true or false, and why.

• Write number sentences, such as these:

$7 - 2 = 5$; $4 + 5 = 8$; $11 - 5 = 3$.

Ask, "Is this number sentence true or false? "Why?"

• Write "$(2 + 2) = (1 + 3)$." Emphasize the use of the ( ). Inside each pair of parentheses is shown another name for a number. Ask, "What is the number that $2 + 2$ names?" "Does $1 + 3$ name the same number?" "Are the two numbers equal?" "Is the number sentence true or false?"

• Write other number sentences and have pupils tell why they are true or false.

Using the Text Page

• Emphasize that the written work at the top of the page is a review of some of the things pupils have been studying. Tell the class that this written work is practice for a test that they soon will be taking.

• Pupils who have difficulty with any of Ex. 1–8 will need more maintenance work.

• The *more capable children* should be able to proceed independently with the written work at the bottom of the page according to the directions given. You may wish to vary the written work so that some pupils write only "True" or "False" for each answer.

For the *slower learners* it probably will be necessary to go over the directions orally and complete examples 2a and 2b on the chalkboard. If the written assignment seems too long for this group have them work columns *a* and *b* only.

Individualizing Instruction

• Permit the *slower learners* to use objects or draw dot pictures to help them with the written assignment, if necessary. This is good practice in comparing two numbers by subtraction, which was recently developed.

• The *more capable children* may work Extra Activities Set 87.

Do You Remember?

[W]

Write what is missing.

1. In the example $9 - n = 2$, $n =$ 7.
2. The first even number after 39 is 40.
3. For "9 tens and 0 ones," we can write 90.
4. In the example $n - 5 = 6$, $n =$ 11.
5. Copy and use $=$ or $\neq$: $8 - 6$ ≠ 3
6. In the number 38 there are 3 tens.
7. The big hand goes around the clock in 60 minutes.
8. In counting by 3's, 18 comes next after 15.

* Equal or Unequal?

True or false sentences [O]

1. $(2 + 3) = (1 + 5)$. Is this true? $2 + 3 =$ 5 and $1 + 5 =$ 6 Are the sums equal or not equal? not equal

[W]

Write your work as shown below for Ex. 2a and 3a.

$(3 + 7) = (6 + 4)$. $3 + 7 = 10$ $6 + 4 = 10$ True.

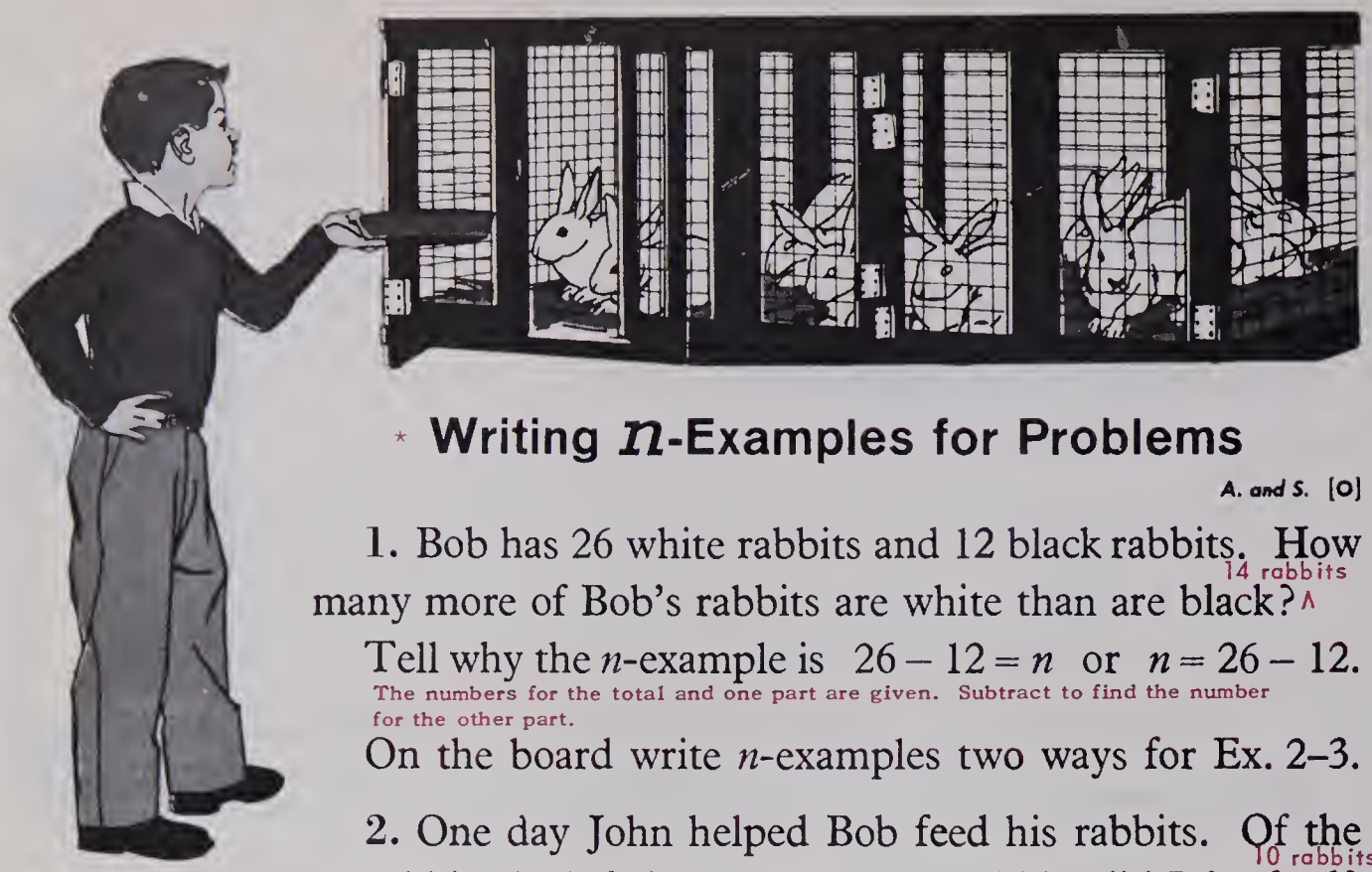
$(8 - 6) = (5 + 3)$. $8 - 6 = 2$ $5 + 3 = 8$ Not true.

	a	b	c
2.	$(3 + 7) = (6 + 4)$ True	$(8 - 5) = (10 - 7)$ True	$(2 + 24) = (39 - 6)$ Not True
3.	$(8 - 6) = (5 + 3)$ Not True	$(27 - 3) = (20 + 4)$ True	$(64 + 3) = (6 + 61)$ True
4.	$(10 + 2) = (7 + 3)$ Not True	$(4 + 7) = (19 - 8)$ True	$(25 + 4) = (2 + 26)$ Not True

Write again each example you labeled as not true. This time write $\neq$ in the sentence to make it true.

● **Extra Activity.** Work Set 87.

* Watch that pupils do not confuse equal and not equal with true and false. It may be hard for them to realize that a number sentence in which two numbers are said to be unequal may be a true sentence.



* Writing *n*-Examples for Problems

A. and S. [O]

1. Bob has 26 white rabbits and 12 black rabbits. How many more of Bob's rabbits are white than are black? 14 rabbits ∧

Tell why the *n*-example is $26 - 12 = n$ or $n = 26 - 12$.

The numbers for the total and one part are given. Subtract to find the number for the other part.

On the board write *n*-examples two ways for Ex. 2–3.

2. One day John helped Bob feed his rabbits. Of the 38 rabbits, Bob fed 28. How many rabbits did John feed? 10 rabbits ∧

38 − 28 = n n = 38 − 28

3. Bob fed his rabbits in 35 minutes and cleaned the cages in 50 minutes. How long was that in all? 85 minutes

35 + 50 = n n = 35 + 50

[W]

On your paper write *n*-examples two ways for Ex. 4–9.

4. Ann lost 6 of her 18 crayons. How many were left? 12 crayons

18 − 6 = n n = 18 − 6

5. Bob is 9 years old. His father is 39 years old. How much older than Bob is his father? 30 years

39 − 9 = n n = 39 − 9

6. How many are 15 fish and 20 fish in all? 35 fish

15 + 20 = n n = 15 + 20

7. Of 45 apples, 23 are red. How many of the apples are not red? 22 apples 45 − 23 = n n = 45 − 23

8. Tom has read all but 10 of the 40 pages in a long story. How many pages has he read? 30 pages

40 − 10 = n n = 40 − 10

9. Ella washed 12 plates, 4 bowls, and 3 cups. How many dishes did Ella wash? 19 dishes

12 + 4 + 3 = n n = 12 + 4 + 3

* Developmentally, pupils now write equations for problems under the teacher's supervision without having first worked the problems.

Teaching Page 78

Pupil's Objectives

(a) To distinguish between the need for addition and the need for subtraction in working problems; (b) to translate written problems into number sentences with n; and (c) to review the different situations that require subtraction.

New Words

fed, feed, years

Pre-Book Lesson

• Use objects and drawings on the board to demonstrate and illustrate the different situations that require subtraction.

• Review the generalizations about the relationships between a total and its parts:

a. If the numbers for two parts are known, we find the number for the total by adding.

b. If the numbers for the total and one of its parts are known, we find the number for the other part by subtracting.

• On the chalkboard write about a classroom situation similar to the following:

There are 29 children in our class. Sixteen of them are girls. How many boys are in our class?

Use the problem as a basis for discussing the information given. Then, for the problem, write an appropriate number sentence with n. Be sure to write the n-example in both of these ways: $29 - 16 = n$ and $n = 29 - 16$.

Using the Text Page

• For Ex. 1–3 have pupils tell whether the numbers given are for parts or for a total. Is the unknown number for a part or for a total?

• Be sure pupils understand the directions for Ex. 4–9 before proceeding with the written work.

• Pupils are not to solve the problems at this time. On page 79, the pupils are directed to return to page 78 and solve these problems.

Individualizing Instruction

• Have *more capable children* write A. and S. problems, using data which you provide (in order to have the pupils avoid using numbers requiring renaming and carrying or renaming to get more ones). Have the remainder of the class write n-examples for those problems. The following information may be used:

a. 17 books, 32 books A.

b. 35 crayons, 20 crayons S.

c. 3 elephants, 4 lions, 12 monkeys A.

d. 18 boys, 12 girls S.

e. 24 cookies, 15 cookies A.

• Have *all pupils* tell for Ex. 4–9 why they wrote an n-example for A. or an n-example for subtraction.

NOTES

Teaching Page 79

Pupil's Objectives

(a) To use a picture of a number line to help identify the numbers for the pairs of parts when the number for a set is 12; and (b) to begin learning the A. and S. facts with sum 12.

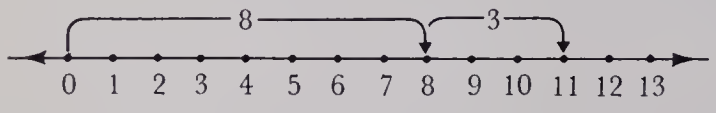
Teacher's Preparation

On the board draw a picture of a number line with points for 0 through 15 labeled, or use a Modern Number Line.* Then write the heading *Parts of a Set of 12* at the side of the board so that it will not interfere with the use of the number-line picture.

Perhaps you will want to ditto a form on which each child will write the tables asked for in Ex. 10, 19, and 20 on the pupil's page. This form should show the three headings *Table of Parts*, *Table of A. Facts*, and *Table of S. Facts*. Children can refer to the tables they make to assist them in writing Ex. 11–18. You may wish to make at the same time enough of these sheets to serve the same purpose later when similar work is assigned for sets of 13, 14, 15, 16, 17, and 18.

Pre-Book Lesson

• Use the number-line picture to identify the numbers for the parts of a set when the number for the set is 11. Draw arrows on the chalkboard above the number-line picture as illustrated below to show adding of the numbers for the parts.



• Use the number-line picture on the chalkboard in order to find the numbers for the parts of a set of 12 members. Under the heading on the board starting with *3 and 9*, show each pair of numbers for the parts when the number for the set is 12. Start with *3 and 9*. Stop at *9 and 3*. Acknowledge that *11 and 1*, and *10 and 2* are pairs of numbers for the parts of a set of 12 but that we are showing only numbers that can be shown by 1-place numerals.

• Use the number-line picture to illustrate adding and subtracting when the sum is 12 and the addends are 8 and 4. Do the same for other addends of the sum 12. Ask particularly how many facts are in the set when the numbers for the parts are 6 and 6. Have them explain their answer.

Using the Text Page

• Use the number-line picture on the board, as well as the one on the pupil's page, for the oral work.

• The written work for Ex. 10, 19, and 20 should be shown on the dittoed forms, if you have prepared them as suggested. Then, making use of the tables they have written, pupils may proceed easily with Ex. 11–18 and Ex. 21–24. Permit any child to use also the number-line picture or representative objects to help him with this work, if needed.

• As pupils finish the written work required on page 79, they are directed to turn back to page 78 and write the work for problems 1–9. However, you may wish to save this assignment for another time.

Individualizing Instruction

• Remind *slower learners* that, in the arithmetic corner, sets of number strips are available for use at any time they are free. They will recall that these number strips, made for use as units for measuring, may also be used to check work with number facts (see Teaching Pages 64–65).

• Have *more capable children* make number-fact cards for all A. and S. facts with sum 12 like those suggested on Teaching Page 76. *All pupils* should now extend their numeral cards through 12.

• Using the number-fact cards which the *more capable children* have made, have for *all pupils* a *Show the Answer* activity (see Teaching Page 1) in connection with A. and S. facts with sum 12.

* See 6, page xix.

Facts with the Sum 12

Addends-sum relationship [O]

On the board, draw a number-line picture like that at the right. Find which numbers for two parts of a set of 12 are shown correctly in Ex. 1–9 and which are not.

1. 12; 3 and 8
2. 12; 4 and 7
3. 12; 9 and 3
4. 12; 5 and 7
5. 12; 8 and 7
6. 12; 6 and 6
7. 12; 8 and 4
8. 12; 6 and 4
9. 12; 5 and 6

Ex. 3, 4, 6, and 7 are correct.

[W]

10. Make a table of numbers for the parts when the number for a set is 12. Go from 3 and 9 to 9 and 3.

3 and 9; 4 and 8; 5 and 7; 6 and 6; 7 and 5; 8 and 4; 9 and 3

Show the set of A. and S. facts about 12 and parts,

11. 4 and 8. **12.** 9 and 3. **13.** 6 and 6. **14.** 7 and 5.

See bottom margin for help.

For each of Ex. 15–18, show a S. fact. Then show the other facts in that same set of A. and S. facts.

15. $12 - 5$ **16.** $12 - 6$ **17.** $12 - 8$ **18.** $12 - 9$

See bottom margin for help.

19. Make the table A. Facts with Sum 12.

See bottom margin for table.

20. Make the table S. Facts with Sum 12.

See bottom margin for table.

Copy and finish each example, showing in place of n the unknown sum or addend it stands for.

	a	b	c	d
21.	$12 - \overset{6}{n} = 6$	$\overset{9}{3} + n = 12$	$\overset{11}{n} - 5 = 6$	$4 + 8 = \overset{12}{n}$
22.	$12 - \overset{4}{n} = 8$	$11 - 9 = \overset{2}{n}$	$12 - \overset{5}{7} = n$	$\overset{4}{n} + 7 = 11$
23.	$4 + 7 = \overset{11}{n}$	$\overset{12}{n} = 5 + 7$	$12 - 4 = \overset{8}{n}$	$9 + 3 = \overset{12}{n}$
24.	$5 + \overset{7}{n} = 12$	$6 + 6 = \overset{12}{n}$	$\overset{9}{n} + 3 = 12$	$\overset{11}{n} - 6 = 5$

Now go back to page 78 and work problems 1–9.

19. $3+9=12$ $7+5=12$ $4+8=12$ $8+4=12$ $5+7=12$ $9+3=12$ $6+6=12$

20. $12-3=9$ $12-7=5$ $12-4=8$ $12-8=4$ $12-5=7$ $12-9=3$ $12-6=6$

* This exercise may well be used as a test on mixed A. and S. facts having sums as great as 12.

* Finding Unknown Sums and Addends

A. and S. facts, sums as great as 12 [W]

Write answers on folded paper.

	a	b	c	d	e	f	g	h	i	j	k
1.	11	12	2	7	11	6	7	12	10¢	3¢	10¢
	−8	−4	+8	+5	−2	+6	+4	−9	−3¢	+7¢	−4¢
	3	8	10	12	9	12	11	3	7¢	10¢	6¢
2.	4	3	6	12	2	8	11	10	12¢	4¢	10¢
	+7	+9	+4	−8	+9	+3	−4	−7	−5¢	+6¢	−5¢
	11	12	10	4	11	11	7	3	7¢	10¢	5¢
3.	8	6	11	4	10	11	9	11	7¢	5¢	8¢
	+4	+5	−7	+8	−6	−5	+3	−3	+3¢	−0	−8¢
	12	11	4	12	4	6	12	8	10¢	5¢	0
4.	12	11	12	5	9	11	3	12	5¢	5¢	10¢
	−7	−9	−3	+6	+2	−6	+8	−6	+7¢	−5¢	−2¢
	5	2	9	11	11	5	11	6	12¢	0	8¢

Do You Like to Try New Things?

Enrichment [W]

For each of Ex. 1–17, find the number for n. Then write the example. For Ex. 1 write, "(78 − 38) = 40." For Ex. 10 write, "(67 − 4) = (60 + 3)."

1. $(78 - \overset{38}{n}) = 40$
2. $(3 + \overset{45}{n}) = 48$
3. $(\overset{17}{n} + 22) = 39$
4. $(\overset{42}{n} + 45) = 87$
5. $(29 + \overset{40}{n}) = 69$
6. $(\overset{87}{n} - 30) = 57$
7. $(\overset{20}{n} + 38) = 58$
8. $(6 + \overset{63}{n}) = 69$
9. $(\overset{37}{n} - 37) = 0$
10. $(67 - \overset{4}{n}) = (60 + 3)$
11. $(\overset{35}{n} + 43) = (70 + 8)$
12. $(88 - \overset{8}{n}) = (70 + 10)$
13. $(\overset{6}{n} + 53) = (46 + 13)$
14. $(8 + \overset{50}{n}) = (52 + 6)$
15. $(77 - \overset{31}{n}) = (23 + 23)$
16. $(18 + \overset{60}{n}) = (45 + 33)$
17. $(\overset{50}{n} - 40) = (23 - 13)$

Teaching Page 80

Pupil's Objective

To have written practice with A. and S. facts.

Pre-Book Lesson

• Review the addends-sum relationship. Write on the board

11 sum	3 addend
− 8 addend	+ 8 addend
addend	sum

Ask, "When we know the sum and one addend, how do we find the other addend?" "When we know two addends, how do we find the sum?"

• Use other examples and corresponding questions to review knowledge of *sums* and *addends*.

Using the Text Page

• Review the folded-paper technique before assigning the written work at the top of the page.

• Remind pupils to watch the signs of operation to see whether they are to find a sum or an addend.

Individualizing Instruction

• *Slower learners* may work with a partner and use number-fact cards (see Teaching Page 79) to learn A. and S. facts which they had wrong in rows 1–4.

• The enrichment material at the bottom of page 80 is designed for the *more capable children*. It is material that can be used at any time. While the *slower learners* are studying number facts, the *more capable children* may well do this work in finding the number n stands for and rewriting each equation. Emphasize that n stands for a number, but there is only one number for n that will make each sentence true. It may be necessary to go over one or two examples orally before pupils proceed independently with written work.

NOTES

Pupil's Objectives

(a) To learn to organize mathematical information in the form of a chart; (b) to learn to read and interpret a chart; and (c) to learn to solve problems based on information given in a chart.

New Words

beach, *crabs*, *snails*

Background

Pairing information about two known variables is a beginning of the *relation concept* in mathematics. The relation concept, and its more narrow application, the *function concept*, are important ideas that permeate most of mathematics.

At this point, it is sufficient for pupils to see that each value of the second variable is related to a value of the first variable (the relation concept). They should not be expected to understand the relation involving one and only one value of the second variable for each value of the first variable (the function concept).

Children encounter pairing experiences very early in the form of pairing spoken words with objects. The child may have a wide range of pairing experiences over a long period of time without being conscious of the pairing idea. In this lesson a chart is produced as a result of pairing objects collected at the seashore with the days of the week.

Many possibilities of pairing values of two variables can be found, such as pupils and weights, time and temperature, pupils and test scores, and number of tosses and bean bag scores. Teachers should utilize every opportunity to provide a background upon which the child can develop a correct understanding of the relation concept. The mathematical usage of the term is very close to its usage in daily conversation. We say that the time it takes to drive a known distance is related to the speed at which we travel, or the amount of money earned is related to the time spent on the job.

Pairings are often recorded and presented in graphs or tables or charts, and these recordings should be explored by pupils to gain a variety of experiences with different kinds of pairings.

Teacher's Preparation

Write the names of the months of the year in a column on the chalkboard with space at the right to record the number of children having birthdays each month. Above and slightly to the right of the column of months, write the heading "Number of Children."

Pre-Book Lesson

• Ask the children with birthdays in January to raise their hands (or stand up). Ask a child to count them. Have another child record the number in the appropriate place on the chalkboard. Perhaps you will want them to use tally marks first, and then write the numeral.

• Do the same for each month of the year. This would be a good opportunity to use the language of "sets." "Will the set of all children with birthdays in June stand?" "How many children are in the set with birthdays in August?"

It is possible that you will have an *empty* set (no pupils with a birthday in one month). Discuss the empty set. What numeral do we use to show that there are no members in a set?

• Discuss the *Table* on the board. It is a record of matching two kinds of information. Have the children tell about the information. Point out that the table could be extended to record the same information by rooms in the school. Then there would be a different column for each room.

Using the Text Page

Read each problem orally with *slower learners* before assigning them as written work.

Individualizing Instruction

• As a part of the science program have *all pupils* observe and record in a chart the following relationships:

a. Temperature at each hour of the day—in the room, outside in the sun and in the shade

b. Heights of different types of flowers each day (week) from time planted to full growth

• *More capable children* may do Extra Activities Set 88.

* Being able to locate information in a table is an important facet of problem-solving.

* Using a Table

Problem-solving [O]

Chuck made a table to show how many shells and other things he found each day he was at the beach.

From the table below, read the answers for Ex. 1–4.

1. Chuck found 5 sea shells on _?_. Wednesday
2. On _?_ he found the greatest number of starfish. Thursday
3. On which 2 days did he find three shells? Monday; Friday
4. On which day at the beach did Chuck find
a. the fewest crabs? Friday **b.** 0 starfish? Wednesday **c.** 9 snails? Wednesday

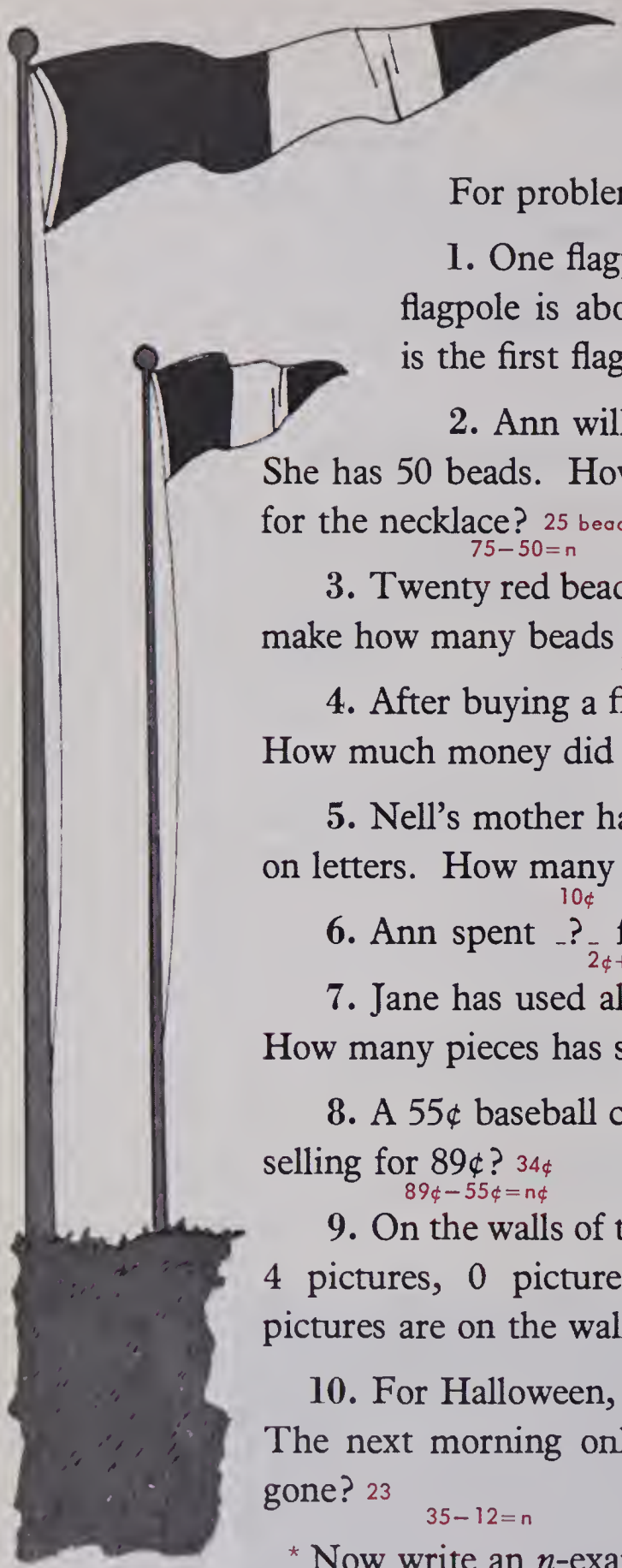
[W]

Write your work for each of Ex. 5–9.

5. How many snails and crabs together did Chuck find
a. on Tuesday? 9 **b.** on Wednesday? 12 **c.** on Friday? 14
6. Chuck found the most crabs on _?_. Monday
7. He found the fewest snails on _?_. Thursday
8. On Thursday and Friday together he found _?_ crabs. 6
9. On the first 3 days Chuck found _?_ shells. 12

● **Extra Activity.** Work Set 88.

	Shells	Snails	Crabs	Starfish
Monday	3	8	6	2
Tuesday	4	6	3	5
Wednesday	5	9	3	0
Thursday	2	3	4	7
Friday	3	12	2	1



A. or S.?

Writing n-examples [W]

For problems 1–10, write only the answers.

1. One flagpole is about 68 feet tall. Another flagpole is about 42 feet tall. How much taller is the first flagpole than the second? 26 feet
68−42=n

2. Ann will use 75 beads to make a necklace. She has 50 beads. How many more beads does she need for the necklace? 25 beads
75−50=n

3. Twenty red beads, 4 black beads, and 5 white beads make how many beads in all? 29 beads
20+4+5=n

4. After buying a fire truck for 29¢, Bob had 20¢ left. How much money did he have at first? 49¢
29¢+20¢=n¢

5. Nell's mother had 36 stamps. She used 12 stamps on letters. How many stamps did she have left? 24 stamps
36−12=n

6. Ann spent 10¢ for stamps costing 2¢, 3¢, and 5¢.
2¢+3¢+5¢=n¢

7. Jane has used all but 25 of her 85 pieces of paper. How many pieces has she used? 60 pieces
85−25=n

8. A 55¢ baseball costs how much less than a baseball selling for 89¢? 34¢
89¢−55¢=n¢

9. On the walls of the third-grade room are 2 pictures, 4 pictures, 0 pictures, and 5 pictures. How many pictures are on the walls of the third-grade room? 11 pictures
2+4+0+5=n

10. For Halloween, Ann's mother bought 35 lollipops. The next morning only 12 were left. How many were gone? 23
35−12=n

* Now write an *n*-example for each of problems 1–10.

* Another step! Pupils, after working the problems, now write number sentences for the problems on their own rather than at the board.

Teaching Page 82

Pupil's Objectives

(a) To use some of the recently-learned A. and S. skills and ideas in the solving of written problems; and (b) to translate written problems into number sentences using *n*.

New Words

grade, *necklace*,

Background

Applying mathematical skills in as many different ways as possible is essential to learning. Solving problems is one important application of mathematical ideas and skills.

There is but one operation we call subtraction. But, there are many applications of the subtraction operation in problem settings. One important objective of the problem-solving program is to give opportunities for differentiating between operations and the uses of any one operation. Thus far, we have identified five different uses for the subtraction operation.

Pre-Book Lesson

• Quickly review the relation between joining sets and the addition idea—that instead of joining two known parts to find the total set, we may add the numbers for the parts to find the number for the total set.

• Review briefly the five situations involving subtraction—situations which require finding

a. how many left b. how many gone

c. how many more needed

d. the other part when both parts are present, but only total and one part known

e. the difference in numbers for two sets

• State a problem within the pupil's mathematical ability. Ask several children to write on the chalkboard an *n*-example for the problem. Remind pupils that an *n*-example may be written in at least two different ways with *n* either first or last in the sentence.

Using the Text Page

• Read the direction line at the top of the page. Emphasize that pupils should try to think the work for each problem and then write only the answer on paper.

• After pupils have written the answers for the problems, have them write an *n*-example for each problem. Accept as correct the *n*-example with *n* written either first or last.

Individualizing Instruction

The *slower learners* may need more practice in matching number-fact cards with answers. For this, group children in pairs to work together or you may wish to work with a particular child in a *Show the Answer* activity.

NOTES

Teaching Page 83

Pupil's Objectives

(a) To identify and start learning the numbers for the parts of a set when the number for a set is 13, and the A. and S. facts for this set and its parts; (b) to use knowledge already acquired as a basis for learning something new; and (c) to extend practice in addition of 4 addends.

Background

In this lesson you will help pupils discover number facts and explore relationships. Do not expect mastery at this time. Using a number line, making a ten, showing sets of A. and S. facts, working with known parts of a set of 12, and using a table of facts—all assist pupils in exploration to discover the parts of a set of 13 and the A. and S. facts for this set and its parts.

Pre-Book Lesson

• Using easily-grouped objects (blocks, discs, flannel-board designs), have children show how to regroup sets of 8 and 5 to make sets of 1 ten and 3 ones. Be sure to emphasize the "renaming" of the 5 in order to use association to make a ten.

The material for Teaching Page 75 should be reviewed as you now develop understanding of facts having the sum 13.

• On the chalkboard, write $8 + 4 = n$. Build an understanding of the relationship between $8 + 4$ and $8 + 5$ somewhat as follows:

Is $8 + 5$ greater than or less than $8 + 4$?
How much greater?
What is the sum of $8 + 4$?
Then, what is the sum of $8 + 5$?

• Have selected children tell the set of A. and S. facts for $13 = 9$ and 4. Repeat with other parts of a set of 13.

Using the Text Page

• You may wish to duplicate a form for the pupils to use for Ex. 5 and also for Ex. 15. (You may have extra forms left after making some, as suggested on Teaching Page 79.) Permit the children to use the table of the parts of a set of 13 to help them in working Ex. 9–14.

• The *more capable children* should be able to proceed independently with Ex. 16–21. Assist the *slower learners* orally with one example before permitting them to go to Ex. 16–21.

• The oral work may be done with small groups or with the entire class. First, give the children a short time to study the examples in a row or a column. Then, ask a child to say just the answers to the examples in that row or column. The other pupils should check his answers mentally.

With the *more capable children* stress both speed and accuracy. Speed should never be emphasized to the detriment of understanding or accuracy. However, as children achieve higher levels of mastery, encouraging speed is a desirable objective.

Individualizing Instruction

• Use as Student Helpers *more capable children* who have mastered the work at the bottom of the page. Arrange the class in groups of two or three with the Student Helpers. The Helpers will listen while others say the answers.

• Have *slower learners* draw pictures to show making a 10 in order to find the sums of these examples: $7 + 5$, $6 + 6$, $4 + 9$, $5 + 8$, $7 + 6$.

• Ask the *more capable children* to draw number-line pictures to illustrate the adding in Ex. 16–21.

• *More capable children* may make number-fact cards (see Teaching Page 76) for all A. and S. facts with sum 13. *All pupils* should extend their numeral cards now through 13.

Facts with the Sum 13

Addends-sum relationship [W]

Copy and finish Ex. 1–3, writing the numeral for n.

1. 12 = 3 and 9, so 13 = n and 9. (4)

2. 12 = 4 and 8, so 13 = n and 8. (5)

3. 12 = 5 and 7, so 13 = n and 7. (6)

4. Use a number-line picture to check Ex. 1–3.

5. Make a table of numbers for the parts when the number for a set is 13. Go from 4 and 9 to 9 and 4.

4 and 9; 5 and 8; 6 and 7; 7 and 6; 8 and 5; 9 and 4

Show the set of A. and S. facts about

6. 13; 5 and 8. **7.** 13; 7 and 6. **8.** 13; 9 and 4.

See bottom margin for help.

Copy and finish Ex. 9–14. For each example, show the other facts in that same set of A. and S. facts. See bottom margin for help.

9.	**10.**	**11.**	**12.**	**13.**	**14.**
13	4	13	6	13	8
−7	+9	−8	+7	−9	+5
6	13	5	13	4	13

15. Make the table A. Facts with Sum 13 and the table S. Facts with Sum 13. See bottom margin for tables.

Copy the numerals in columns, and add.

16. 40, 2, 3, 4 (49) **18.** 2, 4, 3, 4 (13) **20.** 3¢, 4¢, 0, 5¢ (12¢)

17. 21, 0, 6, 2 (29) **19.** 0, 4, 4, 5 (13) **21.** 20¢, 3¢, 3¢, 3¢ (29¢)

[O]

Say the answers by columns, then by rows.

	a	**b**	**c**	**d**	**e**	**f**
22.	5 + 8 (13)	7 + 5 (12)	13 − 4 (9)	12 − 5 (7)	8 + 3 (11)	13 − 7 (6)
23.	12 − 4 (8)	13 − 8 (5)	11 − 5 (6)	8 + 4 (12)	2 + 9 (11)	9 + 4 (13)
24.	13 − 5 (8)	9 + 3 (12)	6 + 7 (13)	11 − 8 (3)	12 − 9 (3)	6 + 5 (11)

4 + 9 = 13 7 + 6 = 13 13 − 4 = 9 13 − 7 = 6
5 + 8 = 13 8 + 5 = 13 13 − 5 = 8 13 − 8 = 5
6 + 7 = 13 9 + 4 = 13 13 − 6 = 7 13 − 9 = 4

Finding Unknown Sums and Addends

A. and S. facts, sums as great as 13 [W]

Write answers on folded paper.

	a	b	c	d	e	f	g	h	i	j	k
1.	13	11	6	12	12	4	7	4	11¢	13¢	10¢
	−4	−9	+7	−8	−9	+7	+5	+9	−7¢	−5¢	−8¢
	9	2	13	4	3	11	12	13	4¢	8¢	2¢
2.	5	8	13	11	3	12	4	2	11¢	8¢	10¢
	+6	+4	−6	−3	+9	−5	+6	+9	−5¢	+5¢	−7¢
	11	12	7	8	12	7	10	11	6¢	13¢	3¢
3.	12	13	3	6	11	7	13	6	10¢	12¢	5¢
	−7	−8	+8	+6	−6	+6	−9	+5	−6¢	−3¢	+8¢
	5	5	11	12	5	13	4	11	4¢	9¢	13¢
4.	11	3	12	10	4	13	9	5	11¢	9¢	12¢
	−8	+7	−6	−4	+8	−7	+4	+7	−4¢	+3¢	−4¢
	3	10	6	6	12	6	13	12	7¢	12¢	8¢

To Keep in Practice

A. and S. [W]

For rows 1 and 2 write the answers on folded paper.

	a	b	c	d	e	f	g	h	i
1.	78	97	13	90	7	24	68¢	29¢	15¢
	−68	−50	+6	−60	+80	+35	−45¢	−7¢	+82¢
	10	47	19	30	87	59	23¢	22¢	97¢
2.	6¢	2¢	30	61	2¢	6¢	3¢	0	40¢
	2¢	2¢	1	3	4¢	1¢	0	4¢	5¢
	4¢	4¢	4	3	2¢	2¢	2¢	1¢	1¢
	12¢	2¢	4	0	3¢	3¢	6¢	7¢	2¢
		10¢	39	67	11¢	12¢	11¢	12¢	48¢

For rows 3–4 copy in columns, and work the examples.

	a	b	c	d	e
3.	0, 3, 4, 5 12	11, 5, 0, 3 19	2, 1, 5, 1 9	68 − 4 64	58¢ − 36¢ 22¢
4.	50, 2, 5, 1 58	3, 3, 3, 3 12	33, 1, 2, 1 37	5 + 33 38	37¢ − 24¢ 13¢

◆**Extra Examples.** Work Set 39.

Pupil's Objectives

(a) To have written practice on A. and S. facts, sums through 13, and also on other recently-taught addition and subtraction skills; and (b) to identify sources of error in order to prepare for the chapter testing program in the near future.

Pre-Book Lesson

Review quickly the addends-sum relationship. Write on the chalkboard examples similar to those below. See if pupils can answer questions about how we find the unknown addend and how we find the sum—that is, what is given and what must we do with it.

13 sum	8 addend
− 5 addend	+ 5 addend
addend	sum

Using the Text Page

• Review the use of folded-paper technique for writing answers. Tell the children that when they correctly complete the work at the top of the page, they will be showing A. and S. facts. Caution them to be sure to watch the operation sign carefully. Encourage them to work quickly, but not at the expense of accuracy.

• You may wish to use the written practice at the bottom of the page for two assignments. Or you may decide that only those pupils who have difficulty with the examples in columns a–d in rows 1 and 2 need to do those examples in columns e–i; and similarly for Ex. a–c in rows 3 and 4.

Individualizing Instruction

• Examine all written work for this page for evidences of (a) weakness in knowledge of facts with sums through 13; (b) weakness in computation; and (c) confusion when using combined addition and subtraction practice.

• *All pupils* could write *n*-examples for problems 1–10 on page 82. You may wish to assign this kind of work to part of the class only.

• Assign Extra Examples Set 39, as necessary.

NOTES

Pupil's Objective

To review *making a 10* as a way of helping to think answers for A. and S. facts with sums greater than 10.

Background

The *Mathematics We Need* program has stressed the importance of offering pupils a variety of ways to arrive at complete mastery of a concept. Learning the basic A. and S. facts has been enhanced through the use of sets of objects in order to *see* the fact, showing sets of related facts, using known facts in order to find unknown facts, and *making a 10* in order to think answers to facts with sums greater than 10. Many teachers have never used this latter procedure; consequently, they do not understand it and find it confusing. The idea of *compensation*, discussed on Teaching Page 36, is fundamental to it.

When we add, compensation allows us to add a number to one of the addends and subtract the same number from the other addend without affecting the sum. Compensation is accomplished, in this instance, by renaming the lesser of the two addends in such a way as to be able to "make 10" out of the other addend. This procedure was again used on page 75 of the pupil's book.

In making the greater addend 10 by using compensation, we are really making use of the Identity Property for Addition. In thinking, "$9 + 6 = (9 + 1) + 5$," we have added 1 to one addend $(9 + 1)$ and subtracted 1 from the other addend $(6 - 1)$. As the renaming $9 + 6 + 1 - 1$ shows, we have really added 0, the Identity for Addition, because $1 - 1 = 0$.

When subtracting, however, compensation makes it possible to subtract an equal amount from both the sum and the known addend without affecting the difference between those two numbers. For example, for $15 - 8$, 15 is renamed $(10 + 5)$. 8 is subtracted from the 10, and 8 is subtracted from the known addend. $(10 - 8 + 5) - (8 - 8) = (2 + 5) - 0 = 7 - 0 = 7$. Thus, $15 - 8 = 7$.

The subtraction may be done without renaming. By using compensation, the number which will change the sum to 10 is subtracted from both the sum and the known addend. $[(15 - 5) - (8 - 5)] = [10 - 3] = 7$

Pre-Book Lesson

• Review *making a 10* as a way of helping to think the answer for an addition fact. Make a dot picture on the chalkboard to show 8 and 6. Then show a set of 10 by drawing a ring around the 8 dots and 2 more dots from the 6. In effect, 2 dots are taken from the 6 dots and put with the 8 dots. The total is not changed, but one part is decreased and the other increased in order to make a 10 and some ones. Show this with numerals: $8 + 6 = 8 + (2 + 4) = (8 + 2) + 4$.

• On the chalkboard, write the sentence $15 - 8 = n$. Show 15 objects on the flannel board. Point out that, in subtracting from a teen number, the greater number (sum) is renamed. Rearrange the 15 objects on the flannel board as 10 and 5 more. Then, remove 8 objects from the group of 10. The flannel board will show 2 objects and 5 objects, or 7 objects. From this demonstration, pupils will see that the answer to $15 - 8$ is 7.

Using the Text Page

Though this page was planned for written work, you may wish to use part of the material for oral work to be done at the chalkboard.

Individualizing Instruction

• Perhaps you will want to have the *more capable children* write the addition and subtraction examples in the following ways:

Addition	*Subtraction*
$n = 8 + 5$	$n = 16 - 7$
$n = 8 + (2 + 3)$	$n = (6 + 10) - 7$
$n = (8 + 2) + 3$	$n = 6 + (10 - 7)$
$n = 10 + 3$	$n = 6 + 3$
$n = 13$	$n = 9$

• The *slower learners* will probably need careful supervision while they complete the written work. Walk among the children to check on their written work. Ask questions of individuals to determine their understanding.

• Assign Extra Examples Set 40 as it is needed

Using 10 in A. and S.

Use of Associative Property [W]

You know how to use 10 as a helper for some A. facts. On this page, you will find how to use 10 as a helper for some S. facts, as well as using 10 for some A. facts.

Addition. First make the greater addend 10.

$9 + 6 = n$ $\quad (9 + 1) + 5 = 15$, so $9 + 6 = 15$

$7 + 8 = n$ $\quad 5 + (2 + 8) = 15$, so $7 + 8 = 15$

Copy Ex. 1–15. Find each sum by first making the greater addend 10.

	(8+2)+3		(7+3)+2		(8+2)+4		3+(2+8)		(9+1)+4	
1.	8 + 5	13	4.	7 + 5	12	7.	8 + 6	14	10.	5 + 8	13	13.	9 + 5	14
	(9+1)+3		(7+3)+3		(9+1)+6		(9+1)+2		1+(2+8)	
2.	9 + 4	13	5.	7 + 6	13	8.	9 + 7	16	11.	9 + 3	12	14.	3 + 8	11
	4+(2+8)		2+(2+8)		3+(3+7)		(7+3)+4		4+(1+9)	
3.	6 + 8	14	6.	4 + 8	12	9.	6 + 7	13	12.	7 + 7	14	15.	5 + 9	14

Subtraction. First subtract from 10. For $15 - 8 = n$, think of 15 as 10 and 5. Make marks: || || || || || and || || |. For $10 - 8$, cover 8 of the 10 marks. Are 2 and 5, or 7, marks left? For $15 - 8 = n$, we can write,

$(10 - 8) + 5 = 7$, so $15 - 8 = 7$.

Copy Ex. 16–35. Find each answer by first subtracting from 10. Write your work for Ex. 16 like this:

$13 - 9 = n$ $\quad (10 - 9) + 3 = 4$, so $13 - 9 = 4$.

	(10−9)+3		(10−6)+5		(10−7)+5		(10−9)+1		(10−6)+2	
16.	13 − 9	4	20.	15 − 6	9	24.	15 − 7	8	28.	11 − 9	2	32.	12 − 6	6
	(10−5)+4		(10−6)+4		(10−8)+3		(10−7)+2		(10−8)+1	
17.	14 − 5	9	21.	14 − 6	8	25.	13 − 8	5	29.	12 − 7	5	33.	11 − 8	3
	(10−6)+3		(10−7)+1		(10−9)+5		(10−7)+4		(10−7)+3	
18.	13 − 6	7	22.	11 − 7	4	26.	15 − 9	6	30.	14 − 7	7	34.	13 − 7	6
	(10−9)+4		(10−5)+3		(10−8)+4		(10−8)+2		(10−9)+2	
19.	14 − 9	5	23.	13 − 5	8	27.	14 − 8	6	31.	12 − 8	4	35.	12 − 9	3

◆ **Extra Examples.** Work Set 40 by using 10.

* Another possible plan is to defer subtracting from 10 until last:
$15 - 8 = (15 - 5) - (8 - 5) = 10 - 3 = 7$.

Facts with the Sum 14 or 15

Addends-sum relationship [W]

Copy and finish Ex. 1–6. For help, use the numbers for two parts of sets of 13. For Ex. 1 *think* "13; 7 and 6, so 14; 7 and 7." Check, using a number-line picture.

1. 14; 7 and n (7)

2. 15; n (7) and 8

3. 14; n (6) and 8

4. 15; 9 and n (6)

5. 15; 7 and n (8)

6. 14; n (5) and 9

7. Show the set of A. and S. facts about each of these:

a. 14; 6 and 8

b. 14; 7 and 7

c. 15; 8 and 7

d. 14; 5 and 9

e. 15; 6 and 9

f. 15; 7 and 8

See bottom margin for help.

Copy and finish Ex. 8–12. For each example, show the other facts in that same set of A. and S. facts. See bottom margin for help.

8. $5 + 9 = 14$

9. $14 - 7 = 7$

10. $15 - 6 = 9$

11. $14 - 8 = 6$

12. $15 - 9 = 6$

13–16. Make tables: A. Facts with Sum 14; S. Facts with Sum 14; A. Facts with Sum 15; S. Facts with Sum 15. See bottom margin for tables.

Copy in columns and add.

17. 30, 3, 4, 2 (39)

18. 0, 5, 3, 5 (13)

19. 4¢, 2¢, 0, 9¢ (15¢, 47¢)

20. 3, 4, 2, 6 (15)

21. 31, 4, 2, 0 (37)

22. 40¢, 4¢, 2¢, 1¢

For rows 23–25, copy each example showing in place of n the unknown sum or addend it stands for.

	a	b	c	d
23.	$n = 8 + 7$ (15)	$n - 9 = 5$ (14)	$15 - 6 = n$ (9)	$13 - n = 6$ (7)
24.	$14 - n = 7$ (7)	$6 + 9 = n$ (15)	$14 - n = 8$ (6)	$15 - 8 = n$ (7)
25.	$n - 6 = 8$ (14)	$n = 13 - 8$ (5)	$n - 9 = 6$ (15)	$n - 7 = 7$ (14)

13. 5+9=14, 6+8=14, 7+7=14, 8+6=14, 9+5=14

14. 14−5=9, 14−6=8, 14−7=7, 14−8=6, 14−9=5

15. 6+9=15, 7+8=15, 8+7=15, 9+6=15

16. 15−6=9, 15−7=8, 15−8=7, 15−9=6

Teaching Pages 86 and 87

Pupil's Objectives

(a) To isolate and begin to learn the numbers for the pairs of parts when the number for the total is 14 or 15 and to learn the associated sets of A. and S. facts with sum 14 or 15; (b) to begin to use the facts with sums 14 and 15 in 3- and 4-addend addition; (c) to work *n*-sentences with the A. and S. facts having sums 14 and 15; and (d) to have written practice on all the A. and S. facts through those with sum 15.

Background

Once again, you are reminded that this is an exploratory lesson for discovering relationships and facts. Do not press for mastery of these facts at this time.

Making Tables of Parts and of A. and S. Facts should lead to an important generalization. *As the sums above 9 get greater, they have fewer A. facts.* Below are the A. facts with sums 11–15:

Tables of A. Facts

Sum 11	*Sum 12*	*Sum 13*	*Sum 14*	*Sum 15*
2 + 9	3 + 9	4 + 9	5 + 9	6 + 9
3 + 8	4 + 8	5 + 8	6 + 8	7 + 8
4 + 7	5 + 7	6 + 7	7 + 7	8 + 7
5 + 6	6 + 6	7 + 6	8 + 6	9 + 6
6 + 5	7 + 5	8 + 5	9 + 5	
7 + 4	8 + 4	9 + 4		
8 + 3	9 + 3			
9 + 2				

Many children are under the impression that when a number is greater, it will have more basic facts associated with it. Actually, just the opposite is the case for numbers greater than 9. Furthermore, if the Commutative Property of Addition is fully understood, a child needs to learn only approximately one half of the facts listed for the sums above. The four A. facts listed under *Sum 15*, for example, result from but two pairs of parts of a set with 15 members.

Pre-Book Lesson

• Use objects whereby children may find the numbers for pairs of parts of a set of 14 members (7 and 7; 8 and 6; 9 and 5) and tell the set of A. and S. facts.

• Follow the same plan for discovering the numbers for pairs of parts of a set of 15 members (8 and 7; 9 and 6).

• Use a number-line picture for "seeing" and confirming the numbers for parts of sets of 14 members and 15 members.

• On the board write the Tables of A. Facts with sums 13, 14, and 15, as shown in the Background section. Use the tables as the basis for a discussion about the relationship between a sum and the number of A. facts for that sum. Also, elicit statements about the relationship between pairs of parts of two different sets (i.e., 7 and 7 are numbers for a pair of parts of a set of 14. Therefore, 6 and 7 would be numbers for a pair of parts of a set of 13; and 8 and 7 would be numbers for a pair of parts of a set of 15.).

Using the Text Pages

• Any part or all of page 86 could be developed orally. If the work is to be written, you may want to make use of dittoed forms for Tables of Parts and Tables of A. and S. Facts (see Teaching Page 79). The *more capable children* should be able to do the work without any difficulty. It may be necessary to prepare the *slower learners* for one or more of the written activities by using an oral presentation. Permit the *slower learners* to draw dot pictures, refer to the Tables of Parts and Tables of A. and S. Facts, and use the technique of making a 10 in working Ex. 1–22. However, encourage them to use their knowledge of related parts, sets of A. and S. facts with sums 14 and 15, and using a 10 as a basis for completing the *n*-sentences in Ex. 23–25.

• Rows 1–5 on page 87 provide practice on the A. and S. facts with sums as great as 15 and may be considered as a progress test. Emphasize the importance of writing the answers as neatly as possible on the folded paper.

Individualizing Instruction

• Read the answers to the examples in rows 1–5 at the top of page 87 while each child checks his

own paper. Try to determine the source of error for each child. (Did not know the fact. Was confused by practice with mixed operation signs. Carelessness.)

- Have *all pupils* show the set of A. and S. facts for each example missed in rows 1–5. For some children this may be an overwhelming task. Let common sense dictate how many sets should be written.

- Have *more capable children* make number-fact cards (see Teaching Page 76) for all facts with sums 14 and 15. *All pupils* should now extend their numeral cards to 15.

- You may wish to assign the enrichment material at the bottom of page 87 to the pupils who had no difficulty with the A. and S. facts at the top of the page.

- Use Extra Problems Set 4 with *all pupils*.

NOTES

*Now a quick test on mixed A. and S. facts having sums as great as 15!

*Finding Unknown Sums and Addends

A. and S. facts, sums as great as 15 [W]

Write answers on folded paper.

	a	b	c	d	e	f	g	h	i	j	k
1.	13	9	15	12	5	7	14	5	13	6	11
	−8	+4	−6	−9	+8	+7	−9	+7	−6	+6	−5
	5	13	9	3	13	14	5	12	7	12	6
2.	6	13	11	8	14	5	7	6	12	12	7
	+9	−9	−7	+5	−7	+6	+8	+8	−5	−4	+5
	15	4	4	13	7	11	15	14	7	8	12
3.	6	15	3	12	8	3	12	14	6	4	2
	+5	−7	+8	−6	+6	+9	−7	−5	+7	+6	+9
	11	8	11	6	14	12	5	9	13	10	11
4.	11	9	14	11	15	5	4	9	4	13	7
	−9	+5	−6	−6	−8	+9	+7	+6	+8	−4	+4
	2	14	8	5	7	14	11	15	12	9	11
5.	14	12	8	13	7	4	11	15	13	8	11
	−8	−8	+7	−5	+6	+9	−8	−9	−7	+4	−4
	6	4	15	8	13	13	3	6	6	12	7

Do You Like to Try New Things?

Enrichment [W]

Make *A.* and *S.* problems for other children to work.

1. Ned worked 13 problems. Sue worked 10. *S.*
2. Joe has 15 red marbles. Sam has 20. *A.*
3. All but 6 of 36 blackbirds flew away. *S.*
4. Of 24 dresses, 12 are new. The others are old. *S.*
5. In two baskets there are 30 eggs and 46 eggs. *A.*
6. Three of 36 glasses were broken. *S.*
7. We have 15 bottles of milk. We need 25. *S.*

◆ **Extra Problems.** Work Set 4.

More work with an A. and S. chart! Pupils will now have had experiences with all the basic A. and S. facts.

Facts with the Sum 16 or 17 or 18

Chart for A. and S. facts; sums as great as 18 [W]

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Use the chart to find addends of sums, as you used the chart on page 33.

1. Find 16. Find the addends (use the blue column and the red row). Show the set of A. and S. facts.

8+8=16 16−8=8

2. Show the set of A. and S. facts for

a. 16.
7+9=16 16−7=9
9+7=16 16−9=7

b. 17.
8+9=17 17−8=9
9+8=17 17−9=8

c. 18.
9+9=18
18+9=9

Copy and finish Ex. 3–7. To go with each fact, show the others in the same set of A. and S. facts. See the bottom margin for help.

3. $\begin{array}{r} 17 \\ -9 \\ \hline 8 \end{array}$

4. $\begin{array}{r} 9 \\ +9 \\ \hline 18 \end{array}$

5. $\begin{array}{r} 16 \\ -7 \\ \hline 9 \end{array}$

6. $\begin{array}{r} 16 \\ -8 \\ \hline 8 \end{array}$

7. $\begin{array}{r} 9 \\ +8 \\ \hline 17 \end{array}$

8. Make tables for A. facts and for S. facts with sum

a. 16.
7+9=16 16−7=9
8+8=16 16−8=8
9+7=16 16−9=7

b. 17.
8+9=17 17−8=9
9+8=17 17−9=8

c. 18.
9+9=18 18−9=9

Teaching Pages 88 and 89

Pupil's Objectives

(a) To identify and start learning the A. and S. facts with sums 16, 17, and 18; (b) to use *n*-sentences with the newly-learned facts; and (c) to *see* all of the A. and S. facts organized in a chart.

Background

The chart on page 88 is one way of summarizing the many relationships between sums and addends as well as pointing up the inverse relationship between addition and subtraction.

With a *column of numerals for addends down the left side of the chart* and a *row of numerals for addends across the top of the chart*, the sum of any two addends can be shown in the appropriate square in the body of the chart. When one addend and a sum are known, the other addend can be determined. Or, when two addends are known, the sum can be found.

Zero was not included as an addend in the chart on page 33 because the principles involved in the 0-facts had not been taught at that time. However, 0 is included in the chart on page 88. Though 0 was not treated formally in the pupil's book with the set of facts for each sum, three important generalizations involving 0 were covered. These three generalizations are based on the use of the Identity Element for Addition (zero):

- When one of two numbers is 0, their sum equals the other number.
- Any number minus 0 equals that number.
- Any number minus the same number equals 0.

A chart of the type shown on page 88 clearly reveals the number of facts associated with each sum. In the lesson on pages 83 and 86 the children explored the number of facts with sums 13, 14, and 15. This type of work should be extended now for the facts with sums 16, 17, and 18. For these three sums there are only six facts, 3 with sum 16, 2 with sum 17, and 1 with sum 18.

For your benefit, summaries of all A. facts and all S. facts with sums through 18 (not including the 0-facts) appear at the end of the suggestions for teaching pages 88 and 89.

Teacher's Preparation

On the chalkboard draw an empty chart of the shape of the one on page 88. Each row and column should be about 3 inches in width. Use blue chalk to show the column of addends on the left side of the chart. Use red chalk to show the row of addends across the top. (You may want to make a more permanent chart by drawing it on tagboard or similar material.)

Pre-Book Lesson

- Use the chart on the chalkboard to review the sums and their pairs of addends studied so far. Questions similar to these should be asked to guide the review:

Two addends I am thinking of are 9 and 2. Who knows where to show the sum on the chart? (Have a pupil write the numeral.)

Is there another place on the chart where we can show the sum for 9 and 2? (Emphasize the Commutative Property—that 2 and 9 have the same sum as 9 and 2.)

Now, I am thinking of the sum 13. One of its addends is 7. Where is the 13 shown on the chart? What is the other addend in the pair?

Is there another place on the chart where we can show the sum 13 for these two addends?

- Continue this type of questioning and the showing of sums until all facts for 13, 14, and 15 have been shown on the chart. Permit children to tell two addends and then ask another child to show the sum in the correct place.
- Point to an empty square where 16 belongs. Ask, "What are the two addends for the sum that should be shown here?" Show the addends on the chalkboard. Do the same for the other pair of addends for 16. Be sure to bring out these points in the discussion about addends with sum 16:

a. If we know two addends and the sum, we can think out the sum of related addends.

b. We can make a 10 to find the sum of two addends.

c. There are only two pairs of parts of a set of 16 members that have numbers greater than 0 and less than 10, so there are only two sets of A. and S. facts with sum 16.

- Use the same type of discussion and analysis

for the sums 17 and 18. Only one set of A. and S. facts will be found for each of these sums. Be sure to have the pupils say all the sets of A. and S. facts for 17 and 18. They should be shown on the chalkboard, also.

Using the Text Pages

- If you have the dittoed forms left from previous lessons (see Teaching Page 79), you may want to use them for the completion of Ex. 8.
- By this time, pupils should be able to write the work easily for rows 9–11. Remind pupils that *n* may stand sometimes for a sum and sometimes for an unknown addend in both A. and S. number sentences.
- Check the answers for rows 12 and 13 to find sources of error in A. and S. facts with sums 16, 17, and 18.
- The oral work for showing how to use a 10 in finding answers for rows 12 and 13 should prove to be profitable for *all pupils*.
- If the chart on the chalkboard does not show all of the sums, ask one of the *more capable children* to fill in the empty squares before the work at the bottom of page 89 is begun. Encourage pupils to refer to the chart on the chalkboard as well as the one in the book to assist with the oral work. Discuss each question. Try to determine the depth of understanding possessed by different children as they discover patterns shown in the chart. Question 2 is intended to reveal informally the nature of zero as the Identity Element for Addition. Because of question 4, some children may notice the relationship between odd-number sums and their addends. Question 5 should bring out the same relationship for even-number sums.

Individualizing Instruction

- *More capable children* may make number-fact cards for facts with sums 16, 17, 18 (see Teaching Page 76).
- *All pupils* now may make cards to extend their sets of numeral cards through 18.
- Use *Show the Answer* with *slower learners*.
- *More capable children* may rewrite each of the examples in rows 9–11 on page 89 to show *n* in three different positions. For Ex. 9a, they would write, "$16 - 7 = n$, $16 - n = 9$, $n - 7 = 9$."

The 81 Addition Facts

1	1	1	1	1	1	1	1	1
+ 1	+ 2	+ 3	+ 4	+ 5	+ 6	+ 7	+ 8	+ 9
2	3	4	5	6	7	8	9	10
2	2	2	2	2	2	2	2	2
+ 1	+ 2	+ 3	+ 4	+ 5	+ 6	+ 7	+ 8	+ 9
3	4	5	6	7	8	9	10	11
3	3	3	3	3	3	3	3	3
+ 1	+ 2	+ 3	+ 4	+ 5	+ 6	+ 7	+ 8	+ 9
4	5	6	7	8	9	10	11	12
4	4	4	4	4	4	4	4	4
+ 1	+ 2	+ 3	+ 4	+ 5	+ 6	+ 7	+ 8	+ 9
5	6	7	8	9	10	11	12	13
5	5	5	5	5	5	5	5	5
+ 1	+ 2	+ 3	+ 4	+ 5	+ 6	+ 7	+ 8	+ 9
6	7	8	9	10	11	12	13	14
6	6	6	6	6	6	6	6	6
+ 1	+ 2	+ 3	+ 4	+ 5	+ 6	+ 7	+ 8	+ 9
7	8	9	10	11	12	13	14	15
7	7	7	7	7	7	7	7	7
+ 1	+ 2	+ 3	+ 4	+ 5	+ 6	+ 7	+ 8	+ 9
8	9	10	11	12	13	14	15	16
8	8	8	8	8	8	8	8	8
+ 1	+ 2	+ 3	+ 4	+ 5	+ 6	+ 7	+ 8	+ 9
9	10	11	12	13	14	15	16	17
9	9	9	9	9	9	9	9	9
+ 1	+ 2	+ 3	+ 4	+ 5	+ 6	+ 7	+ 8	+ 9
10	11	12	13	14	15	16	17	18

The 81 Subtraction Facts

2	3	4	5	6	7	8	9	10
− 1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	− 1
1	2	3	4	5	6	7	8	9
3	4	5	6	7	8	9	10	11
− 2	− 2	− 2	− 2	− 2	− 2	− 2	− 2	− 2
1	2	3	4	5	6	7	8	9
4	5	6	7	8	9	10	11	12
− 3	− 3	− 3	− 3	− 3	− 3	− 3	− 3	− 3
1	2	3	4	5	6	7	8	9
5	6	7	8	9	10	11	12	13
− 4	− 4	− 4	− 4	− 4	− 4	− 4	− 4	− 4
1	2	3	4	5	6	7	8	9
6	7	8	9	10	11	12	13	14
− 5	− 5	− 5	− 5	− 5	− 5	− 5	− 5	− 5
1	2	3	4	5	6	7	8	9
7	8	9	10	11	12	13	14	15
− 6	− 6	− 6	− 6	− 6	− 6	− 6	− 6	− 6
1	2	3	4	5	6	7	8	9
8	9	10	11	12	13	14	15	16
− 7	− 7	− 7	− 7	− 7	− 7	− 7	− 7	− 7
1	2	3	4	5	6	7	8	9
9	10	11	12	13	14	15	16	17
− 8	− 8	− 8	− 8	− 8	− 8	− 8	− 8	− 8
1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
− 9	− 9	− 9	− 9	− 9	− 9	− 9	− 9	− 9
1	2	3	4	5	6	7	8	9

Copy each example, showing the number for n.

	a	b	c	d
9.	$16 - 7 = n$ (9)	$15 - n = 8$ (7)	16 minus n = 9 (7)	$18 - n = 9$ (9)
10.	$15 - 6 = n$ (9)	$n = 9 + 8$ (17)	n less 7 = 8 (15)	$16 - n = 8$ (8)
11.	$n + 8 = 16$ (8)	$17 - 9 = n$ (8)	9 plus 7 = n (16)	$17 - 8 = n$ (9)

For rows 12–13 write answers on folded paper.

	a	b	c	d	e	f	g	h	i	j	k
12.	8	17	9	15	7	14	8	16	9	15	9
	+8 (16)	−8 (9)	+9 (18)	−6 (9)	+9 (16)	−5 (9)	+9 (17)	−8 (8)	+6 (15)	−7 (8)	+5 (14)
13.	16	15	9	17	8	9	16	18	13	8	13
	−9 (7)	−8 (7)	+7 (16)	−9 (8)	+7 (15)	+8 (17)	−7 (9)	−9 (9)	−8 (5)	+6 (14)	−4 (9)

[O]

Show how to use 10 for answers in rows 12–13.

*Finding Patterns from a Chart

[O]

1. The chart, page 88, shows addends of sums 0 through (18).

2. The numerals in the red row (0 row) and the blue column (0 column) are the same as the ones in what row and what column?

3. Find 11 in the chart. See where the other 11's are. Why do the 11's go up the chart as they do?
As one addend decreases by one, the other addend increases by 1.

4. What kind of numbers (odd numbers) are shown in the line of numerals from 17 to 1? Is there another line of numerals like this? Yes

5. What kind of numbers (even numbers) are shown in the line of numerals from 18 to 0? Is there only one line of numerals like this? Yes

* If pupils can find patterned arrangements in tabled material, they have made an important step in arithmetic understanding.

* A possible quick test on mixed A. and S. facts having sums as great as 18!

* Finding Unknown Sums and Addends

A. and S. facts, sums as great as 18 [W]

For rows 1–4 write the answers on folded paper.

	a	b	c	d	e	f
1.	14 − 7 7	15 − 6 9	13 − 5 8	4 + 9 13	3 + 9 12	11 − 8 3
2.	11 − 6 5	8 + 4 12	5 + 7 12	5 + 8 13	14 − 9 5	2 + 9 11
3.	6 + 8 14	12 − 9 3	8 + 3 11	12 − 5 7	14 − 8 6	7 + 7 14
4.	6 + 5 11	7 + 6 13	13 − 7 6	12 − 4 8	12 − 6 6	5 + 9 14

If you missed some answers for rows 1–4, study the facts. Then try rows 5–7.

5.	8 + 5 13	14 − 6 8	16 − 7 9	9 + 4 13	11 − 4 7	11 − 5 6
6.	13 − 9 4	7 + 4 11	7 + 8 15	15 − 9 6	11 − 2 9	8 + 9 17
7.	9 + 5 14	12 − 3 9	15 − 7 8	12 − 8 4	12 − 7 5	6 + 7 13

Greater Than? Less Than?

Inequalities; symbols [W]

First, do the work in the ( )'s. Then copy each example writing "is greater than" or "is less than" where you see _?_.

1. (60 − 10) is greater than > 20
2. (7 + 72) is greater than > 78
3. (42 + 26) is greater than > 58
4. (89 − 34) is greater than > 51
5. (79 − 3) is less than < 78
6. (87 − 27) is less than < 64
7. (32 + 45) is greater than > 67
8. (5 + 30) is greater than > 32
9. (45 + 34) is greater than > 74
10. (58 − 36) is less than < 24
11. (89 − 65) is greater than > 22
12. (49 − 9) is less than < 41

** Just as you may write = for is equal to, you may write > for is greater than and < for is less than. Copy Ex. 1–12 again, writing > or < instead of _?_.

** Now, according to the developmental plan, pupils are introduced to the > and < symbols. It may help to indicate that the sharp end (the "small" part of the symbol) is adjacent to the numeral for the number that is less.

Teaching Page 90

Pupil's Objectives

(a) To have practice in writing the answers to complete A. and S. facts, sums through 18, in preparation for the end-of-chapter tests; and (b) to begin to use the symbols of inequality, $>$ and $<$.

Background

The material at the top of page 90 may well be used as a progress test. The results may be used to diagnose difficulties of the class as well as of each individual in the class.

Class difficulties can be diagnosed by making a record of the number of children in the class who missed each item on the test. This type of *item analysis* identifies items needing further attention by the entire class or groups within the class.

Individual diagnosis can be accomplished by categorizing the items in the test into three or four classifications. In this test, for example, the items may be classified according to the sum involved in each case. Categories may be 11–12, 13–15, and 16–18. An individual's errors may accumulate in one of the categories, thus indicating a specific source of difficulty.

Pre-Book Lesson

• Review briefly the following ways of thinking answers for A. and S. facts:

a. Knowledge of sets of facts using the numbers for a total and a pair of its parts.

b. Relating an unknown fact to a known fact.

c. 'Making 10' and 'subtracting from 10' to help with A. and S. facts having sums greater than 10.

• At the chalkboard, write "(50 − 6) _ ? _ 40." See if pupils remember how to begin work when an example contains parentheses. Most of them will probably remember (see Teaching Page 57) that they must do first the work suggested within parentheses. Then tell them that for "_ ? _" they are first to use the words "is greater than," "is less than" or "is equal to." Then they are to copy Ex. 1–12 again, writing $>$ instead of *is greater than* and $<$ instead of *is less than*. You may need to point out the difference between the two symbols.

Using the Text Page

• Have pupils write answers on folded paper for rows 1–4.

• Rows 5–7 should not be attempted until the child knows his source of errors and has made sure of the facts through use of one of the suggestions in a–c under Individualizing Instruction. This written work well may be considered a progress test on the A. and S. facts.

• After similar work in the Pre-Book Lesson, the children should be able to do the work at the bottom of page 90 without difficulty.

Individualizing Instruction

• Organized review, by groups, on the basis of individual and class diagnosis should make use of one or all of the following:

a. Be sure that pupils can tell the set of A. and S. facts for any given total and two of its parts.

b. Give extra practice on the facts having equal addends. Ask pupils to tell which of those facts help with learning other facts, like $5 + 4 = 9$, $8 + 9 = 17$, $3 + 2 = 5$, $16 - 9 = 7$, and so on.

c. Practice renaming numbers to make a 10. For example, write "8" on the chalkboard. Mention any number from 3 to 8 and ask children how to rename it in order to use it with 8 to make a 10.

• Suggest that *all pupils* who have reasonable command of the A. and S. facts again write answers on folded paper for rows 12 and 13 on page 89.

• *More capable children* may rewrite Ex. 1, 3, 4, 5, 7, 9, 11 (bottom of page) to use "is greater than" instead of "is less than" and "is less than" instead of "is greater than." For Ex. 1, they would write, "20 is less than (60 − 10)."

Teaching Page 91

Pupil's Objective

To work the end-of-chapter test on problem-solving.

New Word

cloth

Background

Beginning on page 91 and extending through page 93 are the regular end-of-chapter tests. These are similar in nature to those given for the previous chapter (see Teaching Page 49 and Teaching Pages 50 and 51).

Pre-Book Lesson

You may wish to set the stage for this test by discussing with the children what they have learned about problem-solving. Review particularly the situations requiring the use of subtraction.

Using the Text Page

After administering this test it might be desirable to discuss it item by item. Ask appropriate questions to check understandings and use illustrations and demonstrations to clarify meanings.

For suggestions about scoring pupils' papers, see Teaching Pages 50 and 51, and Table of Per Cents on Teaching Pages 92 and 93.

NOTES

Can You Solve Problems?

Problem Test 2

For each problem write your work to find the answer.

1. To make a boat, Joe, Bill, and Jack are using 5 big boxes and 10 little boxes, or how many boxes in all? 15 boxes

2. They made a sail from 6 red pieces of cloth, 1 blue piece, and 7 white pieces. They used how many pieces of cloth for the sail? 14 pieces

3. Joe sawed 18 boards. Bill sawed 12 boards. How many more boards did Joe saw than Bill? 6 boards

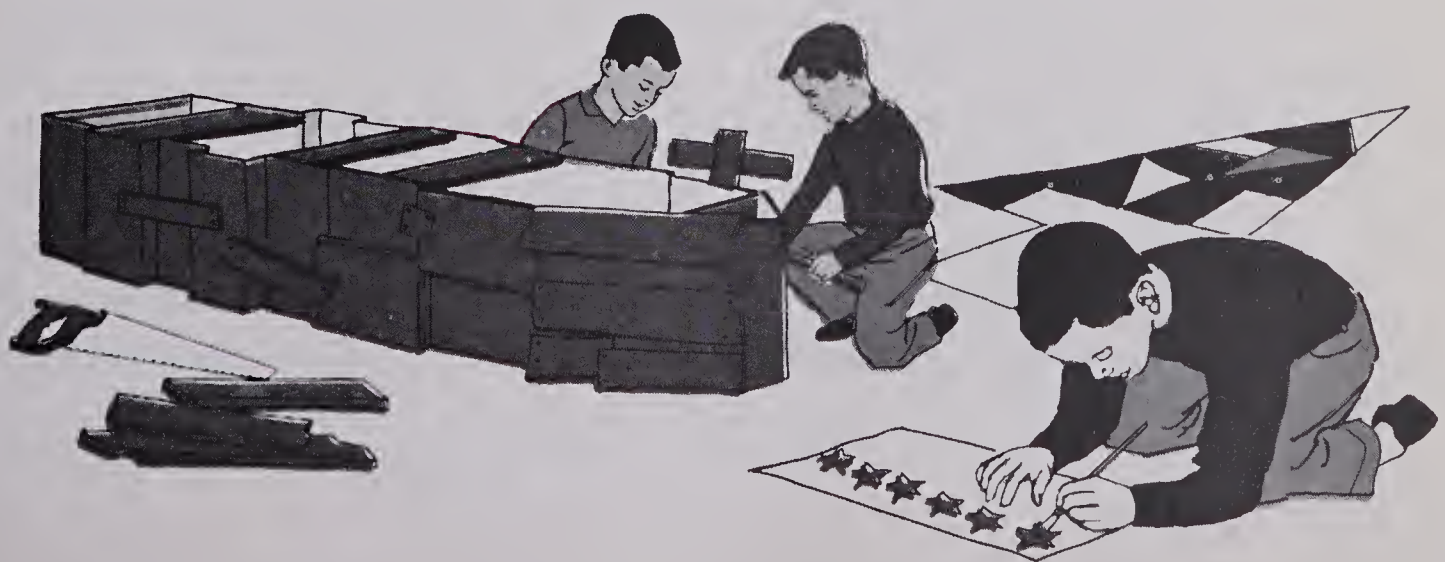
4. Jack brought 25 long nails. Bill brought 21. That made how many long nails in all? 46 nails

5. For rope, Jack spent 75¢ of his 88¢. How much money did he have left? 13¢

6. They cut the rope into 18 pieces. How many pieces did they use if there were 3 pieces left? 15 pieces

7. The picture shows the stars Bill painted. To have 12 stars, how many more stars must be painted? 6 stars

8. The boys have worked 22 hours. Bill thinks it will take 20 more hours to finish. That will be how many hours in all? 42 hours



* If pupils who have trouble do not use Study Pages before practice pages, they may only be getting practice in making the same mistakes.

* Do You Make Mistakes?

Diagnostic Test 2

Write answers on folded paper.

	a	b	c	d	e	Study Pages	Practice Sets
1.	4	16	17	6	15	76, 79, 83, 85, 86, 88	40
	+7 = 11	−9 = 7	−8 = 9	+7 = 13	−6 = 9		
2.	12	9	6	14	3		
	−9 = 3	+9 = 18	+5 = 11	−8 = 6	+8 = 11		
3.	11	52	23	42	60	47, 55	27, 28, 39
	3	4	0	3	2		
	2 = 16	2 = 58	2 = 25	3	4		
				1 = 49	3 = 69		
4.	30	43	5	54	32	52, 53	25, 26, 29, 30
	+60 = 90	+5 = 48	+72 = 77	+10 = 64	+46 = 78		
5.	86	47	60	97	59	58–61, 67, 68	31–38
	−34 = 52	−4 = 43	−30 = 30	−70 = 27	−39 = 20		

Do You Understand?

Test of Information and Meaning 2

Write an *n*-example for each of problems 1–3.

1. Ed spent 9¢. He had 8¢ left. How much did he have at first? $9¢ + 8¢ = n¢$

2. Tom had 8¢. He found 5¢. How much had he then? $8¢ + 5¢ = n¢$

3. Linda has 7¢. Diana has 12¢, or how much more? $12¢ - 7¢ = n¢$

To what clock numeral does the minute hand point

4. at 20 minutes before 2? 8

5. at 6 o'clock? 12

Teaching Pages 92 and 93

Pupil's Objectives

(a) To diagnose difficulties; and (b) to evaluate the achievement other than problem-solving (which has been tested on page 91) on the content of Chapter 2 by examining understandings, and computational skills.

Using the Text Page (page 92, top)

There may be some youngsters who should write the entire example so you can see the source of an error.

Using the Text Pages (pages 92–93)

Be sure the pupils know that they are to write number sentences with n for Ex. 1–3 rather than find the answers to the problems.

Using the Text Page (page 93, bottom)

You may wish to quickly review similar work before administering this test.

Individualizing Instruction

The testing program at the end of each chapter is not merely to seek answers and to award grades, but to measure progress in skills and understandings. By noting strengths and weaknesses, you can help each child as well as improve your own teaching. You should constantly be alert to changing attitudes toward mathematics. Your own attitudes, as well as your instructional procedures, are critical in developing each child's attitudes.

Use the tables below to find the per-cent scores to be entered for the two tests on the individual-record cards (see Teaching Pages 50 and 51).

Table of Per Cents for Chapter 2 Scores

Problem Test		Computation Test					
Score	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*
1	13	1	3	11	37	21	70
2	25	2	7	12	40	22	73
3	38	3	10	13	43	23	77
4	50	4	13	14	47	24	80
5	63	5	17	15	50	25	83
6	75	6	20	16	53	26	87
7	88	7	23	17	57	27	90
8	100	8	27	18	60	28	93
		9	30	19	63	29	97
		10	33	20	67	30	100

Suggestions for Material to Accompany End-of-Chapter 2 Tests

It is normal to find a wide range of achievement and ability in a typical classroom. The tests just administered should indicate sources where additional help or review is needed. Some of the following suggestions may be especially useful for children who do not need review work.

Alternate Uses of Pages in Chapter 2

Page 57. Rewrite each sentence in Ex. 13–21 so that using "is equal to" will make the sentence true. In Ex. 13, $(10 + 8)$ is less than 20. Change any number you wish so that "is equal to" will make the sentence true. Possibilities for this sentence are: $(10 + 10) = 20$; $(12 + 8) = 20$; $(10 + 8) = 18$.

Page 62. Have children rewrite each of the examples in rows 1–5 at the bottom of the page so that n is in three different positions in the sentence. For Ex. 1a, you would write: $n - 9 = 0$; $9 - n = 0$; $9 - 9 = n$.

Pages 66–67. Try rewriting each problem to make it an A. problem if it is now a S. problem and a S. problem if it is now an A. problem:

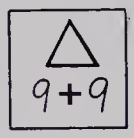
Page 78. Rewrite each problem to make it an A. problem if it is now a S. problem or make it a S. problem if it is now an A. problem.

Page 90. Rewrite each of Ex. 1–12 (bottom of page) so that using "is equal to" will make the sentence true. If the sentence is true as it is written then no change is necessary.

Supplementary Activities

• All pupils may participate in the following game for the purpose of providing drill in a motivating and exciting form.

First, cut 64 cards from tagboard (or similar material) each about 6″ square. Draw a picture of a triangle at the top of 16 of the cards, a picture of a circle at the top of 16 of the cards, a picture of a rectangle at the top of 16 cards, and a picture of a square at the top of 16 cards. Then, on 4 cards (one from each set of 16) show one of the numbers 10 through 25 in renamed form. Rename the number differently on each card. For example, 18 may be renamed (14 + 4), (9 + 9), (19 − 1), and (12 + 6). Each card would look somewhat like the one at the right. Continue in this way until all of the numbers from 10 through 25 have been renamed in four ways.

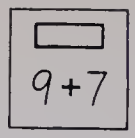


Next, each child makes a 4 × 5 grid like the one shown below. In the top row he draws pictures of the four geometric forms. Inside the other squares he writes the numerals 10 through 25 in any order he wishes. These should not be in a renamed form and they should be placed in random order so that there will be varied experiences.

To play the game, the 64 cards should be shuffled and placed face down on a table. As each card is drawn, it should be held high so all children can see it. Nothing should be said. Each child must then decide what number is named and determine if he has it shown on his grid in the column under the corresponding geometric design.

△	▭	○	□
17	14	11	22
10	25	20	16
21	18	13	24
15	12	23	19

When this card is drawn, the children see that it names 16. In this case 16 must appear in the column under the picture of a rectangle. The pupil who first finds that 16 on his grid appears in the correct column receives a $\frac{1}{2}'' \times \frac{1}{2}''$ card to place in the square. The first child to have four squares covered in any row or column or diagonal wins the game.



• Duplicate copies of a grid similar to the one below for the children to fill in. This is a different way of making the addition chart which they have been studying.

7								
6								
5	6	7	8	9	10	11	12	13	14
4	5	6	7	8	9	10	11	12	13
3	4	5	6	7	8	9	10	11	12
2	3	4	5	6	7	8	9	10	11
1	2	3	4	5	6	7	8	9	10
	1	2	3	4	5	6	7	8	9

In this chart the addends are shown across the bottom and down the left side. This is the first of a series of activities you will find at the end of each chapter dealing with the topic *coordinate geometry*.

Have the children analyze the chart by expressing the location of each sum shown in the chart. By inspection, we see that 7 is shown in several locations in the chart. Have a child describe a particular 7 as follows: "The 7 I am thinking of is shown *over* 5 and *up* 2 in the chart." Another child might say, "The number I am thinking of is shown *over* 7 and *up* 4. What is the number shown?"

It would be a good idea to have a large addition chart drawn on paper or on the chalkboard so that selected children can actually point to the location of the 7 or the 11 just described. The emphasis in this activity should be on the location of a number shown in the chart and a way of expressing its location.

Another use of the chart would be to have children show sums in the chart by giving directions: "Write the numeral that belongs in the square over 4 and up 8 in the chart."

Copy each of Ex. 6 and 7, writing "is greater than" or "is less than" instead of _?_. Copy again using > or <.

6. $(7 + 9)$ _?_ 15 (is greater than; >)

7. $(15 - 7)$ _?_ 17 (is less than; <)

Write only what is missing.

8. Any number minus the same number equals _?_. (0)

9. 1 ft. = _?_ in. (12)

10. 1 wk. and 3 da. = _?_ da. (10)

11. You subtract tens in the same way you subtract _?_. (ones)

12. $42 + 37 = 79$, so $79 - 37 = n$. n = ? (42)

13. To check subtraction, you can _?_ the addends. (add)

14. Use = or ≠. $(5 + 8)$ _?_ $(9 + 7)$ (≠)

15. 29 is between the odd numbers _?_ and _?_. (27; 31)

How Well Can You Compute?

Computation Test 2

Write the answers on folded paper. Check answers.

	a	b	c	d	e	f	g	h
1.	63	80	45	59	78	95	30	17¢
	+5	−50	+42	−9	−66	−75	+64	−9¢
	68	30	87	50	12	20	94	8¢
2.	48	52	15	20	7	96	67	7¢
	−3	+47	−6	+60	+70	−30	−53	+9¢
	45	99	9	80	77	66	14	16¢
3.	16	69	74	23	49	50	85	67¢
	−7	+20	−54	+54	−8	+8	−73	−64¢
	9	89	20	77	41	58	12	3¢

Copy in columns and add. Check your answers.

4. 41, 3, 0, 5 (49)

5. 5, 0, 2, 6 (13)

6. 4, 5, 2, 0 (11)

7. 50, 1, 4, 3 (58)

8. 3¢, 2¢, 1¢, 4¢ (10¢)

9. 20¢, 5¢, 3¢, 1¢ (29¢)

3

Using a Chart

A. and S. problems; making n-examples [O]

Little Beaver wants to be a great hunter like his father. He made a chart to show how many deer, wolves, bears, and foxes his father, Flying Cloud, killed on one trip.

Use the chart. Say the answers for Ex. 1–4.

1. How many animals of each kind did Flying Cloud kill during the trip? 3 deer, 8 wolves, 2 bears, 4 foxes

2. How many more wolves than bears did he kill? 6

3. Did the great hunter, Flying Cloud, kill more wolves than any other kind of animal? Yes

4. Flying Cloud brought the bears and the deer home for his family to eat. How many animals was that? 5 animals

[W]

Write your work for each of Ex. 5–10.

5. How many animals in all did Flying Cloud kill? $3+8+2+4=n$ 17 animals

6. Three other Indians killed 11 more wolves than Flying Cloud did. How many wolves did they kill? $8+11=n$ 19 wolves

Overview — Chapter 3

• In chapter 3, addition using 2-place numerals is extended to renaming of the sum of the ones and carrying a digit to ten's column. Similarly, work in subtraction using 2-place numerals is extended to involve renaming of the sum when the ones cannot be subtracted. The idea of number is extended now to include the set of ordinals through *twentieth*. Work with finding measures of length in this chapter involves inches, feet, yards, miles, with discovery of suitable reference measurements for practical use.

• **Problem-Solving.** Work in problem-solving extends the use of subtraction to include comparing two numbers to find the difference between them. Continued also is the use of data from a chart, writing questions to complete addition and subtraction problems, and the writing of *n*-sentences which show how to work problems. Some work is done also in making pupils aware of tricky words in problems, in judging whether an answer is too great or not great enough, and in working problems orally.

• **Maintenance.** Finding the number for *n* continues to emphasize the addends-sum relationship now in work with 2-place numerals. In work with inequalities, instead of determining only whether a number is greater or less, pupils now must find how much greater or less it is. Thinking other names for numbers has been done casually for some time, but in this chapter, number names are enclosed in parentheses and pupils learn to perform the operation indicated within the parentheses in order to find a simpler number name to use.

Work begun earlier with geometric ideas—paths, simple closed paths, and some polygons—is expanded now to more general use of the names for certain well-known polygons—triangles, rectangles, squares.

• **Enrichment.** The work with parentheses for those pupils who are capable of doing more is expanded to include in a number sentence the use of two parentheses in one of which appears *n* to stand for a number to be operated on. Other work for these pupils involves demonstrating concretely by coins what happens in the renaming of a number in adding or subtracting.

• **Testing.** In addition to the usual end-of-chapter tests—Problem-Solving Test, Diagnostic Test (with suggestions for remedial work), Test of Information and Meaning, and Computation Test—is a mid-unit review or maintenance test titled *Many Things*.

Teaching Pages 94 and 95

Pupil's Objectives

(a) To have more experience in interpreting information recorded in a chart; (b) to differentiate between A. and S. required by problems based on the information in the chart; and (c) to write *n*-examples for problems.

New Words

(page 94) *hunter*, *killed*, *wolves*
(page 95) *arrows*, *geese*

Background

The terminology *number sentence*, *mathematical sentence*, *n-example* or *n-sentence* is used extensively throughout the program. The term *number sentence* can be used any time a mathematical statement is recorded in its symbolic form; i.e., $17 - 8 = 9$ and $17 - n = 9$ are both number sentences. The first of the two sentences above *tells* something, whereas the latter sentence *asks* something. The term *n-example* or *n-sentence* is used to describe any sentence, of this latter type, in which *n* appears. An *n*-sentence is *neither true nor false* and is sometimes called an *open mathematical sentence*. Providing a number for *n* will make it a closed sentence and will determine its truth or falsity.

Pre-Book Lesson

• If any of the suggestions in connection with Teaching Page 81 have materialized for class or

individual activities, use them as a basis for a discussion about keeping a record.

• If you have kept a day-by-day record of weather conditions, holidays, and birthdays for a month on the class calendar, the information can be summarized in a chart somewhat as follows:

October	*Number of Days*
Sunny	_ ? _
Cloudy	_ ? _
Rainy	_ ? _
Holidays	_ ? _
Birthdays	_ ? _

• Perhaps you can make a record on the chalkboard of the different methods of travel by which the children come to school:

Ways of Travel	*Number of Children*
Walking	_ ? _
By bus	_ ? _
By car	_ ? _
By bicycle	_ ? _

• Use a chart you have made and ask, "How many?" for one or more of the categories; make comparisons of categories; using outside information, make problems about one or more of the categories.

• Have pupils make problems using the information in the chart. After a problem has been stated, ask the child to write on the chalkboard an *n*-example for the problem. Remind pupils that an *n*-example may be written in two ways (*n* by itself at the beginning of the sentence or at the end) and it shows how to work the problem.

Using the Text Pages

• As Ex. 1–4 are discussed, emphasize the points developed in the Pre-Book Lesson. Have children make an *n*-example for problem 4.

• First, ask the pupils to work problems 5–10 on their papers. Then, after solving all the problems, have pupils go back and write an *n*-example for each one.

• Before having pupils do the work at the bottom of page 95, write on the board (4 + 7), (5 + 6), (14 − 8), and (48 − 15). Elicit that each is a name for a number. Have pupils tell what number is named by each. Then refer to Ex. 1 in the pupil's book and ask a child to tell another name for (47 + 12).

Remind children that in any number sentence the work in parentheses must be completed before working any other part of the sentence. The written work in Ex. 3 and 4 is similar to that required previously on page 90 but now, instead of writing that one number is greater or less than the other, the pupil must write how much greater or less one number is than the other number.

Individualizing Instruction

• *Slower learners* may need more oral renaming practice for numbers like (38 − 14) and (24 + 53) before doing the written work at the bottom of page 95.

• *More capable children* may write another set of problems, using information from the chart on page 94.

• *All pupils* may work Extra Problems Set 5.

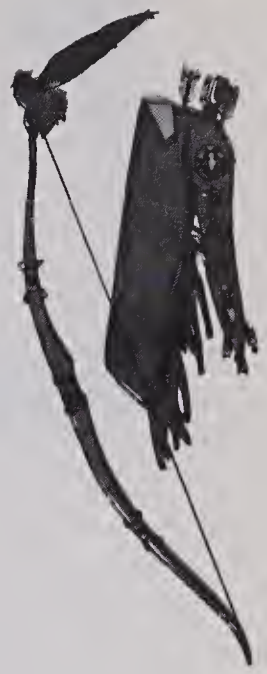
Looking Ahead

In the very near future when adding, children will need to rename the ones and carry a digit to ten's place. As preparation for this skill, you may wish to use the following activities:

• Show with 2-place numerals in a column on the chalkboard 3 or 4 addends the sum of whose ones is greater than 9, as 23 + 14 + 32 + 75. Ask for the sum of the ones and question what should be done to show 14 ones in the answer.

• Pupils should quickly recognize that 14 ones can be renamed 1 ten and 4 ones, so the numeral 4 can be written in one's place and the digit 1 can be *carried* to the ten's column. Note that this gives a precise meaning to *carry*.

• To give practice on the renaming, write on the board several 2-place numerals which can be considered as showing the sums of ones in addition examples. Ask pupils to rename the sum to be tens and ones. Have them use the form, "16 ones = 1 ten and 6 ones."



7. In all, the Indians killed 18 deer on the trip. The chart shows how many Flying Cloud killed. How many deer were killed by other Indians? 15 deer
18 − 3 = n

8. On another trip, the Indians killed 33 ducks and 11 geese. How many more ducks than geese did they kill?
33 − 11 = n 22

9. They used 69 arrows in all. They found 57 of them. How many arrows were lost? 12 arrows
69 − 57 = n

10. Of the 26 Indians on this trip, 3 came home in one canoe. How many used other canoes? 23 Indians
26 − 3 = n

Write *n*-examples for problems 5–10.

♦**Extra Problems.** Work Set 5.

How Much Greater? How Much Less?

Comparisons; inequalities [O]

1. 69 is how much greater than (47 + 12)?
First do the work in ( )'s. 47 + 12 = 59
69 − 59 = 10, so 69 is 10 greater than (47 + 12).

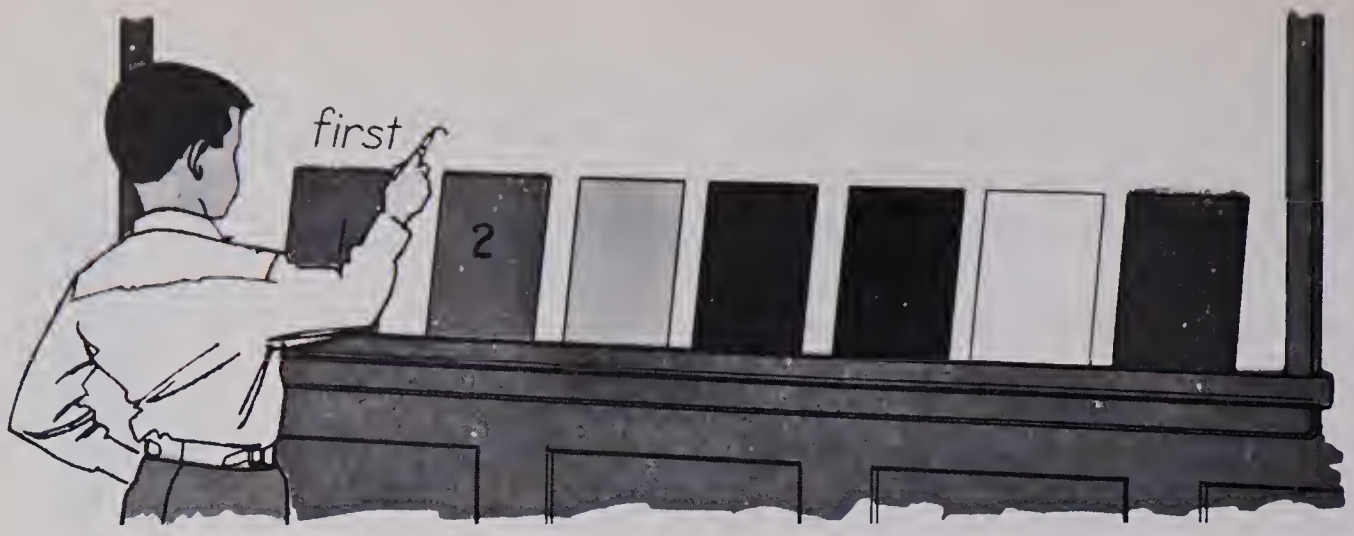
2. (58 − 13) is how much less than 77?
First do the work in ( )'s. 58 − 13 = 45
77 − 45 = 32, so (58 − 13) is 32 less than 77.

[W]

Write your work for Ex. 3–4 as in Ex. 1 or Ex. 2.

3. How much greater is
a. 20 than (77 − 67)? 10
b. (26 + 32) than 35? 23
c. 36 than (57 − 31)? 10
d. (76 − 46) than 10? 20
e. 14 than (97 − 92)? 9

4. How much less is
a. (52 + 15) than 78? 11
b. (69 − 47) than 38? 16
c. 25 than (44 + 24)? 43
d. (69 − 33) than 47? 11
e. 41 than (89 − 33)? 15



Ordinal Numbers

Through tenth [O]

A number may tell how many in a group, or set, as 9 dolls, 27 cents, 4 tens, 0 ones.

A number may mean the order in which a thing comes. Numbers that tell order are called ordinal numbers, or ordinals.

1. Jack placed colored cards in a row at the board.

a. What is the color of the fifth card? ^ black the sixth? ^ white

b. Which card is green? ^ seventh yellow? ^ first pink? ^ third blue? ^ second

* 2. Sue wants to use the red card. This is the _?_ ^ fourth card, or card 4.

3. Joe picked the black card. This is the _?_ ^ fifth card, or card _?_. 5

4. To show order in Ex. 2, you used fourth and you used 4. What two ways did you use in Ex. 3? fifth; 5

5. For Ex. a–i, which tell how many in a set? Which tell about the order in which a thing comes?

a. the sixth example	**d.** Ex. 32	**g.** 15 boys
b. 24 eggs	**e.** page 85	**h.** seat 5
c. the fifth time	**f.** room 10	**i.** 8 birds

Ex. b, g, and i tell how many.
Ex. a, c, d, e, f, and h tell about order.

* Make sure pupils realize that 4th card and card 4 are both names for the ordinal number.

Teaching Pages 96 and 97

Pupil's Objectives

(a) To learn the use of numbers as a means of putting things in order or as a way of identifying objects; and (b) to have practice in adding and subtracting with 1- and 2-place numerals.

New Words

(page 96) *fifth*, *ordinal*, *sixth*
(page 97) *eighth*, *ninth*, *seventh*

Background

When we use numbers to answer the question "How many?" we are using numbers in a *cardinal* sense. When we use numbers to answer the question "Which one?" we are using numbers in an *ordinal* sense.

Children who have used *Mathematics We Need* in previous grades were introduced to the use of ordinal numbers through the extended work accompanying each lesson. This lesson is the first formal presentation of the idea in the grade-3 text.

You will note that the mathematical terms "ordinal numbers" and "ordinals" are used in the child's book. You are encouraged to use the terms frequently, but you may find it useful to use the phrases "Numbers that tell *which ones*" or "Numbers that tell *the order of things*." Grade-level numbers, page numbers, and calendar numbers are all examples of ordinals.

Teacher's Preparation

Have six to ten pieces of construction paper, each a different color, to place in the chalk tray for an activity similar to the activity pictured on page 96.

Write each of the numerals from 1 through 10 on a 5″ × 8″ card. The set of cards will be used for an ordinal-number game.

Pre-Book Lesson

• Tell children that, so far, we have used numbers to tell how many things there are in a set. Identify some sets of things in the classroom and have pupils tell how many there are in each set.

• If you have been identifying rows or tables of children by number, say, "When I say *Table 3*, does the 3 tell how many tables there are in our room?" Emphasize that we say "Table 3" because 3 tells which table we mean. In this instance, 3 is an ordinal number because it tells *which one*.

• Arrange the colored pieces of construction paper along the chalk tray. Have children select the 4th piece of paper from the right (or left). Tell the location of the blue piece of paper by number, not by color. Have children identify a piece of paper by number and have others find it.

• Have six children stand in a row at the front of the room. Give them the numeral cards in random order. Have the pupils arrange themselves correctly in order, from left to right. This activity can be varied in many ways. Place the cards on chairs or desks and have children arrange them in order. Give children the cards to show their turns in an activity.

• Have pupils tell other ways in which ordinal numbers are used.

Using the Text Pages

• Use the experiences and discussion resulting from the Pre-Book Lesson to re-emphasize the important points developed in Ex. 1–7.

• *All pupils* should be ready to proceed with the written work in Ex. 8–18.

• The practice exercises at the bottom of page 97 should cause no trouble, since they constitute review work.

Individualizing Instruction

• Discuss with *all pupils* the distinction between *fourth* as used in the ordinal sense and one fourth as used in connection with a part of a whole or a set (the fractional-number meaning). A spelling lesson in which ordinals and their cardinal-number counterparts are listed would be desirable.

• *More capable children* may be able to do the work at the bottom of page 97 by writing answers only.

• If it is appropriate in your community, call attention to the fact that streets that go in one direction are named *25th Street*, *26th Street*, and so on, and those that run in the other direction are named *First Avenue*, *Second Avenue*, and so on. Sketch on the chalkboard a simple map of the city showing

about six blocks in each direction. Have children write names for the streets and avenues in the abbreviated form: 1st Ave., 2d Ave., 5th St., and so on.

- Some children may need more practice in showing A. and S. facts. Write "13," "14," "15," and "16" on the chalkboard. Have pupils write for each sum shown as many different names as they can using the plus and minus signs. Or, specify that a certain number of different names should be given for each number.
- Assign Extra Examples Set 41 as needed.

NOTES

* The simplified forms 2d and 3d are usually used instead of 2nd and 3rd.

6. In box A the short way to write first through ninth is shown.

a. If Nan is 5th in her row, how many children are ahead of her? 4

b. Tuesday is the 3d day in the week. How many days come before it in the week? 2

A		B
	first 1st	horn
*	second 2d	dime
*	third 3d	cat
	fourth 4th	foot
	fifth 5th	drum
	sixth 6th	boat
	seventh 7th	hand
	eighth 8th	bird
	ninth 9th	pen

10th, 11th, 12th, 13th, 14th, 15th, 16th, 17th, 18th,

7. Count from 10th through 30th.

19th, 20th, 21st, 22d, 23d, 24th, 25th, 26th, 27th, 28th, 29th, 30th [W]

Use the short way to show what ordinals come just before and after

8. 1st - 3d 2d.
9. 3d - 5th 4th.
10. 5th - 7th 6th.
11. 2d - 4th 3d.
12. 7th - 9th 8th.
13. 4th - 6th 5th.

From box B copy the word that is

14. hand seventh. 15. boat 6th. 16. pen ninth. 17. bird eighth.

18. What is the ordinal number for each of these words in box B?

a. 3d cat **b.** 8th bird **c.** 2d dime **d.** 7th hand **e.** 4th foot

To Keep in Practice

A. and S. [W]

Copy in columns. Add or subtract.

1. $40 - 20$ 20	6. $62 + 6$ 68	11. $55¢ - 25¢$ 30¢	16. 20, 2, 4, 3 29
2. $79 - 9$ 70	7. $89 - 6$ 83	12. $65¢ + 4¢$ 69¢	17. 5, 2, 7, 0 14
3. $88 - 27$ 61	8. $43 + 56$ 99	13. $77¢ - 75¢$ 2¢	18. 5, 3, 0, 6 14
4. $7 + 32$ 39	9. $68 - 48$ 20	14. $68¢ - 6¢$ 62¢	19. 31, 2, 3, 2 38
5. $49 - 23$ 26	10. $97 - 80$ 17	15. $49¢ - 47¢$ 2¢	20. 2, 0, 3, 6 11

◆ **Extra Examples.** Work Set 41.

* Requiring pupils to do the adding and subtracting provides further practice in computing.

Greater Than? Less Than? Equal To?

Equalities and inequalities [W]

* Add or subtract. Then copy the example and write "is greater than" or "is less than" or "is equal to."

<	1. $(48 - 25)$ _?_ 30 (is less than)	>	8. $(14 + 1 + 3)$ _?_ 16 (is greater than)
=	2. 15 _?_ $(58 - 43)$ (is equal to)	=	9. $(5 + 2 + 2 + 6)$ _?_ 15 (is equal to)
>	3. 17 _?_ $(68 - 55)$ (is greater than)	<	10. $(40 + 2 + 3 + 2)$ _?_ 50 (is less than)
>	4. $(27 + 30)$ _?_ 52 (is greater than)	<	11. $(2 + 3 + 4 + 5)$ _?_ 18 (is less than)
>	5. $(87 - 40)$ _?_ 30 (is greater than)	>	12. $(2 + 4 + 3 + 9)$ _?_ 16 (is greater than)
=	6. 78 _?_ $(42 + 36)$ (is equal to)	=	13. 69 _?_ $(60 + 3 + 3 + 3)$ (is equal to)
<	7. 25 _?_ $(79 - 43)$ (is less than)	<	14. 12 _?_ $(4 + 5 + 0 + 6)$ (is less than)

Copy each example again, using > for is greater than or < for is less than or = for is equal to.

See symbol before each example.

If . . ., Then . . .

Logic [W]

** Work each example. Then write "T" if what the sentence says is true or "F" if false. *False* means *not true.*

In Ex. 1a, $4 + 9 = 13$, not 14; so write, "1a. F."

	a	b	c
1.	If $n = 4$, then $n + 9 = 14$. F	If $n = 43$, then $41 + 2 = n$. T	If $n = 30$, then $n + 40 = 70$. T
2.	If $n = 12$, then $n - 5 = 8$. F	If $n = 37$, then $n - 11 = 18$. F	If $n = 18$, then $68 - n = 40$. F
3.	If $n = 9$, then $39 - n = 30$. T	If $n = 44$, then $58 - n = 14$. T	If $n = 36$, then $49 - n = 13$. T

● **Extra Activity.** Work Sets 89 and 90.

** The word false is probably familiar to pupils but the idea that a sentence is false when it contains an equals sign may be hard for some pupils to grasp.

Teaching Page 98

Pupil's Objectives

(a) To use expressions and symbols of equality and of inequality; and (b) to use the language and thinking of *logic* to tell whether sentences are true or false.

New Word

false

Background

Practice in using mathematical skills which have been developed may be provided in many different settings. These alternate routes to practice afford the opportunity to explore important mathematical concepts and ideas. The motivational effect of the mathematical idea on the child disguises the drill objective of the exercise. Making drill more palatable by placing it in a new mathematical setting fulfills two important objectives.

Drill in adding and subtracting with 2-place numerals and in adding three and four addends is provided in the written activities on page 98. At the top of the page drill is couched in the language of equality and inequality while at the bottom it is placed in the mathematical setting of logic.

Pre-Book Lesson (page 98 top)

• Show on the chalkboard addends in parentheses for some A. and S. facts. Review the renaming of a number. Then have children tell other names for the numbers shown in the parentheses.

• Have children tell other names for numbers, such as (42 + 15) and (87 − 45).

• Use a number-line picture to find another name for (4 + 2 + 3 + 6). Do the same using other sets of three and four addends.

• Write on the chalkboard, "25 _ ? _ (14 + 12)." Ask a child to rename the number shown in parentheses. Ask another child to compare the renamed number with 25, and then use either *is greater than* or *is less than* to make the sentence true. Repeat the exercise using a symbol until you feel that pupils have mastered the technique.

Using the Text Page (top)

Be sure pupils understand the directions for the written work on the text page. Then they should be able to proceed without any difficulty.

Pre-Book Lesson (page 98 bottom)

• Review *if-then* type of thinking (see Teaching Page 74). Make sure that the children know that n can stand for *any* number.

• Write on the chalkboard, "If $n = 16$, then $n - 8 = ?$" Say, "Who can make this number sentence true?" Ask other children if they agree that the sentence is true.

Ask someone to complete the sentence so that it is not true. Emphasize that there are many ways to make the sentence false (not true), but there is only one number that makes the sentence true.

• Use other examples to make true and false sentences.

• Write on the chalkboard: "If $n = 7$, then $n + 3 = 12$."

Ask, "Is the sentence true or false? Why?"

• Repeat with other examples.

Using the Text Page (page 98 bottom)

You may wish to have your pupils write their work for each example and then write "T" or "F."

Individualizing Instruction

• *Slower learners* may need more oral direction for both written activities on the text page. Perhaps this can be accomplished in small-group work or you may prefer to circulate among the class and help children individually.

• *More capable children* (perhaps others) may be assigned the following written work after completing the work at the top of page 98. Instead of writing "is greater than," or "is less than" for _ ? _, write for each example how much greater or less the first number shown is than the second number. For Ex. 1 they would write, "(48 − 25) is 7 less than 30."

• *More capable children* may do Extra Activities Sets 89 and 90.

Teaching Page 99

Pupil's Objectives

(a) To gain oral and written practice in using other names for numbers; and (b) to find answers for number sentences which involve renaming two numbers before completing an operation.

Background

On page 77, number sentences containing two sets of parentheses (two numbers to be renamed) were introduced. After renaming the two numbers to obtain the standard names, children were to indicate whether the two numbers were equal. They learned that it was often necessary to find the standard names by completing the operation indicated within the parentheses before the equality relationship could be determined.

On page 99, the number sentences contain two sets of parentheses. In this instance, however, an operation is to be performed on the renamed numbers. Once again, it is necessary to complete first the work indicated within the parentheses. Then, the operation indicated by the symbol between the parentheses is performed on the renamed numbers. Because addition and subtraction are binary operations, we add or subtract using only two numbers at a time.

The knowledge and skills required by this material are limited to the basic A. and S. facts. The mathematical setting of the number sentence with the parentheses is highly motivational. Drill which may be presented in a more conventional form may be pursued with vigor and enthusiasm under these circumstances.

Teacher's Preparation

Be prepared to use the number-fact cards (see Teaching Page 76) and the number cards in a *Show the Answer* activity (see Teaching Page 1). All children now should have a complete set of numeral cards for the set of whole numbers from 0 through 18.

Pre-Book Lesson

• Write "(12 − 6)" on the chalkboard. Ask a child to tell what number has been shown. (Or say, "This is one way of naming what number?") Play *Show the Answer* by showing numbers in parentheses on the chalkboard and having the pupils hold up the numeral card to show the number named. Decide on some signal upon which all children will show their answers at the same time.

• Show on the board any of the whole numbers from 0 through 18. Have children rename the number in any way they wish. You may wish to use Student Helpers for this activity. If your class is seated in rows or at tables, a Student Helper at each table can help you by checking the responses in his group.

• Write number sentences on the board in the following steps:

Step 1. Write just this much of the number sentence:

$$(4 + 6)$$

Ask, "This is a name for what number?" Write the numeral beneath the parentheses when it is given.

Step 2. Write the next part of the number sentence:

$$\begin{array}{cc} (4 + 6) & (12 - 5) \\ 10 & 7 \end{array}$$

Elicit the number named by this part of the sentence and write the numeral.

Step 3. Write an operation symbol between the two pairs of parentheses:

$$\begin{array}{ccc} (4 + 6) & + & (12 - 5) \\ 10 & & 7 \end{array}$$

Ask, "What does this sign tell us to do?"

Step 4. Finish writing the number sentence using *n* and then complete the work beneath to make a true number sentence:

$$\begin{array}{ccccc} (4 + 6) & + & (12 - 5) & = & n \\ 10 & + & 7 & = & 17 \end{array}$$

Using the Text Page

• Have pupils use a number-line picture or other aids to assist them in the renaming of numbers in Ex. 1–6.

• Permit *slower learners* to use dot pictures, a number-line picture, or the Arithme-Sticks for help in renaming the numbers in the written work.

* More work with parentheses to point up other names for numbers! Using a variety of names for numbers is an important strand in modern mathematics.

Can You Add and Subtract This Way?

* *Using another name for a number* [O]

1. For $(2 + 2) + (3 + 5) = n$, we can write
   $4 + 8 = 12$. Explain. $2+2=4;\ 3+5=8$

2. For $(10 - 6) + (7 + 2) = n$, we can write
   $4 + 9 = \ ?\ $. Explain. $10-6=4;\ 7+2=9$
   13

3. For $(3 + 5) - (10 - 8) = n$, we can write
   $8 - 2 = \ ?\ $. Explain. $3+5=8;\ 10-8=2$
   6

4. For $(3 + 6 + 8) - (3 + 5 + 9) = n$, we can write
   $17 - 17 = \ ?\ $. Explain. $3+6+8=17$ $3+5+9=17$
   0

Copy on the board and finish each example in rows 5 and 6. Explain your work for each example.

	a	b
5.	$(11 - 7) + (8 + 2) = n$ (14)	$(4 + 7) + (3 + 4) = n$ (18)
6.	$(13 - 9) - (11 - 7) = n$ (0)	$(12 + 3 + 4) + (40 - 30) = \square$ (29)

[W]

Copy and finish the examples in rows 7–14.

7.	$(6 + 4) - (18 - 9) = n$ (1)	$(40 + 3 + 3) - (2 + 4) = n$ (40)
8.	$(5 + 9) + (2 + 3) = n$ (19)	$(3 + 1 + 7) + (76 - 4) = n$ (83)
9.	$(16 - 8) + (4 + 5) = n$ (17)	$(86 - 2) - (3 + 3 + 8) = n$ (70)
10.	$(5 + 6) - (13 - 6) = n$ (4)	$(52 + 2 + 3) - (15 - 8) = n$ (50)
11.	$(4 + 8) + (13 - 9) = \bigcirc$ (16)	$(59 - 46) + (2 + 0 + 2) = n$ (17)
12.	$(17 - 8) - (5 + 3) = n$ (1)	$(3 + 30) + (6 + 1 + 8) = n$ (48)
13.	$(9 + 2) - (11 - 4) = n$ (4)	$(89 - 0) - (2 + 4 + 7) = \triangle$ (76)
14.	$(12 - 5) + (16 - 6) = n$ (17)	$(3 + 2 + 0 + 4) + (28 - 8) = n$ (29)

Reading 3-Place Numerals

1. What page of the book is this? 100 How do we read it? One hundred [o]

2. Other people have used ways different from ours to write numerals. In the picture are other ways of naming 100. Can you tell that they mean 100? No

3. The numeral 100 is called a 3-place numeral. Why? It has 3 digits.

4. A digit in one's place or in ten's place shows the number of ones or tens. What does 5 show in 5? 5 ones in 50? 5 tens

5. In 100, the digit 1 is said to be in hundred's place. What does the 1 show? 1 hundred

6. What digit is in hundred's place in each of these?

a. 470 4 **b.** 608 6 **c.** 293 2 **d.** 881 8 **e.** 362 3 **f.** 502 5

7. 470 is read, "four hundred seventy." 608 is read, "six hundred eight." Read numerals c–f in Ex. 6.

c. two hundred ninety-three
d. eight hundred eighty-one
e. three hundred sixty-two
f. five hundred two

You count by hundreds (100's) in the same way that you count by ones and by tens, 100, 200, 300, and so on.

8. Count by hundreds from 100 to 900. 100, 200, 300, 400, 500, 600, 700, 800, 900

* **100** (one hundred)

* Much extra experience in reading 3-place numerals and the associated words has been provided by the use of both forms for page numbers.

Teaching Pages 100 and 101

Pupil's Objectives

(a) To review the decimal structure of 3-place numerals; and (b) to learn to read 3-place numerals from 100 through 999 and understand what they show.

New Word

(page 101) *renamed*

Background

Pupils who used *Mathematics We Need* in the second grade have been introduced to the reading of 3-place numerals from 100 through 200 and what these numerals show. To understand the importance of positional value in the base-ten numeration system and to apply this knowledge in computation, pupils will need much guidance in learning what 3-place numerals show. The time spent in developing understanding now will be well repaid. Progression from 2-place to 3-place numerals should not prove difficult.

In the work at the bottom of page 101, youngsters are helped to see situations in which the number for the total set of ones is greater than 9.

Teacher's Preparation

Have Ginn Arithme-Sticks* available, if possible, for demonstration work. Have an abacus picture for each of three positions available for each child. (On $8\frac{1}{2}'' \times 11''$ paper ditto three vertical marks about 2 inches apart. Each mark represents a position on the Arithme-Sticks or on an abacus.) Have ready one-inch squares of construction paper for the beads on the Arithme-Sticks.

Pre-Book Lesson

• Give each child an abacus picture and some "beads." Show 29 as 9 ones and 2 tens on the Arithme-Sticks. Have each pupil do the same on his abacus picture. On the chalkboard write the numeral for the number being represented on the Arithme-Sticks and on the abaci pictures. Discuss the relationship between ones and tens on the Arithme-Sticks and also as shown by the numeral. Bring out that a digit in one's place of the numeral tells the number of ones whereas the same digit in ten's place would tell the number of tens.

• Have pupils show numbers like 19, 39, 49, 79 on the abacus picture. When the abacus shows 39, direct children to show 1 more one. Make certain that children understand that because there are then 10 ones, the 10 ones are to be shown as 1 ten on the 10's mark. Have pupils experience removing 10 ones from the 1's mark and placing 1 ten on the 10's mark at the left.

You may wish to refer to the story about the Fisherman's system of keeping a record (See Teaching Page 4). Bring out that the 10-idea in the *system* probably is related to man's 10 fingers.

• Have children show 99 on the abacus picture. Ask, "How much is 1 more than 99?" Then direct the children to show 1 more on the abacus picture. Three types of responses will probably result. Some children will place 1 more bead on the 1's mark to show 10 ones and 9 tens. The majority of the children probably will change the 10 ones to 1 ten and leave the beads for 10 tens on the abacus picture. A small group of children will also remove the beads for 10 tens and place 1 bead on the 100's mark. Make sure that all pupils complete the picture properly to show 1 bead on the 100's mark and no beads on 10's or 1's marks. Ask what the abacus picture then shows. Use the various abacus pictures as a basis for fostering a higher level of understanding about the use of *tenness* in our system of keeping a record. The children now should begin to generalize that, on the abacus, 10 beads on any mark are enough to make 1 more bead on the next mark to the left.

• Direct children to show other numbers of hundreds, tens, and ones on their abaci pictures; i.e., 2 hundreds 4 tens 8 ones. Ask someone to tell what is shown and the number it means. Continue this activity by having one child tell a number and the remainder of the class show it on their abaci pictures. One child may demonstrate on the Arithme-Sticks and another write the numeral on the chalkboard. Discuss what is shown and focus on what each digit in the numeral shows.

* See 3, page xix.

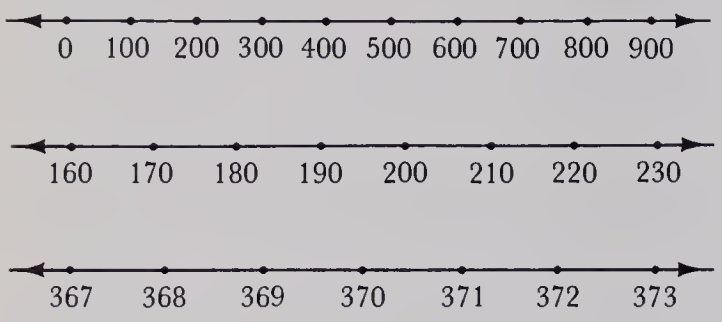
• Review counting by 1's and by 10's. Extend this ability now to counting by 100's. Do not always start at 100.

• Write sets of 2-place numerals on the chalkboard and direct pupils to tell the greatest number and the least number shown in each set. Extend this activity by using sets of 3-place numerals.

Using the Text Pages

• You may wish to use one of the teaching devices mentioned in the Pre-Book Lesson during discussion of Ex. 1–7.

• Draw a number-line picture to show that points on a number line may be used for counting by 100's as well as by 10's and 1's. Use number-line pictures somewhat like these while discussing Ex. 8–13. This will help to develop understanding of the numbers shown by 3-place numerals.



Use a number-line picture to show the whole number just before and just after any given number.

• For the work at the bottom of page 101, have pupils show each number on the Ginn Arithme-Sticks or an abacus picture before completing Ex. 1–4. They will thus be helped to see situations when the number for the total set of ones is greater than 9.

• For the written work (Ex. 5–10) permit the children to use an abacus picture if they need to.

Individualizing Instruction

• *All pupils* may be helped by activities such as these:

a. Examples similar to the following may be used for practice in showing the whole number just before a number, just after a number, and in between two numbers.

_ ? _ 123	175 _ ? _	131 _ ? _ 133
_ ? _ 476	217 _ ? _	199 _ ? _ 201
_ ? _ 500	670 _ ? _	709 _ ? _ 711

b. Have pupils show the set of odd numbers or the set of even numbers for any given set of whole numbers (i.e., 205 through 240).

c. Ditto or have children make a chart like the following. Starting with any given number, the children will count by 1's and write the numerals as far as the space in the chart will permit.

171	172	173	174						

• *More capable children* may do any of the following:

a. Make number-line pictures to show the numbers in Ex. 10–13 at the top of page 101.

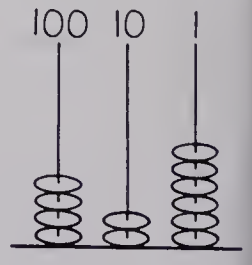
b. Finish examples to demonstrate clear understanding of the structure of the decimal-numeration system. Some examples might be:

100 and 5 and 40 = _ ? _
4 tens and 2 hundreds and 7 ones = _ ? _
14 tens and 6 ones = _ ? _
263 = _ ? _ tens and _ ? _ ones

• *Slower learners* may draw abacus pictures like this.

a. What number is shown on the abacus? Write the numeral.

b. Draw the beads on an abacus picture to show 630.



9. To count by 10's from 180 say, "180, 190, 200, 210," and so on. Count by 10's from

a. 470 to 570. 470, 480, 490, 500, 510, 520, 530, 540, 550, 560, 570

b. 530 to 610. 530, 540, 550, 560, 570, 580, 590, 600, 610

c. 690 to 800. 690, 700, 710, 720, 730, 740, 750, 760, 770, 780, 790, 800

Count by 1's. Say what is missing.

10. 198, _199_, 200, _201_, 202, _203_, 204, _205_

11. 777, _778_, 779, _780_, _781_, _782_, 783, _784_

12. What numbers come just before and just after

a. 580? 579 - 581 b. 961? 960 - 962 c. 699? 698 - 700 d. 400? 399 - 401 e. 701? 700 - 702

13. What is the greatest and the least number shown in each of sets a–c?

a. 609, 424, 599 greatest; 609 least; 424

b. 900, 388, 401 greatest; 900 least; 388

c. 399, 502, 601 greatest; 601 least; 399

Renaming Numbers Greater than 9

*Readiness for carrying to ten's place [O]

A ||||||||||| The number for this set of marks is 12. For set A, 12 can be named 12 ones.

B ϕ || The number for set B is also 12 and for this set, 12 can be renamed 1 ten and 2 ones. Because 12 ones and 1 ten and 2 ones are both names for 12, we can write, "12 ones = 1 ten and 2 ones."

1. 17 ones = 1 ten and _7_ ones
2. 43 ones = _4_ tens and 3 ones
3. 14 ones = _1_ ten and _4_ ones
4. 56 ones = _5_ tens and _6_ ones

[W]

Copy. Finish the work to rename the number.

5. 12 ones = _1_ ten and 2 ones
6. 15 ones = _1_ ten and _5_ ones
7. 18 ones = _1_ ten and _8_ ones
8. 11 ones = _1_ ten and _1_ one
9. 20 ones = _2_ tens and _0_ ones
10. 34 ones = _3_ tens and _4_ ones

* Now pupils begin to use other names for numbers as preparation for work when the sum shown for ones or tens or hundreds is greater than 9.

A

Mary found

Billy found

B

In all

Renaming and Carrying to Ten's Column

[o]

1. Picture A. Mary's shells: 2 tens and 6 ones or 26. Billy's shells: 8 ones, or 8.

2. Picture B. For the total, Mary made as many tens as she could. There were 3 tens and 4 ones.

3. Ex. C. To add 26 and 8, add ones, then tens.

a. Explain the 2 tens and 14 ones. 14 ones = 1 ten and 4 ones, so add 1 ten to the 2 tens.

b. Why can we think of the sum as 3 tens and 4 ones?

2 tens and 14 ones = 3 tens and 4 ones

C	D	E
2 tens and 6 ones + 8 ones 2 tens and 14 ones, or 3 tens and 4 ones, or 34	Tens | Ones 1 2 | 6 + | 8 3 | 4	1 26 + 8 34

4. Ex. D shows carrying when adding the short way.

Ones: 6 + 8 = 14. 14 ones = 1 ten and 4 ones. Write "4" in one's place and write "1" over the ten's digit 2. We say that we have *carried 1 to ten's column.

Tens: 1 + 2 = 3 The sum of 26 + 8 is 34.

5. Ex. E shows the shortest way to add. Explain.

6 + 8 = 14 Carry 1 to ten's column. 2 + 1 = 3

* Note the meaningful use of "carry." For the numeral 14, the digit 1, not the number 1, is carried to the ten's column.

Teaching Pages 102 and 103

Pupil's Objective

To learn the rationale of higher-decade addition with renaming and carrying.

New Word

(page 102) *carrying*

Background

Pages 102 and 103 are the first two pages of a sequence of pages designed to give intelligent mastery of higher-decade addition with renaming and carrying a digit to ten's column. The first step is understanding the process of finding sums for such examples as 46 + 7 and 8 + 63.

Pupils have had partial preparation for this new skill in written and oral looking-ahead activities involving renaming. When 2 tens and 16 ones is renamed 36, the child has actually completed the last step in renaming and carrying in higher-decade addition.

In order to demonstrate the need for renaming and carrying, it is wise to begin by using real or representative objects to solve a classroom problem before proceeding with the material in the pupil's book. In this way the children actually may change objects from a set of ones to a set of tens and a set of ones. Representative objects such as dowels, tongue depressors, or cardboard strips have their special advantage for demonstration purposes, since they can be assembled readily in sets of tens.

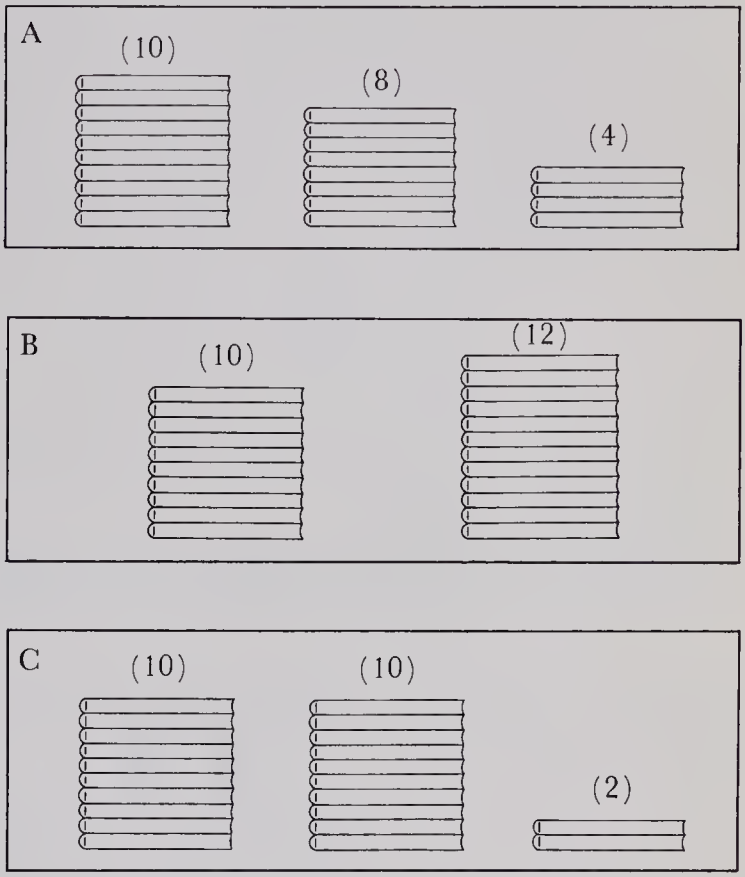
However, when adding numbers, the idea of renaming the number of ones and then carrying a digit to ten's place should be emphasized. Note that we carry a *digit* not a number.

Teacher's Preparation

- Have available sets of objects such as books, pencils, boxes of crayons to use in the Pre-Book Lesson.
- Teaching aids such as pocket charts, an abacus, and Ginn Arithme-Sticks are particularly useful in conjunction with this lesson.

Pre-Book Lesson

- Present a classroom situation similar to the following in which the number for the total of two sets of objects must be found such that regrouping is involved (i.e., 18 books and 4 books).



First, set up the books in a pile of 1 ten, another pile of 8, and another pile of 4, as in picture A. Next, place the pile of 4 books on top of the pile of 8 books to make a total of 12 books, as in picture B. Elicit from the class that two sets have been joined.

Now, ask pupils to suggest ways of finding the number for the total. Associate the renaming of 12 in many different ways with the rearranging of the 12 books in many different ways.

With the books arranged as in picture C, ask why that way of renaming the number 12 will be more useful than any other way. Guide the children to state and see that this is really the way we rename the number for addition. In this activity your pupils will have moved or *carried* a set of 10 to the set of 10 that was already there. Similarly, with numerals, they carry a digit to ten's column.

- Use dowels, pencils, cardboard strips, and so on to illustrate other examples. While some pupils

continue demonstrations with objects for similar examples, have the children at their desks make individual records with ⌀ 's and / 's, with words *tens* and *ones*, and with numerals, as shown below. Emphasize the renaming and eventually the carrying of the digit to ten's column.

29 + 6 = ?		
⌀⌀///// /////	2 tens and 9 ones + 6 ones 2 tens and 15 ones, or renamed, 3 tens and 5 ones, or 35	1 29 + 6 35

Using the Text Pages

• Be sure to stress the regrouping of sets or the renaming and carrying that occurs in each of Ex. 1–6 (e.g., in box C, 2 tens and 14 ones are renamed 3 tens and 4 ones).

• If *slower learners* cannot find sums in the ways suggested for Ex. 7–11, let them use a pocket chart* in the way illustrated in the sequence below for finding the sum of 23 + 8.

Individualizing Instruction

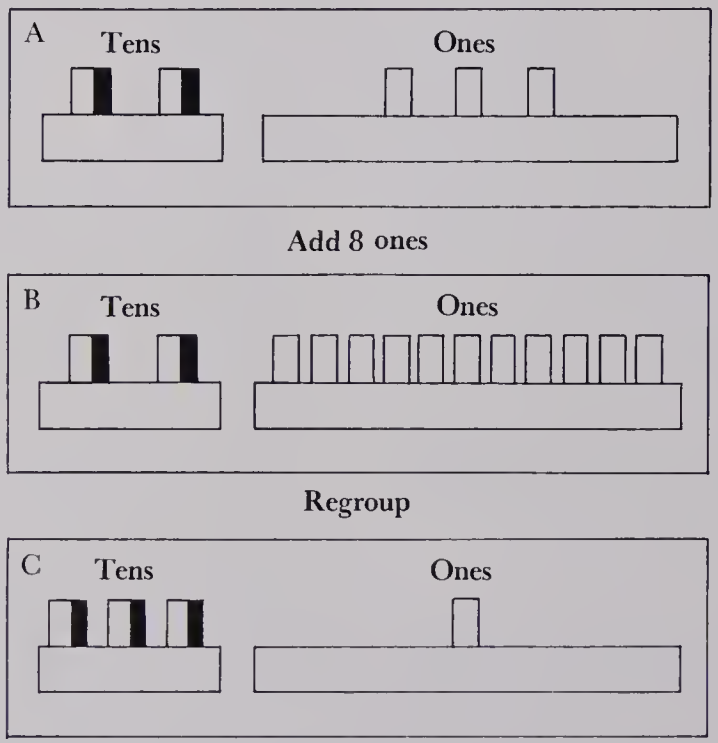
• Give the *slower learners* as much individual attention as possible. Permit them to use any manipulative or visual aids which will help them in their written work. They may need more "renaming" written work of the following types:

15 ones may be renamed _ ? _.
17 ones may be renamed _ ? _.
5 tens and _ ? _ ones may be renamed 62.

• *More capable children* may check their work by drawing number-line pictures to illustrate finding the sums for the examples in rows 8 and 9.

* See 21, page xix.

23



6. Joe found snails at the beach. He counted 9 snails in one can and 36 in the other. How many snails were in the two cans? $9 + 36 = n$

a. Explain and finish the work in Ex. F.

b. Tell how to add the short way in Ex. G; in Ex. H.

F

9 ones
+3 tens and 6 ones

3 _?_ tens and _?_ ones, or 15
4 _?_ tens and _?_ ones, or 45

G

Tens	Ones
1	
	9
+3	6
?	?
4	5

H

$$\begin{array}{r} {\scriptstyle 1} \\ 9 \\ +\,36 \\ \hline 45 \end{array}$$

[W]

Copy the examples in row 7. Find the sums by using tens and ones, as in Ex. C and F.

7. a. $32 + 9$ 41 b. $63 + 8$ 71 c. $2 + 38$ 40 d. $6 + 57$ 63 e. $34 + 8$ 42 f. $9 + 23$ 32

* For rows 8–11 copy the numerals in columns and add the short way, as in Ex. E and H.

	a	b	c	d	e	f
8.	$5 + 46$ 51	$53 + 7$ 60	$56 + 8$ 64	$17 + 5$ 22	$46 + 9$ 55	$5 + 38$ 43
9.	$36 + 5$ 41	$19 + 4$ 23	$39 + 9$ 48	$8 + 75$ 83	$4 + 57$ 61	$37 + 9$ 46

In some of these you do not rename and carry:

10.	$8 + 79$ 87	$44 + 6$ 50	$39 + 5$ 44	$77 + 6$ 83	$28 + 3$ 31	$4 + 51$ 55
11.	$62 + 9$ 71	$32 + 7$ 39	$8 + 74$ 82	$73 + 6$ 79	$9 + 67$ 76	$15 + 9$ 24

When the sum of the ones is greater than 9, write the digit for the ones in one's place and carry the digit for the tens to the ten's column.

* Many children will need to use the crutch of showing the carried digit but, as they become more competent, encourage them to drop this crutch.

More Sets Using Addition Facts

With bridging [O]

A
14 + 9
24 + 9
34 + 9
44 + 9

1. The set of examples in box A use what A. fact? 4 + 9 = 13

a. In each example the sum of the ones is _?_. 13

b. Why is there 1 more ten in the sum than is shown by the ten's digit in the 2-place numeral? The sum of the ones is greater than 9.

c. To add 14 and 9, first add ones, 4 + 9. *Think*, "13." Then for the sum *think*, "23." Explain. The sum of the ones (13) can be renamed 1 ten and 3 ones. Carry 1 to ten's column. 1 ten + 1 ten = 2 tens

Answer to "c"

1 ten + 1 ten = 2 tens

d. For each addition in box A, tell the tens in the sum. Then tell the whole sum. 2, 23; 3, 33; 4, 43; 5, 53

e. Say 3 other examples in this same A. set. Answers will vary. 54 + 9; 64 + 9; 74 + 9

B
35 + 7
5 + 57
75 + 7
5 + 87

2. The set of examples in box B use what A. fact? 5 + 7 = 12

a. For each example, there are 12? ones so there is _?_ 1 more ten in the sum.

b. To add 35 and 7 *think*, "12, 42." Explain. 5 + 7 = 12, so 35 + 7 = 42

c. To add 5 and 57 *think*, "12, 62." Explain. 5 + 7 = 12, so 5 + 57 = 62

d. Tell how to think in adding in the other examples. "12, 82" "12, 92"

e. Say 3 other examples in this same A. set. Answers will vary. 25 + 7; 45 + 7; 65 + 7

Answers to 3, 4, 5, 6 f at bottom of section

For rows 3–6, tell the A. fact used for each example. Say another example in the same A. set. Answers will vary.

	a	b	c	d	e	f
3.	28 + 5 (3; 33) 8 + 5 = 13	78 + 8 (8; 86) 8 + 8 = 16	6 + 16 (2; 22) 6 + 6 = 12	4 + 37 (4; 41) 4 + 7 = 11	6 + 68 (7; 74) 6 + 8 = 14	83 + 9
4.	7 + 14 (2; 21) 7 + 4 = 11	8 + 39 (4; 47) 8 + 9 = 17	74 + 6 (8; 80) 4 + 6 = 10	18 + 2 (2; 20) 8 + 2 = 10	3 + 53 (5; 56) 3 + 3 = 6	52 + 8
5.	89 + 9 (9; 98) 9 + 9 = 18	7 + 37 (4; 44) 7 + 7 = 14	9 + 41 (5; 50) 9 + 1 = 10	5 + 25 (3; 30) 5 + 5 = 10	48 + 7 (5; 55) 8 + 7 = 15	39 + 6
6.	47 + 6 (5; 53) 7 + 6 = 13	55 + 8 (6; 63) 5 + 8 = 13	34 + 8 (4; 42) 4 + 8 = 12	9 + 64 (7; 73) 9 + 4 = 13	4 + 39 (4; 43) 4 + 9 = 13	27 + 9

Answers for 3, 4, 5, 6 f

3f. 3 + 9 = 12 (9; 92)

4f. 2 + 8 = 10 (6; 60)

5f. 9 + 6 = 15 (4; 45)

6f. 7 + 9 = 16 (3; 36)

For each addition in rows 3–6, tell the tens in the sum. Then tell the whole sum.

Teaching Pages 104 and 105

Pupil's Objective

To use knowledge and understanding of the harder basic A. facts to organize new sets of higher-decade addition examples with bridging.

Background

These pages extend the thinking first introduced on pages 45 and 46 of the pupil's book. The purpose of the pages is to help pupils organize their learning around relationships. By this time, pupils have learned principles governing higher-decade addition when no carrying was involved. Also, they have learned to identify sets of examples using addition facts with sums less than 10.

For higher-decade examples with bridging (i.e., adding with a 2-place numeral and a 1-place numeral when the sum of the ones is greater than 9), the skillful pupil will recognize that, for an example like 56 + 8, the sum of the basic fact used, 6 + 8 = 14, is greater than 9. For this reason, the sum of the higher-decade A. example must be 64 because the tens of the sum will be one greater than that shown in the 2-place numeral, and the ones will be the same as the ones in the sum of the basic fact, 6 + 8 = 14. To be proficient in higher-decade addition, with or without bridging, the pupils must know the basic addition facts.

Note the thought pattern described in Ex. 2b for 35 + 7 ("12, 42") and in Ex. 2c for 5 + 57 ("12, 62"). *Slower learners* may need to use an intermediate step in which for 35 + 7 they *think*, "12" (for the ones, 5 + 7), then *think*, "4" (1 more ten), then *think*, "42."

Many children find it more difficult to think the answer for 5 plus 67 than for 67 plus 5. An understanding of the *Commutative Property of Addition* may help the child attack examples of this type so that he will always add the ones to the tens and ones, regardless of the order in which the numerals are written in the example.

Teacher's Preparation

Have some type of place-value teaching aid available for use in the Pre-Book Lesson (see Teaching Pages 102–103).

Pre-Book Lesson

- Write on the chalkboard:

$$\begin{array}{rrrr} 46 & 26 & 76 & 16 \\ +\ 6 & +\ 6 & +\ 6 & +\ 6 \\ \hline \end{array}$$

Ask, "How are all of these examples alike?" You should anticipate several different answers. For example, "They are all adding." "They all have 6 at the bottom." Elicit the important idea that they all use the A. fact 6 + 6 = 12. Then, build the idea that all the examples belong to the set using the addition fact 6 + 6 = 12.

- Ask, "In what way are the examples on the board different?" Be sure to elicit the important idea that each shows a different number in the ten's place.

Use bundles of sticks, a place-value chart, or Ginn Arithme-Sticks to illustrate that, for each example, the number for the tens in the sum is one greater than the number for the tens given. (Use this illustration to develop the thinking suggested in the Background section.)

- Have the children tell other examples that belong to the set of examples using the same A. fact. Have others explain how to *think* the answers.

Using the Text Pages

- You may wish to use representative materials while discussing Ex. 1–2 in order to clarify further the idea that the ones to be added are the same for each example of the set. It should be made clear, too, that in these examples the sum of the ones is 10 or more, so the ten's digit in the answer must show 1 more ten than the ten's digit in the example.

- For Ex. 3–6, have pupils tell the addition fact used for each example. Then, have them tell another example in the same set. It may not be necessary to have the *more capable children* go through all of these examples in this manner. Perhaps after completing two rows they should be assigned the written work in Ex. 7–13 to do independently.

Next, go through these examples (rows 3–6) a second time and have the children tell only the ten's digit in the answer. *Slower learners* may need many step-by-step experiences to help them think

answers for higher-decade addition examples. If you deem it desirable, have *slower learners* use manipulative objects to see what happens and build additional understanding of the steps involved. It may be helpful to write the following steps on the chalkboard as youngsters are using objects to demonstrate them:

a. 55 + 8; A. fact used is 5 + 8 = 13.

b. 13 is more than 9, so the sum is in the 60's.

c. Sum is 63.

Be sure to emphasize the use of the Commutative Property of Addition where appropriate for the examples in rows 3–6.

• Select children to read each direction orally for Ex. 7–13 to be sure that all directions are understood. The first example for each part of the lesson may be written on the chalkboard as a guide.

• The purpose of the material at the middle of page 105 is to help pupils learn the shortest, most economical way to give sums for higher-decade addition examples with bridging. Do not hurry the children into adding the quick way, particularly the *slower learners*, but encourage them toward this goal.

• The practice material at the bottom of page 105 may be used as a progress test to check the pupils' work in the higher-decade addition and subtraction that has been introduced thus far. Use your own judgment as to how many of these examples should be assigned to the different ability groups in your class. Even though the material is provided, it may not be necessary for all pupils within your class or within any ability group in your class to do all of these examples.

Individualizing Instruction

• Duplicate or put on the chalkboard mixed higher-decade A. examples, with and without bridging, as shown below. Without saying the sums, children may distinguish between examples that require renaming of the ones in the sum and those that do not. Have them explain their answers.

4	36	18	6	47	3	52
+65	+ 7	+ 4	+21	+ 9	+58	+ 4

• Number-line pictures may be used in the following ways:

a. Your permanent number-line model or one drawn on the board may be used with the *slower learners* as one more way to help them understand the thinking invoved in these examples.

b. Have *more capable children* draw number-line pictures to illustrate the adding for examples in any row on page 105. Have them sense the Commutative Property of Addition by drawing the 2-place numeral for all examples. In this example, 8 + 64, the 64-arrow would be drawn first on the number-line picture, even though the 8 appears first in the example.

• Use Extra Examples Set 42 as deemed necessary.

NOTES

[W]

In which A. set is each of these?

6 + 9 = 15
7. 36 + 9 **8.** 5 + 26 **9.** 24 + 7 **10.** 2 + 58
5 + 6 = 11 4 + 7 = 11 2 + 8 = 10

Write 4 examples in the A. set using

Answers will vary.
11. 3 + 8 = 11. **12.** 9 + 6 = 15. **13.** 4 + 9 = 13.
23 + 8; 3 + 18 29 + 6; 9 + 26 24 + 9; 34 + 9
43 + 8; 3 + 38 9 + 46; 59 + 6 4 + 49; 4 + 29

Copy the examples in rows 3–6 on page 104 so that they show the order of the addends changed. Find the sums. For Ex. 3a write, "5 + 28 = 33."

A Quicker Way to Add

[W]

In the examples 4 + 45 and 44 + 5, you can *think*, "9, 49" but it is quicker to *think* "49" at once.

In the examples 5 + 36 and 35 + 6, too, it is quicker to think the sum, "41" at once.

Turn to page 103. Copy the examples from rows 7–11 in columns. Find the sums the new, quick way.

To Keep in Practice

A. and S. [W]

Add or subtract. Use folded paper. Check your work.

	a	b	c	d	e	f	g	h	i
1.	43	98	7	63	98	9	77¢	38¢	68¢
	+50	−60	+48	+35	−76	+90	−50¢	+9¢	−37¢
	93	38	55	98	22	99	27¢	47¢	31¢
2.	16	89	72	39	67	69	2¢	65¢	65¢
	+8	−53	+7	+7	−63	−5	+76¢	−45¢	+23¢
	24	36	79	46	4	64	78¢	20¢	88¢
3.	57	94	89	3	87	3	87¢	8¢	38¢
	+42	−84	−80	+76	−7	+47	−26¢	+57¢	−34¢
	99	10	9	79	80	50	61¢	65¢	4¢

◆ **Extra Examples.** Work Set 42.

More Adding of 3 and 4 Numbers

Using higher-decade A. with bridging [O]

The boards to the tent are 6 feet, 5 feet, 7 feet, and 6 feet long. About how long is the board walk?

Study Ex. 1a. Add downward. *Think*, "11, 18, 24." 2 + 19 = 21, 21 + 3 = 24, 24 + 2 = 26

For Ex. 1b *think*, "21, 24, 26." Explain. ^Tell how to find the sums in Ex. 1c and 1d.

1c. Think, "13, 22, 30." 1d. Think, "12, 19, 28."

	a	b	c	d	e	f	g	h
* 1.	6	2	5	5	6	5	11¢	8¢
	5	19	8	7	3	9	4¢	10¢
	7	3	9	7	8	7	6¢	5¢
	6	2	8	9	6	5	6¢	6¢
	24	26	30	28	21 (23)	26	22¢ (27¢)	29¢

To check addition, change the order of adding. Check Ex. 1e–1h and tell which sums shown are not correct.

[W]

Copy each set of numerals in a column. Add and check.

2. 7, 5, 0, 9 21
3. 4, 9, 7, 4 24
4. 8, 7, 5, 0 20
5. 10, 3, 8, 6 27
6. 9, 8, 8 25
7. 3, 42, 0, 7 52
8. 6¢, 8¢, 9¢, 7¢ 27¢
9. 17¢, 3¢, 2¢, 5¢ 27¢
10. 9¢, 9¢, 8¢ 26¢

◆ **Extra Examples.** Work Set 43.

* Remind pupils that addition is a binary operation, so it is possible to add only two numbers at a time, no matter how many numbers there may be to add.

Teaching Page 106

Pupil's Objective

To use column addition with four addends and harder bridging.

Background

New on this page is bridging in the 20-decade in column addition. Now that your pupils have had practice in higher-decade addition with bridging, they should be ready for the extension of column addition encountered in this lesson.

Addition is a binary operation, so only two numbers are added at a time. However, you will recall that the Associative Property of Addition, first discussed on Teaching Page 40, permits us to begin with any two addends and add a third to their sum. For consistency in approach to performing the operation, we teach the child to begin with the first addend and repeatedly add in sequence to the last addend and, as a check for the work, we teach him to begin with the last addend and retrace his steps to the first addend.

In an example like the one at the right, many children recognize instantly the 10 for the 7 + 3. Then, they proceed to add the remaining addends. Conceivably, such children will arrive at the sum as follows: They *see* the 10 immediately. Then they commute 10 and 9 and think "13" for 4 + 9. Then, 13 and 10 are added to get the sum 23. The steps in thinking the answer this way are

$$\begin{array}{r} 4 \\ 7 \\ 3 \\ +9 \\ \hline \end{array}$$

1. 4 + (7 + 3) + 9
2. 4 + 10 + 9
3. Commute 10 and 9.
4. (4 + 9) + 10

The kind of thinking shown above is not recommended as an initial method of attacking addition with several addends. However, pupils should not be discouraged from its use as long as they are not having difficulty with it. In fact, the commutative and the associative properties should be introduced to these children as a rationale for their way of thinking to find the sum.

Pre-Book Lesson

• Use objects such as blocks*, discs*, or paper squares if help is needed in thinking the bridging.

* See 13 and 12, page xix.

Some children may need help with the idea of joining three or more sets of objects before they can find the sum of three or more addends.

• Have a pupil show sets of objects being joined. Then have another pupil join the sets in reverse order. At the same time, a third pupil may write a number sentence on the chalkboard to represent the joining taking place. A fourth pupil may check the sum by adding in reverse order.

Using the Text Page

• As pupils do the oral work, check to see that they are using higher-decade addition sets and are thinking the answers.

• Use a number-line picture to help pupils *see* the *unseen numbers* in each example.

• Let pupils who do not have difficulty with the oral work write their work for Ex. 2–10. For others, more oral work should be provided.

Individualizing Instruction

• If any children are having difficulty adding an unseen number, try the following activity.

Write a 1-place numeral on the board which all children can see. Say any number from 1 through 9. The children are to find the sum of the *seen number* shown on the board and the *unseen number* spoken.

• *Slower learners* may be helped by the following exercise which may be placed on the chalkboard.

a.
$$\begin{array}{r} 9 \\ 7 \\ +5 \\ \hline \end{array}$$

9 + 7 = 16
16 + 5 belongs to the set using 6 + 5 = 11.
6 + 5 = 11 so 16 + 5 = 21.

b.
$$\begin{array}{r} 6 \\ 8 \\ +8 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 9 \\ 7 \\ +9 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 8 \\ 3 \\ 5 \\ +7 \\ \hline \end{array}$$

e.
$$\begin{array}{r} 9 \\ 4 \\ 6 \\ +5 \\ \hline \end{array}$$

• *More capable children* may try making up their own examples of 4 addends in which bridging to the 20-decade is necessary. Suggest that they write 3 examples with a sum of 21, 3 with a sum of 22, and so on. Let them check each other's work. Then select some examples to present to the class.

• Assign Extra Examples Set 43 as needed.

Teaching Page 107

Pupil's Objective

To extend the use of *if-then* thinking in order to find the solutions for number sentences in which n is known.

Background

The use of logic as an approach to developing precision in mathematical reasoning was discussed first on Teaching Page 74. The written work on page 107 fulfills important objectives of the mathematics program. First, it extends the use of logic in reasoning. The child must determine what information is given and then use the information in solving some number sentences. Second, all of the number sentences necessitate use of skills which just recently have been learned. The work on this page affords an opportunity for drill in higher-decade addition and subtraction and 3- and 4-addend addition.

Pre-Book Lesson

• It is possible that some *slower learners* will find this lesson too abstract at this time. You may prefer to have them make use of the first suggestion under Individualizing Instruction.

Another alternative is to have the *slower learners* turn back to the written work on page 74 (Ex. 4–9) after the Pre-Book Lesson is finished.

• Review the ways in which n has been used in the arithmetic program. Be sure to stress the idea than n can be used to represent *any* number.

In a previous lesson, page 74, the pupils had to solve number sentences containing n when the number n stood for was given. On page 98, the number n stood for was given and then the child had to determine whether the number sentence was true or false. This lesson is a more advanced application of the written work on page 74 because pupils sometimes must use 3 or 4 addends.

• Write on the chalkboard, "If $n = 6$, then $n + 4 = ?$" After discussing the solution to this number sentence, write, "$n + n = ?$" Be sure to point out that n may be used more than once in a number sentence, but when it is so used, it stands for the same number each time.

• Write other *if-then* sentences on the chalkboard requiring either addition or subtraction and sometimes using n more than once in the number sentence to be solved.

Using the Text Page

• For Ex. 1–6, have the children write the entire number sentence, substituting the value given for n in each case.

• For Ex. 7–15, ask the pupils to read each sentence, think the work, and write only the answer on their papers. Some children may need to write the number sentence before they can arrive at a solution. They should be permitted to do so.

Individualizing Instruction

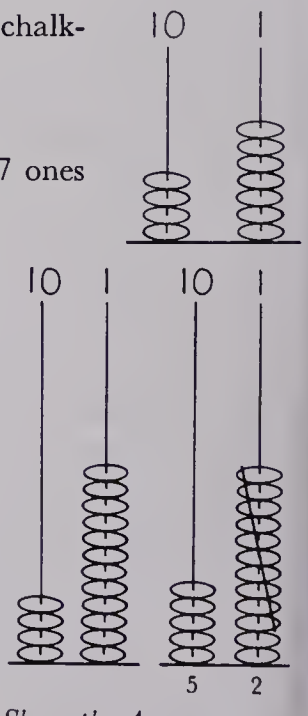
• *All pupils* may use the number-fact cards prepared previously for sums 13 through 18 and match examples with answers.

• Have *all pupils* draw abacus pictures for the following examples which require renaming of ones.

47	26	59	63	35
+ 5	+ 8	+ 4	+ 7	+ 8

Illustrate one example on the chalkboard as follows:

$$47 + 5 = n$$



Step 1. Show the 4 tens and 7 ones on the line abacus.

Step 2. Then, show 5 more ones. Be sure to count the ones to find how many ones in all.

Step 3. If there are 10 or more ones, cross out 10 ones and draw 1 more ten.

Step 4. Show the number of tens and ones by numerals below the marks for tens and ones.

• Have *all pupils* play the *Show the Answer* game in small groups with some of the *more capable children* holding up cards for which other children in the groups show answers.

If . . ., Then . . .

Logic [W]

Work Ex. 1–6. For Ex. 1a write, "5 + 68 = 73."

1. If $n = 68$, then
a. $5 + n = ?$ 73
b. $n - 30 = ?$ 38
c. $n + 9 = ?$ 77

2. If $n = 7$, then
a. $n - n = ?$ 0
b. $n + 0 = ?$ 7
c. $n + 58 = ?$ 65

3. If $n = 45$, then
a. $8 + n = ?$ 53
b. $n - 40 = ?$ 5
c. $n + 9 = ?$ 54

4. If $n = 75$, then
a. $n - 10 = ?$ 65
b. $n + 7 = ?$ 82
c. $n - n = ?$ 0

5. If $n = 6$, then
a. $6 + 9 + n = ?$ 21
b. $89 - n = ?$ 83
c. $n + 10 + 6 = ?$ 22

6. If $n = 47$, then
a. $n + 30 = ?$ 77
b. $n - 30 = ?$ 17
c. $9 + n = ?$ 56

Work Ex. 7–15. Write only the answers on your paper.

7. If $n = 9$, then
a. $n + 75 = ?$ 84
b. $4 + 3 + n + 7 = ?$ 23
c. $59 - n = ?$ 50

8. If $n = 4$, then
a. $n + 9 + 8 + 3 = ?$ 24
b. $87 - n = ?$ 83
c. $n + 0 = ?$ 4

9. If $n = 5$, then
a. $9 + n + 8 = ?$ 22
b. $68 + n = ?$ 73
c. $79 - n = ?$ 74

10. If $n = 8$, then
a. $n + n + n = ?$ 24
b. $79 + n = ?$ 87
c. $7 + 3 + n + n = ?$ 26
d. $n + 67 = ?$ 75

11. If $n = 22$, then
a. $4 + n + 7 + 2 = ?$ 35
b. $n + n = ?$ 44
c. $n + 6 + 4 = ?$ 32
d. $8 + n = ?$ 30

12. If $n = 8$, then
a. $22 + n + n = ?$ 38
b. $78 - n = ?$ 70
c. $n + 7 + 0 + n = ?$ 23
d. $39 + n = ?$ 47

13. If $n = 46$, then
a. $n - 41 = ?$ 5
b. $6 + n + 6 = ?$ 58
c. $20 + n = ?$ 66
d. $89 - n = ?$ 43

14. If $n = 6$, then
a. $n + 75 = ?$ 81
b. $10 + n + 8 = ?$ 24
c. $98 - n = ?$ 92
d. $52 + n = ?$ 58

15. If $n = 57$, then
a. $n + 9 = ?$ 66
b. $n - 52 = ?$ 5
c. $88 - n = ?$ 31
d. $40 + n = ?$ 97

Our Money

Coins; dollar sign; cent point [O]

1. Starting at the left, name the coin which is

a. 5th. (half dollar) b. 3d. (dime) c. 6th. (dollar) d. 4th. (quarter) e. 2d. (nickel) f. 1st. (cent)

2. Which of the coins shown is worth

a. 10¢? (dime) b. 50¢? (half dollar) c. 5¢? (nickel) d. 25¢? (quarter) e. 1¢? (cent)

* The dollar, the coin shown at the right, is worth 100¢, but we write "$1.00" most of the time. The sign $ is the dollar sign. The dot in $1.00 is the cent point.

$1.04: one dollar and four cents. We say "and" for the cent point when we read dollars and cents.

$4.53: four dollars and fifty-three cents

$0.61: sixty-one cents $0.07: seven cents

3. Read these:

a. $3.42 (three dollars and forty-two cents) ∧ b. $0.74 (seventy-four cents) ∨ c. $6.09 (six dollars and nine cents) ∨ d. $8.16 (eight dollars and sixteen cents) ∧ e. $0.03 (three cents) ∨

[W]

4. Write these with the dollar sign and cent point:

a. Two dollars and ten cents $2.10
b. Five dollars and two cents $5.02
c. Six dollars and eleven cents $6.11
d. Seventy cents $0.70
e. Four cents $0.04
f. 87¢ $0.87
g. 9¢ $0.09
h. 62¢ $0.62
i. 13¢ $0.13
j. 127¢ $1.27

* Help pupils to realize that we use $, the cent point, and ¢ with numerals only to indicate that we are showing a number of dollars and cents.

Teaching Page 108

Pupil's Objectives

(a) To learn to recognize the coins in our money system, as well as know their values; (b) to learn the technical name for each coin, as well as the terms "coins," "dollar sign," and "cent point" and to use the symbol $; and (c) to learn to read and write numerals representing money that require two and three digits.

New Word

coin

Teacher's Preparation

Have ready one or more of each coin to show to the class and to use concretely in checking answers to the first suggestion under Individualizing Instruction. You may wish to use educational toy money,* as well as have some children make patterns of coins from tagboard, noting the value on each coin.

Pre-Book Lesson

Show coins and discuss their values. Note whether there are any children who seem uncertain of values. Try to provide some time to help them individually before requiring them to do written work. Use real or toy money, establishing simple situations that are easily comprehended by all children.

Using the Text Page

• Take advantage of the opportunity to review the ordinal-number idea when you are discussing Ex. 1 and 2.

• During oral discussion of Ex. 3, compare these numerals representing money with other 2- and 3-place numerals.

• You may wish to have Ex. 4 done on the chalkboard by some pupils while others work at their desks.

* See 16, page xix.

Individualizing Instruction

• Make lists of coins having equal values, as in the following illustration for 25¢:

25¢
a quarter
two dimes and a nickel
three nickels and a dime
twenty-five cents
and so on

• Paste several pupil-made coin patterns on a chart to show arrangements of coins having equal values.

• Duplicate or write on the chalkboard the following questions for written work to be done by *all pupils*:

a. How much money is a dime and a nickel?
b. How much money do these sets of coins make:
   (1) A half dollar and a quarter?
   (2) A dime and three cents?
   (3) A dime, a nickel, and three cents?
c. How much would be left if you had
   (1) a quarter and spent 4¢?
   (2) 28¢ and spent 5¢?
d. How much less is
   (1) 5¢ than a quarter?
   (2) 7¢ than 17¢?

• *More capable children* may look up in an encyclopedia: *Mint* to find out how our coins are made; *Money* to find out about money used in earlier days; or *Coins* to find out about coin collections.

Reminder

Continue to maintain the following:

a. Ability to tell time and to use the calendar intelligently
b. Ability to record information in chart form
c. Ability to read and write 3-place numerals

Teaching Page 109

Pupil's Objective

To use the recently learned skills in addition and subtraction when solving problems containing money numbers.

New Words

delivers, *errands*, *newspapers*

Background

Putting newly-taught skills to work in solving problems is one way of showing their usefulness. Application of mathematics is an essential part of the learning of mathematics. Through applications, pupils are introduced to many concepts and understanding of these concepts deepens as they are applied to a variety of situations. Interest and motivation are increased as pupils see widespread uses of mathematics.

Pre-Book Lesson

• Review quickly the parts-total relationship. Be sure children know these two important generalizations:

a. When the numbers for the parts are known, the number for the total can be found by addition.

b. When the numbers for the total and one of its two parts are known, the number for the other part can be found by subtraction.

• Review the various problem situations for which subtraction is used. Emphasize that in each instance the numbers for a total and one of two parts are known.

• Discuss ways in which different pupils in the class have earned money. Use this opportunity to develop any vocabulary on the text page that may be strange to your pupils.

Using the Text Page

• First, have the children solve problems 1–7. They should write their work to show how they found the answers.

• After they have found the answers to all problems, have them go back and write an *n*-example for each problem to show how the problem was solved. For Ex. 2, any of the following number sentences would be acceptable:

$49¢ + 10¢ = n¢$ $\qquad$ $10¢ + 49¢ = n¢$
$n¢ = 49¢ + 10¢$ $\qquad$ $n¢ = 10¢ + 49¢$

• After completion of the written assignments, have an oral lesson based on the problems. The discussion should be centered on the reason for selecting addition or subtraction as the operation used. The reasons should be stated in terms of the parts-total relationship.

Ask questions of this type:

Does this problem give us numbers for parts, or numbers for the total and one of its two parts?

How do you know the number is for a part of the total?

Which number is for the total?

Individualizing Instruction

• *All pupils* may copy and finish these examples:

a.	47	84	6	28	55
	$+ 5$	$+ 9$	$+35$	$+ 7$	$+ 9$
b.	17	n	n	8	n
	$+ n$	$+49$	$+ 6$	$+ n$	$+37$
	23	55	62	84	41

• The role of *translation* in solving problems has been discussed in connection with lessons on Teaching Page 43 and Teaching Page 66. The writing of *n*-examples for Ex. 1–7 on page 109 involved translating information in a problem to a number sentence with *n*.

Write the following number sentences on the chalkboard and have the *more capable children* write for each number sentence a problem that the sentence could represent. This is translating from the mathematical to the problem situation.

$n = 8 + 6$	$50 - 30 = n$	$25 - n = 15$
$n = 35 + 8$	$n - 6 = 5$	$49 - n = 44$

Earning, Saving, and Spending Money

A. and S. problems; writing n-examples [W]

Write your work for problems 1–7.

1. Joe delivers newspapers. Each day he leaves 8 papers, 7 papers, and 9 papers at houses on three different streets. How many newspapers in all does Joe deliver on these streets? 24 papers $8+7+9=n$
The numbers for the parts are given. Add to find the sum.

2. Ella had 49¢. For washing the dinner dishes she earned 10¢. How much money did Ella have then? 59¢ $49¢+10¢=n¢$
The numbers for the parts are given. Add to find the sum.

3. Tom has 78¢ in his bank. Ed has 6¢ more than that in his bank. How much does Ed have in his bank? 84¢ $78¢+6¢=n¢$
The numbers for the parts are given. Add to find the sum.

4. Ted's father gave Ted 75¢ one week for helping him. Ted spent 32¢. How much of the 75¢ did Ted keep? 43¢ $75¢-32¢=n¢$
The numbers for the total and one part are given. Subtract to find the number for the other part.

5. Nell had 80¢ when she went into the store. When she came out, she had 60¢. How much did Nell spend? 20¢ $80¢-60¢=n¢$
The numbers for the total and one part are given. Subtract to find the number for the other part.

6. Jack has earned 58¢. He wants to buy a ball that costs 89¢. How much more money does Jack need? 31¢ $89¢-58¢=n¢$
The numbers for the total and one part are given. Subtract to find the number for the other part.

7. For doing errands Ron earned a nickel, a dime, and 5 cents. How much money did Ron earn doing errands? 20¢ $5¢+10¢+5¢=n¢$
The numbers for the parts are given. Add to find the sum.

Now write an *n*-example for each problem.

[O]

Tell why you added or subtracted in each problem.

Renaming in Addition with 2-Place Numerals

Carrying to ten's place [O]

1. The 25 children in Grade 3 and 17 from Grade 4 saw a movie. That was _?_ (42) children in all. $25 + 17 = n$

Explain how to find the sum by using

a. Ex. A. **b.** Ex. B.

Explain why finding the sum in Ex. C is really like using Ex. A; using Ex. B. 5 ones + 7 ones = 12 ones, or 1 ten and 2 ones. 1 ten + 2 tens + 1 ten = 4 tens

A	B	C	D
⊕ ⊕ || ||| | ⊕ || || ||| |	2 tens and 5 ones +1 ten and 7 ones 3 tens and 12 ones, or 4 tens and _?_ (2) ones, or _?_ (42)	Tens | Ones 1 2 | 5 +1 | 7 4 | 2	1 25 + 17 42

c. Ex. D shows how to add 25 and 17 the shortest way. First add ones, rename and carry, then add tens.

Ones: $5 + 7$. *Think*, "12." 12 ones = 1 ten and 2 ones, so write "2" in one's place and carry the 1.

Tens: $1 + 2 + 1$. *Think*, "4." Write "4" in ten's place.

When pupils have conquered renaming, the carrying required in addition of this type will not seem difficult.

Teaching Pages 110 and 111

Pupil's Objective

To discover the rationale for addition with two 2-place numerals requiring renaming and carrying.

Background

Preceding lessons have led toward the mastery of higher-decade addition involving renaming and carrying. Therefore, addition with two 2-place numerals involving renaming and carrying is an extension of earlier learning.

In the sequence of presentation, children first see the need of solution; then they may use semi-concrete objects; then ⌀'s and /'s; and then the words tens and ones before they learn the abbreviated thought-process and work with digits.

Teacher's Preparation

Have place-value teaching aids available (see Teaching Page 53). Be sure to have objects that children can bundle together (dowels or tagboard strips) as well as a pocket chart or a similar teaching device. You may wish to duplicate and have ready pictures of a line abacus and discs or paper squares for use as suggested in Individualizing Instruction on Teacher's Page 111.

Pre-Book Lesson

• Present a situation, preferably a local one, such as the following:

Betty's class sold tickets for the school carnival. They sold 28 tickets to children and 17 tickets to adults. How many tickets did they sell in all?

Have the children suggest at the chalkboard ways in which the solution might be found. After various ways have been explored, show the advantage of adding the ones first. Then, rename the 15 ones to be 1 ten and 5 ones, write the digit for the ones (5) in one's place, and carry the digit for the tens (1) to the ten's column. Illustrate with tickets (3 bundles of ten and 15 ones) what has happened, so pupils may see the operation concretely. From the total ones, make a bundle of 10 ones into a 10-bundle. Place the new 10-bundle with the other tens so the children can "see" the 4 tens and 5 ones. Point out that, when we record the work, we rename the ones and carry a digit to ten's column instead of making a 10-bundle to put with the tens.

Use dowels, tongue depressors, tagboard strips, and so on to illustrate other examples of this type. Note those youngsters whose understanding of the work appears to be weak.

• Use a vertical chart of two columns labeled *Tens* and *Ones*. With ⌀'s and /'s, demonstrate what we show when we rename the ones and carry the digit for the tens to ten's column. That is, encircle 10 of the /'s, cross them out and show another ⌀ in the ten's column. Following the use of this chart and possibly the pocket chart (see Teaching Page 53), the work should be carried to a more abstract level by adding with the words *tens* and *ones* and finally with digits only.

• Have children make records at the board of examples with ⌀'s and /'s, words, and numerals. The other members of the class should make corresponding records individually at their desks.

Using the Text Pages

• For Ex. 1, some children may show on the chalkboard different ways of adding 25 and 17. Other children should follow the procedures, using their books. Be sure to emphasize that the work in Box D is the *shortest way* or the *fastest way* of writing the example.

• Make certain the children understand the directions for each of rows 2–4 before administering that work. Continue to refer to the work in box D as the *shortest way* of writing and thinking the examples. Have the pupils use the shortest way to explain how the sums were found in row 4.

• The *slower learners* probably will need more work with concrete and semi-concrete materials before they proceed with writing the work for Ex. 5–8. While the *more capable children* proceed with this written work independently, help the *slower learners*, using the vertical charts or the abacus picture described on Teacher's Page 111.

Individualizing Instruction

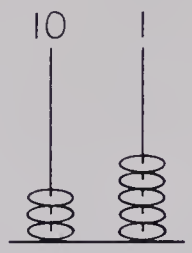
• The transition from a concrete demonstration to the illustration of the mathematical process by

use of ⌀'s and / 's and then to the more abstract recording may be made in several stages. The use of the vertical chart suggested in the Pre-Book Lesson recommended the encircling of 10 ones to make a ten. *Slower learners* need a great deal of help in making the transition to the abstract.

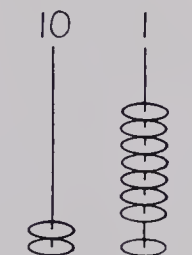
• Have *all pupils* use the abacus picture as another way of illustrating what happens when we carry. Each child should have a piece of paper with three vertical marks shown on it. He will need about 25 discs or 1-inch squares of construction paper.

$$35 + 27 = ?$$

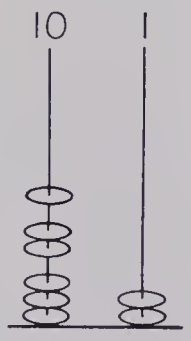
Step 1.
Have each child show 35 on his abacus picture by placing the necessary discs or squares on the appropriate marks. Ask a child to tell what he has shown on the marks of his abacus.



Step 2.
Show 27 more on the abacus picture. When the children place the 7 discs on the mark for 1's they may exceed the length of the vertical mark and even go off the paper. It is possible to show any number of ones on the abacus picture. However, when we write the numeral we must rename the ones to be tens and ones.



Step 3.
With appropriate discussion, have the children remove 10 ones and place 1 disc on the 10's mark to substitute for the 10 ones just removed.



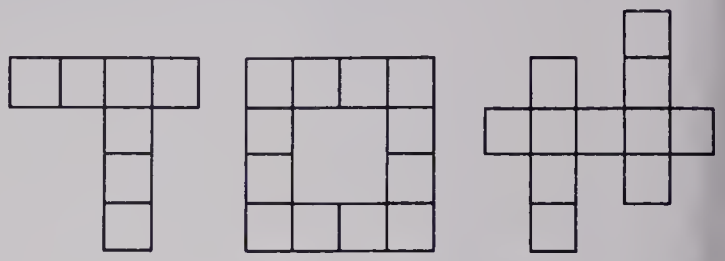
Step 4.
Emphasize what was done to get the total 6 tens and 2 ones.

Use ⌀'s and / 's, the words *tens* and *ones*, and then digits as developed in the Pre-Book Lesson as well as on the text page, in order to make the complete transition from the concrete to the abstract.

• *All pupils* may show the additions in row 3 on page 111 as follows:

$$\begin{array}{r} 17 = 10 + 7 \\ +25 = 20 + 5 \\ \hline 30 + 12 = 42 \end{array}$$

• Duplicate drawings like those below on paper or place them on the chalkboard. *All pupils* may fill in the drawings as directed.



They are to write a numeral in each square so that the sum for each row and column in the figure will be

a. 25. b. 21. c. 26.

• Other possible activities for *slower learners* are:

a. Crossing out examples with incorrect sums, using rows of examples like those below, on the chalkboard or duplicated on paper

46	31	53	24	18	33
+42	+49	+16	+36	+43	+49
88	80	79	50	61	82

b. Encircling examples in which *no* renaming of ones and carrying a digit to ten's column is required; then adding in the other examples

35	27	38	41	18	75
+41	+16	+36	+28	+37	+19

c. Copying the example and writing the missing digit

48	34	20	57	16	42
+37	+45	+46	+18	+47	+3?
?5	?9	?6	?5	??	??

• Assign Extra Examples Set 44 as needed.

For row 2, copy on the board and work

Ex. a as in Ex. A, page 110. Ex. d–e as in Ex. C.

Ex. b–c as in Ex. B. Ex. f–g as in Ex. D.

	a	b	c	d	e	f	g
2.	27	42	17	59	74	48	17
	+34	+18	+39	+28	+18	+46	+48
	61	60	56	87	92	94	65

Tell how the sums in row 3 were found. We add for columns f and g just as for columns a-e.

3.	29	16	38	18	46	53¢	$0.47
	45	25	18	14	38	19¢	0.42
	74	41	56	32	84	72¢	$0.89

Which of the sums shown in row 4 are not correct?

4.	29	55	34	37	12	72¢	$0.48
	57	35	23	14	67	18¢	0.29
	86	80	67	51	79	80¢	$0.77
		90	57			90¢	

[W]

Copy and add the shortest way. Check your work.

	a	b	c	d	e	f	g	h	i
5.	21	69	17	15	27	56	14	29¢	$0.53
	29	13	53	68	56	15	78	64¢	0.37
	50	82	70	83	83	71	92	93¢	$0.90
6.	14	48	19	36	26	14	37	53¢	$0.36
	77	25	37	54	19	76	34	28¢	0.26
	91	73	56	90	45	90	71	81¢	$0.62
7.	18	42	34	48	15	67	23	67¢	$0.35
	42	39	18	47	26	27	65	28¢	0.39
	60	81	52	95	41	94	88	95¢	$0.74
8.	29	36	34	15	35	45	16	49¢	$0.27
	32	47	29	17	43	35	58	16¢	0.45
	61	83	63	32	78	80	74	65¢	$0.72

◆ **Extra Examples.** Work Set 44.

Try These

[W]

For rows 1–4 show the sums on folded paper. Check your work. In some examples you carry, but not in all.

	a	b	c	d	e	f	g	h	i
1.	10	19	21	26	28	37	54	18¢	$0.23
	80	36	67	2	25	7	19	69¢	0.56
	90	55	88	28	53	44	73	87¢	$0.79
2.	87	79	4	59	8	25	48	25¢	$0.29
	6	10	65	34	73	38	26	34¢	0.57
	93	89	69	93	81	63	74	59¢	$0.86
3.	50	37	73	3	45	36	78	73¢	$0.47
	7	18	24	29	46	47	18	25¢	0.44
	57	55	97	32	91	83	96	98¢	$0.91
4.	19	58	46	5	26	68	9	19¢	$0.70
	52	31	6	30	55	4	65	49¢	0.18
	71	89	52	35	81	72	74	68¢	$0.88

Copy each set of numerals in a column. Add and check.

5. 5, 7, 0, 6 18
6. 3, 7, 8, 9 27
7. 25, 5, 3, 4 37
8. 6, 9, 7, 2 24
9. 7, 43, 2, 5 57
10. 9, 0, 9, 8 26

◆ **Extra Examples.** Work Set 45.

Working with *n*

Addends-sum relationship [W]

Work each example. Then copy it, showing the sum or addend which *n* stands for.

1. $40 + n = 70$ 30
2. $n - 32 = 37$ 69
3. $n = 9 + 58$ 67
4. $78 - n = 36$ 42
5. $n + 48 = 98$ 50
6. $67 - n = 40$ 27
7. $21 + n = 34$ 13
8. $n - 51 = 38$ 89
9. $52 + n = 72$ 20
10. $n - 41 = 38$ 79
11. $33 + n = 59$ 26
12. $86 - n = 51$ 35

Teaching Page 112

Pupil's Objectives

(a) To practice finding sums in various types of addition with 2-place numerals with and without carrying; and (b) to practice finding the unknown number for *n* in number sentences involving higher-decade addition and subtraction.

Background

In order to develop competence in the newly-taught skills, and to develop the ability to discriminate between examples which require renaming and carrying and those which do not, the practice set has a mixture of examples some of which do not require renaming and carrying.

Teacher's Preparation

Have paper ready for folded-paper practice.

Pre-Book Lesson

- Review with all children adding with two 2-place numerals requiring renaming and carrying.
- Help children select some examples, either from the chalkboard or from the pupil's book, which require no renaming and carrying and which they may do orally. Check to determine whether they have trouble discriminating between examples with no renaming and carrying and examples with renaming and carrying.
- Write some simple number sentences on the board using *n*. Have some for A. and some for S. with *n* standing for a sum or an addend. Review addition and subtraction in terms of addends-sum relationships and make sure that pupils know how to find the number *n* stands for in both A. and S. examples, when it is a sum and when it is an addend.

Using the Text Page

- Before doing rows 1–4 at the top of the page, review the folded-paper technique (see Teaching Page 10), if necessary.
- Review how sums are shown when adding with numerals representing money (¢ and $).
- The *more capable children* should be able to proceed independently with the work at the bottom of the page. It may be necessary to go over the examples orally with the *slower learners* before they write the work.

Individualizing Instruction

- After the written work at the bottom of the page is finished, go over each example orally. Have pupils tell why they added or subtracted. For each example, have a child tell whether *n* in the sentence stands for a sum or an addend.
- *Slower learners* may need additional practice with basic A. and S. facts. Have them match the number-fact cards with answer cards (see Teaching Page 76).
- Assign Extra Examples Set 45 as needed.

NOTES

Teaching Page 113

Pupil's Objectives

(a) To gain experience in supplying questions that may be used in addition and subtraction problems; and (b) to gain experience in translating the information given in written problems to number-sentence form.

Teacher's Preparation

To assist children in the spelling of words to use in the problems, write on the chalkboard the words listed below and encourage children to find other words needed. The so-called "cue-words" for addition and subtraction are purposely mixed in this list to avoid dependence upon them as such.

all together	in all	need
cost	left	spend
farther	money	total
gone	more	

Pre-Book Lesson

- Consider a situation such as the following:

  Today, there are 17 girls and 14 boys here.

Ask the children to think about the information given and then to make-up a question for which they would need to use addition to find the answer.

As different children ask their questions have them tell why they would need to use addition to find the answer.

- Ask each child to first state and then write on the chalkboard a number sentence that tells how to find the answer to the question he asked.
- Discuss the list of words on the board, adding to it as needed, and analyze any of the words for possible spelling difficulties.

Using the Text Page

- After reading the information given in each exercise, the child should write only the question on his paper, not the entire problem. Possible questions are

(1) In all, how many people did Don count when he looked back?

(2) How many miles farther did Don have to ride on the train?

(3) How many more people did Don count on one side of the car than on the other side?

(4) How many people were left on the train after seven people left?

(5) How much money did Don have left after buying milk and peanuts?

(6) How many peanuts did Don eat?

- After the pupils have written a question for each problem, have them find the answer for each problem.
- Finally have the pupils write an *n*-example for each problem to show how they worked it.

Individualizing Instruction

- Discuss with the class some of the completed problem questions. Have the class try to detect and analyze errors. When necessary, use such aids as quick sketches on the board, dot pictures, and ϕ's and /'s to help clarify meanings.
- *All pupils* may turn to Extra Problem Set 5 and write an *n*-example for each problem. They should have worked the problems before completing this activity. The purpose of this written work is to write number sentences only, not to solve the problems.

Writing Questions to Make Problems

Writing n-examples [W]

Make problems about Don's train ride. The *A.* means to write an addition question. The *S.* means to write a subtraction question. Questions will vary.

1. Looking back, Don counted 5 men, 3 women, and 5 children, counting himself. *A.* 13 people $5+3+5=n$

2. Don was to ride 76 miles in all. The train stopped the first time after going 30 miles. *S.* 46 miles $76-30=n$

3. At one time, Don counted 28 people on his side of the car and 17 people on the other side of the car. *S.* 11 people $28-17=n$

4. Once when the train stopped, there were 37 people in the car. Seven people got off. *S.* 30 people $37-7=n$

5. On the trip, Don spent 20¢ of his 50¢ for milk and peanuts together. *S.* 30¢ $50¢-20¢=n¢$

6. Don counted the peanuts in the bag. There were 59. He ate all but 10 of them. *S.* 49 peanuts $59-10=n$

Work the problems. Then write an *n*-example for each.

To Keep in Practice

[W]

Write the answers on folded paper. Check your work.

	a	b	c	d	e	f	g	h	i
1.	89	19	78	8	29	86	4	20¢	$0.27
	−72	+74	+20	+69	−8	−43	+88	+60¢	+0.55
	17	93	98	77	21	43	92	80¢	$0.82
2.	27	48	39	76	59	89	57	18¢	$0.06
	−26	+18	−9	+10	+23	−60	−53	−5¢	+0.75
	1	66	30	86	82	29	4	13¢	$0.81
3.	67	60	13	77	83	36	85	49¢	$0.19
	−47	+17	+19	−14	+4	+59	+7	−6¢	+0.59
	20	77	32	63	87	95	92	43¢	$0.78
4.	98	78	89	8	57	69	98	28¢	$0.48
	−30	−8	−21	+73	+33	−65	−41	+67¢	+0.41
	68	70	68	81	90	4	57	95¢	$0.89

Do You Like to Try New Things?

Enrichment [W]

Set A		
6	19	14
+3	+8	+29
?	?	?

1. In set A are the addends even or odd numbers? one odd and one even number in each example

2. Are all the sums for set A even or odd numbers? odd

3. Copy and finish: When an odd number and an even number are added, the sum is an _?_ number. odd

4. Test the sentence in Ex. 3. Make three examples like 8 + 17, and three others like 25 + 46, and add.

Set B		
8	27	48
−5	−6	−21
?	?	?

5. In set B are even or odd numbers used? one add and one even number in each example.

6. Are the answers for set B odd or even numbers? odd

7. Copy and finish: When an even number is subtracted from an odd number or an odd number from an even number, the answer is an _?_ number. add

8. Test Ex. 7. Make three examples of both kinds.

Here and on pages 30, 31, 63, and 69, help pupils discover that for two counting numbers under the operations of A. and S., if both are even or both odd, the answer is an even number; if one is even and the other odd, the answer is an odd number.

Teaching Page 114

Pupil's Objective

To practice computation in addition and subtraction examples with 2-place numerals of all types that have been taught.

Background

At the top of the page in the pupil's book is mixed practice in higher-decade addition and addition with two 2-place numerals (both with and without carrying) and in higher-decade subtraction and subtracting with 2-place numerals without renaming—that is, in all kinds of addition and subtraction thus far taught, except addition of three and four addends.

It is a general principle of teaching, first to develop each new skill to the point of mastery or near mastery, giving isolated practice on it in order to avoid confusion with other skills; then after competence has been established to mix the new skill with others previously taught. Mixed practice obviously calls for discrimination among skills and for recognition of the correct procedure to employ in each exercise.

The enrichment material at the bottom of the pupil's page is an extension of material presented earlier in which the following generalizations were developed:

a. The result of adding 1 to an odd number is an even number.

b. The result of adding 1 to an even number is an odd number.

c. The sum of two even numbers is an even number.

d. The sum of two odd numbers is an even number.

e. The difference between two even numbers is an even number.

f. The difference between two odd numbers is an even number.

Two more generalizations are presented in this lesson:

a. The sum of an odd and an even number is an odd number.

b. The difference between an odd and an even number is an odd number.

Teacher's Preparation

Have paper available for folded-paper written practice.

Using the Text Page

• Use the mixed practice as a diagnostic test to discover the various kinds of errors needing attention. Many children make *accidental* errors for one reason or another. However, if an error of a particular type occurs regularly, it can be assumed that the child is having difficulty with that type of example. As you check a child's work, look for errors that persist.

• The children who do not appear to be making errors persistent in nature, may try the enrichment exercises independently.

• Help children with persistent errors. Especially aid those who have difficulty in distinguishing between addition and subtraction, and those who still are not sure when renaming and carrying are needed in addition examples.

Children whose primary difficulty stems from lack of mastery of the basic facts will need a variety of activities to help them master the facts. See some suggestions in the Individualizing-Instruction section below.

Individualizing Instruction

• *Slower learners* who need help with the A. and S. facts may be assigned any of the following activities:

a. *Renaming game.* Write 14 on the chalkboard. Have the pupils rename 14 in as many ways as they can using A. facts only. Write 8 on the chalkboard. Have 8 renamed in as many different ways as possible using S. facts only. Repeat for other numbers.

b. *Comparison game.* Write exercises of the following type on paper or on the board:

$$(9 + 5) _ ? _ (16 - 7)$$

Have the children write "is greater than" or "is less than" to make the sentence true. Be sure to use an A. or S. fact in each equation.

• Use visual-instruction materials to help children having difficulty with renaming and carrying.

Teaching Page 115

Pupil's Objectives

(a) To check understandings of important ideas studied to this point; and (b) to use higher-decade addition and subtraction skills in comparing two numbers.

Background

The measurement of pupil understandings is difficult, to say the least. A quick automatic response by a child is often accepted as an indication of understanding. The questions at the top of page 115 are illustrative of the type that seek to determine the extent to which children understand vocabulary, concepts, and generalizations in mathematics.

It is in the day-by-day developmental lessons that you have the greatest opportunity to observe each child's level of understanding. No substitute has been devised for the individual-interview approach in which the teacher asks leading questions of the child in order to identify the specifics of his understandings as well as his difficulties.

The work with sequences in this lesson (Ex. 5–8) is more advanced than in previous lessons, as the counting here begins with a number that is not a multiple.

Pre-Book Lesson

• Review the idea of other names for a number. Write on the chalkboard, "(5 + 6), (17 − 4), (42 + 7)." Have the children tell for what number each of these is another name.

• Write "(48 — 24)" on the chalkboard. Ask for what number this is another name. Then, write "29" on the board. Ask, "Is 29 greater or less than (48 − 24)?" Have them then tell how much greater it is.

• Repeat the same activity in comparing numbers—some greater and some less than another number.

Using the Text Page

• You may wish to assign this small mid-chapter test to all the children immediately (See first item, Individualizing Instruction). Allow children who find difficulties to ask questions or to omit troublesome examples temporarily.

• Be sure the pupils understand the directions for the work at the bottom of the page. Remind them to *rename* the number in the ( ) first and then make the comparison.

Individualizing Instruction

• You may wish to have *all pupils* use the written work at the top of the page as a progress test. If so, the work should be corrected item by item, with each child marking his own paper. The discussion of meanings and the observation of procedure will augment learning in this type of review. Information gained by noting questions asked and examples omitted during the test, plus errors found in the correcting, will indicate to you where clarification or reteaching is needed.

• *All pupils* may try the written work at the bottom of page 115. Permit them to proceed with this work independently. Make sure that they understand what is required. Not only are two numbers to be compared and one to be found greater (or less) than the other, but the final sentence must tell how much greater (or less) one number is than the other.

• Have *all pupils* write answers on paper for incomplete A. and S. facts administered orally. Use the incomplete A. and S. facts at the top of page 90 for oral presentation.

• Have *slower learners* match number-fact cards with answer cards for those facts most frequently missed when working the above oral assignment.

* Once or twice within Chapters 3-8, the authors present mid-chapter review tests of information and meaning under this heading. This is the first of these tests.

* Many Things

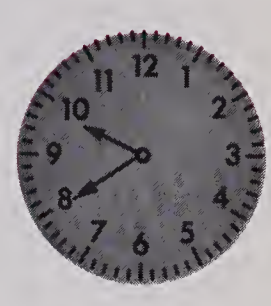
Review of information and meaning [W]

Write the answers.

1. What odd number comes next after 33? 35

2. $63 + n = 77$. To find the number for n, you subtract.

To what numeral does the clock minute hand point at



3. 20 minutes before 10? 8 **4.** 15 minutes past 5? 3

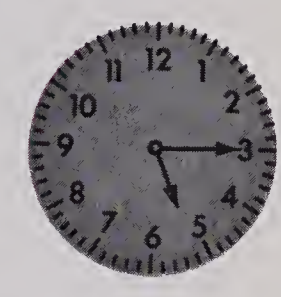
Write the numerals to show counting

5. by 3's from 19 to 40. 19, 22, 25, 28, 31, 34, 37, 40

6. by 4's from 17 to 53. 17, 21, 25, 29, 33, 37, 41, 45, 49, 53

7. by 5's from 12 to 42. 12, 17, 22, 27, 32, 37, 42

8. by 10's from 170 to 220. 170, 180, 190, 200, 210, 220



9. Copy and use $=$ or $\neq$: $(16 + 7) \stackrel{?}{\neq} (19 + 5)$.

Copy the examples in which you carry to ten's place.

10. $32 + 37$ **11.** $43 + 9$ (43 + 9) **12.** $18 + 45$ (18 + 45) **13.** $54 + 5$

14. To compare two numbers, do you add or subtract? subtract

Write each of these the short way:

15. third 3d **16.** seventh 7th **17.** sixth 6th **18.** eighth 8th

How Much Greater? How Much Less?

Inequalities [W]

For Ex. 1a write, "$48 - 16 = 32$. $35 - 32 = 3$.
35 is 3 greater than $(48 - 16)$."

1. How much greater is

a. 35 than $(48 - 16)$? 3

b. $(38 + 49)$ than 25? 62

c. 45 than $(4 + 27)$? 14

d. 76 than $(9 + 63)$? 4

e. $(38 + 37)$ than 65? 10

2. How much less is

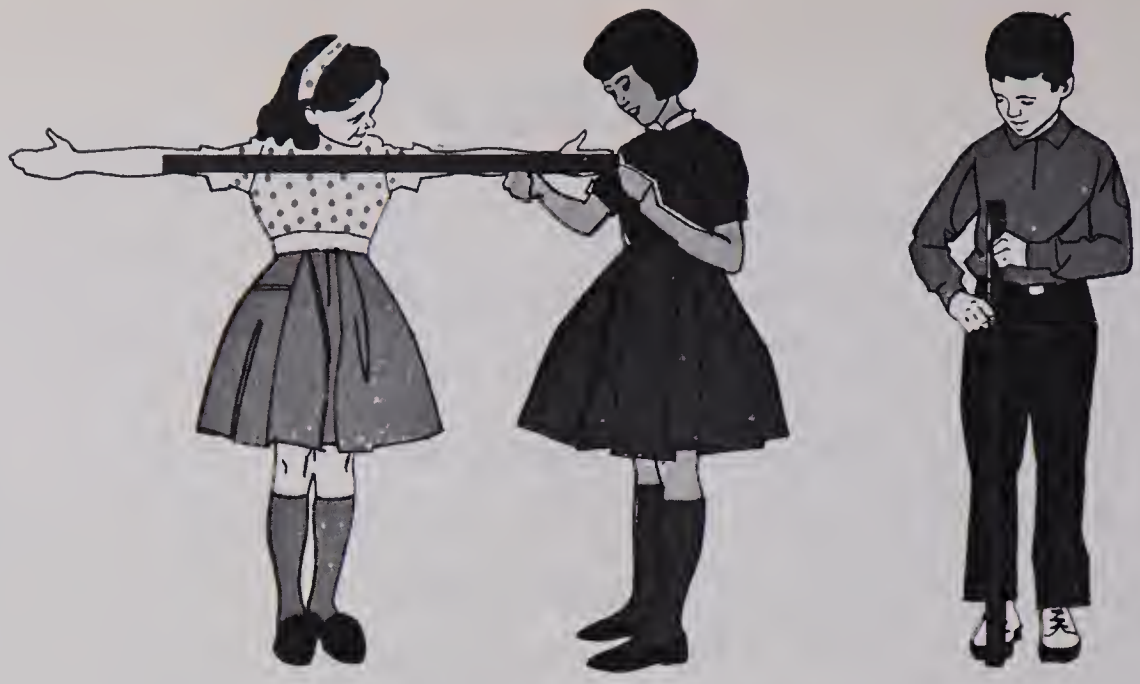
a. $(47 - 12)$ than 39? 4

b. 23 than $(18 + 19)$? 14

c. $(40 - 20)$ than 38? 18

d. 52 than $(35 + 48)$? 31

e. $(15 + 46)$ than 78? 17



Measuring in Yards and Miles

Discovering reference measures; mental procedures [O]

We measure some things in yards.

1. Do what Tom is doing. See how far up your body the yardstick comes. This distance is your body yard.

2. Do what Linda and Betsy are doing. See how far the yardstick reaches. Call this your arm yard.

3. Look at a yardstick. How many feet long is it? 3 How many inches long is it? 36

3 feet (ft.) = 1 yard (yd.) **36 inches (in.) = 1 yd.**

4. On the board, draw a mark you think is 1 yard long. Measure the mark to check it.

5. Can a man be 2 yards tall? Yes 4 yards? No 1 foot? No

We measure very long distances in miles.

6. Tell a place that is 1 mile from your school. Think of this distance when you hear about 1 mile.

Here are some other reference measurements—the body yard, the arm yard, and the school mile.

Teaching Pages 116 and 117

Pupil's Objectives

(a) To acquire the concepts of *yard* and *mile*; (b) to acquire a functional concept of *yard* and to learn the relationship between feet and yards; (c) to establish a *body yard* or an *arm yard* to use as reference measures; and (d) to write questions to make subtraction problems and then work the problems.

New Words

(page 116) *body*, *distances*

Background

The primary purpose of these pages is to put meaning into terms used in linear measurement; in this case, *yards* and *miles*. Just as for the *foot* and the *inch*, a "reference measure" for the yard is suggested. You may also want to suggest certain distances in your community that are about one mile. Pupils will be able to use these to help them understand 3 miles, 7 miles, and so on.

If the unit *yard* is to have more meaning than merely 36 inches or 3 feet, pupils must have some immediate concrete way of thinking and using it. Consequently, the *body yard* and the *arm yard* are developed.

As children become familiar with more units of linear measurement they should begin to select the most appropriate unit for describing a particular distance. Remind pupils again that measurements are only approximations and that the measure of a length is the *number* of times the unit is applied to the item to be measured. See the discussion in more detail on Teaching Pages 64 and 65.

Teacher's Preparation

Have as many yardsticks available as possible.

Have in mind two or three well-known locations that are about one mile from the school.

Pre-Book Lesson

• Review the ideas of the *finger inch* and the *arm foot* (see Teaching Pages 64 and 65).

• Have the children estimate the lengths of different objects in the room. Then have them measure the lengths with rulers. Ask for the answers in such a way that you receive a number in reply. Then say that we have to know what unit was used in finding that the measure of the length is 3 or 7 or whatever. So, we may say that the measure of the length is 3 in inches or 7 in feet, and so on.

Be sure to again stress the idea that measurements are approximations as was noted on teaching pages 64 and 65. Have the pupils, when giving their answers, say "just a little more than" or "just a little less than."

• Have pupils measure something that is about 3 feet in length. Describe its length in feet and then introduce the *1 yard* as another unit of measurement for the same distance.

Using the Text Pages

• As the material in Ex. 1–6 is being developed, be sure to allow for many experiences in (a) estimating distances in inches, feet, and yards; (b) checking the estimate with a body *reference* measure; (c) checking the estimate with a more precise instrument for measuring.

• The important idea to be stressed in Ex. 7–13 is that of appropriateness. Encourage pupils to select correct units of measurement with such comments as, "Does the sentence make sense?" "Is that kind of answer reasonable?"

• Make certain that the pupils understand the directions for the work at the bottom of page 117. They are to write *questions* only to make S. problems from the information given in each exercise. Then they are to work the problems.

Individualizing Instruction

• Help *all pupils* to become acquainted with the use of common tools of measurement for yards and miles, such as the tape measure, yardstick, surveyor's tape, and automobile odometer.

• *More capable children* may do Extra Activities Set 91.

• Write on the chalkboard or duplicate on paper the following examples for *all pupils*:

$$\begin{array}{r} 58 \\ -\ n \\ \hline 46 \end{array} \qquad \begin{array}{r} 22 \\ +\ n \\ \hline 59 \end{array} \qquad \begin{array}{r} n \\ -63 \\ \hline 15 \end{array} \qquad \begin{array}{r} 87 \\ -42 \\ \hline n \end{array}$$

$n + 18 = 48$ $\quad 83 + n = 31$ $\quad n + 64 = 97$

• For *all pupils*, write on the chalkboard or duplicate on paper sentences like the following in which the child is to write the correct operation sign:

$32 \square n = 83$ $\quad n \square 46 = 12$

$15 \square 14 = 29$ $\quad 74 \square 12 = 86$

Reminders

Continue to maintain the following:

a. Ability to use the clock and the calendar with increasing efficiency

b. Ability to recognize commonly-used coins and understand their relative values

c. Ability to record information in appropriate chart form

d. Ability to read and write 3-place numerals

NOTES

Say the right word—feet, inches, yards, or miles.

7. Ann's crayon box is 3 _inches_ wide.
8. The flagpole is 75 _feet_ high.
9. It is 15 _miles_ from one town to the next.
10. It takes 3 _yards_ of cloth to make a coat.
11. The house for my dog is 4 _feet_ long.
12. My finger is about 2 _inches_ long.
13. On a long trip, Mr. Green went 472 _miles_ in one day.

● **Extra Activity.** Work Set 91.

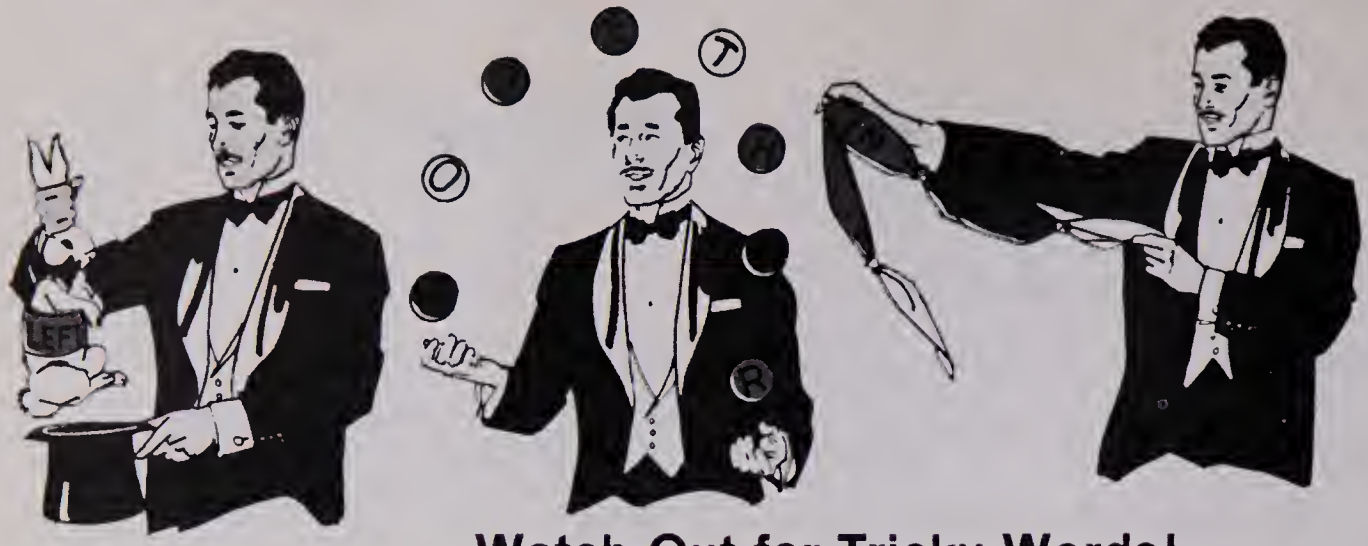
Making and Working Problems

Completing S. problems [W]

Questions will vary.

Write subtraction questions for Ex. 1–10. Then work the problems. Write your work on your paper.

1. You have 59¢. Tom has 47¢. — 12¢
2. We had 48 bags. We used 6 of them for sandwiches. — 42 bags
3. Ann found that 12 of the 16 girls were at school. — 4 girls
4. There were 27 apples. All but 7 were eaten. — 20 apples
5. Joan picked 25 red roses and 12 pink roses. — 13 roses
6. Sue read 38 stories. 5 were about horses. — 33 stories
7. I have 5¢ in my pocket. I want to buy a 29¢ game. — 24¢
8. A 75¢ book is now selling for 45¢. — 30¢
9. Mother made 48 cookies. Only 10 are left. — 38 cookies
10. A big doll costs 98¢. A smaller doll costs 65¢. — 33¢



Watch Out for Tricky Words!

* *Language difficulties in problem-solving* [O]

Some words in problems are tricky. You may think they tell you to add or to subtract when they do not.

Read each problem to find out if you are to add or to subtract. Do not work these problems now.

Does in all always mean to add? Study Ex. 1 and 2.

1. In all, there were 7 boys and 10 girls in a school play. How many children were in the play? 17 children A.

2. In all, there were 19 children at Ann's party. Six were boys. How many were girls? 13 girls S.

Does left always mean to subtract? Study Ex. 3 and 4.

3. Tom lost some of his crayons. He had 8 left in one box and 12 in another. How many crayons were left?
20 crayons A.

4. Tom lost 8 of his 28 crayons. How many were left?
20 crayons S.

Does together always mean to add? Study Ex. 5 and 6.

5. On Saturday, Sam earned 15¢ and Joe earned 20¢. How much did they earn together that day? 35¢ A.

6. Joe and Bill together earned 46¢. If Joe earned 22¢, how much did Bill earn? 24¢ S.

* An important lesson for pupils! These are catchy words which often mislead pupils in problem-solving work.

Teaching Pages 118 and 119

Pupil's Objectives

(a) To see the necessity of reading the entire problem carefully, and of understanding what is asked for, before attempting a solution; and (b) to practice *thinking* the solution to problems without actually writing anything on paper.

New Words

(page 119) *booklet*, *pumpkins*, *sheets*

Background

It is a common practice among pupils to try to solve problems by looking for cues—words or short phrases—which by themselves appear to indicate a certain process. Consideration will reveal that the supposed addition cues *in all* and *together* may appear in subtraction problems; while the supposed subtraction cues *left*, *gone*, and *more* may actually be used in addition problems. The paired problems on pages 118 and 119 will disclose to pupils the unreliability of depending on such word cues.

Pre-Book Lesson

• Review the interrelationships between addition and subtraction in terms of a total and its parts. Make certain that children understand what operation to use when numbers for two parts are given and the number for a total is wanted; also, what operation to use when numbers for a total and one of its two parts are known and the number for the other part must be found.

• Review quickly the various situations in which numbers for a total and a part are known. These different subtraction-problem situations need to be discussed frequently.

Using the Text Pages

• Elaborate upon the idea of not getting caught by *tricky* words. For each pair of problems, make certain that pupils see that the only sure way to determine the operation is to find whether parts are being joined together, and therefore addition is used, or an unknown part is being sought, and therefore subtraction is used.

• After the oral work has been completed, have the children write their work for each problem.

• Read aloud Ex. 1 at the bottom of page 119. Ask children to raise their hands if they know the answer. Have a child tell the answer. Have the child explain how he thought the answer. Analyze the problem for *tricky* words. Determine whether the numbers given in the problem are for parts or for a total and one of its two parts.

Proceed with the rest of the problems in the same manner.

Individualizing Instruction

• Give particular attention to the difficulties of *slower learners* as revealed by the class discussion. They may be asked to illustrate some examples on the chalkboard or on paper with dots or other symbols.

• You may like to have timed tests with part or all of the class. Use incompleted A. and S. facts for material and have pupils write answers on folded paper.

• Have timed tests using the higher-decade A. examples in Extra Examples Set 42.

• Review A. and S. facts having sums 13 through 18.

• Have pupils write a set of A. and S. facts for each sum and two addends given in Ex. a–d. They may write the examples for *a* and *b* in vertical form and those for *c* and *d* in horizontal form.

a. $28 = 12 + 16$ b. $69 = 42 + 27$
c. $45 = 30 + 15$ d. $80 = 60 + 20$

Looking Ahead

• Many lessons to come will use examples and problems involving money numbers and coins. To be certain that the pupils recognize and understand the values of various coins the following review is recommended:

a. If you prepared them, use the lists of coins having equal values and the charts showing relationships between coins for review of coins and their relative values (see Teaching Page 108).

b. Discuss with the class that adding and sub-

tracting money numbers is just like adding and subtracting other numbers.

• Very soon in subtraction, renaming a number to show more ones will be introduced. The following activity will be good preparation for the thinking involved. Show with 10-bundles and single sticks that

a. 43 can be renamed 3 tens and _ ? _ ones.
b. 52 can be renamed _ ? _ tens and 12 ones.
c. _ ? _ can be renamed 6 tens and 15 ones.

NOTES

Does more always mean to subtract? Study Ex. 7 and 8.

7. Peg worked 7 problems. Then she did 6 more. How many problems did Peg work? 13 problems A.

8. Jean worked 14 problems. That was 8 more than Ann worked. How many problems did Ann work? 6 problems S.

Does gone always mean to subtract? Study Ex. 9–10.

9. Tom cannot find 7 of his old marbles and 9 of his new marbles. How many of his marbles are gone? 16 marbles A.

10. Rusty can find only 43 of his 48 marbles. How many of his marbles are gone? 5 marbles S.

[W]

Now write your work for problems 1–10.

Working Problems in Your Head

A. and S. problems [O]

Think the work and give only the answers.

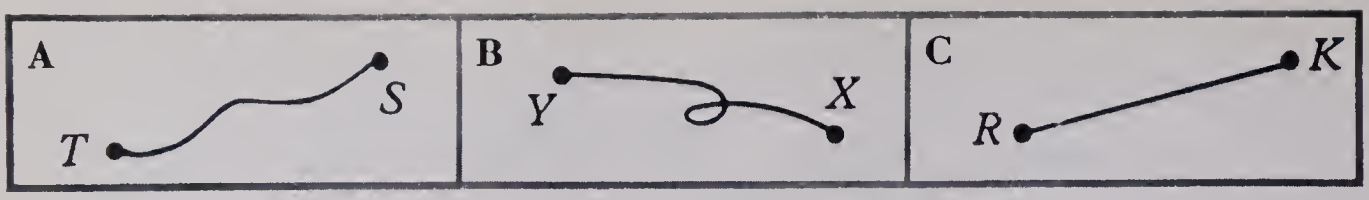
1. May has read all but 8 of the 38 pages in her story. How many pages has she read? 30 pages

2. Mike has 39¢. 30¢ of it is in dimes. How much money has he in other coins? 9¢

3. Tom's father is 39 years old. Tom just had his 8th birthday. How much older is his father than Tom? 31 years

4. One booklet takes 18 sheets of paper. Another takes 10. For both booklets how many sheets are needed? 28 sheets

5. Three pumpkins weigh 8 pounds, 2 pounds, and 7 pounds. How much do the pumpkins weigh together? 17 pounds



Paths

Geometry [0]

1. Box A shows a path from point T to point S or from point S to point T. Could there be other paths from point T to point S? Yes

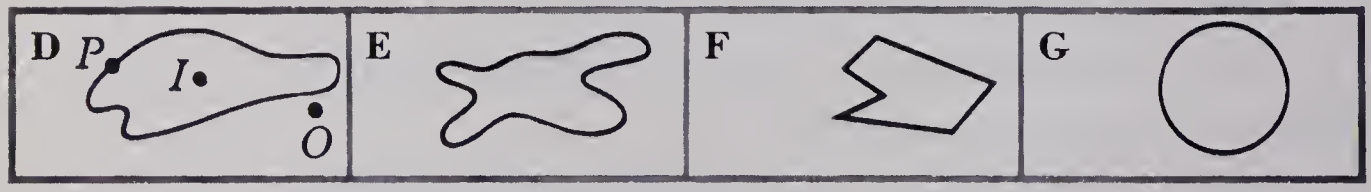
Yes

2. Does the path pictured in box B cross itself? Must every path from point Y to point X cross itself? No

3. In box C, is there a shorter path from point R to point K than the path shown? No

A straight path like the one pictured from the end point R to the end point K is called a line segment. A line segment is a straight path and has two end points.

4. At the board, draw pictures of 3 line segments and label their end points.



5. In these boxes are pictures of simple closed paths. Does it look as if a simple closed path crosses itself? No

6. Box D shows point P in the path, point I inside the path, and point O outside the path. Can you get from I to O and not cross the path? No

7. In any of boxes E–G, can you get from a point inside a path to a point outside and not cross the path? No

The fact that geometric ideas are introduced early in the elementary-school curriculum so that the ideas will be thoroughly understood when more formal geometry is taught later helps make the earlier program mathematics rather than arithmetic.

Teaching Pages 120 and 121

Pupil's Objective

To begin to learn about some of the basic ideas in geometry: points, lines, and polygons.

New Words

(page 120) *closed*, *simple*
(page 121) *polygons*, *rectangles*

Background

Studying about numbers and their properties constitutes but a small part of the discipline of mathematics. In this "space age" we hear a great deal about points, lines, and space in general. Geometry is the study of ideas like these.

A point in geometry is suggested by the "point" of a pencil. Actually, mathematicians do not define a point. They say a point is an undefined term and has certain properties. We may represent points, lines, and other geometric ideas by concrete models, such as a pencil point, or by picture models such as dots. The point is most often represented by a *dot* on a piece of paper or on the chalkboard. Such a dot is thought of as being so small that is has *no* size. The dot represents a point in that it indicates a location. A point may be described as a precise location in space.

Thus far, children have been learning about sets of numbers. Now, sets of points become important. A *line* is thought of as an infinite set of points. The term *line* means *straight line*. It is understood in geometry that all lines are straight. A line is suggested by the edge of a table, the edge of a ruler, the intersection of the ceiling and a wall.

A line is without limit. It extends in each of two directions. A line is usually represented as shown at the right. The arrowheads indicate that the line extends indefinitely in both directions. (This idea has already been introduced through the use of the number line.) It is customary to assign a letter to the representation of a point and then refer to *point D* or *point E*. The letter assigned to a particular point is written near the dot which represents the point.

When we consider only the part of a line between two given points, including the two points, that portion of the line is called a *line segment*. The part of the line between points D and E and the *end points* D and E is a line segment.

Think of two points D and E in space. Think of moving a pencil tip from the location of D to the location of E. The *path* followed by your pencil tip from E to D is generally known as a *curve*. It is possible to follow many paths between the two points. The paths all may be in the same plane. It is possible, however, to follow a path that goes outside this plane. Each path is a set of points and each path is called a *curve*. Even that which we call a *line* is a special case of a *curve*. However, at this time in the *Mathematics We Need* program we shall avoid use of the term "curve" in this sense. Use of the word "path" at this time is adequate to build the geometric concepts introduced.

A minimum of three points in the same plane are required for a *closed curve*. What the children already know as a *rectangle*, a *square*, or a *triangle* is a simple closed curve determined by four or three points which are the end points for four or three line segments. A simple closed curve does not cross over itself at any time but a closed curve may do so.

Polygons are simple closed paths made by line segments (sets of points) all in the same plane.

For the part of the plane within a polygon, we use the words "*inside* the polygon" and for all the plane outside the polygon itself, we use the words "*outside* the polygon." The inside and outside do not include the polygon itself.

Teacher's Preparation

The idea of a *square corner* is introduced on page 121. You may wish to have materials available that children can use for measuring corners of polygons in pictures to determine whether they are square. The corners of any paper you use in class may be assumed to be square.

Cut the paper into four pieces so that each

corner of the original piece becomes a measure of a square corner. It will probably be a good idea to have the children mark the corner with an X or in some way to identify it as the *square-corner measure*.

Pre-Book Lesson (pages 120–121)

• Discuss the idea of a *point* with the class. Discuss as many different ways of representing a point as possible; i.e., sharpened end of pencil, end of a pin or styllus, where two walls and the ceiling meet. Bring out the idea that points exist all around us, because they are locations.

• Introduce children to the practice of using a dot (on the chalkboard or on paper) to represent a point. Make representations of points on the chalkboard and label them with capital letters.

• Place two representations of points on the chalkboard. Label them K and W. Have different children make pictures of paths between the two points. (Intuitively, children are building ideas in terms of experiences with picturing paths in the same plane. It is not necessary to discuss the possibilities of paths outside the plane represented by the chalkboard.)

Using the Text Page (page 120)

There are enough new geometric ideas, terms, and activities on this page to consume all the time normally given to an arithmetic lesson. Consider each of the following very carefully and thoroughly:

a. Paths may be demonstrated with the use of string. It can be demonstrated that the string will be shortest between two points when it is taut. All other paths will require longer pieces of string.

b. Have children draw pictures of simple closed paths on the chalkboard. Encourage many different shapes. Emphasize that a simple closed path begins and ends at the same point and does not cross over itself.

c. Identifying points *in* the path, *inside* the path, and *outside* the path as in Ex. 6 should be stressed and extended.

Using the Text Page (page 121)

Note that the definition of the *rectangle* is such that the *square* is included. Then, the square is introduced as a special kind of rectangle.

You may wish to extend pupil thinking and understanding through questions like these:

a. Is every rectangle also a polygon?
b. Is every square also a rectangle?
c. Is every rectangle also a square?
d. Is every triangle also a polygon?
e. Is every square also a polygon?

Individualizing Instruction

• Have *all pupils* identify representations of polygons in the room. Make sure that they understand that the polygon is the closed curve only and does not include the space inside the curve. Use the names *triangle*, *rectangle*, *square*, and *closed path* as frequently as possible.

• Have *all pupils* draw pictures of open and closed paths. The closed paths should include some polygons. Be sure children can tell what each of their drawings represents and why.

• Prepare for *all pupils* a duplicated sheet picturing line segments. Give directions orally similar to the following:

a. In the first segment pictured mark two points. Label them F and P.

b. In the second segment pictured show a line segment with end points Y and Z.

c. Show two line segments in line segment 3. Label the end points of one H and J. Label the end points of the other M and N.

d. In line segment 4 show a line segment with end points A and B. Show point G so that line segment AG is represented as longer than line segment AB.

It is somewhat difficult for pupils to realize that points, line segments, lines, polygons, and so on are only ideas and what we see are only pictures representing the ideas.

Polygons

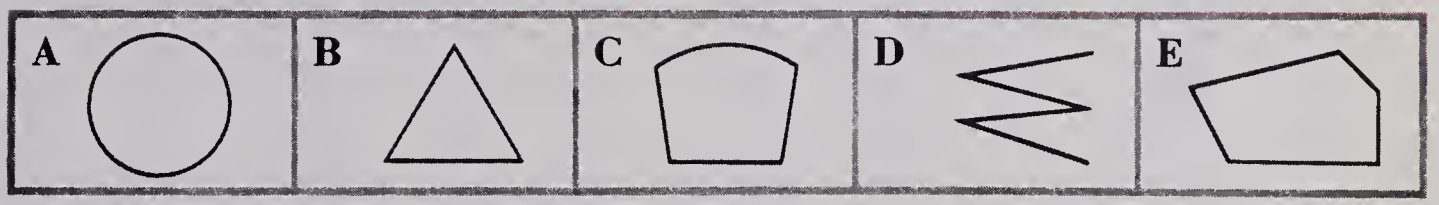
Geometry [O]

Simple closed paths made of line segments only are called polygons. The line segments are the sides.

1. Which figures pictured in A–E below are polygons? B and E

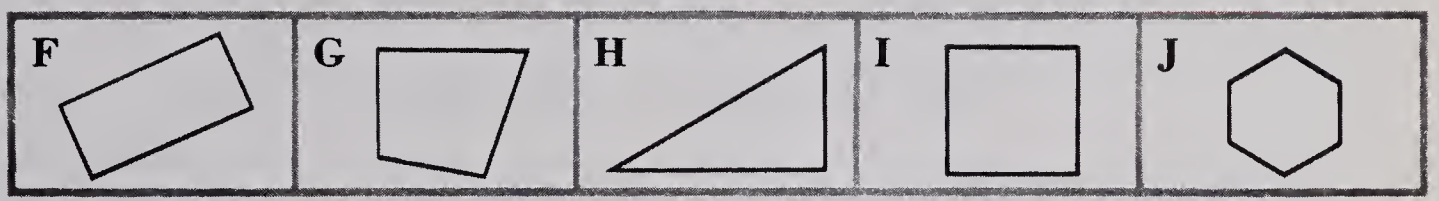
2. Why is a circle not a polygon? It is not made of line segments.

3. Tell how many sides each polygon has. B has 3; E has 5



A polygon with only 3 sides is called a triangle.

4. Which polygons pictured on this page are triangles? B and H



We may call a corner of this page a square corner.

5. Cut a square corner from a piece of paper and use it to test each corner in polygons F–J. Which polygons have one square corner or more? F, G, H, I

6. Tell how many square corners each polygon has. F has 4, G has 1, H has 1, I has 4

A polygon that has only 4 sides and 4 square corners is called a rectangle.

7. Which polygons shown on this page are rectangles? F and I

When the sides of a rectangle all have the same length, the rectangle is called a square.

8. Which polygon pictured on this page is a square? I

* A quick test on subtraction facts having sums as great as 18!

* Finding Unknown Addends

S. facts, sums as great as 18 [W]

Subtract. Write the answers on folded paper.

	a	b	c	d	e	f	g	h	i	j
1.	14	16	13	12	15	11	13	17	15	11
	5	8	7	6	7	9	5	8	6	7
	9	8	6	6	8	2	8	9	9	4
2.	12	11	13	10	18	12	14	11	11	13
	7	8	9	8	9	3	7	5	2	6
	5	3	4	2	9	9	7	6	9	7
3.	16	10	14	12	11	15	10	12	11	12
	9	6	6	9	6	8	9	5	4	8
	7	4	8	3	5	7	1	7	7	4
4.	12	13	17	11	14	13	16	15	10	14
	4	8	9	3	8	4	7	9	7	9
	8	5	8	8	6	9	9	6	3	5

Do You Like to Try New Things?

Enrichment [W]

In the example, $(4 + 3) + (12 - n) = 11$, what number does n stand for? $4 + 3 = 7$ and $7 + 4 = 11$. $12 - n$ must equal 4, so $n = 8$.

What does n stand for in $(13 - 6) - (n + 4) = 0$? $13 - 6 = 7$. $n + 4$ must equal 7, so $n = 3$.

Work Ex. 1–10. On your paper write only the numeral to use in place of n.

1. $(6 + 8) - (18 - 12\,n) = 8$
2. $(9 + 8) - (11 - 0\,n) = 6$
3. $(17 - 8) - (7\,n - 1) = 3$
4. $(8 + 7) - (9 + 3\,n) = 3$
5. $(14 - 8) + (2 + 2\,n) = 10$
6. $(16 - 7) + (3\,n + 7) = 19$
7. $(3 + 6) - (9 - 2\,n) = 2$
8. $(5 + 12) - (11\,n - 3) = 9$
9. $(14 - 9) + (2 + 3\,n) = 10$
10. $(16 - 9) - (8\,n - 7) = 6$

Teaching Page 122

Pupil's Objective

To check progress on the S. facts with sums 10 through 18.

Background

The assignment at the top of page 122 will serve as a progress test on the S. facts. The activity suggested in the enrichment portion on this page (probably useful at this time for the *more capable children* only) falls into the category of *palatable drill*. As discussed previously, drill can be placed in a setting which is motivating and mathematically useful. To complete the following number sentence, $(4 + 2) + (8 - n) = 9$, a knowledge of the A. and S. facts is all that is necessary. However, the activity involves using another name for a number and is preparation for 2-step and 3-step problem-solving. First, the child must think of $(4 + 2)$ as another name for 6. Then, he must think, "6 plus what number equals 9?" After determining that 3 is the number needed to complete the sentence, he must then think, "Eight minus what number is another name for 3?" Consequently, he finds that n in this sentence stands for 5.

Teacher's Preparation

Have paper available for the pupils to use for the folded-paper progress test at the top of page 122.

Using the Text Page

• Without review or discussion, have the children write answers on folded paper. You may wish to use the results of this work as a test of each pupil's progress.

• For children who have no apparent difficulties with the S. facts, you may wish to assign the enrichment material at the bottom of the page. Some children may be able to proceed independently with this written work. However, for others you may wish to have a short oral activity in which you discuss the following:

a. Review the use of another name for a number.

b. Review the addends-sum relationship in terms of A. and S. facts.

c. Illustrate the three steps involved in arriving at the solution of each number sentence (see suggestions in the Background section).

d. Indicate the way in which you wish the pupils to write their work. Some may be able to *think* the answer and merely write the solution on paper. Others may need to write each step on paper.

Individualizing Instruction

• The following types of activity may be useful for *slower learners* who do not have mastery of the S. facts:

a. In an oral activity, give a sum and a pair of its addends and have one of these children tell the set of A. and S. facts (i.e., "The sum I am thinking of is 12. The addends are 7 and 5. Say the set of A. and S. facts for 12; 7 and 5.").

b. These pupils may match A. and S. number-fact cards with answer cards for the sums 10 through 18.

c. Show on the chalkboard some sums with pairs of addends and have pupils show on their papers the set of A. and S. facts for each.

• Have one of the *more capable children* direct a *Show the Answer* game. He may write number sentences on the board and other pupils may show the solution upon a signal from the director of the activity.

• An adaptation of the enrichment material at the bottom of the page for the *slower learners* is the following. Provide several sentences of the type

$$(5 - 2) + (7 + 1) = n$$

In this activity the child must provide other names for two numbers and then either add or subtract to find what n stands for.

• *All pupils* may write answers for the examples in rows 1–4 at the top of the page as though they were examples in higher-decade A. Encourage them to use the "quick" way.

Pupil's Objectives

(a) To make use of the addends-sum relationship in finding what n stands for in number sentences involving A. and S. with 2-place numerals; and (b) to practice renaming tens and ones in order to show 10 more ones.

Background

Both activities on this page are preparation for renaming in subtraction to show 10 more ones. If pupils have no major difficulty in solving the number sentences at the top of the page, then they are ready to go on to more advanced stages of subtraction.

The material at the bottom of the page is readiness for renaming in subtraction examples. Note that for subtraction we have renaming before we compute. In addition, we rename after, and then have to *carry* the digit to the ten's column, as an added step.

Pre-Book Lesson

• Many children have complete mastery of the A. and S. facts, but, the inverse relationship between addition and subtraction of a number has escaped them. These number sentences with 2-place numerals should help bring this relationship into fuller focus. Review the generalizations, with illustrations on the chalkboard, in preparation for the written work at the top of the page:

a. When you know two addends, you can find the sum by adding.

Write on the chalkboard, "$n - 23 = 15$." Have pupils tell whether each number shown is an addend or a sum. Then have them tell what operation to use to find the solution.

b. When you know the sum and one addend, you can subtract to find the other addend.

Write $58 - n = 24$ on the chalkboard. Ask similar questions about the numbers shown in this number sentence.

• In preparation for the written work at the bottom of page 123, have some review of renaming (see the Looking-Ahead section on Teaching Page 119). In the discussion, bring out the following:

a. A number can be named in many different ways.

b. An equals sign may be used between any two number names if they name the same number.

c. There are times when it is important to rename a number in a certain way in order to make it easier for us to solve an example. This particular principle is especially helpful in subtraction examples when we cannot subtract the ones or the tens, and so on.

Using the Text Page

• Before doing the written work at the top of the page you may wish to have your pupils tell whether n stands for a sum or for an addend in each example.

• Emphasize that just one way of renaming the number is used in each exercise at the bottom of the page. It is a special way that will be used when we want to show 10 more ones. Refer to the $\not{0}$'s and $/$'s in the illustrative examples. For each of Ex. 1–8, you may wish to have some children use $\not{0}$'s and $/$'s to show the number before it is renamed and then afterward.

Individualizing Instruction

• *More capable children* may write sentences similar to the following:

a. 6 tens and 14 ones is another name for the number _ ? _.

b. 8 tens and 7 ones can be renamed _ ? _ tens and _ ? _ ones to show 10 more ones.

After they have written 10 sentences of this type, they may exchange with someone and complete the other person's sentences.

• *Slower learners* will probably profit from a more objective development leading to renaming tens and ones. Use a place-value teaching aid* to illustrate what has happened when the number shown in each of Ex. 1–3 is renamed. You may wish to provide each of these children with an abacus picture so that he can illustrate each exercise on paper.

* See 21, page xix.

*More work in using other names for numbers—this time in preparation for work in subtraction when renaming is needed to show more ones or tens or hundreds.

Working with *n*

n standing for an unknown addend or sum [W]

Add or subtract to find the number *n* stands for.

1. $n - 13 = 16$ (29)
2. $48 - n = 25$ (23)
3. $22 + n = 67$ (45)
4. $n + 12 = 42$ (30)
5. $33 + n = 79$ (46)
6. $n - 22 = 55$ (77)
7. $36 + 24 = n$ (60)
8. $66 - n = 34$ (32)
9. $59 - 27 = n$ (32)
10. $n + 32 = 96$ (64)
11. $n - 14 = 43$ (57)
12. $51 + n = 84$ (33)

Renaming to Show 10 More Ones

Readiness for renaming sums for subtracting [O]

A ⏀ ⏀ ⏀ ⏀ ⏀ | | | We can think of these as a total set of 53 sticks, with the parts 50 and 3. Then for set A, why can 53 be named 5 tens and 3 ones?

B ⏀ ⏀ ⏀ ⏀ || || || || || ||| We can think of these also as a total set of 53 sticks, with the parts 40 and 13. For set B, can 53 be renamed 4 tens and 13 ones? Can we write, "5 tens and 3 ones = 4 tens and 13 ones"?

[W]

Copy and finish Ex. 1–8 to show other names.

1. 69 = _?_ (6) tens and 9 ones = 5 tens and 19 _?_ (ones)
2. 75 = 7 tens and _?_ (5) ones = 6 tens and _?_ (15) ones
3. 48 = 4 _?_ (tens) and 8 _?_ (ones) = 3 _?_ (tens) and 18 _?_ (ones)
4. 80 = _?_ (8) tens and _?_ (0) ones = 7 tens and _?_ (10) ones
5. 92 = 9 _?_ (tens) and _?_ (2) ones = 8 tens and _?_ (12) ones
6. 36 = _?_ (3) tens and 6 ones = _?_ (2) tens and 16 _?_ (ones)
7. 51 = 5 _?_ (tens) and _?_ (1) one = _?_ (4) tens and 11 _?_ (ones)
8. 60 = _?_ (6) tens and 0 _?_ (ones) = _?_ (5) tens and 10 _?_ (ones)

Renaming Sums for Subtracting

To show 10 more ones [O]

1. After using 6 of her 24 pictures, Sue wanted to know how many she had left. Could she subtract?

a. Is the number sentence $24 - 6 = n$? Yes

b. Before the renaming in Ex. A to show more ones in the sum, could you subtract the ones? No Why? There were not enough ones.

c. Explain the renaming in Ex. A.

2 tens and 4 ones = 1 ten and 14 ones

A	B	C
1 ten and 14 ones ~~2 tens and 4 ones~~ − 6 ones _?_ and _?_	Tens | Ones 1 | *(14) ~~2~~ | ~~4~~ − | 6 1 | 8	1 (14) * ~~2~~ ~~4~~ 6 18

d. Is Ex. B really like the work in Ex. A? Yes Has the sum, 24, been renamed to show more ones? Yes

To subtract in Ex. B, first subtract ones, then tens.

Ones: $14 - 6 = 8$. Write "8" in one's place.

Tens: Write "1" in ten's place. Why? There is no ten to subtract.

e. Ex. C is the shortest way to subtract. Explain. They must think place-value names.

* Circling the 14 helps emphasize that the 2-place numeral actually is showing that many ones.

Teaching Pages 124 and 125

Pupil's Objective

To learn the rationale of higher-decade subtraction with renaming the sum.

Background

These pages and the next few are designed to develop understanding and skill in higher-decade subtraction when renaming the sum is required for subtracting. The teaching plan is similar to those recently given for higher-decade addition with bridging (see Teaching Pages 102 and 103) and adding with two 2-place numerals requiring renaming and carrying (see Teaching Pages 110 and 111).

It may be well to recall certain general features of the teaching program:

a. An emphasis at the outset upon an application of the new skill, thus drawing attention to its usefulness.

b. A progression in materials, from the use of objects to pictures of objects to abstract symbols.

c. A corresponding progression in thought processes, from verbal accounts of what is done in regrouping objects to explanations of the operation and the renaming necessary, and finally to a way of *thinking* the subtraction and showing an answer.

d. The importance of teaching carefully, to the end that the generalizations and the relationships will be learned meaningfully and understood thoroughly.

e. The need to vary instruction for differences in learning rates.

Note on pupil's page 124 that the result of renaming the sum in box C is kept in the proper place-value columns and that the digits showing the ones are encircled.

The same method of encircling is used again later when two renamings are involved (see pupil's page 192). This may be seen for 842 − 97 in steps *a* and *b* below.

a.
$$\begin{array}{r} 3\,(12) \\ 8\,\not{4}\,\not{2} \\ -\ 9\,7 \\ \hline 5 \end{array}$$

b.
$$\begin{array}{r} 7\,(13)\,(12) \\ \not{8}\,\not{4}\,\not{2} \\ -\ 9\,7 \\ \hline 7\,4\,5 \end{array}$$

Of course, in an example such as 205 − 47, where 20 tens are renamed (see pupil's page 196), only the digits for the ones require encircling as shown in the following written work:

$$\begin{array}{r} 1\,9\,(15) \\ \not{2}\,0\,\not{5} \\ -\ 4\,7 \\ \hline 1\,5\,8 \end{array}$$

Be sure this encircling plan is understood by your pupils, because its use will be continued until thorough understanding has been attained.

Teacher's Preparation

Make certain that appropriate materials are available to use in demonstrating regrouping to show why we rename the sum in order to subtract.

Pre-Book Lesson

• Institute activities with objects in order that the pupils may discover for themselves the reason why we rename a sum in order to subtract. For example, write on the chalkboard:

Lucy has passed out 8 pencils from the box of 24. She wants to know how many pencils remain in the box.

a. Use rubber bands to bundle together 24 pencils (or dowels or splints or tagboard strips) so the pupils may see the arrangement of 2 bundles of ten and 4 single pencils.

b. Ask your pupils how 8 pencils (representing those distributed by Lucy) are to be taken away from this group. The solution will probably be suggested.

Since 8 pencils cannot be taken from the 4 individual pencils, then something must be done so the 8 pencils can be taken away. At this point, be sure to demonstrate the necessary regrouping of 24 pencils. As the pencils were arranged, the child saw 2 bundles of ten and 4 single pencils and decided that he could not take away 8 pencils. After one of the bundles of ten had been unbound and changed to single pencils, he knew that he then could take away 8 pencils.

Have a child remove 8 pencils and tell how many pencils are left by *reading* the arrangement of pencils as 1 bundle of ten and 6 single pencils, or 16 pencils.

• A solution to the problem having been found, try several examples of the same type. While demonstrations are in progress with bundles of objects, records to show the renaming and subtracting may be kept on the chalkboard and at each child's desk. ⌀ 's and / 's, words, and numerals should be used for keeping records, as illustrated below in *a–c* for the example 35 − 7.

a. ⌀'s and / 's

Step 1. ⌀ ⌀ ⌀ ///// = ⌀ ⌀ ////////////////

Step 2. ⌀ ⌀ ////////////

b. Using words

Step 1. 3 tens and 5 ones = 2 tens and 15 ones

Step 2.

	2 tens and 15 ones
−	7 ones
	2 tens and 8 ones

c. Using numerals

Step 1. Sum renamed in order to subtract in one's column

35	2 (15)
− 7	− 7

Step 2.

2 (15)
− 7
28

Using the Text Pages

• As Ex. 1–4 are discussed, emphasize the need for renaming. Make certain that understanding is being developed.

• Ex. 5–12 may be done at the chalkboard. You may give one example (as Ex. 5) for pupils to write in the two ways specified (Ex. D and F). This will serve as a check to determine which pupils are able to proceed independently with written work. Continue to work with *slower learners*, using objects if necessary, in order to help them overcome specific weaknesses.

Individualizing Instruction

• *Slower learners* may need to continue work with materials in order to *see* that renaming is necessary. They should be permitted to use the crutches illustrated as long as needed.

It is suggested that, immediately following the working with crutches, the same example be done without crutches, so that progress will be made toward omitting them entirely. You are cautioned, however, against forcing a child to abandon the crutch until he feels secure without it.

• *More capable children* may demonstrate regrouping with other means, such as tickets in a pocket chart, dimes and cents, stacked cardboard circles, "beads" on an abacus picture, Ginn Arithme-Sticks.* Have them at the same time keep a running record on the chalkboard to show the renaming and subtracting. These children should be able to do this, whereas the *slower learners* might have to work with objects only.

In each form of manipulation along with the accompanying record, the transformation of the total and the renaming of the sum are clearly demonstrated. The use of alternate forms of demonstration reinforces the understandings built in the developmental lesson.

• *All pupils* may make number-line pictures to illustrate Ex. 5–10. Perhaps you will want to duplicate number-line pictures for the children to use.

• *Slower learners* may need more renaming practice of the following type:

a. 57 = _ ? _ tens and 7 ones = 4 tens and 17 _ ? _.

b. 63 = 6 tens and _ ? _ ones = 5 tens and _ ? _ ones.

c. 68 = 6 _ ? _ and 8 _ ? _ = 5 _ ? _ and 18 _ ? _.

d. 90 = _ ? _ tens and _ ? _ ones = 8 tens and _ ? _ ones.

e. 72 = 7 _ ? _ and _ ? _ ones = 6 tens and _ ? _ ones.

f. 56 = _ ? _ tens and 6 ones = _ ? _ tens and 16 _ ? _.

g. 71 = 7 _ ? _ and _ ? _ one = _ ? _ tens and 11 _ ? _.

• *All pupils* may do Ex. 5–10 as follows:

$$\begin{array}{rrrrr} 43 & = & 40 + 3 & = & 30 + 13 \\ -\ 4 & = & 4 & = & 4 \\ \hline & & & & 30 + 9 = 39 \end{array}$$

* See 3, page xix.

Encourage children to abandon the crutch as soon as they can do so securely.

2. Nine of Sue's 24 pictures are colored. How many of her pictures are not colored?

a. Is the number sentence $24 - 9 = n$? Yes

b. Why must you rename the sum? To show more ones

24 = 2 tens and 4 ones, or 1 ten and 14 ones.

c. Tell how to finish the work for Ex. D; for Ex. E. Write 5 in one's place. Write 1 in ten's place.

3. $43 - 8 = n$

a. Can you subtract the ones? No Why? There are not enough ones.

b. Tell how to finish the work for Ex. F; for Ex. G. 4 tens and 3 ones can be renamed 3 tens and 13 ones. 13 − 8 = 5. Write 5 in one's place and 3 in ten's place.

D	E
1 ten and 14 ones ~~2 tens and 4 ones~~ − 9 ones ? 1 ten and 5 ones	1 (14) ~~2 4~~ − 9 ? 15
F	**G**
4 tens and 3 ones − 8 ones ? 3 tens and 5 ones	43 − 8 ? 35

4. For which of these examples must you rename the sum? Why? a, b, c, d, and e You cannot subtract the ones.

a. 26 − 8	**d.** 90 − 9
b. 44 − 9	**e.** 52 − 6
c. 30 − 6	**f.** 78 − 5

Copy Ex. 5–7 on the board and subtract. Use tens and ones as in Ex. D and F.

5.	6.	7.	8.	9.	10.
43	62	70	65	74	57
− 5	− 8	− 5	− 9	− 6	− 8
38	54	65	56	68	49

Copy Ex. 8–10 on the board and subtract. Use the shortest way, as in Ex. E and G.

11. Write 8 subtraction examples on the board, 5 in which you rename the sum and 3 in which you do not.

12. Finish the examples you wrote for Ex. 11.

When you cannot subtract the ones, you rename the sum to show 10 more ones.

More Sets Using Subtraction Facts

With renaming [O]

A
32 − 7
82 − 7
22 − 7

1. The examples in box A are in the set using the S. fact, 12 − 7 = 5.

a. How do you know this is so? Each sum is renamed to show 12 ones.

b. The one's digit in each answer is 5. Why? 12−7=5

c. Each time, why does the ten's digit in the answer show 1 fewer ten than the ten's digit in the sum? Each sum is renamed showing 1 less ten.

d. Say 3 other examples in the S. set using 12 − 7 = 5.
Answers will vary. 32−7; 42−7; 52−7

B
51 − 8
21 − 8
91 − 8

2. The set of examples in box B use what S. fact? 11−8=3

a. How can you tell? Each sum is renamed to show 11 ones.

b. Explain the digit in one's place in each answer. 11−8=3

c. Explain the digit in ten's place in each answer. Each sum is renamed showing 1 less ten.

d. Say 3 other examples in this S. set.
Answers will vary. 31−8; 61−8, 71−8

Say only the ten's digit in each answer for Ex. 3–11.

3. $80 - 6 \overset{7}{=} ?4$
4. $30 - 6 \overset{2}{=} ?4$
5. $90 - 6 \overset{8}{=} ?4$
6. $24 - 8 \overset{1}{=} ?6$
7. $46 - 8 \overset{3}{=} ?8$
8. $92 - 3 \overset{8}{=} ?9$
9. $67 - 9 \overset{5}{=} ?8$
10. $45 - 7 \overset{3}{=} ?8$
11. $63 - 7 \overset{5}{=} ?6$

Write on the board 3 examples in the same S. set as
Answers will vary.

12. 12 − 9.
13. 23 − 7.
14. 30 − 7.
15. 15 − 6.
16. 25 − 8.
17. 36 − 9.
18. 18 − 9.
19. 24 − 6.
20. 33 − 5.
21. 16 − 7.
22. 22 − 5.
23. 37 − 8.

24. Say only the ten's digit in the answer for each example you wrote.

25. Now say only the one's digit in the answer for each example.

Pupil's Objective

To use knowledge and understanding of the harder basic S. facts to organize new sets of higher-decade subtraction examples when renaming is necessary.

Background

Identifying sets of examples using addition and subtraction facts assists pupils in the development of generalizations. They have learned the principles governing higher-decade addition with and without bridging and in subtraction in which no renaming is involved.

In examples like 45 − 7, 95 − 7, 35 − 7, the set of examples using the subtraction fact 15 − 7 = 8 can be identified. To be proficient in higher-decade subtraction, with and without renaming, the pupils must know the basic subtraction facts.

Samples of the shortened way of dealing with each example are:

42 − 7, *think*, "(12 − 7), 5 (ones); 3 (tens); 35 (answer)." Or, briefer still, for 54 − 6, *think*, "8, 48."

Teacher's Preparation

Draw three number-line pictures on the chalkboard to show points for numbers (a) 55 through 64; (b) 25 through 34; (c) 85 through 94.

Arrange and label the points on each number-line picture so that the one's digits which are alike are one above another. Leave room between each of the three for pupils to draw arrows.

Pre-Book Lesson

• Write these three number sentences on the chalkboard: 63 − 6 = *n*, 33 − 6 = *n*, 93 − 6 = *n*.

Ask, "In what ways are these number sentences alike?" Anticipate answers like: "They all have 3 in the one's place." "They are all subtraction sentences." "Each has 6 to be subtracted."

• With appropriate discussion, illustrate the subtracting in each sentence on one of the number-line pictures prepared in advance.

• Have pupils tell ways in which the moves on the number-line pictures were similar.

Be sure to elicit these two important generalizations:

a. The examples are all related to the basic S. fact, 13 − 6 = 7.

b. For each example, the ten's digit in the answer shows 1 fewer ten than the number of tens in the sum.

Using the Text Page

Other representative objects, such as bundles of dowels, paper strips, Arithme-Sticks, an abacus, may be used simultaneously with the discussion of the examples in box A and box B, to clarify the ideas that the digit in one's place is the same for all examples in each set and that the digit in ten's place shows 1 fewer ten than the number of tens in the sum.

Encourage pupils to *think* the answers by sets, so as to minimize dependency upon the crutch—rewriting the ones. Not all pupils will be able to think the answers at this time.

Individualizing Instruction

Slower learners may understand renaming but still may be weak on the more difficult subtraction facts. Use this opportunity to strengthen their skills by having them make ⌀'s and /'s and complete the following sets:

a. 13 − 5 = 8, so 23 − 5 = ?8 and 43 − 5 = ?8
b. 12 − 5 = 7, so 22 − 5 = ?7 and 62 − 5 = ?7
c. 10 − 6 = 4, so 40 − 6 = ?4 and 70 − 6 = ?4
d. 11 − 3 = 8, so 31 − 3 = ?8 and 61 − 3 = 5?
e. 14 − 5 = 9, so 24 − 5 = ?9 and 74 − 5 = 6?
f. 17 − 8 = 9, so 47 − 8 = ?9 and 87 − 8 = ?
g. 13 − 6 = 7, so 53 − 6 = ? and 93 − 6 = ?

Write three examples in the same set, with answers.

h. 11 − 9	i. 15 − 6	j. 13 − 7
k. 14 − 7	l. 10 − 7	m. 12 − 8

Teaching Pages 127 and 128

Pupil's Objectives

(a) To learn the shortest, most economical way of finding answers for higher-decade S. examples when renaming of the sum is necessary; and (b) to practice finding answers in higher-decade subtraction with and without renaming the sum.

Background

Do not expect all pupils to respond immediately to the *quick way*. Helpers and crutches may be needed temporarily and intermittently, but try to have children avoid fixing patterns that later must be changed. Help children to keep in sight the goal of immediate response by encouraging immediate but accurate answers.

Note the thought pattern described in Ex. 1 for 27¢ − 8¢ (*think*, "9¢, 19¢"). *Slower learners* may need to use an intermediate step in which for 27 − 8 they *think*, "9" (for the ones) "1" (for the renamed tens), and then *think*, "19."

The practice material on page 127, both oral and written, includes only those examples whose sums must be renamed. Mixed practice, both with and without renaming, is provided at the bottom of page 128.

Pre-Book Lesson

• Demonstrate with objects the regrouping that occurs and consequently the renaming of the sum for subtracting in 45 − 8. In the ensuing discussion be sure to elicit these ideas:

a. the subtraction fact 15 − 8 = 7 is identified.

b. the ten's digit in the answer shows 1 fewer ten than the ten's digit in the sum.

• Illustrate subtracting for the same example on a number-line picture on the chalkboard.

Using the Text Page (page 127)

• As children give the answers orally for Ex. 1–6, you will see which pupils can think the answers quickly and are ready to proceed with the written work.

• Ask pupils who are slow in recording answers to do a few orally for you privately so that you may follow their thought processes and give remedial help.

Using the Text Page (page 128)

• For *slower learners* you may be able to anticipate and prevent confusion in the written work at the top of the page by having them first do some of the work suggested in the Individualizing Instruction section below.

• The written work at the bottom of the page may be adapted in many different ways as suggested in the following section.

Individualizing Instruction

• For *slower learners* particularly, duplicate or put on the board examples like those below. Explain the purpose of the assignment and give as much help as you think necessary.

a. Find and work examples in which renaming of the sum is not necessary.

39	52	28	65	76	46
−6	−8	−4	−5	−7	−2

b. Find and work examples in which renaming of the sum is necessary.

21	62	56	23	49	32
−7	−2	−9	−4	−3	−8

c. Write the missing digit. Be careful!

44	36	82	57	63	25
−5	−1	−7	−8	−3	−4
?9	?5	?5	?9	?0	?1

• *All pupils.* To help develop the quick way of giving the answer in this type of higher-decade subtraction, let children play the following game:

Place an empty box on a desk. Hold up a group of splints in bundles of tens and also some singles. Have children note how many there are, and then hide the splints in the box. Withdraw a few single splints (any number from 1 to 9), show them, and ask how many are in the box. Repeat, using any number from 20 to 99, and withdrawing any number from 1 to 9. Points may be given for correct

A Quick Way to Subtract

[O]

Sam has 27¢, Bob has 20¢, and Mike has 32¢. If each boy buys an 8¢ ice cream, each one will have how much money left?

1.	2.	3.
27¢	20¢	32¢
−8¢	−8¢	−8¢
19¢	12¢	24¢

Will the S. sets using $17 - 8 = 9$, $10 - 8 = 2$, and $12 - 8 = 4$ help you?

For Ex. 1 *think*, "9¢, 19¢." Explain.

Now, think the answers for Ex. 2 and Ex. 3 the same quick way.

Find the answers in rows 4–6 the quick way.

	a	b	c	d	e	f
4.	42 − 7 35	54 − 7 47	24 − 5 19	37 − 8 29	72¢ − 5¢ 67¢	41¢ − 8¢ 33¢
5.	76 − 9 67	83 − 4 79	40 − 4 36	86 − 7 79	21¢ − 5¢ 16¢	63¢ − 7¢ 56¢
6.	60 − 7 53	61 − 9 52	65 − 9 56	90 − 5 85	64¢ − 9¢ 55¢	44¢ − 6¢ 38¢

[W]

For rows 7–9, write the answers on folded paper. Subtract the quick way. We subtract for column f in the same way that we subtract other numbers.

7.	93 − 6 87	55 − 7 48	70 − 3 67	67 − 9 58	32 − 6 26	\$0.30 − \$0.09 \$0.21
8.	82 − 6 76	41 − 3 38	62 − 9 53	51 − 7 44	81 − 4 77	\$0.73 − \$0.09 \$0.64
9.	84 − 6 78	75 − 6 69	43 − 8 35	70 − 1 69	46 − 9 37	\$0.62 − \$0.04 \$0.58

Copy in columns. Subtract the quick way. Check.

10.	40 − 2 38	46 − 8 38	63 − 4 59	75 − 8 67	53 − 6 47	\$0.42 − \$0.08 \$0.34
11.	81 − 2 79	57 − 8 49	41 − 4 37	23 − 5 18	48 − 9 39	\$0.30 − \$0.06 \$0.24

A Quicker Way to Subtract

[O]

$61 - 8 = n$. The ten's digit in the answer will show 1 fewer ten. So, *think* the answer at once, "53."

Rows 1–2. Think the answer at once. Then say it.

	a	b	c	d	e	f
1.	35 − 8 27	35 − 7 28	21 − 4 17	60 − 3 57	53 − 7 46	34¢ − 6¢ 28¢
2.	81 − 9 72	43 − 4 39	46 − 7 39	62 − 7 55	86 − 9 77	22¢ − 5¢ 17¢

[W]

For rows 3–5, think the answers at once. Write your answers on folded paper, and check your work.

3.	38 − 9 29	21 − 5 16	83 − 9 74	50 − 6 44	42 − 6 36	83¢ − 8¢ 75¢
4.	80 − 4 76	74 − 7 67	26 − 8 18	47 − 8 39	74 − 5 69	61¢ − 8¢ 53¢
5.	52 − 9 43	65 − 9 56	92 − 8 84	71 − 6 65	60 − 7 53	44¢ − 9¢ 35¢

◆ **Extra Examples.** Work Set 46.

Try These

[W]

Use folded paper. Check your work.

	a	b	c	d	e	f
1.	80 − 7 73	49 − 8 41	81 − 3 78	32 − 3 29	53 − 7 46	$0.78 − $0.08 $0.70
2.	37 − 2 35	53 − 5 48	84 − 5 79	56 − 9 47	38 − 6 32	$0.60 − $0.09 $0.51
3.	63 − 8 55	71 − 2 69	39 − 6 33	46 − 9 37	41 − 7 34	$0.55 − $0.07 $0.48
4.	50 − 8 42	86 − 8 78	58 − 9 49	74 − 6 68	84 − 8 76	$0.38 − $0.05 $0.33
5.	37 − 7 30	97 − 8 89	63 − 4 59	67 − 9 58	62 − 5 57	$0.93 − $0.09 $0.84
6.	83 − 6 77	65 − 8 57	82 − 6 76	49 − 7 42	40 − 2 38	$0.84 − $0.09 $0.75

◆ **Extra Examples.** Work Set 47.

responses, so that the game may be played by teams or by individual groups of partners.

Slower learners may be asked to check answers by counting backward.

• You may wish to use the Try These section with the *more capable children* in one or more of the following ways:

a. Write only the answers on folded paper, checking each answer as it is written.

b. Conduct a "speed" contest. Have a child say the answers only to a row or a column. Keep a record of both speed and accuracy. Teams may be formed or competition may be on an individual basis. Permit children to enter the contest as they wish.

• The Try These section may be adapted for *all pupils* as follows:

a. Have the children show the number renamed in order to complete the subtraction. For example, in 1c they may write 81 = 7 tens and 11 ones. Select just certain examples to be worked in this manner. Use numeral and letter as a means of identifying the examples in order to give the children more experience in reading and following directions of this type.

b. Have the children either tell or write another example that belongs to the same subtraction set as the example selected.

• As needed, assign Extra Examples Sets 46 and 47.

NOTES

Teaching Page 129

Pupil's Objectives

(a) To have practice in computational skills learned so far; and (b) to find further applications of logic as a setting for practice in addition and subtraction skills.

Background

The practice at the top of the page includes in mixed order all of the addition and subtraction skills that have been taught:

a. column addition with as many as 4 addends

b. higher-decade addition with and without renaming and carrying

c. adding with 2-place numerals with and without renaming and carrying

d. higher-decade subtraction with and without renaming

e. subtracting with 2-place numerals without renaming

The material affords opportunity for a preliminary survey of achievement—preliminary in the sense that pupils will still have time before the chapter computation test to bring their skills up to the level of their capacity.

Pre-Book Lesson

• Ask children to select from the set of examples at the top of page 129 one example of each kind:

a. Higher-decade addition with no renaming and no carrying

b. Higher-decade addition with renaming and carrying

c. Higher-decade subtraction with no renaming

d. Higher-decade subtraction with renaming

e. Addition with 2-place numerals requiring no renaming and no carrying

f. Addition with 2-place numerals requiring renaming and carrying

g. Subtraction with 2-place numerals requiring no renaming

h. Column addition with three and four addends

You may want to mix the order in which you ask for each skill to be identified.

• Ask the children to study the *if-then* sentences at the bottom of page 129 to see whether there are any apparent difficulties. Answers only are to be given. Written work of this type has been presented several times previously (see pupil's page 107).

Using the Text Page

• For the exercises at the top of page 129, you may wish to assign only one row at a time to *slower learners*. Give them a *score* (number right) for each row. After helping them, encourage improvement of the score for the next row.

• The *more capable children* should be able to proceed independently with the work at the bottom of the page. You may prefer to assign this work to the *slower learners* instead of all of the written exercises at the top of the page. This is drill work, too, but is placed in a setting that is more motivating to children. Circulate among the children while they are doing this written work and discuss any problems with them individually.

Individualizing Instruction

• Have *all pupils* compile lists of words frequently used in arithmetic. These lists may include words with which you will want your children to be familiar for vocabulary and writing purposes. They may write on newsprint or butcher paper classifications such as *Numbers*, *Money*, *Fractions*, *Shapes*, *Measures*, and so on, and list their words in the appropriate place.

• Assign Extra Examples Set 48, as deemed necessary.

To Keep in Practice

A. and S. [W]

Write answers on folded paper. Check your work.

	a	b	c	d	e	f	g	h	i
1.	96	46	89	64	60	48	37	53	\$0.80
	−7	−6	−83	+35	+7	−9	+45	−9	−0.06
	89	40	6	99	67	39	82	44	\$0.74
2.	7	82	78	35	82	36	65	46	\$0.57
	+52	+5	−2	+40	−7	+32	−8	+38	−0.09
	59	87	76	75	75	68	57	84	\$0.48
3.	80	78	45	68	26	40	23	68	\$0.97
	−9	−58	−7	−45	+31	−30	−6	−34	−0.08
	71	20	38	23	57	10	17	34	\$0.89
4.	2¢	3¢	12¢	5¢	9¢	4¢	3¢	7¢	\$0.81
	4¢	4¢	0	2¢	5¢	3¢	2¢	4¢	0.01
	5¢	8¢	9¢	6¢	2¢	9¢	4¢	5¢	0.06
	11¢	5¢	2¢	8¢	5¢	9¢	6¢	4¢	0.05
		20¢	23¢	21¢	21¢	25¢	15¢	20¢	\$0.93

Extra Examples. Work Set 48.

If . . ., Then . . .

Logic [W]

Work each of Ex. 1–9. Write only the answers.

1. If $n = 65$, then
   a. $n + 8 = ?$ 73
   b. $n - 8 = ?$ 57

2. If $n = 8$, then
   a. $72 + n = ?$ 80
   b. $72 - n = ?$ 64

3. If $n = 9$, then
   a. $33 - n = ?$ 24
   b. $n - 7 = ?$ 2

4. If $n = 50$, then
   a. $20 + n = ?$ 70
   b. $n - 8 = ?$ 42

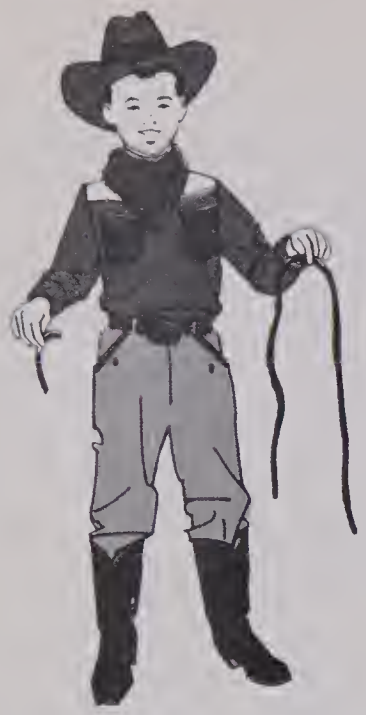
5. If $n = 24$, then
   a. $n + 18 = ?$ 42
   b. $n - 6 = ?$ 18

6. If $n = 55$, then
   a. $17 + n = ?$ 72
   b. $n - 8 = ?$ 47

7. If $n = 33$, then
   a. $n + 30 = ?$ 63
   b. $n + n = ?$ 66

8. If $n = 68$, then
   a. $n + 20 = ?$ 88
   b. $99 - n = ?$ 31

9. If $n = 25$, then
   a. $n - 6 = ?$ 19
   b. $30 + n = ?$ 55



Which Answer Cannot Be Right?

Help in problem-solving: a reasonable answer in S. [O]

1. Sam's rope was 52 inches long. He cut off 8 inches. Then it was 44 _?_ inches long.

Joe thought that the rope would be 60 inches long. Why was he wrong? The part left must be smaller than the whole rope.

When you subtract from a whole-number sum and the answer is a whole number, can that answer ever be greater than the sum? No Why? The number for the part left will be less than the number for the total.

2. Mary missed 8 of 34 examples. Did she work 42 examples correctly? No Why, or why not? The number for the other part will be less than the number for the total.

For each of problems 3–8 tell which of the two numbers shown must be the wrong answer. Explain why.

20¢ 3. A large ball cost 57¢. A book cost 37¢. How much less did the book cost than the ball? **20¢** **94¢** 94¢
94¢ is greater than the sum 57¢

26 4. Of 58 toy soldiers, 32 were broken. How many soldiers were not broken? **90** **26** 90
90 is greater than the sum 58.

24 5. Nan cut out all but 6 of her 30 paper dolls. How many did she cut out? **36** **24** 36
36 is greater than the sum 30.

12 6. Our room is 37 feet long and 25 feet wide. How many feet longer is the room than it is wide? **12** **62** 62
62 is greater than the sum 37.

44¢ 7. If you spend 6¢ from a half dollar, how much money would you have left? **56¢** **44¢** 56¢
56¢ is greater than the sum 50¢.

14 8. One boat is 12 feet long. Another is 26 feet long. How many feet longer is the larger boat? **38** **14** 38
38 is greater than the sum 26.

[W]

Write the work for problems 1–8.

 Correct answers shown at left of problem numeral.

Teaching Page 130

Pupil's Objective

To learn to note the relationship in subtraction between the numbers for the sum and the answer, as a check against answers in subtraction showing numbers which are far too great.

New Words

soldiers, wrong

Background

Reasonableness is an important ingredient in mathematical thinking. Children should be able to use the understandings, the principles, and the generalizations they have been building as a basis for accepting or rejecting an answer to a number sentence or a problem. They should begin to ask questions like, "Does the answer make sense?" If the child has no basis for saying "Yes" or "No" to the question, then he lacks in understanding.

Teacher's Preparation

You may have noted several subtraction examples from pupil's work in which answers showed numbers greater than the sums subtracted from. If possible, use such cases (without embarrassing a child) in the Pre-Book Lesson.

Pre-Book Lesson

Write on the chalkboard examples with answers similar to the following:

34	29	40	56	25
−6	−9	−5	−23	−14
38	29	45	57	39

Have the children analyze each example and tell why the answer obviously is wrong.

Using the Text Page

- First have pupils tell which of the two answers given after each problem is wrong and why they know that it is.
- After doing the oral work, pupils should be able to proceed easily with the written work on the page.

Individualizing Instruction

- For *slower learners*, prepare exercises like those below, in which one of the three answers given—showing so great a number as to be obviously wrong—is to be crossed out or encircled.

24 − 9 (33 5 15)	75 − 7 (59 82 68)
44 − 6 (38 17 52)	37 − 15 (22 16 62)
49 − 24 (74 19 25)	64 − 13 (51 80 44)

Be sure to point out that the correct answer may not be present, but the child is to select the answer that does not "make sense" because it shows a number greater than the sum shown in the example.

- *All pupils* may continue with the list of words used in arithmetic.
- *More capable children* may write and solve *if-then* sentences of their own, using any of the skills they have learned.

Teaching Page 131

Pupil's Objectives

(a) To use given information to make questions for addition and subtraction problems; and (b) to find the number *n* stands for in addition and subtraction number sentences.

New Words

bulbs, stage

Background

The role of *translation* in solving problems has been discussed previously. In this lesson the emphasis is not upon making a complete translation from the written words to the symbolic sentence, or vice versa, but upon using given information as a basis for stating or writing a problem question of the type (A. or S.) designated. Note that in each exercise the child is told the type of problem to make—either A. or S.

Pre-Book Lesson

Write some interesting problem information on the board that the children can use for either an addition problem or a subtraction problem.

At the beach Billy found 6 starfish and his sister found 9 starfish.

Ask children to think of an A. question that could be asked about the information. Then ask for a S. question. Ask several pupils to state their questions.

Emphasize the fact that very often either an addition or a subtraction question can be asked about the same information.

Using the Text Page

• Have pupils state orally either an A. question or a S. question to make a problem, as directed for each exercise. Choose the best question stated for each problem and write it on the chalkboard. These questions will be referred to later in the written part of the lesson.

• Have pupils write their work for the solution of each problem. Urge them to read the information written in the book as well as the question written on the board.

• Before doing the written work at the bottom of page 131 have children tell why they would add or subtract to find the number *n* stands for in each example.

Individualizing Instruction

• Ideally, your pupils now should know all the A. and S. facts, but actually you may find that this is not the case. Listen as *slower learners* do orally some of the examples at the top of page 129. You may find specific weaknesses (which are easily overcome) or general weaknesses (which indicate the necessity for constant review of beginning learnings). Have these pupils continue to use study cards for facts needing practice.

• Have *more capable children* check all sets of Study Cards being used to see that they are complete and correct for all A. and S. facts with sums through 18.

• Have *more capable children* make on cards sets of higher-decade examples using A. facts (without and with carrying) and sets using S. facts which do and do not require renaming.

• *All pupils* may write *n*-examples for the problems whose questions are written on the board.

• *All pupils* may make an addition-subtraction chart for the sums 10 through 18 and their pairs of addends, beginning with 5 and ending with 9.

	5	6	7	8	9
5	10	11			
6					
7					
8					
9					

Reminder

Continue to maintain the following:

a. *Our Subtraction Book* (see Teaching Page 73) in which pupils record illustrations of different situations requiring subtraction

b. Ability to recognize commonly-used coins and understand their relative values

Making Questions for Problems

Completing A. and S. problems [O]

Make a question for each problem. *A.* means to make an addition question. *S.* means to make a subtraction question.

Questions will vary.

1. In the school play there are 18 girls, 8 boys, and 4 older people. *A.* 30 people

2. Joe is on the stage 9 times. Bob is on the stage 21 times. *S.* 12 times

3. In her part, Betty must say 35 lines. She has learned 25 lines. *S.* 10 lines

4. The play has two acts. The first act lasts 28 minutes. The second lasts 17 minutes. *A.* 45 minutes

5. Fifty-two tickets for the play have been sold. There are 38 more tickets to be sold. *A.* 90 tickets

6. Of the 50 light bulbs, 8 bulbs are no good. *S.* 42 bulbs

[W]

Write your work to find answers for problems 1–6.

Working with *n*

n in any position in A. or S. [W]

Work each example to find the number *n* stands for.

1. $\overset{45}{n} + 6 = 51$

2. $79 - \overset{72}{n} = 7$

3. $\overset{58}{n} - 22 = 36$

4. $72 = 3 + \overset{69}{n}$

5. $\overset{10}{n} = 76 - 66$

6. $\overset{37}{n} + 8 = 45$

7. $63 + \overset{22}{n} = 85$

8. $\overset{50}{n} - 30 = 20$

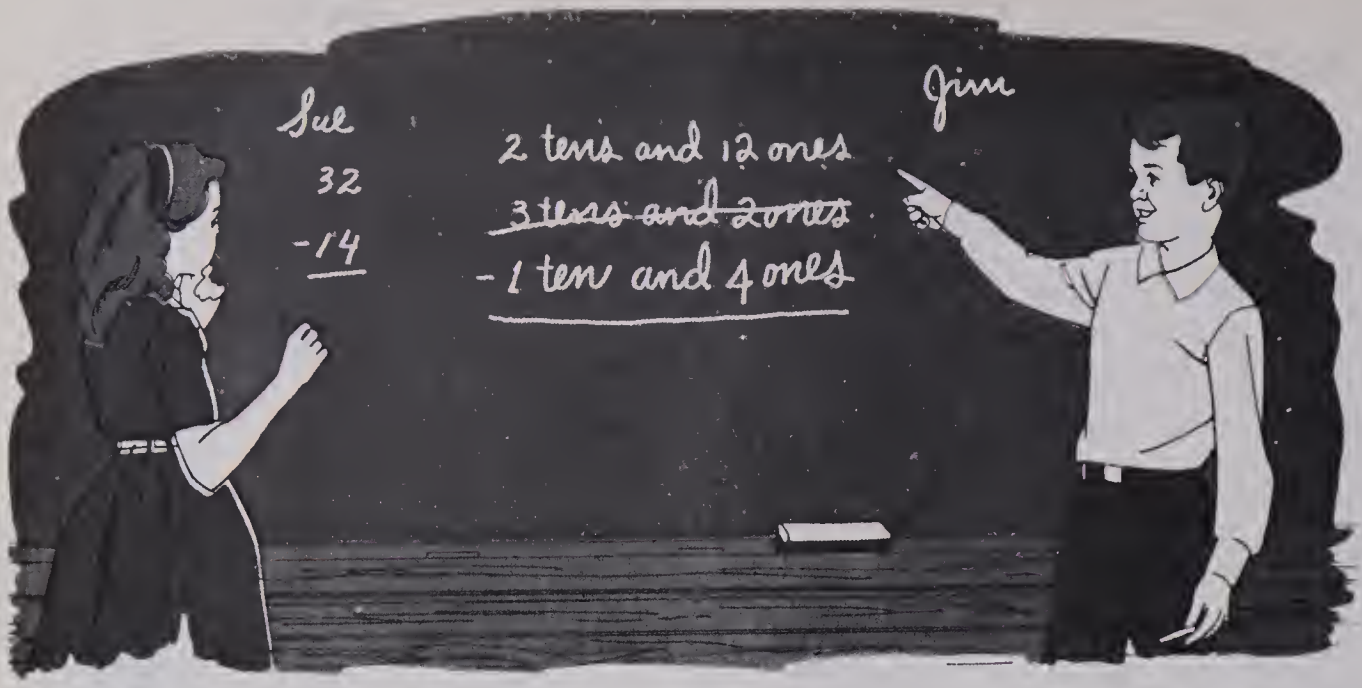
9. $80 = \overset{71}{n} + 9$

10. $55 + 28 = \overset{83}{n}$

11. $\overset{59}{n} - 12 = 47$

12. $6 + \overset{28}{n} = 34$

●**Extra Activity.** Work Set 92.



More Renaming Sums for Subtracting

Both sum and known addend greater than 9 [O]

Miss Snow told the children that each must have 32 sheets of paper to make a new picture book. Sue had 14 sheets. How many more sheets did she need?

Sue wrote the example on the board, but she could not subtract the ones. Why did she have to rename 32? To show more ones

1. On the board Jim showed Sue how to find the answer with tens and ones. Explain his work. 32 is first shown as 3 tens and 2 ones. 3 tens and 2 ones is renamed 2 tens and 12 ones

2. Miss Snow showed the children a shorter way to subtract. She said to work with ones first, and then with tens. Study the work at the left.

2 tens − 1 ten = 1 ten 12 ones − 4 ones = 8 ones

$$\begin{array}{r} {}^{2}\,\cancel{3}\,{}^{(12)}\,\cancel{2} \\ 1\,4 \\ \hline 1\,8 \end{array}$$

Ones: *Think*, "I cannot subtract 4 from 2, so I rename the sum to be 2 tens and 12 ones. Then $12 - 4 = 8$." Write "8" in one's place in the answer.

Tens: *Think*, "$2 - 1 = 1$." Write "1" in ten's place in the answer.

Check by adding upward: $8 + 4 = 12$; $1 + 1 + 1 = 3$.

When pupils are able to rename the sum to show 10 more ones, they should not have too much difficulty with this kind of subtraction. Many children will need to lean for a while on the crutch of showing the renaming.

Teaching Pages 132 and 133

Pupil's Objectives

To develop intelligent skill in subtracting with 2-place numerals when renaming of the sum is required.

New Words

(page 133) *carefully*

Background

If higher-decade subtraction (subtracting with a 2-place numeral and a 1-place numeral) is thoroughly understood when renaming of the sum is necessary, and if skill in that process has been carried to a high level of proficiency, subtracting with two 2-place numerals when renaming of the sum is required should present little difficulty.

Pre-Book Lesson

• Present an interesting problem situation, preferably a local one, such as the following, for the children to solve:

Betty has to work 34 examples in all. She has finished 18. How many more examples must she work?

Ask children to suggest ways in which a solution of the S. example may be found. Because of their previous experience with renaming in higher-decade subtraction, they may quickly suggest renaming the sum in order to subtract the ones.

• Provide other examples for discussion and demonstration. Have children demonstrate with bundles of dowels, place-value charts, Ginn Arithme-Sticks or other concrete materials so all pupils can see the changing of 1 ten to 10 ones. At the same time, have children make records at the chalkboard and at their desks.

NOTE. A frequent mistake in demonstrating subtraction in a place-value chart or a similar device is the attempt to show two different numbers on the teaching aid before doing the subtracting. Only the total should be shown on the device. Then, proceed to remove the part that is known.

The procedure for recording the subtracting for 42 − 27 is illustrated in the second column for three different methods:

a. Using Φ's and /'s

Step 1. ΦΦΦΦ //= ΦΦΦ //////////

Step 2. ΦΦΦ//// ~~||||||~~

b. Using numerals and words

Step 1. 4 tens and 2 ones = 3 tens and 12 ones

Step 2. 4 tens and 2 ones = 3 tens and 12 ones
− 2 tens and 7 ones = 2 tens and 7 ones
1 ten and 5 ones, or 15

c. Using numerals

Step 1. Sum renamed in order to subtract in one's column.

$$\begin{array}{r} 42 \\ -27 \\ \hline \end{array} \qquad \begin{array}{r} 3\,(12) \\ \not{4}\,\not{2} \\ -2\,7 \\ \hline \end{array}$$

Step 2.

$$\begin{array}{r} 3\,(12) \\ \not{4}\,\not{2} \\ -2\,7 \\ \hline 1\,5 \end{array}$$

• Use one of the examples on the chalkboard to review the addends-sum relationship and how checking in subtraction makes use of this relationship: The sum of the two addends in the subtraction should equal the sum given in the subtraction example. Remind pupils how they may check without copying the example.

$$\begin{array}{r} 42 \text{ sum} \\ -27 \text{ addend} \\ \hline 15 \text{ addend} \end{array} \quad \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \uparrow \text{ add upward}$$

Since carrying in addition may be involved, do some of the checking orally with the class.

Using the Text Pages

• During the oral work, let some pupils use objects to demonstrate and others write *tens* and *ones* as frequently as is necessary in order to "see" the need for renaming.

• *Slower learners* may need to work the examples in rows 6–9 by writing each example using *tens* and

ones and rewriting it with *tens* and *ones* to show the renaming. You may wish to assign only a part of these exercises at one time. Some children find a great number of examples overwhelming. This is especially true of *slower learners*. On the other hand, it is neither equitable nor necessary to require the *more capable children* to do all of these examples merely because they can. They should not have to pay a penalty for being competent. Instead, give them something to do that will extend their learnings.

Individualizing Instruction

• *Slower learners* may profit from exercises like the following:

a. Cross out examples in which you need not rename the sum.

68	71	43	75	24	36
−35	−18	−41	−16	−14	−28

b. Copy examples above in which renaming of the sum is necessary, and then work them.

c. Copy examples below which have wrong answers, and then work them correctly.

62	87	56	43	89	50
−60	−58	−47	−25	−65	−37
2	39	9	28	14	13

d. Copy each example and write the missing digits.

46	35	94	36	42	71
−29	−28	−47	−25	−26	−53
?7	?7	?7	??	??	?8

• *All pupils* may write the answers for examples in any one or all of rows 3, 4, and 5 on page 133.

• Use the pocket chart (see Teaching Page 53) with *slower learners* who need help on renaming the sum. The chart may be used for either addition or subtraction. Pupils may use their sets of numeral cards in the pockets of the chart. Be sure to have cards to indicate the operations. For a subtraction example one pupil may show any 2-place numeral using the top pockets and do the same in the bottom pockets. He should show the operation sign in the pocket made for that purpose. Then other pupils may tell how many tens, how many ones, and rename the sum if necessary for subtracting.

• *All pupils* may do some of the examples on page 133 as follows:

$$\begin{array}{r} 52 = 50 + 2 = 40 + 12 \\ -38 = 30 + 8 = 30 + \ 8 \\ \hline 10 + \ 4 = 14 \end{array}$$

• As needed, assign Extra Examples Set 49.

NOTES

In which examples must you rename the sum before you subtract? a, c, e, f, and g Why? You cannot subtract the ones.

	a	b	c	d	e	f	g	h
3.	54	55	82	66	80	43	82	35
	26	23	64	36	48	14	48	20

Say the missing digit, then the whole answer.

4.	76	53	60	82	52	84	37	83
	−29	−28	−37	−36	−34	−34	−19	−52
	4?7	2?5	2?3	4?6	1?8	5?0	1?8	3?1

In row 5, find the wrong answers. Explain why they are wrong. Use tens and ones on the board to help you.

5.	30	65	28	70	54	36	61	83
	−26	−47	−19	−35	−38	−17	−49	−46
	14	18	9	45	17	19	22	37
	4			35	16		12	

[W]

Copy and subtract. Check. Work carefully.

	a	b	c	d	e	f	g	h	i	j
6.	27	42	60	71	85	41	65	90	78¢	82¢
	18	23	21	56	38	37	27	29	29¢	24¢
	9	19	39	15	47	4	38	61	49¢	58¢
7.	70	81	94	35	52	97	95	33	63¢	53¢
	24	25	56	19	39	38	48	27	58¢	45¢
	46	56	38	16	13	59	47	6	5¢	8¢

Sometimes you do not rename the sum, so be careful!

8.	94	60	76	44	55	71	43	52	64¢	47¢
	24	25	56	19	26	38	29	15	47¢	29¢
	70	35	20	25	29	33	14	37	17¢	18¢
9.	70	81	45	62	35	89	42	67	74¢	96¢
	42	24	42	58	17	37	37	54	38¢	29¢
	28	57	3	4	18	52	5	13	36¢	67¢

♦ **Extra Examples.** Work Set 49.

Try These

[W]

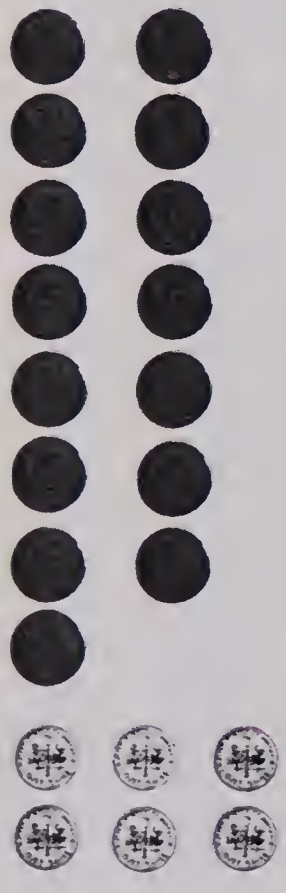
Copy in columns and subtract. Watch for renaming!

1. 89 − 8 81
2. 95 − 77 18
3. 30 − 9 21
4. 98 − 36 62
5. 77 − 50 27
6. 30 − 8 22
7. 59 − 16 43
8. 94 − 8 86
9. 67 − 46 21
10. 52 − 29 23
11. 95 − 28 67
12. 46 − 9 37
13. 74 − 65 9
14. 20 − 7 13
15. 57 − 38 19
16. 41 − 36 5
17. 76 − 7 69
18. 63 − 27 36
19. 58 − 53 5
20. 38 − 8 30
21. 75 − 39 36
22. \$0.23 − \$0.19 \$0.04
23. \$0.84 − \$0.06 \$0.78
24. \$0.85 − \$0.53 \$0.32

◆ **Extra Examples.** Work Set 50.

Do You Like to Try New Things?

Enrichment [W]



Use these coins to work Ex. 1–5: 6 dimes, 15 cents.

1. $43¢ + 9¢ = n¢$.

Lay out two rows of coins for 43¢ and 9¢. Put together the cents. Do you get 12? 12 = 1 ten and 2 ones, so use 1 dime in place of the 10 cents. Then the total is _?_ dimes and _?_ cents, or _?_¢.

2 52

2. $36¢ + 28¢ = n¢$.

Make two rows of coins. Do you use 1 dime in place of 10 cents in the total? Yes

Use your coins to work Ex. 3–5. For each example you will need only one row of coins for the total.

3. 63¢ − 5¢ 58¢
4. 42¢ − 9¢ 33¢
5. 54¢ − 29¢ 25¢

Did you show carrying in Ex. 1–2? Yes Did you show renaming in Ex. 3–5? Yes

Be ready to show others what you have done.

pil's Objectives

(a) To practice writing answers for subtraction amples as quickly as possible; and (b) to have actice in differentiating between subtraction examples that involve renaming of the sum and those it do not.

ckground

All four kinds of subtraction that have been ght are included in the written practice at the of the page but, of course, the section is gned largely for practice in subtracting with 2-place numerals when renaming of the sum ecessary.

he material at the bottom of page 134 is sug- ed as an enrichment activity in which *more ble children* are to use coins or pictures of coins he basis for demonstrating objectively the re- ing of ones to be tens and ones. These pupils to be prepared to demonstrate the technique he remainder of the class. It is merely one way of developing the idea objectively.

r Book Lesson

Review with the class the four kinds of sub- on examples that have been taught this year:

Higher-decade subtraction with no renaming

Higher-decade subtraction with renaming of m to show more ones

Subtracting with two-place numerals with no ning

Subtracting with two-place numerals with ing of the sum to show more ones

sk children to identify an example of each rom the set of examples at the top of the page.

the Text Page

work at the top of this page can be handled y different ways. The *more capable children* hink" the answers and write them on folded Or, you may have *all pupils* write the s for the first two columns on folded paper en copy the last two columns in vertical d write the answers.

You may wish to assign only one column at a time to *slower learners* and give them a score (number right) for each column. After they have completed all columns, have them check their scores to determine whether they showed improvement as they progressed from column to column.

Individualizing Instruction

- The subtraction examples at the top of the page may be used for diagnostic purposes. Have the children check their own work to identify the types of errors made as follows:

a. Write on the chalkboard names for the four kinds of subtraction and label them A, B, C, D (see the Pre-Book Lesson).

b. Each child writes the letters A, B, C, and D down the side of a paper. After each letter, he writes the example number for each one he missed under that category. (For example, if a child missed Ex. 14, he would write "14" after the B to indicate that he had made an error on an example in higher-decade subtraction with renaming.)

The headings and the examples of each are as follows:

A. Higher-decade subtraction, no renaming:
1 and 20

B. Higher-decade subtraction with renaming:
3, 6, 8, 12, 14, 17, 23

C. Two-place subtraction, no renaming:
4, 5, 7, 9, 19, 24

D. Two-place subtraction, renaming:
2, 10, 11, 13, 15, 16, 18, 21, 22

Use each child's tabulations of his own errors as a basis for determining the kind of remedial work or extra help he needs.

- The *more capable children* may be able to proceed independently with the enrichment activity. You may prefer to provide toy money or cardboard coins made by the children to use in working the exercises. Each child should have 6 *dimes* and 15 *cents* to use.

Another alternative is to check the work of the *more capable children* and then permit them to help the remainder of the class in this activity.

- Assign Extra Examples Set 50.

Teaching Page 135

Pupil's Objectives

(a) To check progress in all addition and subtraction skills that have been taught; and (b) to have practice in solving addition and subtraction number sentences with *n* standing for a sum or an addend.

Background

Nothing new is presented on this page. It provides an opportunity for each child to check on his strengths and weaknesses in the skills that have been taught to date. It is, also, an opportunity for the teacher to do some group checking. The practice at the top of the page may well be considered a progress test because it covers all types of work in A. and S. studied so far this year.

The items in this practice material can be classified according to the various types of addition and subtraction, with and without carrying or renaming. Group checking may reveal either general or specific areas of weakness possessed by the children.

Using the Text Page

• Without any advanced preparation, ask the children to proceed with the written assignment at the top of the page. You may permit a child to copy the examples if he wishes to do so. If you wish to shorten the assignment somewhat, you may leave out columns a, b, and c without eliminating any of the example types.

• Make the assignment at the bottom of the page, too, without advance preparation or review. Results on this written activity should indicate understanding (or lack of it) of the relationship between addition and subtraction.

Individualizing Instruction

• The following information should be useful to you if you wish to have the children participate in self-diagnosis. (See page 134.)

A. Higher-decade addition, renaming and carrying: 1b, 3d, 4a
B. Two-place addition, no renaming and carrying: 1e, 4c
C. Two-place addition, renaming and carrying: 2b, 2d, 2f, 3b, 3g, 3h, 3i, 4h, 4i
E. Higher-decade subtraction, no renaming: 1c
F. Higher-decade subtraction, renaming: 1c 2e, 2i, 3f, 4e
G. Two-place subtraction, no renaming: 2g 3c, 4b, 4f
H. Two-place subtraction, renaming: 1a, 1 1g, 1h, 1i, 2a, 2c, 2h, 3a, 3e, 4d, 4g
I. Column addition, no bridging: 5g, 5h
J. Column addition, bridging: 5a, 5b, 5c 5d, 5e, 5f, 5i

• *All pupils* may write a problem for each number sentence in Ex. 1–5 at the bottom of page 135.

• Assign Extra Examples Set 51 and Extra Problems Set 6 as they may be necessary at this time.

• *More capable children* may rewrite each Ex. 1–15 so that it shows *n* in three different positions. For Ex. 1 they may write,

$$60 - n = 41 \text{ or } 60 - 41 = n$$
$$\text{or } n - 19 = 41 \text{ or } n - 41 = 19$$

To Keep in Practice

A. and S. [W]

Write your answers on folded paper. Check your work.

	a	b	c	d	e	f	g	h	i
1.	94	9	69	50	60	92	73¢	42¢	$0.73
	−37	+56	−6	−6	+38	−29	−27¢	−15¢	−0.46
	57	65	63	44	98	63	46¢	27¢	$0.27
2.	81	57	96	19	87	26	57¢	45¢	$0.73
	−57	+29	−37	+47	−9	+59	−54¢	−28¢	−0.08
	24	86	59	66	78	85	3¢	17¢	$0.65
3.	74	25	76	7	85	58	37¢	47¢	$0.19
	−68	+48	−54	+44	−37	−9	+53¢	+47¢	+0.55
	6	73	22	51	48	49	90¢	94¢	$0.74
4.	87	90	30	91	74	86	54¢	76¢	$0.46
	+8	−70	+40	−68	−6	−60	−25¢	+15¢	+0.34
	95	20	70	23	68	26	29¢	91¢	$0.80
5.	6	1	40	7	32	8	2¢	40¢	6¢
	7	8	7	0	3	1	2¢	3¢	8¢
	2	24	6	5	0	9	3¢	1¢	0
	9	6	5	8	6	4	2¢	4¢	7¢
	24	39	58	20	41	22	9¢	48¢	21¢

♦ **Extra Examples.** Work Set 51.

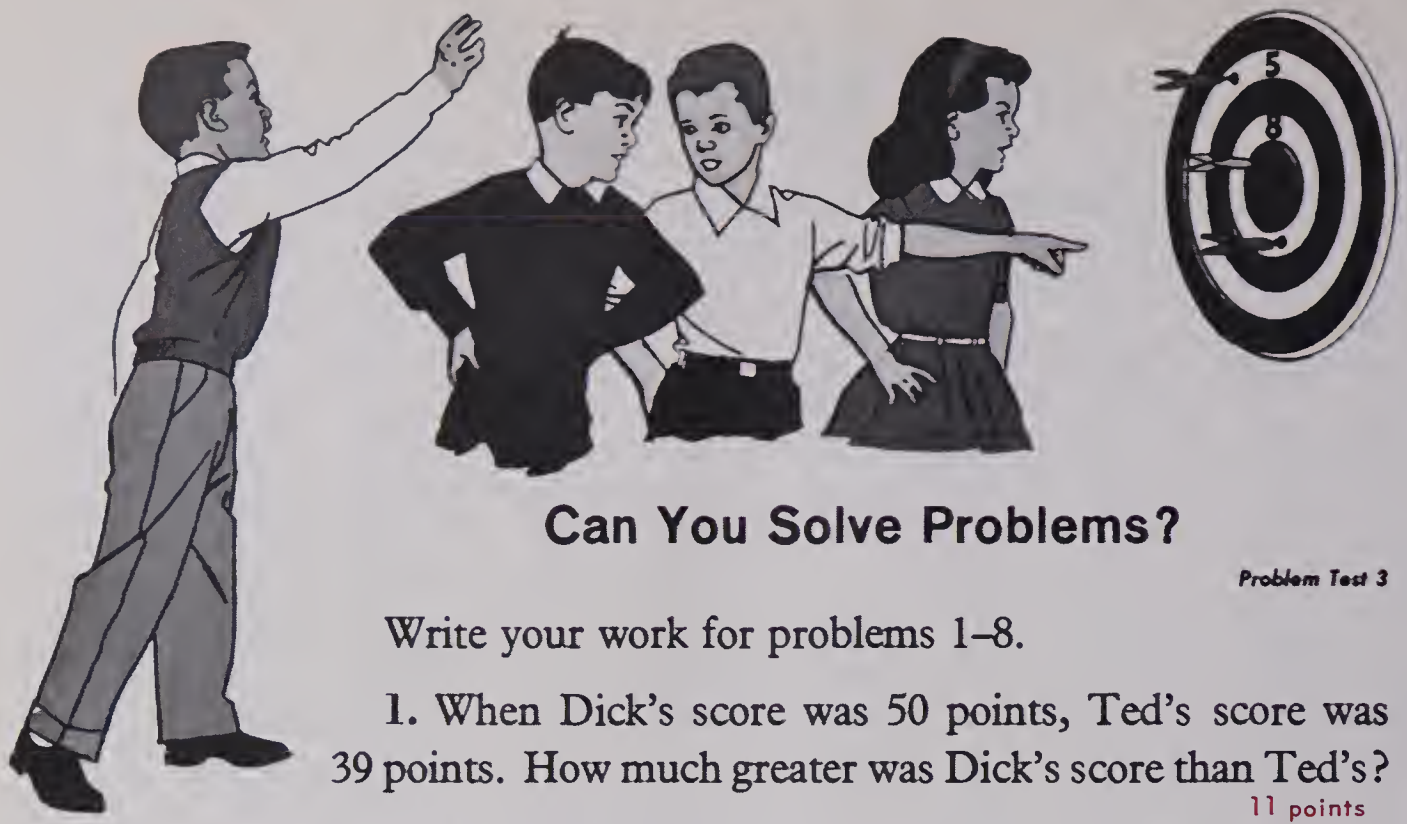
Finding the Unknown Sum or Addend

Addends-sum relationship [W]

Work each example to find the unknown number.

1. $60 - n = 41$ (19)
2. $n = 72 - 19$ (53)
3. $n + 35 = 72$ (37)
4. $46 + 29 = n$ (75)
5. $63 = 80 - n$ (17)
6. $n - 18 = 68$ (86)
7. $63 - n = 59$ (4)
8. $\triangle + 15 = 67$ (52)
9. $45 = 36 + n$ (9)
10. $n - 24 = 67$ (91)
11. $n = 20 + 37$ (57)
12. $n - 13 = 48$ (61)
13. $75 - n = 0$ (75)
14. $\square - 28 = 50$ (78)
15. $47 = n + 29$ (18)

♦ **Extra Problems.** Work Set 6.



Can You Solve Problems?

Problem Test 3

Write your work for problems 1–8.

1. When Dick's score was 50 points, Ted's score was 39 points. How much greater was Dick's score than Ted's? 11 points

2. Ruth's score was 32 points. Then with her next 3 darts she made 6 points, 8 points, and 0. What was her total score then? 46 points

3. When Ella had 39 points, Sue had 53. How many points did Ella need to make her score the same as Sue's? 14 points

4. A boys' team made 78 points in a game. A girls' team made 16 points more. What was their score? 94 points

5. Joe's score was 35 points. He made 21 points in his last turn. How many points had he made before that? 14 points

6. When they finished their game, Mary's score was 80 and Betty's was 66. How many points ahead was Mary? 14 points

7. The highest score Jane ever made was 95. When she had 76 points, how many more points did she need to match her highest score? 19 points

8. With four darts Jim made 10 points, 8 points 3, points, and 6 points. What was his total score? 27 points

Pupil's Objective

To work the end-of-chapter test on problem-solving.

New Words

darts, score, team

Background

Beginning on page 136 and extending through page 139 are the regular end-of-chapter tests. These are similar in nature to those given for the previous chapters (see Teaching Pages 49, 50 and 51; 91, 92 and 93).

Pre-Book Lesson

You may wish to set the stage for this test by having pupils recall when they should add and when they should subtract. Make sure that your pupils realize that, though this test deals with more difficult addition and subtraction than in the problem tests for previous chapters, the reasons for adding and subtracting are the same.

Using the Text Page

Help slower readers with the pronunciation of any words they may find difficult. Most pupils should be able to proceed independently with the work on the page.

- After administering this test, it might be desirable to discuss it with your pupils item by item. For per-cent scores for use in continuing the individual records, see Teaching Pages 138 and 139.
- For suggestions about scoring pupils' papers, see Teaching Pages 50 and 51, and Table of Per Cents on Teaching Pages 138 and 139.

NOTES

Pupil's Objective

(a) To diagnose difficulties in the addition and subtraction skills taught in Chapter 3: and (b) to have oral practice on the harder higher-decade addition and subtraction.

Background

The diagnostic test appearing at the end of each chapter in this text is a valuable means of finding out and remedying what difficulties present themselves at the particular stage of development in the work.

Using the Text Page

- The diagnostic test is of the same type as th presented for Chapters 1 and 2. Your pupils accustomed by now to using folded paper, so th should have no trouble with this technique. T suggested text pages for review and the prac sets for more work of the same type should acc plish the necessary remedial treatment.
- An excellent exercise in oral practice on harder higher-decade addition and subtractio provided at the foot of the page. This work ma used for groups particularly needing it.

NOTES

Do You Make Mistakes?

Diagnostic Test 3

Write answers on folded paper.

	a	b	c	d	e	Study Pages	Practice Sets
1.	5 6 4 7 22	3 18 2 4 27	10 3 9 6 28	4 6 6 11 27	6 6 5 8 25	106	39, 41, 43
2.	33 +9 42	8 +44 52	75 +8 83	37 +9 46	5 +46 51	102–105	42
3.	37 +25 62	56 +38 94	14 +46 60	48 +18 66	23 +58 81	110	44, 45
4.	27 −8 19	62 −6 56	44 −9 35	56 −8 48	31 −4 27	124–128	46, 47
5.	74 −38 36	81 −25 56	78 −29 49	55 −26 29	70 −42 28	132	49, 50

Oral Practice

Say the answers quickly.

1–8. Add 8 to each of these numbers:

62 70 50 58 83 91 75 83 47 55 36 44 28 36 49 57

9–16. Subtract 9 from each number shown above.

53 41 74 66 38 27 19 40

17–24. Add 7 to each of these numbers:

33 40 80 87 21 28 74 81 46 53 68 75 55 62 92 99

25–32. Subtract 6 from each number shown above.

27 74 15 68 40 62 49 86

Do You Understand?

Test of Information and Meaning 3

Write "Yes" or "No" for answers to Ex. 1–13.

1. Is a 4-foot rope longer than a rope 1 yard long? Yes

2. If $n = 30$, does $n + n = 60$? Yes

3. In counting, does 301 come just after 299? No

4. Do some months have five whole weeks? No

5. Is a quarter worth more than 2 dimes and a nickel? No

6. Do we measure a truck in miles? No

7. Has a yard fewer than 30 inches? No

8. If $n = 42$, does $n - n = 0$? Yes

9. In 475, is 4 in ten's place? No

10. Are there as many as 70 minutes in an hour? No

11. Must you rename and carry to ten's place in adding 75 and 16? Yes

12. In 153, is the digit 5 in hundred's place? No

13. Can you get from a point inside a simple closed path to a point outside and not cross the path? No

Write an *n*-example for each of problems 14–15.

14. Farmer Brown has 23 white hens and 19 red hens. How many more white hens than red hens has he?
23 − 19 = n

15. Ted Brown collected 33 eggs. He collected 19 in the hen house. How many did he find in other places?
33 − 19 = n

16. Make a row of 10 large dots. Put X on the 9th dot from the left and draw a box around the 6th one.

Teaching Pages 138 and 139

Pupil's Objectives

To measure the extent to which the content of Chapter 3 has been learned.

New Word

(page 138) *collected*

Using the Text Pages (pages 138–139)

All children should be able to do the work for Ex. 1–25 in this Test of Information and Meaning according to the directions given for each group of questions. It would be a good idea to discuss the test, item by item, after it has been completed by all pupils.

Using the Text Page (page 139)

This Computation Test will provide much information about the abilities of your class to perform the additions and subtractions called for.

Individualizing Instruction

Use the tables below to find the per-cent scores to be entered on the individual record cards (see Teaching Pages 50 and 51).

Table of Per Cents for Chapter 3 Scores

Problem Test	
Score	*Per Cent*
1	13
2	25
3	38
4	50
5	63
6	75
7	88
8	100

Computation Test

Score	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*
1	3	9	28	17	53	25	78
2	6	10	31	18	56	26	81
3	9	11	34	19	59	27	84
4	13	12	38	20	63	28	88
5	16	13	41	21	66	29	91
6	19	14	44	22	69	30	94
7	22	15	47	23	72	31	97
8	25	16	50	24	75	32	100

Suggestions for Material to Accompany End-of-Chapter 3 Tests

hildren who encounter no serious difficulties in tests just administered may be assigned any of following activities to supplement and extend mathematical skills and understandings. scuss suggestions for alternate use carefully they differ from directions on the page.

Alternate Uses of Pages in Chapter 3

Page 94. Have children make a chart about some collections they have made—kinds of leaves or wild flowers found or birds seen, and so on. The *more capable children* may make up some A. and S. problems about numbers of things shown in their charts.

Page 95. Have pupils rewrite each example in Ex. 3 and 4 at the foot of page 95, so that, for Ex. 3, the question asked is, "How much less?" and, for Ex. 4, the question asked is, "How much greater?" They may write each answer after the example as reworded.

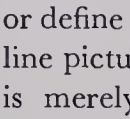
Page 99. Have children write and complete some number sentences of their own like the ones at the bottom of page 99. Emphasize that you want sentences that are true.

Page 107. More capable children may write and complete *if-then* sentences of their own. They may exchange with each other to check whether the sentences written are true.

Page 122 (bottom of page). More capable children may write number sentences of their own modeled after Ex. 1–10 on page 122. Then each may exchange with someone else and work the examples the other pupil wrote.

Supplementary Activities

• Some numbers are called *triangle numbers* because a set of objects for that number can be arranged so that their perimeter is in the shape of a triangle. 1 is considered to be the first triangle number. The next triangle number is 3. Have children find n for each of the number sentences below and also arrange objects for the triangle number named. For example, the arrangement for $1 + 2 + 3$ should be something like the picture at the right. Children may use discs, paper squares or even make X's on paper.



$n = 1$
$n = 1 + 2$
$n = 1 + 2 + 3$
$n = 1 + 2 + 3 + 4$
$n = 1 + 2 + 3 + 4 + 5$
$n = 1 + 2 + 3 + 4 + 5 + 6$

Let children go as far as they can in identifying triangle numbers. Help children generalize about how triangle numbers are determined.

• Some numbers are called *square numbers* because the arrangement of a set of objects for that number is such that their perimeter is in the shape of a square. 1 is also considered to be the first square number. The next square number is 4. Have children find n for each of the number sentences below and then use objects to form a square for the number.

$n = 1$
$n = 1 + 3$
$n = 1 + 3 + 5$
$n = 1 + 3 + 5 + 7$
$n = 1 + 3 + 5 + 7 + 9$

Permit the children to continue to find square numbers. Help them form a generalization as to how a square number may be determined.

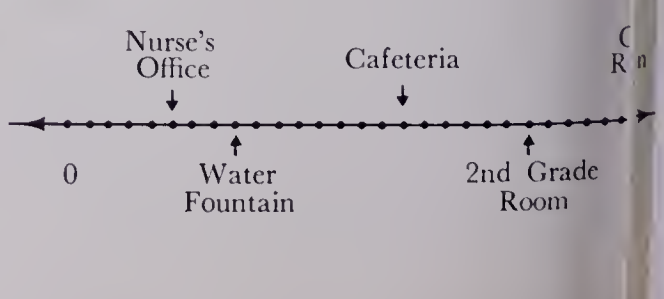
• The two preceding activities should be done by the pupils before doing this one. It is dependent upon concepts formed in the other activities.

Have the pupils add any two consecutive triangular numbers. Try several possibilities. What generalization can be made?

• In the coordinate-geometry activity suggested at the end of Chapter 2 (see Teaching Pages 92 and 93), the children were introduced to the idea of using numbers to indicate a *location* on the addition chart. It is the purpose of this activity to use numbers in connection with the number-line picture as a means of indicating location.

Up to this time, the points shown on the number-line picture have been used primarily for counting purposes. They can just as well be used to indicate or define limits of distances. In fact, some number-line pictures are used for just that. A scale of inches is merely a number-line picture which describes distance.

Let pupils make number-line pictures and assign each point on the number-line picture to represent a child's step. For example, they may let 0 represent some starting point like the school office. Let the pupils ask questions such as the following and label the correct points, somewhat as in the diagram below: How many steps is it from the office to our room? What points of interest between the office and our room can be labeled on the number-line picture?



Copy Ex. 17–20, putting in what is missing.

17. 48 = 3 tens and 18 ones

18. In 527, the digit 5 is in hundred's place.

19. 603 is read, "6 hundred 3."

20. In counting by 10's, 460 comes just before 470.

Do your work for Ex. 21–25 on another piece of paper. For each of Ex. 21–23 find the number for n.

21. $8 + n = 25$ (17) **22.** $n - 8 = 23$ (31) **23.** $40 - 9 = n$ (31)

Copy Ex. 24 and 25. Write > for is greater than or < for is less than or = for is equal to.

24. (58−26) = (48−16) **25.** (18+30) > (86−56)

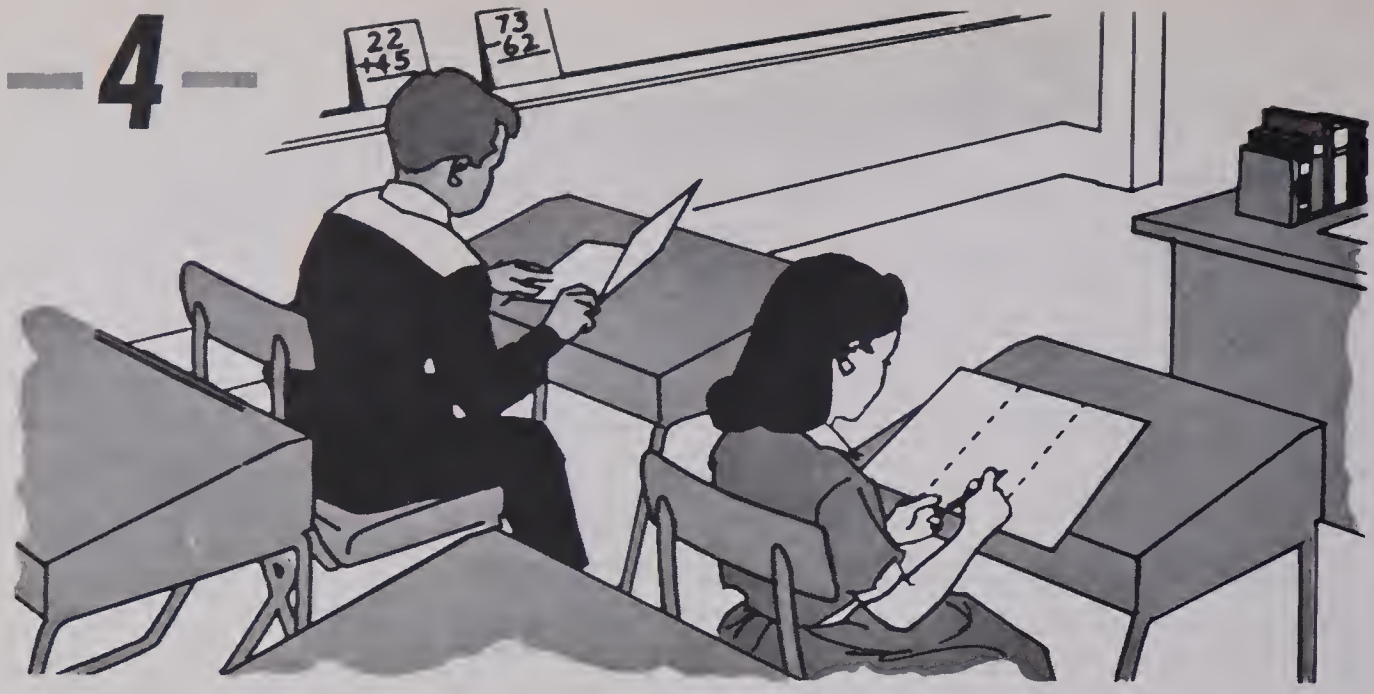
How Well Can You Compute?

Computation Test 3

Write answers on folded paper.

	a	b	c	d	e	f	g	h
1.	90	8	81	50	47	65	94¢	$0.52
	−70	+70	−37	−5	+49	−25	−85¢	+0.27
	20	78	44	45	96	40	9¢	$0.79
2.	9	79	47	30	28	53	63¢	$0.55
	+69	−8	−29	+49	+69	+37	−26¢	−0.38
	78	71	18	79	97	90	37¢	$0.17
3.	62	36	27	53	89	87	53¢	$0.84
	−8	+44	+60	−48	−9	−68	+29¢	−0.51
	54	80	87	5	80	19	82¢	$0.33
4.	11	2	30	8	7	42	9¢	5¢
	8	8	5	3	2	7	7¢	7¢
	6	7	8	7	8	3	9¢	5¢
	0	6	2	8	4	3	3¢	7¢
	25	23	45	26	21	55	28¢	24¢

4



Discovering Fraction Numbers

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ [O]

You have been using sets of counting numbers and whole numbers. There are sets of fraction numbers, too.

1. See how Joe is folding his paper to show parts of the same size. How many parts of the whole are there? 2

When there are 2 parts of the same size, for each part we can think of the fraction number, one half. To name this fraction number, we can write the fraction $\frac{1}{2}$.

2. May will fold her paper along the lines she has drawn. How many parts of the same size will she show? 3

For each of these parts, we can write "$\frac{1}{3}$." Why?

There are 3 parts of the same size.

3. The inside of this rectangle is shown in _?_ 4 parts of the same size.

For each of these parts, we can write "$\frac{1}{4}$." Why?

There are 4 parts of the same size.

Numerals like 7, 3, and 9 name whole numbers. Numerals that we call fractions, like $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, name fraction numbers.

* Help children to realize the difference between the terms fraction number (the number idea) and fraction (the numeral which names the number).

Overview — Chapter 4

• In chapter 4, you will sense a big step forward in the arithmetic in Grade 3—beginning work in this grade with the set of fraction numbers. Pupils will discover the concepts for the fraction numbers one half, one third, and so on through one tenth and learn to write the fractions. They will experiment with relative sizes of all halves, all thirds, and so on.

Since, in this chapter, the pupil is thinking about what a 3-place numeral shows, addition and subtraction are both extended to include work with 3-place numerals to the extent of renaming the ones when the sum is greater than 9 and renaming the sum when the ones cannot be subtracted.

Pupils learn in this chapter how to round numbers to the nearest ten in order to estimate answers for addition and subtraction.

Another topic new to this grade is the work in this chapter providing readiness for multiplication of whole numbers.

• **Problem-Solving.** In addition to increasingly difficult work with *n*-sentences, using data from tables, differentiating between the need for addition or for subtraction, oral problem-solving, deciding by its size whether an answer is too great or not great enough, the work in problem-solving now requires estimating answers by using rounded numbers.

• **Maintenance.** Several of the topics begun in earlier chapters are now expanded. Measurements are extended to include weight and dozens. Logic in the *if-then* sentences and inequalities are extended to the use of 2-place numerals. Number sentences containing *n* require more difficult work—addition and subtraction which require renaming.

• **Enrichment.** For those children who learn quickly and have time for enrichment, there are several exercises to excite them in addition to the reservoir of suggestions at the end of the pupil's text.

• **Testing.** The usual chapter tests and the mid-unit test (*Many Things*) appear within the pages of this chapter.

Teaching Pages 140 and 141

Pupil's Objectives

(a) To review and extend knowledge of unit fraction numbers (fraction numbers with numerators 1) as applied to a part of a single object; and) to use the terms *fraction*, *half* (*halves*), *third* (*thirds*), and *fourth* (*fourths*) and the symbols $\frac{1}{2}$, $\frac{1}{3}$, d $\frac{1}{4}$.

Background

Thus far in Grade 3, study has been limited to the s of *whole numbers* and *counting numbers*. Pages)–141 constitute an introduction to the set conaing the positive *fraction numbers* (the positive ional numbers) and zero. The positive rational nbers come into existence as a result of forming a io of a whole number and a counting number. h numbers as $\frac{1}{3}$, $\frac{5}{8}$, and $\frac{7}{4}$ all express ratios. e set of all possible such ratios make up the set positive rational numbers.

Research studies reveal that children know a at deal about fraction numbers when they enter de 3. They are very familiar with the idea of halves (though they may refer to the *smaller half* or the *larger half*), somewhat less familiar with the idea of fourths, and still less familiar with the idea of thirds—all applied, as in this lesson, to part of a single object. At a later time, fraction numbers as applied to part of a group of objects will be studied.

The following ideas should be stressed: (a) when an object is separated into parts of the same size, we can think of a certain fraction number; (b) numerals (fractions) to show fraction numbers are written in a special way; (c) the part of the numeral below the fraction bar helps to name the particular fraction number because it tells into how many parts of the same size an object has been separated.

Some very important distinctions used throughout the program in *Mathematics We Need* should be understood by the teacher. Use of the term *fraction number* will always be in reference to the abstract idea of number. In fact, it is used in the same way that *number* is used as differentiated from *numeral*. The term *fraction* is used in the same sense as the word *numeral*. It is considered a numeral.

Teacher's Preparation

• Locate tagboard or heavy paper for cutting out shapes.

• Have a standard measuring cup available. It is preferable to have one with sides perpendicular to the bottom rather than one with sloping sides.

• Collect or prepare the necessary parts of *pies* or discs or cut-outs for use on a flannel board.*

Pre-Book Lesson

• Have pupils tell about parts of a whole with which they are familiar: one half apple, one half cooky, one half gallon of milk, one fourth pound of candy. Through guided questioning, lead pupils to mention in each case that the term "one half" names one of two parts of the same size. Do the same for other fraction names that may be mentioned.

• On the flannel board show different items that obviously have been cut into two parts of the same size. Show others that have been cut in two parts which are not the same size. Ask pupils to identify those parts for which they can think, "one half" and to prove that the parts are the same size by laying one part on top of the other part. Call attention to the names for the fraction numbers "one half" and "$\frac{1}{2}$."

Be sure to emphasize what the digit below the fraction bar shows—the number of parts of the same size into which the whole has been separated. Any fraction with 2 below the fraction bar names halves. The digit above the bar names the number of parts of that particular size.

• Proceed in a similar manner with fourths and with thirds.

• Have different children go to the chalkboard and write the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

Using the Text Pages

• You may wish to have pupils produce parts of the same size, as the children in the pictures are doing.

* See list of items on page xix.

• Before assigning the written work, discuss th various shapes pictured on page 141. The childre will probably know *circle*, *square*, and *triangl* However, *rectangle* may be new to them. In passing idcntify shape D as an *ellipse*, but do not expec mastery of this term. In this incidental work wit geometric forms, you may want to mention agai that the shape is the curve only, and does nc include the space inside it.

Individualizing Instruction

• For experimentation by *all pupils*, have avai able in the arithmetic corner some whole dis and also parts of the same size (halves, third fourths).

• Allow children to use a measuring cup f experimenting and discovery in connection wi halves, thirds, and fourths. Let them pour wat grains of rice, or sand.

• *Slower learners* may need further help with t flannel board and appropriate parts of the same si Be certain that the pupils understand that a p takes value in terms of a particular whole. U a whole and related parts so the children can *see* th

4 fourths are the same quantity as 1 whole.

3 thirds are the same quantity as 1 whole.

2 halves are the same quantity as 1 whole.

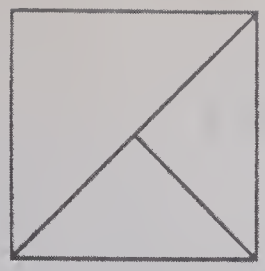
1 fourth of a whole is not necessarily the sa size as 1 fourth of another whole.

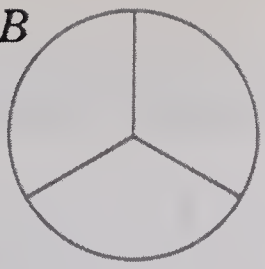
All fourths of the same thing are the same size

• For additional oral work, have *all pupils* why each of shapes F through J do or do not pict halves or thirds or fourths.

• *All pupils* may identify $\frac{1}{2}$-inch and $\frac{1}{4}$-i marks on a ruler. Elicit that each of those ma may indicate an end point of a certain line s ment. The length of the line segment is $\frac{1}{2}$ or the length of a 1-inch line segment.

• *More capable children* may look up the mean of the word "fraction" and explain it to the o members of the class. They may find that definition disagrees with the use in this lesson s many dictionary definitions do not make the n ber-numeral distinction.

A 

B 

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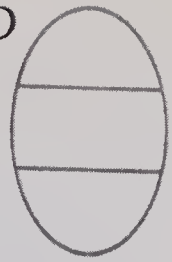
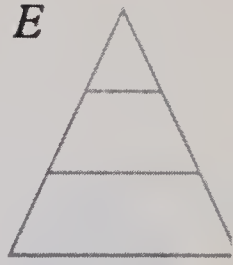
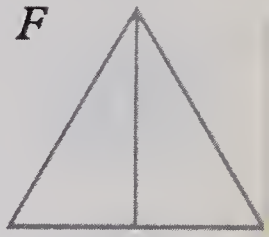
D 

E 

3

4. When a whole is in thirds, it is in _?_ parts of the same size. For each part, you can think of the fraction number, one third.

F 

Change Ex. 4 to make a sentence

When a whole is in fourths, it is in 4 parts of the same size.

5. about fourths. ^ **6.** about halves. v

When a whole is in halves, it is in 2 parts of the same size.

7. Which of *A*–*E* show thirds of a whole? v Why? v

B and C

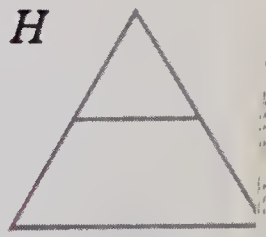
Each whole is in 3 [w] parts of the same size.

G

Write the letter of the picture *F*–*J* that shows

F I,J G

8. halves. **9.** fourths. **10.** thirds.

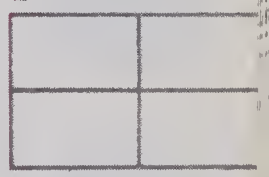
H 

Show three circles like *K*. Label them *X*, *Y*, and *Z*.

11. Circle *X*. Color $\frac{1}{4}$ of the inside.

12. Circle *Y*. Color $\frac{1}{3}$ of the inside.

13. Circle *Z*. Color $\frac{1}{2}$ of the inside.

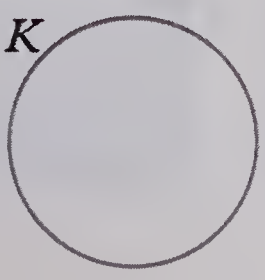
I 

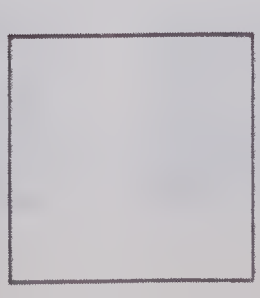
14. Draw pictures of two squares like *L*. Show the inside of each square in thirds, in a different way each time.

15. Draw pictures of three more squares. Use these to show fourths of a whole in three different ways.

J

16. Draw pictures of two rectangles like *M*. Show the inside of one rectangle in thirds. Write the fraction $\frac{1}{3}$ on each part. Show the inside of the other rectangle in fourths. Write the fraction $\frac{1}{4}$ on each part.

K 

L 

M

More Fraction Numbers

$\frac{1}{5}$ through $\frac{1}{10}$ [O]

A

1. The inside of circle *A* is shown in 2 parts of the same size. For each part you can write "$\frac{1}{2}$." The fraction number, one half, means 1 of the 2 parts.

B

2. The inside of circle *B* is shown in 4 parts of the same size. For each part you can write "$\frac{1}{4}$." The fraction number, one fourth, means 1 of the 4 parts.

C

3. The inside of circle *C* is shown in 3 parts of the same size. For each part you can write "$\frac{1}{3}$."

Here are some other fractions:

$\frac{1}{5}$ (one fifth)	$\frac{1}{7}$ (one seventh)	$\frac{1}{9}$ (one ninth)
$\frac{1}{6}$ (one sixth)	$\frac{1}{8}$ (one eighth)	$\frac{1}{10}$ (one tenth)

4. To give $\frac{1}{5}$ of a pie to each boy, you must cut the pie into 5 parts of the same size. Explain.

5. Into how many parts of the same size would a pie be cut to give each boy $\frac{1}{7}$? 7 $\frac{1}{9}$? 9 $\frac{1}{8}$? 8 $\frac{1}{10}$? 10 $\frac{1}{6}$? 6

D

6. The inside of circle *D* is shown in eight parts. For each part can you write "$\frac{1}{8}$"? No Explain. All parts are not the same size.

E

7. The inside of circle *E* is shown in eight parts. Why can you write "$\frac{1}{8}$" for each part? All parts are the same size.

8. If a cake is cut into 10 pieces of the same size, will each piece be one tenth? Yes Write the fraction on the board.

9. The inside of each square is shown in parts of the same size. Name the fraction number for each black part. $\frac{1}{10}$

F G H I J

$\frac{1}{6}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{7}$

Teaching Pages 142 and 143

Pupil's Objectives

(a) To extend understandings about fraction numbers; and (b) to learn the meaning and the numerals for the fraction numbers—one fifth, one sixth, one seventh, one eighth, one ninth, and one tenth.

New Words

(page 142) *tenth*
(page 143) *bar*, *share*

Background

The purpose of these pages is to extend the experiences and understandings developed in connection with the fraction numbers $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. The activities of the previous pages should have prepared the pupils for generalizing about all fraction numbers named by unit fractions.

It is difficult for children to cut objects accurately into 5, 6, 7, 8, 9, or 10 parts of the same size; so, instead of making their own representations for fraction numbers, children should use pictured representations like those in the pupil's book. You may wish to have teacher-prepared or commercially-produced materials (pies, discs, and so on) that are cut into parts of the same size to be used for demonstrations.

This introduction to fraction numbers (positive rational numbers) is to acquaint children with ways of expressing parts of a whole and with the special way of writing numerals for the fraction numbers being shown. In the third-grade book of this series there will be no operations performed on fraction numbers since operations should be preceded by thorough development of the idea of fraction numbers.

Teachers frequently refer to "the top number" of the fraction. You are cautioned against this type of terminology. A fraction has three parts—a bar, a numeral above it, and another numeral below it. These numerals are symbols for the numbers that we later refer to as the *numerator* and the *denominator*.

It is not necessary to use the term *rational numbers* at this time. If you wish, you may use the terms *denominator* and *numerator*, but do not insist upon mastery of these words.

Teacher's Preparation

- Have available parts for demonstrating the fraction-number ideas, $\frac{1}{2}$ to $\frac{1}{10}$ (see Teaching Pages 140 and 141). Use a variety of shapes and materials, taking care that the parts can be checked for size. Many kindergartens are equipped with a set of blocks of many different shapes which are excellent for showing parts of the same size.

Drawings on the chalkboard or on large sheets of paper may be used for identification and reference. Smaller items may be duplicated and given to the pupils to manipulate.

- You may wish to duplicate a page with several number-line pictures on it to use for the number-line activity suggested in the Individualizing Instruction section.

Pre-Book Lesson

- Some children may observe and then participate in the folding and cutting into halves and fourths (as review) and into eighths (as extension of learning) strings, ribbons, and circular and rectangular shapes.
- Use the flannel-board wholes and parts or other prepared materials to demonstrate the fraction-number ideas one fifth, one sixth, one seventh, one ninth, and one tenth.
- Review the special way of writing fractions and the meanings of each of the digits in the numeral. Have children go the chalkboard and write fractions as you direct. Ask pupils to tell what the numerals express.

Using the Text Pages

- Use your supplementary fraction materials to help clarify the concept of the fraction numbers from $\frac{1}{5}$ through $\frac{1}{10}$.
- The *more capable children* should be able to proceed with the written work independently. You may wish to go through the exercises orally with the remainder of the class before assigning the written work. Permit pupils to use concrete mate-

rials (if they need them) to assist them with the written exercises.

Individualizing Instruction

• *Slower learners* may profit from some problems in sharing objects equally. Use an actual *object*—paper, candy bar, apple, ribbon, and so on—and have a given number of children share it equally. Be sure to discuss the importance of sharing equally. "How many parts of the same size will there be if 5 children share equally? What part of the apple will each child get?"

• *More capable children* may extend their concepts about fraction numbers by performing the following number-line activity:

a. Use a page with several number-line pictures on it.

b. In each of three of them carefully identify two points that are two inches apart (or a multiple of two inches, as four inches, six inches, or eight inches). Then in each of three of them identify two points which are an odd number of inches apart (as three inches or five inches, and so on).

c. Have children use their rulers to locate the point for $\frac{1}{2}$ on each line segment shown. Emphasize that two line segments of the same length will be shown when the point halfway between the end points is identified.

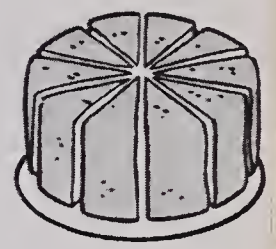
d. Some children may be able to locate points for $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{5}$, and so on, but this is more difficult to do. If the distance between the two points identified is 5 inches, then ask the children to identify the points for fifths of the line segment. See if anyone thinks of one half of one half for a fourth.

NOTES

[W]

Write the missing numeral for each of Ex. 10–15.

10. Sam broke his candy bar into 6 parts of the same size. He gave Sue one part, or _?$\frac{1}{6}$ of the candy bar.



11. At Cora's party, each girl got $\frac{1}{10}$ of the cake. Cora had cut it into 10 parts of the same size.

12. Mrs. Ball cut a ribbon into 5 parts of the same size. Each part was $\frac{1}{5}$ of the whole ribbon.

13. Mr. Carr cut off $\frac{1}{8}$ of a long board. He could get 7 more pieces of that same size from what was left of the board.

14. If 6 girls share equally a quart of ice cream, each girl's share will be $\frac{1}{6}$ of the quart.

15. A group of boys shared a pie equally. Tom got $\frac{1}{9}$ of the pie. 8 other boys in the group got pieces of the same size.

Write in words

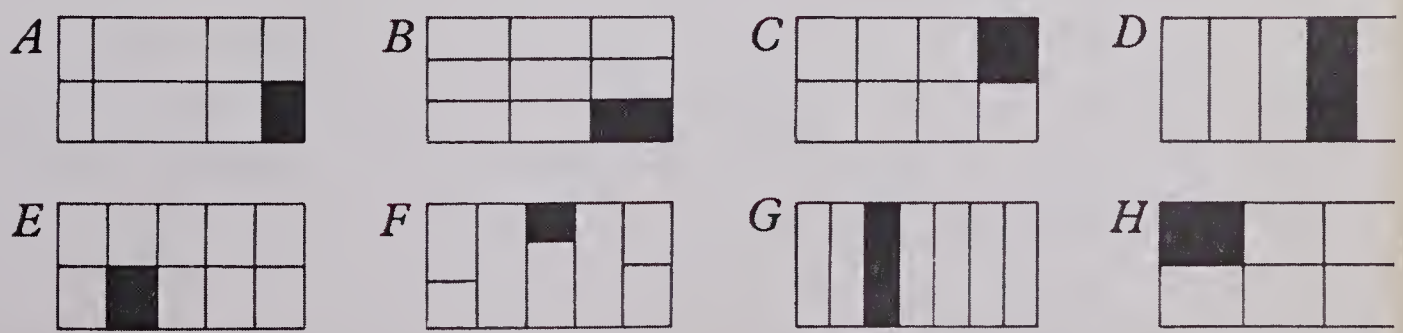
16. $\frac{1}{5}$. one fifth **17.** $\frac{1}{8}$. one eighth **18.** $\frac{1}{4}$. one fourth **19.** $\frac{1}{9}$. one ninth **20.** $\frac{1}{3}$. one third

Write with digits

21. one sixth. $\frac{1}{6}$ **22.** one tenth. $\frac{1}{10}$ **23.** one seventh. $\frac{1}{7}$

For the colored part, in which picture below can you think of

24. $\frac{1}{9}$? B **25.** $\frac{1}{6}$? H **26.** $\frac{1}{8}$? C **27.** $\frac{1}{10}$? E **28.** $\frac{1}{7}$? G **29.** $\frac{1}{5}$? D



To Keep in Practice

A. and S. [W]

Write your answers on folded paper.

	a	b	c	d	e	f	g	h	i
1.	78	85	80	96	18	86	94	67¢	56¢
	−9	−47	+9	−16	+49	−7	−59	−20¢	+27¢
	69	38	89	80	67	79	35	47¢	83¢
2.	60	45	52	57	7	94	53	43¢	83¢
	+8	−9	+26	−48	+82	−77	−24	+34¢	−28¢
	68	36	78	9	89	17	29	77¢	55¢
3.	56	98	84	9	25	46	87	90¢	84¢
	−8	−80	−68	+68	−8	+8	−39	−40¢	+8¢
	48	18	16	77	17	54	48	50¢	92¢
4.	64	8	92	51	27	43	71	36¢	82¢
	−56	+78	−67	−7	+55	−37	−48	+49¢	−56¢
	8	86	25	44	82	6	23	85¢	26¢

If . . ., Then . . .

Logic [W]

Work each of Ex. 1–9 to find the answer. Write only the answers on your paper.

1. If $n = 49$, then
   a. $70 - n = ?$ 21
   b. $99 - n = ?$ 50

2. If $n = 35$, then
   a. $n + 16 = ?$ 51
   b. $n - 16 = ?$ 19

3. If $n = 54$, then
   a. $n - 16 = ?$ 38
   b. $n - 40 = ?$ 14

4. If $n = 67$, then
   a. $14 + n = ?$ 81
   b. $n - 57 = ?$ 10

5. If $n = 28$, then
   a. $n + n = ?$ 56
   b. $n - n = ?$ 0

6. If $n = 76$, then
   a. $n - 39 = ?$ 37
   b. $18 + n = ?$ 94

7. If $n = 60$, then
   a. $n - 13 = ?$ 47
   b. $n - 55 = ?$ 5

8. If $n = 72$, then
   a. $n + 19 = ?$ 91
   b. $n - 47 = ?$ 25

9. If $n = 43$, then
   a. $n - 27 = ?$ 16
   b. $38 + n = ?$ 81

Teaching Page 144

Pupil's Objectives

(a) To check the ability to find answers to higher-decade A. and S. examples and to examples in A. and S. with 2-place numerals; and (b) to use A. and S. with 2-place numerals in a setting of logic.

Background

The practice section at the top of page 144 includes mixed presentation of the following skills and may well serve as a progress test:

a. higher-decade addition with and without renaming and carrying

b. addition with 2-place numerals with and without renaming and carrying

c. higher-decade subtraction with and without renaming the sum

d. subtraction with 2-place numerals with and without renaming the sum

Developing precision in mathematical reasoning through logic has been developed rather extensively so far (see Teaching Pages 74, 98, 107, 129). The *if-then* exercises at the bottom of page 144 afford the opportunity to develop precision in thinking as well as to practice using the addition and subtraction skills reviewed in the material at the top of the page.

Pre-Book Lesson

• Have selected children go to the chalkboard and write an example of a type which will be encountered at the top of page 144. After the child has written the example have him tell why it is the particular type he was requested to write.

• Have each of the addition and subtraction types written on the chalkboard and discussed. If necessary, use materials or draw pictures to help pupils with any example where confusion is apparent.

Using the Text Page

• Children should be able to proceed independently with the examples at the top of the page after the preliminary explanations. As each child finishes this portion of the page, have him go on to the written work at the bottom of the page.

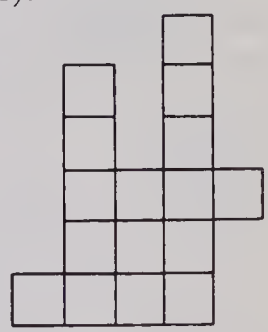
• Some children may need to write the examples at the bottom of the page vertically in order to find the answers. Where possible, encourage pupils to write them in horizontal form and solve them mentally.

Individualizing Instruction

• Select for *all pupils* appropriate activities and experiences from among the familiar suggestions given earlier in suggestions on numerous teaching pages for individualizing instruction. Any of the following activities should be appropriate:

a. Timed tests for rows of A. and S. facts (answers written on folded paper).

b. Diagrams filled in so that the sums of all numbers shown in rows or columns will be the same.



c. Number lines used to show adding or subtracting for number sentences.

d. Use of any of the materials available in the arithmetic corner.

• *More capable children* may use rulers or other materials to explore fraction numbers independently while you work with others.

Teaching Page 145

Pupil's Objective

To review and extend the concept *dozen*.

New Word

peaches

Background

This material on *dozen* will probably require very little developmental teaching, since your pupils will undoubtedly be familiar with the term and its meaning. On page 145 they will have an opportunity to apply their knowledge in many varied ways. It should be noted that in using the expression "dozen," we always use it to mean a dozen things.

Pre-Book Lesson

- Ask different children to explain what *dozen* means. Bring out that *dozen* is a word used very frequently and is always associated with one particular number. Ask pupils to give examples of ways in which they have used the word "dozen."
- Ask if the children know any other words we use to identify a particular number. Third-grade children should be familiar with terms like "pair," "couple," "twins," "triplets," "quartet," and so on.

Using the Text Page

Ex. 4, 5, and 10 are new in a sense. They are not unlike previous exercises in which the child determined how much *greater than* or *less than* one number was in comparison with another.

Individualizing Instruction

- Ex. 4, 5, and 10 may be changed to written work in which the child is to describe the relationship as well as write the correct answer. Ex. 4a would be written:

20 apples are 8 more than 1 dozen.

- *All pupils* may study advertisements and catalogues, listing items that can be bought by the dozen.
- Discuss with *more capable children* the term *cheaper by the dozen* as an introduction to certain advantages in quantity buying. Although you may need to tell pupils the product of 12 times the cost of one item, the children should be able to find the amount saved by buying by the dozen rather than individually. If the total cost for the latter is $0.99 or less, problems of the following type may be considered:

If you buy 12 candy bars at 5¢ each, the total cost is 60¢. A box holding 1 dozen costs 50¢. How much do you save when you buy by the dozen?

- Have children work exercises such as the following in relation to the idea of the dozen:

17 books is 1 dozen and _ ? _ more
21 children is 1 dozen and _ ? _ more
1 dozen and 7 more = _ ? _ apples
1 dozen and 11 more = _ ? _ houses
1 dozen is _ ? _ fewer than 15 chairs
1 dozen is _ ? _ fewer than 20 desks

- *More capable children* may do Extra Activities Set 93.

Reminder

Continue to maintain the following:

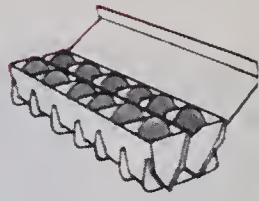
a. Ability to know the serial order of numbers and use rote and rational counting by 1's, by 2's, and by 10's to 900

b. Ability to recognize by sight, without recourse to counting, standard-pattern arrangements for groups as large as 10

c. Ability to understand and use ordinals as far as fiftieth

* This is a good time to bring out that often we use other numbers than ten as a basis for grouping.

* Dozens of Things



Measurements [O]

1. If you buy one dozen eggs, how many eggs would you get? Is it 10? 18? 12? 12

2. Are there 12 in one dozen chairs? Yes in one dozen plates? Yes in one dozen oranges? Yes

12 things = 1 dozen (doz.) **1 doz. = 12 things**

3. Which of these things can be sold by the dozen?

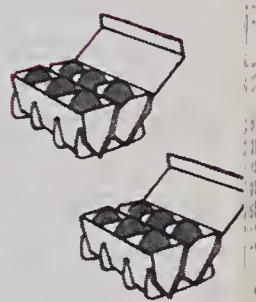
cookies √	oranges √	ice cream	pears √
cups √	gasoline	flowers √	paint
flour	honey	milk	cupcakes √

4. How many more or fewer than 1 dozen are

a. 20 apples? 8 more **c.** 8 peaches? 4 fewer **e.** 17 stamps? 5 more
b. 9 pencils? 3 fewer **d.** 14 potatoes? 2 more **f.** 10 rolls? 2 fewer

5. How many eggs in all are there in

a. 1 dozen and 5 eggs? 17 **b.** 1 dozen and 9 eggs? 21



6. How many eggs are in each of the boxes at the right? 6 in the two boxes? 12

7. 12; 6 and 6. Is the number for each of these two parts of a set of 12 the same? Yes

8. Then are there 6 things in $\frac{1}{2}$ dozen? Yes

9. How many are there in $\frac{1}{2}$ dozen peaches? 6 in $\frac{1}{2}$ dozen pencils? 6 in $\frac{1}{2}$ dozen baseballs? 6 in $\frac{1}{2}$ dozen roses? 6

10. How many more or fewer than $\frac{1}{2}$ dozen are

a. 8 apples? 2 more **b.** 5 cookies? 1 fewer **c.** 9 oranges? 3 more

● **Extra Activity.** Work Set 93.

Weighing Things

Measurements: pound and ounce; reference measures [O]

1. Ann's mother is buying 5 pounds of apples at the store. What is Ann using to weigh the apples? scale

2. Has Ann put in too many apples or too few? too many

3. Find a book that weighs about 1 pound. Find some other things in the room that weigh about 1 pound.
Answers will vary. Meat, butter, potatoes, coffee

4. Name four things that are sold by the pound.

5. How many pounds do you weigh?

6. Guess how much each of three other children weigh. Then ask them, to see if you have made good guesses.

Things weighing less than 1 pound are weighed in ounces. Most school pencils weigh about 1 ounce.

16 ounces (oz.) = 1 pound (lb.) **1 lb. = 16 oz.**

7. Which things weigh 1 pound or more?

a dog √ a ruler a table √ a pencil an orange

In Ex. 8–12, say the missing word, pounds or ounces.

8. The doll weighs 25 ounces.

9. The morning paper weighs 5 ounces.

More reference measurements—the book pound and the pencil ounce.

Teaching Pages 146 and 147

Pupil's Objective

To learn the concepts *pound* and *ounce*, their abbreviations *lb.* and *oz.*, and their relationships.

New Words

(page 146) *ounces*
(page 147) *gain*, *weights*

Background

These pages are intended to give meaning to some units of measurement used in weighing. Pupils have heard of the unit *pound* many times in connection with their own weight. Greater comprehension and understanding comes from associating the correct terms with common objects—dogs, books, chairs, apples, and the like. The table of measures of weight in the middle of page 146 should be learned, but it is even more important that your pupils at this time have functional concepts for the two units, pound and ounce. These concepts are best developed from direct experiences with objects. Only you, the teacher, can be sure that this phase of instruction receives adequate attention.

Once again it is important to point out that measurements are always approximations. The smaller the unit of measurement selected, the more accurate the approximation. Be sure to guide children to this kind of understanding as these two units of weight are used. Refer back to Teaching Pages 64 and 65 for further discussion of this idea.

Children now should be aware that the measure of a thing is the number of times the unit is applied to the particular item to be measured. The measure may be 3 and to make clear the unit we are using, we may add "in pounds" or "in ounces," and so on. In everyday life however, we are quite likely to indicate the measurement by saying "3 pounds" or "3 ounces." The measure and the unit constitute the measurement.

A *reference measure* for weight is somewhat more difficult to identify and to use than is a reference measure for length. However, it may be possible to establish a *book pound* or some other weight reference for use in the classroom.

Teacher's Preparation

- If possible, have a scale available for weighing light objects (2 to 10 pounds) as well as objects that weigh from 1 ounce to 1 pound. If a scale of this type is not available, perhaps you can proceed satisfactorily with a scale from the nurse's office used for weighing children.
- You may wish to weigh different objects in the room in advance of the lesson to find some things which may be used as reference measures for a pound and an ounce.

Pre-Book Lesson

- Elicit from children information about objects measured in pounds, such as food that comes in pound containers (butter, coffee), food that is weighed at the store (meat, fruit), feed for animals, building materials (nails), and weights of baby brothers and sisters. Introduce the term *ounce* by discussing how light objects are weighed. Pieces of mail and many food items in the store are measured in ounces.
- Weigh several objects on a scale. To extend the idea of measurements as approximations, have children express weights as *a little more than* or *a little less than* in terms of the nearest unit of measurement being used.
- Compare weights of unknown value with those of known values; i.e., allow pupils to make judgments based on the weight of objects chosen as standard weights. Refer to the *book pound* as a useful way of estimating the weight of another object.

Using the Text Pages

- Where possible, use materials in the room and a scale in connection with the oral activities.
- Make it known to the class that we do not use plural abbreviations for units, since we don't think of more than one of a unit but rather the number of times (the measure) the unit is applied to the particular item to be measured. For example, six pounds is written "6 lb."
- Before assigning the written work, have pupils write and work some examples on the chalkboard of the type 1 lb. 7 oz. = _ ? _ oz. and some of the type 18 oz. = _ ? _ lb. _ ? _ oz.

Individualizing Instruction

• *All pupils* should watch for occasions when they see something weighed in ounces. They may keep a list of objects weighed in ounces. Perhaps a child has a letter scale at home which could be brought to school.

• *All pupils* may note labels on packages of groceries at home. Lead them to notice the differences between net weight (contents only) and gross weight (contents plus packaging).

• *More capable children* may obtain information at the post office about the weights of letters and packages and postage rates.

• Have *all pupils* work exercises such as the following in relation to the idea of pounds and ounces:

19 oz. is the same as 1 pound and _ ? _ ounces
22 oz. is the same as _ ? _ pound and _ ? _ ounces
1 lb. _ ? _ oz. is the same as 27 oz.
_ ? _ lb. 7 oz. is the same as 23 oz.
1 lb. is _ ? _ oz. lighter than 26 oz.
30 oz. is _ ? _ oz. heavier than 1 lb.

• *More capable children* should be able to work independently on the following mixed practice involving several different measurements:

18 eggs is _ ? _ more than 1 doz.
1 lb. _ ? _ oz. is the same as 25 oz.
21 in. is the same length as 1 ft. _ ? _ in.
25 oz. is _ ? _ oz. heavier than 1 lb.
_ ? _ ft. _ ? _ in. is the same length as 20 in.
1 _ ? _ 14 _ ? _ is the same weight as 30 oz.
1 hr. _ ? _ min. is the same as 83 min.
97 min. is _ ? _ min. more time than 1 hr.

• *More capable children* may do Extra Activities Set 94.

Looking Ahead

On page 153 is a lesson which introduces the use of a table as a means of comparing information for two different dates. At this time you may wish to gather information which may be used in connection with the Pre-Book Lesson for Teaching Page 153. Weights of children a year ago and now would be very useful information and easy to use in table form of this kind.

NOTES

10. Mrs. Burns bought 8 _?_ of potatoes. pounds

11. Joe's bicycle weighs about 40 _?_. pounds

12. It would take many apples to weigh 15 _?_. pounds

[W]

Write answers for Ex. 13–22.

Scale A measures weights in ounces.

13. How many ounces does the box weigh? 11 oz.

14. How much less than 1 pound (16 oz.) does it weigh? 5 oz.

Scale B measures weights up to 300 pounds.

15. How much does the boy in the picture weigh? 63 lb.

16. To weigh 71 pounds, how much must he gain? 8 lb.

17. 1 lb. = 16 oz. How many ounces in all are
a. 1 lb. 4 oz.? 20 oz. **b.** 1 lb. 15 oz.? 31 oz. **c.** 1 lb. 9 oz.? 25 oz.

18. How many ounces more than a pound are
a. 17 oz.? 1 oz. **b.** 23 oz.? 7 oz. **c.** 19 oz.? 3 oz. **d.** 30 oz.? 14 oz.

19. One bag of flour weighs 45 pounds. Another weighs 50 pounds or how many pounds more? 5 lb.

20. For 40¢ Sue can buy 12 ounces of one kind of candy or 20 ounces of another kind. How many more ounces can she buy of the second kind than of the first? 8 oz.

21. Last year Tom weighed 57 pounds. Now he weighs 62 pounds. How many pounds did he gain? 5 lb.

22. Ned weighs 60 pounds. His brother Ken weighs 7 pounds less. How many pounds does Ken weigh? 53 lb.

● **Extra Activity.** Work Set 94.

Another mid-chapter review!

Many Things

Maintenance [W]

Make up these addition examples. Then work them.

1–4. In four examples, add with a 1-place numeral and a 2-place numeral, having two with renaming and carrying to ten's place and two with no renaming.

Make up these subtraction examples. Then work them.

5–8. In four examples, subtract with two 2-place numerals, having two where you must rename the sum to show more ones and two where you do not need to rename.

9. Write the fraction you can use for one part when a ribbon is cut into 9 parts of the same size. $\frac{1}{9}$

10. Draw a picture of a line segment with end points A and B.

11. How many minutes more or fewer than an hour are
a. 38 minutes? 22 fewer **b.** 72 minutes? 12 fewer **c.** 90 minutes? 30 more

12. How many inches more or fewer than a foot are
a. 22 inches? 10 more **b.** 17 inches? 5 more **c.** 10 inches? 2 fewer

Practice in Column Addition

[W]

Copy each set of numerals in a column, and add.

1.	40, 5, 7, 3 55	**5.**	3, 33, 7, 6 49	**9.**	7¢, 6¢, 4¢, 8¢ 25¢
2.	5, 0, 9, 8 22	**6.**	22, 0, 8, 5 35	**10.**	2¢, 0, 5¢, 8¢ 15¢
3.	8, 70, 5, 6 89	**7.**	8, 0, 2, 3 13	**11.**	4¢, 50¢, 7¢, 9¢ 70¢
4.	4, 2, 0, 6 12	**8.**	9, 5, 0, 9 23	**12.**	2¢, 7¢, 70¢, 8¢ 87¢

Teaching Page 148

Pupil's Objectives

(a) To review important ideas and skills; (b) to have practice in writing examples for which a description is given; and (c) to have practice in column addition with four addends.

Background

Practice in translation is an essential part of the building power in problem-solving. The written exercises in the upper portion of page 148 form a progress test of the extent to which certain understandings have been developed. If children really understand renaming and carrying and renaming a sum to show 10 more ones, then they will have little difficulty writing examples of each kind.

Many children understand arithmetical concepts but, because of a reading difficulty, they are at a disadvantage in this type of exercise. Adequate provision must be made for these youngsters to be successful within the limits of their abilities.

Pre-Book Lesson

Discuss each of the following addition and subtraction types with the class. If there is any question or confusion about a given type, have a child write an illustrative example on the chalkboard:

a. adding with a 1-place numeral and a 2-place numeral *without* and *with* renaming and carrying to ten's place

b. adding with two 2-place numerals *without* and *with* renaming and carrying to ten's place

c. subtracting with a 2-place numeral and a 1-place numeral *without* and *with* renaming the sum

d. subtracting with two 2-place numerals *without* and *with* renaming the sum

Using the Text Page

• If there is a group of children in your class who will have difficulty reading Ex. 1–12 at the top of the page, you may wish to read the directions with them.

• As pupils finish the work at the top of the page, they should go on to the exercises at the bottom of the page. The directions say to copy each example into column form. However, you may wish to permit some children to do the work mentally and write answers only.

Individualizing Instruction

• *More capable children* may exchange papers and check as the top portion of the page is completed. Greater emphasis should be placed upon the type of examples written rather than the answer for the example.

• *All pupils* who made mistakes in writing the appropriate types of examples for addition and subtraction may profit from the following activity:

a. You as teacher may duplicate or write on the chalkboard the four A. and S. types listed in the Pre-Book Lesson section.

b. Have pupils identify the type of each example in one or more of the rows at the top of page 144.

• If you have not already done so, perhaps for *more capable children* you will want to use the fifth suggestion in Individualizing Instruction on Teaching Page 147.

Teaching Page 149

Pupil's Objectives

(a) To write appropriate questions to complete A. and S. problems and then to work the problems; (b) to write *n*-examples for problems; and (c) to find numbers for three parts of a set.

New Word

piano

Background

All types of translation are involved in the activities at the top of page 149. First, the child must write either an addition or a subtraction problem using information which is given. At this particular time in building problem-solving skills, this program requires the child to solve the problem at a different time than when he translates the problem information into a number sentence. You will notice that, at this grade level, it is not the intent of the authors to have pupils write number sentences as helpers for solving the problems since it is crucial that pupils thoroughly understand problem relationships first. In a later grade, of course, the pupils will be directed to write the number sentence and then use it as a means of helping in the solution of the problem.

Finding parts of a set has been a very important phase of this program. So far, the emphasis has been upon naming the number for a set in terms of the numbers for two parts. The purpose in emphasizing only two parts has been to focus as completely as possible upon the mastery of the addition and subtraction facts.

Now, children need to begin to think of ways of naming the number for a set in terms of numbers for more than two parts. Though column addition has been a facet of the program for some time, the emphasis has not been upon the use of three or more addends as a means of *naming* a number. At the bottom of page 149 is a short activity which introduces the idea of naming numbers for three parts of a set.

Using the Text Page

• Be sure the children understand the three directions for the work at the top of the page. Have them proceed independently with this assignment. Circulate among the pupils and help individuals who are having difficulty.

• The following preliminary preparation is recommended before assigning the written work at the bottom of the page:

a. Discuss with the class the way in which parts of a set have been shown (two parts). Also, discuss naming a number in different ways by showing *two* addends.

b. Explain that the number for a set can be named by showing *three* addends. Have different children show a set of 10 by using three parts and then tell the number for the set and the numbers for the three parts.

• Have pupils turn to the text page and continue the lesson as it is developed there. Be sure to emphasize that there are different ways of showing the numbers for a set and its parts. In Ex. 2, for example, the child is asked to show the numbers for three parts of a set of 14 in four different ways.

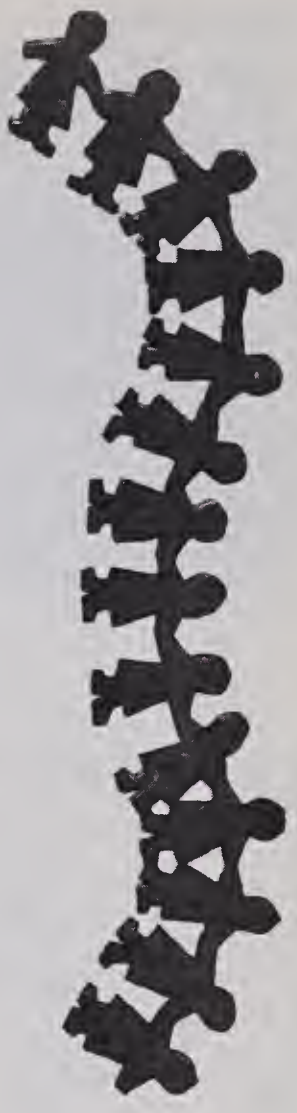
Individualizing Instruction

• *All pupils* may use a number-line picture to illustrate each of the four ways they used to name numbers for three parts of a set in Ex. 2–7 at the bottom of page 149.

• *More capable children* may write number sentences using various ways of naming the numbers for a set and its parts. You may wish to extend the writing of number sentences by having these children use ( ) in the sentences they write to indicate which two addends they will add first before adding the third to get the sum. For example, if a child named the number for a set of 14 by showing the addends, 4 and 2 and 8, he would then write a number sentence. The Associative Property would permit him to write the sentence either as $(4 + 2) + 8 = 14$ or as $4 + (2 + 8) = 14$.

Writing Questions for Problems

A. and S.; making n-examples [W]



Write a question to make an addition problem where you see *A.* and a subtraction problem where you see *S.*

1. Nan cut out 14 paper dolls. Jean cut out 18. *A.*
32 paper dolls 14+18=n ✓

2. One size of oranges costs 49¢ a dozen. Another size costs 59¢ a dozen. *S.* 10¢ 59¢−49¢=n¢

3. Monday Jane played the piano 45 minutes. Tuesday she played the piano 38 minutes. *A.* 83 minutes 45+38=n

4. In a third grade there are 20 boys and 14 girls. *S.* ✓
6 boys 20−14=n

5. Joe has read 27 pages in his book. There are 96 pages in the book. *S.* 69 pages 96−27=n

6. Sam has 29¢. He wants to buy a 45¢ book. *S.* ✓
16¢ 45¢−29¢=n¢

Work the problems on your paper.

Write an *n*-example for each problem.

Three Parts of a Set

[W]

1. For each of Ex. a–d at the right, the number for the set is 12. The numbers for two of the three parts of each set are shown. Copy and finish each example so that the numbers for the three parts are shown. Each time use digits from the set 1 through 9.

a. 3 and 5 and 4 ?
b. 1 ? and 9 and 2
c. 2 and 7 ? and 3
d. 4 and 2 and 6 ?

For each of Ex. 2–7, the number for a set is shown. Show numbers for three parts of each set in four ways, as in the box. Use digits from the set 1 through 9. ✓

Answers will vary.

2. 14
3; 2; 9
4; 7; 3
5; 2; 7
6; 1; 7

3. 11
3; 2; 6
4; 3; 4
5; 3; 3
6; 3; 2

4. 15
8; 4; 3
7; 5; 3
3; 5; 7
6; 6; 3

5. 18
9; 5; 4
7; 6; 5
6; 3; 9
8; 5; 5

6. 13
4; 7; 2
5; 2; 6
9; 1; 3
7; 5; 1

7. 17
8; 4; 5
7; 6; 4
4; 5; 8
3; 8; 6

Adding with 3 or 4 Two-Place Numerals

[o]

A
32
20
16
68

B
1
31
18
22
26
97

1. At the Old Clothes Sale there were 32 pairs of men's shoes, 20 pairs of women's shoes, and 16 pairs of children's shoes. How many pairs of shoes were on sale?

$$32 + 20 + 16 = n$$

a. What is new in Ex. A? Adding with three 2-place numerals.

b. Explain the work. ∧ Check the sum. 2+0=2, 2+6=8. Write 8 in one's place 3+2=5, 5+1=6 Write 6 in ten's place.

2. On sale, too, were 31 dolls' dresses, 18 babies' dresses, 22 girls' dresses, and 26 women's dresses. How many dresses of all kinds were on sale?

$$31 + 18 + 22 + 26 = n$$

a. In Ex. B why must you carry 1 to ten's column? There are more than 9 ones. ∧

b. Explain the addition for this example.

1+8=9 9+2=11, 11+6=17, 17=1 ten and 7 ones. Write 7 in one's place. Carry 1 to ten's column. 1+3=4, 4+1=5, 5+2=7, 7+2=9 Write 9 in ten's place.

Explain the additions for Ex. 3–5.

3.	4.	5.	6.	7.	8.
42	8¢	40	30	14	$0.22
10	43¢	6	14	28	0.16
22	26¢	19	3	50	0.29
74	77¢	65	12	2	0.32
			59	94	$0.99

Tell how to find the sums for Ex. 6–8.

Teaching Pages 150 and 151

Pupil's Objective

To learn to work column addition with three and our 2-place numerals when renaming and carrying f 1 to ten's place is required.

New Word

(page 150) *sale*

Background

Your pupils have added with three and four igits; they have bridged the 20-decade in some of hese additions; they have added with two 2-place umerals; and they have carried 1 to ten's place in ddition. This lesson, therefore, is a combination nd extension of previous learnings.

For purposes of consistency in developing a nethod of attack in working column-addition xamples, this program has pupils start at the top f the column and add each succeeding number hown as he moves down the column. Then, to void the possibility of repeating mistakes, it is sug-ested that the result be checked by adding upward. nsofar as accuracy, speed, or anything else is oncerned, the direction in which a pupil adds in olumn addition doesn't make any difference. In act, the Associative Property of Addition tells us hat it doesn't matter whether the first and second f three numbers or the second and third of those umbers are grouped together first. Con-ider the example at the right. For most hildren, 2 + 2 is readily recognized as 4. The Commutative Property of Addition tells s we may commute 2 + 6 and commute 9 + to make it possible to associate 2 + 2. (2 + = 6 + 2, 9 + 2 = 2 + 9). Now the ex-mple is 6 + 2 + 2 + 9. Then, 6 + 4 is quickly hought of as "10." Finally, 10 + 9 becomes 19, he sum.

2
6
9
2

In this instance we did not begin either at the top r the bottom of the column, but selected two umbers shown elsewhere in the column, because f the ease with which they could be renamed. The ext numbers selected for grouping made a 10. tudies have shown that children identify combina-ions with the sum 10 more readily than other combinations. They also find it easier to add to a 10 than to most other numbers. The steps in thinking the answer this way then are:

1. Commute 2 + 6 and 9 + 2.
2. 6 + (2 + 2) + 9
3. (6 + 4) + 9
4. 10 + 9
5. 19

If you have children who are inclined to seek combinations in column addition which are instantly recognized, they should not be discouraged unless it is interfering in some way with the results they get. Children who look to make 10's in column addition are applying the Commutative and the Associative Properties at an advanced level.

Teacher's Preparation

Have a place-value teaching aid (Arithme-Sticks, place-value chart, or abacus picture) available for demonstrating the carrying of 1 to ten's place in column addition.

Pre-Book Lesson

Write on the chalkboard examples like the following to review and extend previous learnings:

$0.25	40	40	12
0.30	6	12	41
0.14	19	34	23
		11	8

a. First consider an example with no carrying. Using the skills which have already been developed, have the pupils add the numbers of ones, then add the numbers of tens.

b. Now, consider an example which will involve carrying 1 to the ten's place. Add the numbers of ones. Discuss the similarity between renaming the sum of the ones and the resulting carrying of 1 to the ten's column in column addition and in adding with two 2-place numerals.

c. Use your place-value teaching aid, and, at the chalkboard, ϕ 's and / 's, and column headings Tens and Ones to help illustrate carrying 1 to ten's place.

Using the Text Pages

• Use your place-value teaching aid during oral discussion of Ex. 1–8.

• Most children should be able to proceed independently with the written work on page 151. You may wish to give *slower learners* additional help with teaching aids and then ask them to write their work as suggested in item 1a under Individualizing Instruction.

Individualizing Instruction

• *Slower learners* may do examples with labels *Tens* and *Ones* and then compare their answers with those for the abstract work:

a. 3 tens and 6 ones	b. 36
5 ones	5
4 tens and 2 ones	42
7 tens and 13 ones,	83

or 8 tens and 3 ones, or 83

• *All pupils* may profit from renaming practice of the following type:

74 = _ ? _ tens and 14 ones
_ ? _ = 5 tens and 17 ones
68 = 5 tens and _ ? _ ones

• Have *slower learners* complete examples like those below. They may work in small groups, some writing on the chalkboard and some on paper. Perhaps you will want to have some of the *more capable children* help when needed.

Write the missing digits.

a.	b.	c.
25	19	7
13	6	26
+ 6	+42	+14
?4	?7	?7

d.	e.	f.
30	28	$0.47
41	1	0.28
+ 9	+53	+ 0.13
??	??	$?.??

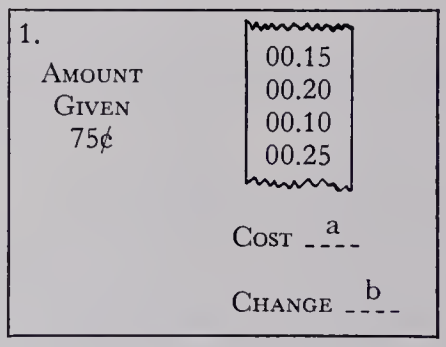
• Ask children to bring sales tapes from home. Select appropriate sales tapes for the following activity (in general, the column should not be too long, should not involve carrying more than 1 to ten's column, and should not have more than 2-place numerals):

a. Each sales tape should be fastened to a heavy cardboard or tagboard with the "total" cut off.

b. Each board should then be numbered in the upper left corner.

c. Indicate on the chalkboard the amount of money given the clerk.

d. Write "Cost" and "Change" below the sales-tape items.



e. Prepare an Answer Card on which you write the answers for all sales-tape cards by numbers.

f. Place the sales-tape cards in a location available to all children. In using the set of cards there are two kinds of practice: (1) finding the sum, and (2) finding the change. (When you first make the cards you may want to write only "Cost" on the card. Then, some time later write "Change.")

g. Since each card is numbered, the child can record his answers by number. After he has completed the work for any one or any set of cards, the child may then check his work by referring to the answer card.

• *More capable children* should enjoy the enrichment activity at the bottom of page 151. It is not just an activity in renaming numbers. First, the child must determine what number has been named. Then, he must write two other names for the same number, one using A and the other using S. After pupils have completed the work, you may have them check each other's work.

• Assign Extra Examples Set 52 as needed.

[W]

Copy each set of numerals in a column. Add. Check your answer.

	a	b	c
9.	2, 35, 33 70	24, 13, 25, 36 98	32¢, 19¢, 26¢ 77¢
10.	39, 24, 30 93	11, 9, 6, 15 41	4¢, 36¢, 40¢ 80¢
11.	40, 12, 34, 11 97	31, 28, 16, 3 78	30¢, 8¢, 9¢, 2¢ 49¢
12.	3, 51, 12, 22 88	12, 8, 40, 17 77	\$0.17, \$0.46, \$0.04 \$0.67
13.	15, 30, 6, 13 64	55, 21, 8, 5 89	\$0.08, \$0.18, \$0.23 \$0.49
14.	20, 18, 1, 23 62	7, 8, 4, 60 79	\$0.30, \$0.49, \$0.17 \$0.96

◆ **Extra Examples.** Work Set 52.

Do You Like to Try New Things?

Enrichment [W]

For each of Ex. 1–12, write, as shown for Ex. 1, two other examples with the same answer. Let one example be A. and the other S. Use 2-place numerals. Answers will vary.

In Ex. 1, $48 - 19 = 29$. Your A. example might be $19 + 10$ or $12 + 17$ or something else. Your S. example might be $72 - 43$ or $64 - 35$ or something else.

If, for Ex. 1, you used $19 + 10$ and $72 - 43$, you would write

$(48 - 19) = (19 + 10)$ and $(48 - 19) = (72 - 43)$.

1. $(48 - 19)$	**4.** $(14 + 59)$	**7.** $(60 - 40)$	**10.** $(31 + 19)$
2. $(33 + 27)$	**5.** $(82 - 28)$	**8.** $(27 + 28)$	**11.** $(80 - 26)$
3. $(52 - 17)$	**6.** $(29 + 23)$	**9.** $(63 - 17)$	**12.** $(18 + 38)$

More about Carrying

Carrying a digit showing more than 1 ten [O]

1. Miss Brown's class is collecting old magazines for people in other lands. At four houses they collected 38, 16, 4, and 7 magazines. How many magazines did the class collect there? $38 + 16 + 4 + 7 = n$

Study the work in the box. Add downward.

Ones: *Think*, "14, 18, 25." 25 can be renamed 2 tens and 5 ones, so write "5" and carry 2. Sometimes you carry a digit showing more than 1 ten.

Tens: *Think*, "5, 6." Write "6."

To check the sum 65, add upward.

$$\begin{array}{r} {\scriptstyle 2\ \ } \\ 38 \\ 16 \\ 4 \\ 7 \\ \hline 65 \end{array}$$

Which sums for Ex. 2–7 are shown wrong? Why?

2.	3.	4.	5.	6.	7.
34	36	40	7¢	\$0.11	\$0.18
6	20	17	28¢	0.28	0.07
18	9	19	6¢	0.16	0.07
25	27	8	25¢	0.18	0.08
83	92	64	66¢	\$0.63	\$0.20

Answer for Ex. 4 is 84; 2 is carried to ten's column.
Answer for Ex. 6 is \$0.73; 2 is carried to ten's column.
Answer for Ex. 7 is \$0.40; 3 is carried to ten's column. [W]

Copy each set of numerals in a column. Add and check.

8. 34, 7, 16, 16 73
9. 16, 17, 53, 9 95
10. 30, 12, 47, 5 94
11. 25, 2, 40, 4 71
12. 14, 16, 13, 32 75
13. 17, 8, 33, 8 66
14. 12, 14, 5, 42 73
15. 5, 28, 16, 44 93
16. 7¢, 49¢, 6¢, 5¢ 67¢
17. \$0.27, \$0.25, \$0.38 \$0.90
18. 9¢, 7¢, 38¢, 34¢ 88¢
19. 5¢, 10¢, 22¢, 7¢ 44¢

Extra Examples. Work Set 53.

Children sometimes get the idea that 1 is always the digit that is carried. Use this lesson to emphasize that the carried digit may be a numeral other than 1.

Teaching Page 152

Pupil's Objective

To learn how to carry 2 or a digit for a greater number to ten's column in column addition.

New Words

class, magazines

Background

Up to this point, the only digit that has been carried is 1. In this lesson, children should be led to discover for themselves the generalization that many times it is necessary to carry 2, 3, 4, and so on to the ten's column.

An application of the Associative Property of Addition provides a means of fostering greater understanding of column addition as well as a way of checking the answer. Consider Ex. A. The number of ones are sequentially associated and then added. Their sum is shown. Then, the numbers of tens are associated and added. Then the sum of the tens is shown below the sum of the ones. The 5 tens is shown as 50. Each of the sums (for the ones and for the tens) thus recorded is an addend of the sum for the example. The two addends (partial sums) are then added to find the final sum.

A

```
  49
  15
   7
   8
  29  partial sum (ones)
+50   partial sum (tens)
  79  sum
```

The Associative Property of Addition also permits us to show partial sums by allowing us to show the sum of the numbers of tens first. Consider Ex. B. In this example, the numbers of tens are associated and added first. The partial sum is shown. Then, the numbers of ones are sequentially associated and then added. The partial sum is shown below the sum of the tens. The two partial sums are added to find the final sum.

B

```
  8
 27
  5
 46
 60  (partial sum, tens)
+26  (partial sum, ones)
 86  sum
```

Pre-Book Lesson

• Have pupils use a pocket chart or Arithme-Sticks or an abacus picture to find the answer for a problem similar to the following:

If 15 girls, 12 boys, 9 mothers, and 6 fathers go to the puppet show, how many chairs will be needed?

Each of the numbers should be represented on the teaching aid. The sum for the one's column will be 22.

• Write the column headings Tens and Ones on the chalkboard. Write an example like the one at the left. Have a child add the numbers shown in the one's column and write the sum as a number of ones at the bottom of the column. Add the numbers shown in the ten's column and write the sum as a number of tens. Then, rename the number shown so that ones will be shown by a single digit.

Tens	Ones
3	8
1	6
	2
2	7
6 tens	23 ones
or, 8 tens	3 ones

Using the Text Page

• You may wish to have pupils work Ex. 2–7 on the chalkboard.

• Observe the children's work as they do Ex. 8–19 to be sure that they copy the examples correctly, and carry to ten's column.

Individualizing Instruction

• One or more of the following activities may be considered for the *slower learners:*

a. Review higher-decade examples using addition facts, as taught on pages 45–47, 104–105.

b. For column addition, allow temporary use of the crutch of writing the digit carried.

c. Provide practice in renaming:

4 tens and 25 ones = _ ? _
1 ten and 31 ones = _ ? _

• *More capable children* may check their answers to Ex. 8–11 by showing partial sums.

Pupil's Objectives

(a) To learn to read and interpret information given in another kind of table; (b) to solve problems based on information given in a table; and (c) to become familiar with different ways of recording information.

Background

In a previous lesson (see Teaching Page 81), the relation concept was introduced by using a chart as a means of recording and reporting paired information. In this lesson, recorded rainfall is paired with location (city). The same information could have been organized in graph form. At this point, however, the table form is somewhat easier for the child to read and interpret, and is just as useful for extending the relation concept.

Teacher's Preparation

If, as was suggested, you gathered information about weights of children a year ago and now at the time of the lesson on pounds and ounces (see Teaching Pages 146–147), it may well be organized for use in this lesson.

Pre-Book Lesson

• Write on the chalkboard in table form the names of several children and their weights a year ago and now. Use this information as the basis for questions like the following:

How much does Fred weigh this year?
How much did Fred weigh last year?
Which person weighs 63 pounds?
Is Jan's weight this year greater or less than her weight last year?
How many more pounds does Tom weigh this year than last?

• Make certain that all children know how to read the table. Ask questions like:

What do these numerals show?
Why do we have the names of children written on the board?

Using the Text Page

• Call the pupils' attention to the table. Ask questions about the table to determine their understanding about it (i.e., "What do the numbers shown under *Last Year* mean?").

• You may wish to read each problem aloud with children having reading difficulties before asking them to go ahead with the written work. All others should be able to proceed independently.

Individualizing Instruction

• As a part of the health, social-studies, and science programs, have pupils observe and record paired information like the following:

a. Temperature in different cities throughout the country which has been reported on television or in the newspaper.

b. A record of children's weights and heights in the fall and at midyear.

• *More capable children* may look in encyclopedias for information related to the social-studies topic under consideration which can be reported in table form.

*Help pupils in interpreting this double heading.

Using a Table

A. and S. problems [O]

Use numbers shown in the table to answer Ex. 1–12.

1. Name the city that had 39 inches of rain this year. Brookside

2. Name the city that had the least rainfall last year. Cactus Point

3. Name the cities in order, from that with the most rainfall to that with the least rainfall last year. Brookside, Fern Valley, Hillsdale, Rock Town, Sunnyvale, Ford Creek, Cactus Point.

4. Do the same for this year. Fern Valley, Brookside, Hillsdale, Rock Town, Sunnyvale, Ford Creek, Cactus Point.

5. Last year Hillsdale had just the same rainfall as what two other cities together? Sunnyvale and Rock Town.

City	*Inches of Rainfall	
	*Last year	*This year
Fern Valley	38	45
Sunnyvale	12	14
Brookside	40	39
Cactus Point	2	4
Hillsdale	24	28
Rock Town	12	15
Ford Creek	7	9

[W]

Write your work for Ex. 6–12.

How many fewer inches of rain fell last year than this
6. in Fern Valley? 7 inches 7. in Hillsdale? 4 inches 8. in Rock Town? 3 inches

9. This year which had the most rainfall—Sunnyvale, Brookside, Cactus Point, or Hillsdale? Brookside

How many more inches of rain have fallen this year
10. in Fern Valley than in Ford Creek? 36 inches

11. in Brookside than in Rock Town? 24 inches

12. The most rain ever to fall in one year in Cactus Point was 21 inches. How many fewer inches fell there all of last year? 19 inches

Should You Use A. or S.?

Writing n-examples [W]

Write the numerals 1–8 on your paper. After each, write "A." if you are to use addition or "S." if you are to use subtraction. Do not solve any problems now.

1. Joe likes to count birds. On different days he counted 12, 10, 14, and 9 birds. How many birds in all did Joe count? A. 45 birds
$12+10+14+9=n$

2. Joe's class made 30 pictures of birds. They painted 16 of them. They colored the others with crayons. How many did they color with crayons? S. 14 pictures
$30-16=n$

3. Sam made a bird feeder. He spent 48¢ for wood, 19¢ for nails, and 29¢ for paint. How much money did he spend for these things? A. 96¢
$48¢+19¢+29¢=n¢$

4. It took Sam 48 minutes to paint his bird feeder. How many minutes less than an hour did he work? S. 12 minutes $60-48=n$

5. Sam had 70¢. He spent 35¢ for bird seed. How much money did he have left? S. 35¢
$70¢-35¢=n¢$

6. Sue's father bought 50 pansy plants. That night he put 28 of them in the garden. How many pansy plants does he still have to put in the garden? S. 22 plants
$50-28=n$

7. Sue had 80¢. She spent some of it for flower seeds. She had 41¢ left. How much did the seeds cost? S. 39¢ $80¢-41¢=n¢$

8. Jane counted 14 red flowers, 8 blue flowers, and 23 white flowers in her garden. How many flowers did Jane count? A. 45 flowers $14+8+23=n$

*Now solve the problems. Then write each *n*-example.

* Note that experience with three important problem-solving skills is provided on this page.

Pupil's Objectives

(a) To have practice in selecting the appropriate operation (A. or S.) to use for solving a problem; (b) to have practice in solving A. and S. problems; and (c) to have more practice in writing *n*-examples for problems.

New Words

feeder, *pansy*, *solve*

Background

The emphasis upon translation as an important phase of problem-solving is seen again in this lesson. First, the child is to read the problem and then decide what operation (A. or S.) should be used. Next, he goes through the problems a second time and writes the work for each problem. Finally, he goes through the problems once more and writes an *n*-example for each one.

The writing of an *n*-example for a problem reveals the extent to which a child can translate the written information into a number sentence. It is one way of determining a child's understanding of the information in a problem and how it is related to the addition and subtraction skills he has been learning.

Pre-Book Lesson

• Review the addends-sum relationship in terms of addition and subtraction sentences. Write each of the following sentence types on the board and have selected pupils tell whether each numeral and *n* in the sentence stands for an addend or a sum and why.

$42 + 17 = n$	$68 - 25 = n$	$74 - n = 42$
$74 + n = 95$	$n + 36 = 72$	$n - 18 = 55$

• Perhaps you will want to have children state a problem for each sentence above.

Using the Text Page

• Be sure to emphasize the following order in completing the assignment:

First. Read each problem and write on paper "A." or "S." for the operation that should be used.

Second. Read each problem again and then write the solution for it.

Third. Read the problem once more and write a number sentence with *n*.

• Children with reading difficulties will need individual or small-group attention.

Individualizing Instruction

• After the written assignment has been completed, you may wish to have an oral lesson in which children tell why they added or subtracted in each problem.

• In a follow-up lesson, children may write their *n*-example for each problem on the chalkboard. Have all the different *n*-examples for each problem written on the chalkboard and have the children tell why each number sentence is or is not correct for the problem.

• *All pupils* may use a number line to show the adding or subtracting for each of the problems.

• *More capable children* may turn to the table on page 153 and write five problems about the information in the table. They may use the problems at the bottom of page 153 as a guide. After the problems are written *all pupils* may find solutions.

Teaching Page 155

Pupil's Objectives

(a) To have written practice on all the addition and subtraction skills which have been learned; and (b) to have oral practice on higher-decade addition and subtraction examples.

Using the Text Page

- After the pupils have worked Ex. 1–25 you may ask them to check their work in ways they have learned.
- The suggestion in the middle of the page for Ex. 1–12 may be used as an optional activity.
- The oral practice may be used in several different ways:

a. Pupils may work in pairs; one saying the answers to the examples in a column while the other pupil checks.

b. Have timed races between teams. Keep a record of the time required by each team to say the answers for all rows.

c. Folded-paper written timed trials may be held for each column. Inform children of their time for writing answers for a given column and encourage them to improve upon it while working the next column. (Caution: Don't emphasize speed at the expense of accuracy.)

Individualizing Instruction

- For *slower learners*, you may use an abbreviated assignment of the written work at the top of page 155. Assign the following examples which will indicate understanding of operations and will reveal any need for help: Ex. 1-6, 11, 16, 18, and 20. This plan will enable you to correct and give help after each assignment as well as locate sources of difficulty and need for increased concentration.
- You may wish to use *more capable children* to assist in checking all written work.
- *More capable children* may write original examples as an extension of *turning examples around* as suggested in the middle of the page.
- You may wish to have some children work sets of Extra Examples that have not been used previously.

NOTES

To Keep in Practice

A. and S. [W]

Copy each of Ex. 1–25 in a column and work.

1. $55 + 27$ 82	5. $90 - 7$ 83	9. $7 + 42$ 49	13. $46 \overset{85}{+} 39$
2. $74 - 37$ 37	6. $89 - 36$ 53	10. $97 - 29$ 68	14. $45 \overset{69}{+} 24$
3. $36 + 53$ 89	7. $39 + 36$ 75	11. $98 - 78$ 20	15. $92 \overset{45}{-} 47$
4. $72 - 25$ 47	8. $71 - 65$ 6	12. $53 + 37$ 90	16. $58 \overset{22}{-} 36$
17. $\$0.76 - \$0.35$ $0.41	20. $\$0.81 - \$0.46$ $0.35	23. $16 + 18 + 9 \overset{61}{+} 18$	
18. $\$0.14 + \$0.48$ $0.62	21. $\$0.28 + \$0.62$ $0.90	24. $\$0.78 + \$0.18$ $0.96	
19. $\$0.44 - \$0.26$ $0.18	22. $\$0.72 - \$0.29$ $0.43	25. $53 + 7 + 9 \overset{96}{+} 27$	

Now use your answers for Ex. 1–12 and turn the examples around. Write your work like this:

Ex. 1. Write, "55 + 27 = 82, so 82 = 55 + 27."

Ex. 2. Write, "74 − 37 = 37, so 37 = 74 − 37."

Oral Practice

Say the answers by rows and by columns.

	a	b	c	d	e	f
1.	$62 - 9$ 53	$88 - 7$ 81	$40 + 8$ 48	$51 + 7$ 58	$79 - 5$ 74	$64 \overset{60}{-} 4$
2.	$46 + 5$ 51	$32 + 6$ 38	$27 - 6$ 21	$37 - 8$ 29	$84 + 5$ 89	$44 \overset{41}{-} 3$
3.	$35 - 6$ 29	$68 - 5$ 63	$72 + 5$ 77	$60 - 8$ 52	$42 + 7$ 49	$36 \overset{30}{-} 6$
4.	$63 + 8$ 71	$52 - 8$ 44	$63 + 9$ 72	$71 - 4$ 67	$19 + 4$ 23	$86 \overset{79}{-} 7$
5.	$49 + 7$ 56	$25 + 9$ 34	$35 - 9$ 26	$53 - 7$ 46	$25 - 7$ 18	$83 \overset{75}{-} 8$
6.	$33 - 5$ 28	$62 - 4$ 58	$36 + 7$ 43	$75 - 8$ 67	$51 + 9$ 60	$46 \overset{55}{+} 9$
7.	$29 + 5$ 34	$83 - 4$ 79	$87 + 6$ 93	$24 - 8$ 16	$36 - 9$ 27	$28 \overset{35}{+} 7$

* More practice in judging whether answers to A. and S. examples are reasonable! Developing this ability is a great help in getting accurate answers.

* Which Answer Must Be Wrong?

A. and S.; estimating [O]

When addends are whole numbers, the sum is never less than any of its addends.

When you subtract from a whole-number sum and the answer is a whole number, it is never greater than the sum.

Correct answers are encircled.

Which of the answers must be wrong? Tell why.

1. $37 - 18$ **20** **19** **42** (19)
   42 It is greater than the sum 37.
2. $19 + 45$ **32** **57** **64** (64)
   32 It is less than the addend 45.
3. $28 + 35$ **68** **63** **19** (63)
   19 It is less than bath addends.
4. $15 + 63$ **78** **58** **86** (78)
   58 It is less than the addend 63.
5. $41 - 23$ **15** **52** **18** (18)
   52 It is greater than the sum 41.
6. $61 - 37$ **40** **98** **24** (24)
   98 It is greater than the sum 61.
7. $29 + 19$ **50** **48** **18** (48)
   18 It is less than bath addends.
8. $52 - 36$ **88** **16** **34** (16)
   88 It is greater than the sum 52.
9. $19 + 25$ **44** **6** **70** (44)
   6 It is less than both addends.
10. $50 - 39$ **11** **22** **89** (11)
    89 It is greater than the sum 50.

[W]

Copy Ex. 1–10 and find the correct answers. Check.

Do You Like to Try New Things?

Enrichment [W]

Right or wrong? Explain why.

1. A hen weighs 5 pounds standing on one leg. So, standing on both legs, she weighs 10 pounds. Wrong
   The hen still weighs 5 pounds.

2. Jean earned 55¢ and spent 50¢ of it for a cake for her mother's birthday. Right. The sum is not less than the addend.

3. Ida weighs 60 pounds. Her brother Jim weighs 15 pounds more, or 75 pounds. Right. 60+15=75

4. Sue bought 2 pounds of butter and 12 ounces of beans. The beans weighed more than the butter. Wrong
   2 pounds is the same weight as 32 ounces.

5. The one road from Big Town to Little Town is 12 miles going to Little Town and 14 miles coming back.
   Wrong. The distance would be the same.

Teaching Page 156

Pupil's Objective

To learn to judge whether an answer in A. or S. is greater or less than it should be.

New Word

leg

Background

Note the two generalizations at the top of page 156. First, a word about the emphasis on whole numbers in the two statements. These generalizations are true only for the set of *whole numbers* and the set of *counting numbers*. Children will eventually encounter other sets of numbers for which these statements would not always be true (for example the *integers* and *rational numbers*). Consequently, it is necessary to include the qualifying words, *whole numbers*, in the generalizations at this time. Whatever children learn should be mathematically accurate and not have to be unlearned at a later time.

Perhaps you will want to paraphrase the statements in order to help children understand their full meaning. For example:

When we are subtracting *whole numbers*, the answer will never be greater than the number subtracted from.

When a *whole number* is added to a *whole* number, the answer will always be as great as or greater than the greater addend. (When 0 is added to a whole number, the sum is equal to (as great as) the other addend.)

On an earlier page (see Teaching Page 130), the importance of having a basis for estimating was discussed. The two generalizations at the top of page 156 help pupils to make an initial judgment as to the reasonableness of an answer. If the result of an addition or subtraction operation falls within the limits of these two generalizations, then the answer *makes sense*.

Pre-Book Lesson

Discuss the addends-sum relationship. It would be a good idea to use children or objects in the room to demonstrate.

a. Have 4 children stand in one place. Have 5 other children stand in another location. Ask a question that would involve joining the two sets of children. Then, have a child write a number sentence on the chalkboard for the problem situation, using the numbers involved.

b. Join the two groups. Ask questions about the sum of the two addends as compared to each addend. (You may want to demonstrate the type of example when the sum is equal to one of the addends.)

c. Use the same approach for developing the generalization about subtraction. You may want to demonstrate the subtraction using two or three of the subtraction ideas (take-away, how-many-gone, difference).

Using the Text Page

- During the oral work stress the use of the two generalizations as the basis for identifying an answer that must be wrong. Do not permit children to work the examples mentally. Ask questions such as, "Which answer cannot be right?" "Why doesn't that answer make sense?"
- After doing the written work have children check their work for each example by changing the order of the addends (adding upward).
- *More capable children* should enjoy finding the ridiculous aspect in each problem in the Enrichment section.

Individualizing Instruction

All pupils may write examples and three suggested answers, one of which cannot be right because of the generalizations developed in this lesson.

Teaching Page 157

Pupil's Objectives

(a) To differentiate between the need for addition and the need for subtraction in problems; (b) to solve addition and subtraction problems orally without the use of pencil and paper; and (c) to have practice in working addition and subtraction examples with 2-place numerals.

New Word

sister

Background

Not all people are alike in their ability to form mental images. All too often teachers say "Think" or "Think hard" and pupils frequently respond with some physical evidence of straining—the frown or wrinkled brow. In this lesson children are to read a problem and then *think* a solution without the use of pencil or paper. Two distinct thought acts are involved. First, the pupil must decide what operation is necessary. Then, he must perform the operation and arrive at a reasonable answer. Some children will have completed both thoughts before others have completed the first one. For this reason, slower thinkers should be taught separately in this lesson. It should be noted that those who think slowly are not necessarily poor students.

Using the Text Page

- First, read the page heading. Point out that pupils are to *think* the answers without using pencil and paper.
- Read each problem and ask for answers. Ask if other children agree with the answer given. Have the pupils tell *why* they added or subtracted.
- You may wish to assign only some of the written work at the bottom of the page to some or all of the children, instead of assigning the whole section. However, this section may well serve as a progress test at this time if you wish to make the entire assignment.
- *More capable children* may not need to copy the numerals for doing either the addition or the subtraction examples.

Individualizing Instruction

- *Slower learners* may begin the written assignment at the bottom of the page, while you develop the oral lesson at the top of the page with the rest of the class. Then, go through the oral work separately with the *slower learners*.
- *More capable children* may write *n*-examples for the problems at the top of the page.

Reminder

Continue to maintain and extend the following:

a. Ability to recognize common geometric forms and shapes

b. Ability to recognize common measuring instruments and occasions for their use

c. Ability to recognize commonly-used coins and understand their relative values

d. Ability to tell time to the minute

No Pencils, Please!

Oral problem-solving [O]

1. Three fish Tom caught measured 9 inches, 12 inches, and 8 inches. Put end to end, they measured _?_ inches. 29
Numbers for the parts are given. Add to find the number for the total.

2. Jim is 64 inches tall. His sister is 9 inches shorter. How tall is his sister? 55 inches Numbers for the total and one part are given. Subtract to find the number for the other part.

3. Tom had 49¢. He paid 39¢ for a baseball. How much money has he left? 10¢ Numbers for the total and one part are given. Subtract to find the number for the other part.

4. A 9-ounce rock weighs _?_ ounces less than a pound. 7
Subtract to find the difference.

5. Tom ate all but 6 of the 48 peanuts in the bag. How many peanuts did Tom eat? 42. Numbers for the total and one part are given. Subtract to find the number for the other part.

6. Nan has 8 roses. She wants to have 18 roses for her mother. How many more roses does she need? 10
Numbers for the total and one part are given. Subtract to find the number for the other part.

7. Tom scored 20 points on a test. On the same test Bob scored 9 points more, or _?_ points. 29 Numbers for the parts are given. Add to find the number for the total.

Tell why you added or subtracted in each problem.

To Keep in Practice

A. and S. [W]

Copy each set of numerals in a column. Add and check.

1. 39, 35 74	5. 28, 26 54	9. 40, 7 47	13. 23¢, 10¢ 33¢
2. 33, 29 62	6. 43, 35 78	10. 54, 17 71	14. 55¢, 20¢ 75¢
3. 46, 19 65	7. 44, 36 80	11. 23, 17 40	15. 35¢, 28¢ 63¢
4. 80, 9 89	8. 32, 18 50	12. 57, 28 85	16. 69¢, 19¢ 88¢

17. 8, 7, 27, 9 51 18. 17, 29, 8, 39 93 19. $0.39, $0.28 $0.67

Copy each set of numerals in Ex. 1–16 and subtract.

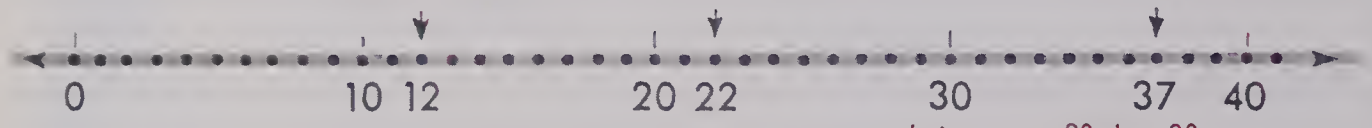
1. 4 2. 4 3. 27 4. 71 5. 2
6. 8 7. 8 8. 14 9. 33 10. 37
11. 6 12. 29 13. 13¢ 14. 35¢ 15. 7¢ 16. 50¢

Rounding Numbers to Tens

Discovering mental procedures [O]

There were 19 boys in Joe's class, but he told his mother that there were about 20. Joe had rounded 19 to the nearest ten.

1. On the number-line picture, is 12 nearer to 10 or to 20? 10 Then should we round 12 to 10 or to 20? 10



It is nearer 20 than 30.

2. Should we round 22 to 20 or to 30? 20 Why?

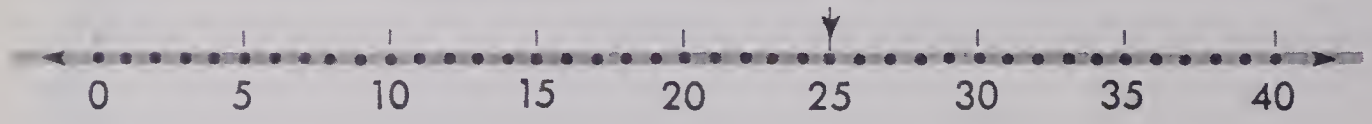
It is nearer 40 than 30.

3. Should we round 37 to 30 or to 40? 40 Why?

4. How should we round 18? 20 33? 30 27? 30 Explain.

18 is nearer 20 than 10; 33 is nearer 30 than 40; 27 is nearer 30 than 20.

Twenty-five is halfway between 20 and 30. Should we round 25 to 20 or to 30? Study the number-line picture.



In this book for all numbers shown by 2-place numerals ending in 5, through 85, we shall round to the next greater ten. So, we round 25 to 30 and not to 20.

5. Round each of these numbers:

a. 35 40 **b.** 65 70 **c.** 85 90 **d.** 55 60 **e.** 75 80

[W]

Show the rounded number for

6. 24. 20	**10.** 45. 50	**14.** 15. 20	**18.** 41. 40	**22.** 64. 60	**26.** 46. 50
7. 79. 80	**11.** 83. 80	**15.** 66. 70	**19.** 11. 10	**23.** 72. 70	**27.** 38. 40
8. 51. 50	**12.** 48. 50	**16.** 92. 90	**20.** 57. 60	**24.** 26. 30	**28.** 87. 90
9. 43. 40	**13.** 25. 30	**17.** 33. 30	**21.** 18. 20	**25.** 75. 80	**29.** 14. 10

Sometimes an estimated answer is all that is needed for arriving at a number wanted. Rounded numbers provide a basis for work in estimating.

Teaching Page 158

Pupil's Objective

To learn to round tens and ones to the nearest number of tens.

New Word

should

Background

Skill in estimating is extended in the next three lessons. Estimating is essential for determining the reasonableness of an answer.

Have you observed the unhappy behavior of many adults when, for some practical reason, they need to know the approximate sum of two numbers such as 47 and 32? Many of them resort to the use of pencil and paper or they guess or trust to luck or undertake some painful method of mental computation. All this is unnecessary, for there are simple and efficient ways of estimating. The estimated sum or difference is usually not exact but, for many purposes, it is good enough.

Up to this point, the only basis the child has for estimating the sum of 47 and 32 is that the result must be greater than 47. Rounding numbers permits greater precision; thus, 47 becomes 50 (or 5 tens), and 32 becomes 30 (or 3 tens), and the estimated sum is 80 (or 8 tens). Now, the child knows that the actual answer to 47 + 32 must be near 80. So, if the actual computation shows a sum very far from 80 (10 or more), there is reason to suspect a mistake.

Teacher's Preparation

You will need a number-line picture to refer to in this and subsequent lessons. If you have a permanent number-line model showing whole numbers to 100 (see Teaching Pages 39 and 52), it should be mounted so all pupils can see it. Otherwise, draw a number-line picture near the top of the chalkboard which may be used for a few days. Label only the points for 5, 10, 15, and so on.

Pre-Book Lesson

• Engage the class in a discussion concerning the use of the words *about* and *almost*. For example, "What do I mean when I say there are about 30 children in our class?" Have children give other illustrations of the use of *about* or *almost* in reference to a number of things.

• When we say "about 40" or "almost 40" we mean near 40 but not exactly 40. We call this *rounding* a number to the nearest number of tens. Point to numbers shown on the number-line picture and have pupils tell the nearest number of tens.

• Continue to point to the number-line picture and have children tell if they are *rounding up* or *rounding down*.

• Teach the rounding of a number with 5 in one's place as a special case. In this program, a number ending in 5 is *rounded up* to the next greater number of tens.

Using the Text Page

• Refer to the number-line picture on the board as well as to those in the child's book, as pupils answer Ex. 1–5 orally.

• While the pupils are writing their work for Ex. 6–29, circulate among the children to determine the extent to which they are having difficulty. Some children may need help of the type suggested for *slower learners* in the Individualizing Instruction section.

Individualizing Instruction

• It may be necessary to work with *slower learners* as a group, using bundles of tens and ones or using ⌀'s and /'s on the chalkboard to help these children get the idea of *about*.

• *All pupils* may draw number-line pictures to illustrate some of the examples in the written work. Or, you may wish to duplicate number-line pictures on paper and have children label a point for each of Ex. 6–13 to help them in visually identifying the nearest number of tens.

Looking Ahead

Very soon, computation with 3-place numerals will be introduced. Children should be given as many opportunities as possible now to read and write 3-place numerals and sense the meaning of numbers involving hundreds, tens, and ones.

Pupil's Objectives

(a) To learn to estimate the sum of numbers involving tens and ones by rounding each addend to the nearest number of tens; and (b) to have practice in estimating sums.

New Words

beside, estimate

Background

Estimating in examples like 42 + 26 only rarely gives an exact sum, but in many life situations estimated sums satisfy our needs—they are near enough. (Of course, we have to know *when* they are good enough and when, on the other hand, we must have accurate sums.) Up to now, your pupils usually have shown their addends on paper and added by columns. Now they will learn how, by rounding, they can approximate sums for two numbers involving tens and ones. In the work on this page, each number is to be rounded to a number of tens. Then the pupils estimate sums by adding the numbers of tens. Very often, the estimated answer will not be the actual sum. There are occasions, however, when the estimated sum and the actual sum are the same.

Pre-Book Lesson

• Many uses for rounding to a number of tens in estimating a sum can be found in problems encountered daily by children. Consider a problem similar to the one below:

> The school nurse will need about how many health charts for the two third-grade classes? There are 28 children in one class and 31 in the other.

Ask children if they know a quick way of estimating the number of health cards needed without actually working the problem. Some pupils should suggest using rounded numbers.

• Refer to the number-line picture to find that 28 is nearer 30 than any other number of tens. Do the same with 31. Show the two rounded numbers on the board. Guide the children to see that adding two numbers involving tens only is easy.

• Add on the chalkboard 28 + 31. Then, add 30 + 30. Point out the closeness of the two answers. Use the words *about*, *almost*, and *approximately* to stress the idea that usually the estimate is not exact.

• Use other examples of rounding to estimate the sum. Each example should be checked by addition in order to show that the estimated answer, while close to the actual sum, is not always the same. But, for many purposes, the estimate is adequate. It is particularly useful to determine whether or not an answer makes sense.

Using the Text Page

• While answering Ex. 1–6 orally permit children to refer to the number-line picture on the chalkboard if they need help.

• Make sure the pupils understand the directions for each of the written exercises. They may refer to the number-line picture if they wish.

• After the written work has been completed, you may have an oral lesson in which pupils tell *why* they chose a particular answer, or *how* they arrived at a particular estimate.

Individualizing Instruction

• *Slower learners* may need to do Ex. 7–20 orally before they are assigned as written work.

• As *slower learners* do the written exercises, observe their work to identify sources of error. If they are having difficulty in rounding these numbers, use a place-value chart, ⌀'s and /'s, and the number-line picture to help them understand that rounding means to find the nearest number of tens.

• *More capable children* may find challenge and excitement in the following:

a. Try to decide, in advance of finding the answer, whether the estimated answer will be greater than, less than, or equal to the exact answer.

b. Some children will be able to go one step further by determining in advance how much greater or less the estimated answer will be than the exact answer. For example, their thinking may be something like this: "One rounded number is *up* 2; the other is *down* 3; so, the estimate will be 1 less than the exact answer.

Using Rounded Numbers to Estimate Sums

[o]

1. Ex. A. We can round 32 to 30 and 47 to 50.
a. $30 + 50 = {}^{80}?$ So, our estimate of the sum is $^{80}?$.
b. 80 is not the true sum. At the board add 32 and 47 to find the true sum. 79

A
32 47 ?

2. Ex. B. Round 48 to $^{50}?$ and 29 to $^{30}?$. What is the estimated sum? $^{80}\wedge$ the true sum? 77

B
48 29 ?

Estimate sums. Then find true sums at the board.

3. $41 + 32 = {}^{70}73$ 4. $35 + 33 = {}^{70}68$ 5. $41 + 39 = {}^{80}80$ 6. $24 + 65 = {}^{90}89$

[w]

For each of Ex. 7–10 show the estimate you think nearest to the true sum. Do not work the example. Estimates are shown above the sign of operation.

7. $23 + 37 = {}^{60}60$ **70 60 80**
8. $36 + 51 = {}^{90}87$ **60 80 90**
9. $52 + 18 = {}^{70}70$ **80 70 60**
10. $45 + 24 = {}^{70}69$ **80 90 70**

Show your estimates of the sums for Ex. 11–18.

11. $41 + 31 = {}^{70}72$ 13. $29 + 28 = {}^{60}57$ 15. $32 + 35 = {}^{70}67$ 17. $45 + 23 = {}^{70}68$
12. $23 + 55 = {}^{80}78$ 14. $26 + 43 = {}^{70}69$ 16. $63 + 21 = {}^{80}84$ 18. $34 + 46 = {}^{80}80$

Beside each of your estimates for Ex. 7–18, work the example to see if your estimate was a good one.

Do not work Ex. 19–20. Show your estimate of each sum. Then beside it write "Yes" or "No" for the answer.

19. Do the 28 children in Grade 3 and the 27 children in Grade 4 make more than 50 children in all? 60 Yes

20. If Mr. Burns ties together a 30-foot rope and a 28-foot rope, will he have the 80-foot rope he needs? 60 No

Estimating Answers in Subtraction

Using rounded numbers [O]

A	B	C
72 −41 ?	81 −39 ?	45 −28 ?

1. Explain these estimated answers:

Ex. A. 30 (70 − 40 = 30) Ex. B. 40 (80 − 40 = 40) Ex. C. 20 (50 − 30 = 20)

Estimate the answers for Ex. 2–5.

2.	3.	4.	5.
65	92	57	38
−39	−48	−39	−22
30	40	20	20

[W]

For each of Ex. 6–9, show the estimate you think nearest to the true answer. Do not work the example.

Estimates are shown above the sign of operation.

6. $38 \overset{10}{-} 32$ **20 30 10**

7. $57 \overset{30}{-} 28$ **20 10 30**

8. $82 \overset{10}{-} 68$ **0 20 10**

9. $76 \overset{20}{-} 55$ **20 0 40**

For Ex. 10–17, write only the estimated answers.

10. $71 \overset{20}{-} 52$

11. $69 \overset{10}{-} 57$

12. $85 \overset{60}{-} 28$

13. $64 \overset{20}{-} 41$

14. $48 \overset{30}{-} 23$

15. $78 \overset{40}{-} 36$

16. $92 \overset{30}{-} 59$

17. $83 \overset{50}{-} 32$

Beside each of your estimated answers for Ex. 6–17, work the example to see if your estimate was a good one.

Do not work Ex. 18–20. Show your estimates. Then write "Yes" or "No" for your answers.

18. Eighteen of Dick's 32 pigeons are gray. Are 26 pigeons some other color? 10 No

19. After driving 45 miles of the 78 miles on his trip, does Mr. Fry still have 50 miles more to drive? 30 No

20. Eighteen of the 35 children in our class were at home. Were 30 children in school? 20 No

Teaching Page 160

Pupil's Objectives

(a) To learn how to estimate the answers when subtracting with two 2-place numerals; and (b) to have practice in rounding to the nearest number of tens to estimate the answers in subtraction.

New Words

driving, eighteen, pigeons

Background

Rounding numbers to the nearest number of tens is used, also, for estimating the answer when subtracting with two 2-place numerals. In the example 57 − 32, 57 becomes 60 (6 tens) and 32 becomes 30 (3 tens), and the estimated answer is 30 (3 tens).

Pre-Book Lesson

• Discuss the rounding of numbers to the nearest number of tens and how rounding has been used in estimating a sum. Tell the class that it is often useful to use approximate answers in subtraction, too. See if they can tell some times when estimated answers would be adequate. If they have trouble, suggest situations such as:

a. A mileage chart shows that it is 62 miles from Central City to South Park and 24 miles from Central City to Richland. About how many miles is it from South Park to Richland?

b. Fred has 92 papers to deliver on his paper route. If he has delivered 48 papers, about how many papers does he have left?

• Estimate the answers to the problems that have been stated by writing on the chalkboard the two forms below:

a.	9 tens	b.	90
	−5 tens		−50
	4 tens, or 40		40

It may be necessary to refer to the number-line picture for help in finding the nearest number of tens.

Using the Text Page

• Have Ex. 2–5 done on the chalkboard using two forms like those that were written on the board in the Pre-Book Lesson. Following Ex. 5, ask pupils to formulate their own rule for estimating in subtraction. *Any* statement, offered by the children, even if it is lengthy, is acceptable provided that it covers the procedure adequately. The important thing is that the children understand what they are doing. Be sure to stress the fact that when answers must be accurate, they must be obtained by actually doing the computational work.

• Direct pupils to write their answers for Ex. 6–17 on paper in such a way that there will be room to work the complete example beside the estimated answer. Stress the importance of estimating to find an estimated answer. (Some children will be tempted to find the actual answer and then show the nearest number of tens.)

• You may want to read problems 18–20 aloud. Some children may have difficulty thinking the estimate. Permit them to write their estimates if they need to. However, encourage them to try to think of a number of tens and to subtract the numbers of tens only.

Individualizing Instruction

• *Slower learners* may need help of the following type:

a. As these children work Ex. 6–17, notice their difficulties. If errors occur in rounding to the nearest number of tens, give oral practice with the number-line picture or with representative materials. Give sufficient help and practice so that the process is truly understood.

b. As soon as there is facility in rounding to a number of tens, give practice at the board on Ex. 6–17 and then on paper. Both forms suggested in the Pre-Book Lesson should be written. Should errors occur in subtraction, reteaching and practice in subtraction facts may be necessary.

• *More capable children* may try to decide, in advance of finding the actual answer, whether the estimated answer will be greater than, less than, or equal to the exact answer.

Teaching Page 161

Pupil's Objectives

(a) To have written practice in estimating sums and unknown addends; (b) to have written practice in renaming a number and comparing it with another number; and (c) to have practice in finding the number for n in number sentences.

Background

This page reviews many different mathematical skills and concepts that have been developed to date.

The *rounding* of numbers to the nearest number of tens in estimating answers is the first part of the written work at the top of the page. Then, children are to find the actual answers. These examples involve all of the skills that have been taught in adding and subtracting with two 2-place numerals.

The material dealing with inequalities in the middle of the page also emphasizes renaming a number, using parentheses. A number involving tens and ones has been named by using A. or S. The children are to determine whether that number is greater than or less than another number.

At the bottom of the page are some number sentences with n. n stands for either a sum or an addend in the sentence and children must use their understanding of the addends-sum relationship to determine the number for which n stands.

Pre-Book Lesson

Each of the following activities may be used.

a. Review the rounding of a number involving tens and ones to the nearest number of tens. Discuss how rounding may be used for estimating the answers to addition and subtraction examples.

b. Review renaming numbers and the ways in which numbers may be renamed. Review the concepts *greater than* and *less than* and the symbols $>$ and $<$ to be used. Use the number-line picture to help build more complete understanding of these ideas.

c. Review the writing of number sentences with n. Have children read sentences with n in different positions in the number sentence. As the different forms of the addition and subtraction number sentences are being read and discussed, review the addends-sum relationship.

Using the Text Page

- Be sure children understand the directions for both parts of the written work at the top of the page.
- *Slower learners* may need to show each of the rounded numbers as well as the estimated answer for each of Ex. 1–16.
- Some children may need to write the examples in vertical form to find the renamed number for Ex. 1–10 in the middle of the page.
- Have children show how they found the number which n stands for in each of Ex. 1–12 at the bottom of the page by writing their work in either the horizontal or the vertical form, as they prefer.

Individualizing Instruction

- *Slower learners* may be engaged in an oral follow-up lesson in which they tell why they added or subtracted to find the number for n in Ex. 1–12 at the bottom of the page.
- *All pupils* may rewrite the sentences at the bottom of the page so that n will appear at each of three positions in the number sentence. For Ex. 1, the child would write, "$62 + n = 80$; $n + 18 = 80$; $62 + 18 = n$."
- *More capable children* may do the following activities:

a. Write the sentences in Ex. 1–10 at the middle of the page so that each sentence can be written with the equals sign (=). Ex. 1 would be written, "$(80 - 63) = 17$."

b. An alternate activity for Ex. 1–12 at the bottom of the page is to have the children estimate the number for n in each sentence.

- *All pupils* may write problems for some of the number sentences at the bottom of the page.

Practice in Estimating Answers

A. and S. [W]

Do not work these examples now. Instead, show only your estimates of the answers. Use folded paper. Estimates are shown above the sign of operation.

1. $73 \overset{50}{-} 21$ 52
2. $48 \overset{80}{+} 32$ 80
3. $82 \overset{30}{-} 54$ 28
4. $39 \overset{70}{+} 27$ 66
5. $94 \overset{50}{-} 42$ 52
6. $65 \overset{30}{-} 36$ 29
7. $43 \overset{90}{+} 47$ 90
8. $87 \overset{40}{-} 51$ 36
9. $27 \overset{70}{+} 38$ 65
10. $56 \overset{30}{-} 28$ 28
11. $75 \overset{40}{-} 35$ 40
12. $34 \overset{70}{+} 37$ 71
13. $94 \overset{60}{-} 31$ 63
14. $63 \overset{10}{-} 49$ 14
15. $59 \overset{20}{-} 36$ 23
16. $81 \overset{50}{-} 29$ 52

Now copy each of Ex. 1–16 and find the true answer.

> (Is Greater Than) or < (Is Less Than)?

Inequalities [W]

First do the work in ( )'s. For Ex. 1 write,

* $80 - 63 = 17$, so $(80 - 63) \overset{<}{?} 41$.

1. $(80 - 63) \overset{<}{?} 41$
2. $(23 + 42) \overset{<}{?} 70$
3. $(73 - 25) \overset{<}{?} 50$
4. $(87 - 49) \overset{>}{?} 48$
5. $(36 + 52) \overset{>}{?} 78$
6. $(18 + 58) \overset{<}{?} 70$
7. $(52 - 37) \overset{<}{?} 25$
8. $(38 + 48) \overset{>}{?} 87$
9. $(93 - 29) \overset{>}{?} 60$
10. $(26 + 28) \overset{<}{?} 44$

Working with *n*

Addends-sum relationship [W]

Write each example with a numeral in place of *n*.

1. $62 + \overset{18}{n} = 80$
2. $75 - \overset{36}{n} = 39$
3. $\overset{51}{n} - 15 = 36$
4. $65 = \overset{29}{n} + 36$
5. $\overset{72}{n} - 35 = 37$
6. $\overset{44}{n} + 41 = 85$
7. $27 + 29 = \overset{56}{n}$
8. $51 - \overset{18}{n} = 33$
9. $\overset{75}{n} - 37 = 38$
10. $\overset{63}{n} = 37 + 26$
11. $70 - \overset{27}{n} = 43$
12. $\overset{60}{n} - 32 = 28$

* To be certain pupils get computation practice, have them actually show the work each time as shown for Ex. 1.

n-Examples for Problems

A. and S. [W]

Choose and copy the correct *n*-example, or number sentence, for each problem.

1. Anne must play her piano an hour a day. After 42 minutes how many minutes does she still have to play?

$42 + 60 = n$ $\quad n = 60 + 42$

$60 - 42 = n$ 18 minutes

2. Sunday the wind blew 25 miles an hour. Monday it blew 42 miles an hour, or _?_ miles faster.

$n = 42 + 25$ $\quad 25 + 42 = n$ $\quad n = 42 - 25$

17 miles

3. Joe spent 38¢. He had 55¢ left. How much did Joe have at first?

$38¢ + 55¢ = n¢$ $\quad n¢ = 55¢ - 38¢$ $\quad n¢ = 38¢ - 55¢$

93¢

4. Jane has learned all but 11 of the 30 spelling words. How many spelling words has she learned?

$n = 11 + 30$ $\quad 30 + 11 = n$ $\quad 30 - 11 = n$

19 words

5. In 15 years, Ruth will be 24 years old. How old is she now?

$24 - 15 = n$ $\quad n = 15 + 24$ $\quad n = 24 + 15$

9 years

6. Farmer Brown has 11 brown cows and 22 black cows. How many cows does he have all together?

$n = 22 - 11$ $\quad 11 + 22 = n$ $\quad n = 11 - 22$

33 cows

7. Jim Brown has milked 20 of his father's 33 cows. How many cows does he still have to milk?

$33 + 20 = n$ $\quad 33 - 20 = n$ $\quad n = 20 + 33$

13 cows

◆ **Extra Problems.** Work Set 7.

Teaching Pages 162 and 163

Pupil's Objectives

(a) To have practice in selecting appropriate *n*-examples for problems; (b) to have practice in writing appropriate A. and S. questions for problem information that is given; (c) to estimate the answers to the resulting problems; (d) to write *n*-examples for the written problems; and (e) to have practice in column addition with four addends.

New Words

(page 162) *spelling*
(page 163) *jelly*

Background

Some of the skills involved in solving problems are emphasized on these two pages. The primary purpose of page 162 is to determine the extent to which children can read a problem and interpret correctly the information that is given. The ability to translate problem information into a number sentence is an important facet of solving problems. In the exercises on page 162 the children are merely to select the best sentence given as a model for the problem.

The first activity on page 163 is for the children to write A. or S. questions for the information that is given. In each problem they are told what kind of problem to write. Frequently, children think the correct operation, but they have difficulty stating the question. In this activity, the stress is solely upon writing the questions properly.

For additional practice in using rounded numbers for estimating, it is suggested that the children make an estimate of the answer for each problem before they actually solve the problem. Finally, it is recommended that *n*-examples be written for each of the problems. In this activity the children are to write number sentences of their own creation rather than select one created by someone else.

Pre-Book Lesson

• Use blocks, discs, or some other objects to demonstrate a *how-many-gone* subtraction situation similar to the following:

The bread man placed 35 loaves of bread on the shelf at the grocery store. The next day when he returned there were 12 loaves of bread still on the shelf. How many loaves of bread had been sold?

After discussing the problem and the way of demonstrating it, ask a child to write on the chalkboard an *n*-example for the problem with *n* standing for the answer. Ask children to tell why the number sentence is or is not a correct one. Ask other children to write other *n*-examples that are also correct.

• Ask children to state generalizations in their own words about when to add and when to subtract in terms of what they know about the numbers for a total and its parts.

Using the Text Pages

• Before they do the written work on page 162, point out to the class that there are three number sentences given below each problem. The pupils are to copy the one that is best for the problem but are not to solve the problems now. On page 165, there is a direction for pupils to solve these problems.

• There are four exercises in connection with the problem work at the top of page 163.

a. Have pupils write good questions for the information that is given. *Slower learners* may need help with reading each problem.

b. Ask the class to show estimates of the answers for the problems.

c. Have *all pupils* work their problems to see if each estimate was a good one.

d. Ask the children to write an *n*-example for each problem. Suggest that *n* be written in place of the answer.

• Some of the *more capable children* may be able to add mentally in each example at the bottom of page 163 without copying it into column form. They should be permitted to write answers only in this case.

Individualizing Instruction

• Duplicate a blank addition-subtraction chart (see Teaching Page 33) for use with *all pupils*. Show down the left side and across the top addends 0 through 9.

Direct the children in using the chart with questions like the following:

a. If one addend is 4 and the other addend is 2, what is the sum? Where will we show that sum in the chart? (There will be two squares in the chart in which to show the sum of the addend 4 and the addend 2.)

b. The sum I am thinking of is 8. One of two addends is 5. Where is the sum shown in the chart? (There are two squares.)

c. 0 and 7 are two addends. What is the sum? Where is it shown in the chart? (There are two squares.)

d. One of two addends is 6. The sum is 12. What is the other addend? Where is the sum shown in the chart?

• Have the children fill in the remainder of the chart. After the chart has been completely filled in by *all pupils*, ask questions like the following to review certain generalizations:

a. When 0 is one of two addends, what can we say about the sum? (It equals the other addend.)

b. When 1 is one of two addends, what can we say about the sum? (It is one greater than the other addend.)

c. When one of two addends is an odd number and the other addend is an even number, what kind of number is the sum? (Odd)

d. When two addends are even numbers, is the sum an odd number or an even number? (Even)

e. When two addends are odd numbers, is the sum an odd number or an even number? (Even)

f. When the sum is an even number and one of two addends is an odd number, will the other addend be an odd number or an even number? (Odd)

g. When the sum is an odd number and one of two addends is an odd number, is the other addend an odd number or an even number? (Even)

• For *all pupils* who are still having trouble with some of the A. and S. facts, a review of the following *helpers* may be profitable:

a. Use known facts that have an addend 1 greater or 1 less than an addend of the unknown fact. For example, for $8 + 5$, the child can relate it to either $7 + 5 = 12$ or $8 + 4 = 12$ (each has an addend 1 less than an addend of $8 + 5$). Facts with an addend 1 greater than an addend of $8 + 5$ are $9 + 5 = 14$ and $8 + 6 = 14$. Because one addend is 1 less than an addend in the known fact the sum will be 1 less; and, similarly, if an addend is 1 greater, the sum will be 1 greater. Children should be encouraged to relate something unknown (or about which there is doubt) to something they know.

b. Use facts with equal addends to help with facts having sums one greater or one less. Facts like $6 + 6 = 12$, $7 + 7 = 14$, $8 + 8 = 16$ often are more readily mastered than others. A fact like $7 + 6 = 13$ can be related to either $6 + 6 = 12$ or to $7 + 7 = 14$.

c. Making a 10 is another valuable helper for many children. Some children are able to learn $8 + 5$, the child can think, "$8 + 2 = 10$, plus hesitation. It would probably be a mistake to force them to change their thought pattern. However, some children will find *thinking through 10* easier than any other thought pattern. For the example, $8 + 5$, the child can think, "$8 + 2 = 10$, plus $3 = 13$." Many *slower learners* are able to think through 10 in finding the sum, but they have difficulty writing the number sentence for the same operation.

d. Knowing one fact in a set of A. and S. facts may be used to help with other facts in that set: $5 + 7 = 12$, so $12 - 7 = ?$

• *More capable children* may write number sentences in which the Associative Property is used to assist in adding with two 2-place numerals:

$$\begin{aligned} 47 + 16 &= n \\ 47 + (10 + 6) &= n \\ (47 + 10) + 6 &= n \\ 57 \quad + 6 &= n \\ 63 &= n \end{aligned}$$

Children can learn this way of adding with two 2-place numerals very easily. It is sometimes difficult for them to see the relationship between what they are writing and what they are thinking.

• Assign Extra Problems Set 7 to *all pupils*.

Making Problems

Writing questions; making n-examples [W]

Write a question to make an addition problem where you see *A.* and a subtraction problem where you see *S.*

1. Last week Mike bought 35 pounds of dog food. He has fed 8 pounds of it to his dog. *S.* 30 27 pounds 35−8=n

2. Ed has read 46 pages in his book. He has 44 more pages to read. *A.* 90 90 pages 46+44=n

3. Sam has 43 marbles. Ted has 68 marbles. *S.*
30 25 marbles 68−43=n

4. Mr. Brown has fed 30 of his 75 hens. *S.*
50 45 hens 75−30=n

5. Mrs. Brown has given away 18 of the glasses of jelly she made. She has 47 glasses of jelly left. *A.*
70 65 glasses 47+18=n

6. Our school bought 86 bottles of milk. There are 9 bottles left over. *S.* 80 77 bottles 86−9=n

Show estimates of the answers for problems 1–6.

Next, work all six problems.

Then write an *n*-example for each problem.

To Keep in Practice

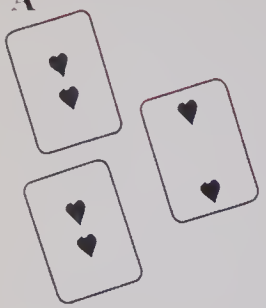
Column A. [W]

Copy each set of numerals in a column. Add and check.

	a	b	c
1.	40¢, 9¢, 18¢ 67¢	52, 6, 6, 28 92	\$0.60, \$0.06, \$0.07, \$0.15 \$0.88 ∧
2.	6, 8, 50, 15 79	9, 15, 8, 38 70	\$0.03, \$0.39, \$0.29, \$0.18 \$0.89 ∧
3.	38, 17, 6, 7 68	7, 6, 60, 19 92	\$0.24, \$0.27, \$0.04, \$0.28 \$0.83 ∧
4.	9¢, 27¢, 19¢ 55¢	48¢, 9¢, 29¢ 86¢	\$0.15, \$0.59, \$0.06, \$0.15 \$0.95 ∧

* All sets which have the same number of members are equivalent sets. The numbers of these sets are equal.

A



Another Way to Find How Many in All

Readiness for multiplication [O]

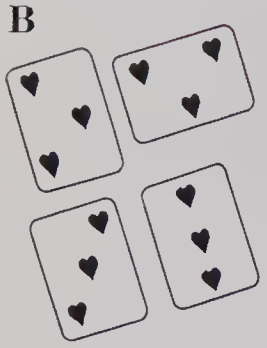
1. Jean pasted a set of 2 hearts on each of 3 cards. Use picture A to find how many hearts she used.

a. Count by 2's. How many hearts in all? 6

b. Add 2's. $2 + 2 + 2 = ?$ 6

c. Now you know that three 2's = ? 6

B



2. Kay put a set of 3 hearts on each of 4 cards. Use picture B to find how many hearts she used in all.

* a. Because there were 3 hearts in each set, we say these sets of hearts are equivalent. How many equivalent sets did Kay use? 4

b. To find how many hearts in all, you can add:

$$3 + 3 + 3 + 3 = ?$$

c. Now you know that four 3's = ? 12

On the board write the work for each of Ex. 3–5. Find how many hearts you will need in all. First show an addition example with its sum. Then write the short way as in Ex. 1c and 2c.

3. You put a set of 3 hearts on each of 3 cards. 3+3+3=9
three 3's = 9

4. You put a set of 4 hearts on each of 3 cards. 4+4+4=12
three 4's = 12

5. You put a set of 2 hearts on each of 5 cards. 2+2+2+2+2=10
five 2's = 10

6. For Ex. 1–5, were the sets in each example equivalent? Yes Each time, did you find how many in all? Yes

Which of Ex. 7–9 can you write the short way as in Ex. 1c and 2c? Ex. 8 and 9 Tell why. The sets are equivalent.

** 7. $3 + 2 + 7$ 8. $5 + 5 + 5$ 9. $6 + 6 + 6$

** When the numbers for subsets are not equal, we add to find the number for the total set; when they are equal, we usually multiply.

Teaching Pages 164 and 165

Pupil's Objectives

(a) To recognize equivalent sets of objects; (b) to begin to notice the number of members in each set and the number of sets; and (c) to learn a new way of reading and stating a number sentence when the sets are equivalent.

New Word

(page 164) *equivalent*

Background

Earlier you were introduced to addition as a binary operation; that is, that you operate on two numbers and get a unique third number called the *sum*. In these lessons, your pupils will receive readiness for a new operation. Multiplication is also a binary operation; that is, an operation on two numbers (called *factors*) which yields a unique third number (called the *product*). When we think of the numbers 5 and 2 and get 7, we are adding. When we think of the numbers 5 and 2 and get 10, we are multiplying.

Since numbers are a property of sets of objects, it is natural for us to go back to the idea of sets in order to prepare youngsters for this new operation. The activities on these two pages require attention to a number of equivalent sets—three equivalent sets with 2 in each set, four equivalent sets with 3 in each set, and so on. Equivalent sets are pictured on page 165 as patterns of dots. They may be demonstrated in your room with children, chairs, blocks, discs, and so on. Sets that are equivalent have the same number of members. Thus, a set of 3 children is equivalent to a set of 3 chairs. Concentrate on the use of equivalent sets and discourage counting by 1's at this time.

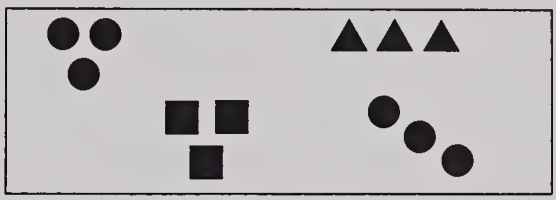
The exercises on these two pages call for the identification of a given number of equivalent sets and eventually the thinking of each such combination in terms of multiplication. The written work on page 165 should show whether your pupils have grasped the relationship between addition and multiplication. Notice that the terms "multiply" and "multiplication" are deliberately deferred, as is the organization of M. facts into tables, until they can be fully comprehended.

Teacher's Preparation

Consider objects in the classroom that are arranged in groups of 2, 3, 4, or 5. These may include chairs, collections, bulletin-board arrangements, objects from the arithmetic corner, blocks, tables, and children themselves, and the like.

Pre-Book Lesson

• On a flannel board show four sets, each with 3 things.



Ask questions of the following type:

a. What is the size of each set (group)? Stress that all of these sets have the same number of members so the sets are equivalent.

b. How many sets of 3 are there? (Or, How many 3's are there?)

c. How many circles (objects, squares, etc.) are shown in all?

• Have children tell the different ways they arrived at 12. Write their responses on the board. You may get the following types of answers:

a. Counting by 1's

b. Counting by 3's

c. Adding 3's $3 + 3 + 3 + 3 = 12$

d. Two 3's at the top is 6; two 3's at the bottom is 6. $6 + 6 = 12$

As pupils give their responses, relate each one to previous learning. When adding 3's is mentioned, discuss the four 3's that are written on the board. Tell the children that "Four 3's" is a special way of reading and writing this kind of number sentence. This special way can be used only when all the sets are the same size, or equivalent.

• Use children or objects in the room to show sets of 2, 3, 4, or 5. Have children tell the number for each set (the number of members in each set), the number of sets, and how many members in all.

They may use counting (by 2's, 3's, etc.) or they may add (2 + 2 + 2, etc.). If counting by 1's is suggested, show advantages of other methods.

• Encourage pupils to summarize in their own words what they have been doing (i.e., finding the number for the total of several sets that are equivalent).

Using the Text Pages

• When appropriate, it may be helpful to use a flannel board* and cut-outs** or objects in the room to demonstrate the ideas in Ex. 1–12.

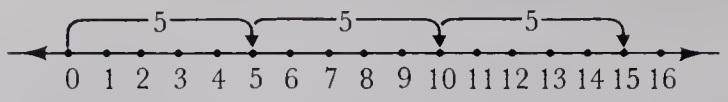
• *More capable children* should be able to proceed independently with the written work. You may want to go through each row in the chart orally with the *slower learners* before assigning the work as written work. In Ex. 13–21 have *slower learners* read each sentence before writing it as directed.

• Since the problems on page 162 have not been worked, have pupils turn to that page now and solve problems 1–7.

Individualizing Instruction

• *All pupils* may make a chart similar to the one on page 165 in which each row calls for three different ways of expressing the same idea. Column A should show geometric designs (circles, triangles, squares) picturing equivalent sets. Column B should express the idea as an addition sentence, and column C should show the new way children are learning to state the idea as a multiplication number sentence. The side-by-side arrangement in each row stresses the meaning.

• Duplicate a page of number-line pictures on which pupils may illustrate the examples in the chart on page 165. Show pupils how Ex. 12a can be worked by using a number-line picture.



• Duplicate or put on the chalkboard for *all pupils* an exercise such as the following:

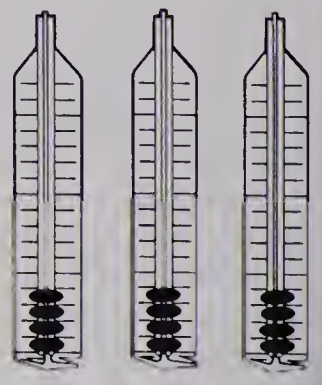
Draw a ring around each example that indicates equivalent sets.

4 + 4 + 4	2 + 5 + 4	5 + 5 + 1
3, 6, 9, 12	2, 5, 7, 9	4, 8
five and two and three		three 4's

During discussion of correct answers for the work above, have children tell the number for each set and how many sets there are.

• *Slower learners* may need practice in counting by 2's, 3's, 4's, 5's. You may wish to have them make a chart like the one suggested on Teaching Page 5. Other suggestions for counting by 2's, 3's, and so on found in that same lesson may be adapted for developing readiness for multiplication.

• Two children may work together using Ginn Arithme-Sticks* as follows:



One child shows a certain number of equivalent sets on the Arithme-Stick. The second child gives the number sentence that has been illustrated and then tells the number for the total.

* See 2, page xix.
** See 17, page xix.

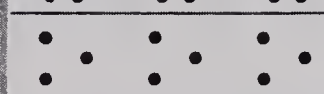
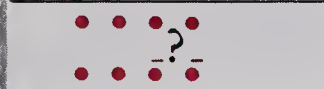
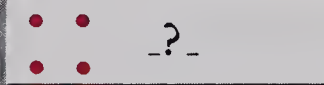
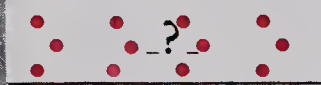
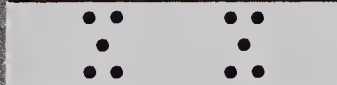
* See 3, page xix.

10. In row a, Ex. 12, each box shows three 5's. Explain.

11. In row b, tell what is missing.

[W]

12. Copy and finish rows c–l.

a.		$5 + 5 + 5$	three 5's = 15
b.		$3 + 3 + 3$	three 3's = 9
c.		$2 + 2 + 2 + 2$	four 2's = 8
d.		$2 + 2$	two 2's = 4
e.		$4 + 4 + 4$	three 4's = 12
f.		$3 + 3$	two 3's = 6
g.		$3 + 3 + 3 + 3$	four 3's = 12
h.		$5 + 5$	two 5's = 10
i.		$2 + 2 + 2$	three 2's = 6
j.		$6 + 6$	two 6's = 12
k.		$8 + 8 + 8$	three 8's = 24
l.		$4 + 4$	two 4's = 8

Write each of Ex. 13–21 as in the last column of the chart. For help, add or make dot pictures.

13. $4 + 4$ (two 4's = 8)

14. $6 + 6$ (two 6's = 12)

15. $5 + 5$ (two 5's = 10)

16. $2 + 2 + 2$ (three 2's = 6)

17. $4 + 4 + 4$ (three 4's = 12)

18. $3 + 3 + 3$ (three 3's = 9)

19. $1 + 1 + 1 + 1$ (four 1's = 4)

20. $3 + 3 + 3 + 3$ (four 3's = 12)

21. $2 + 2 + 2 + 2$ (four 2's = 8)

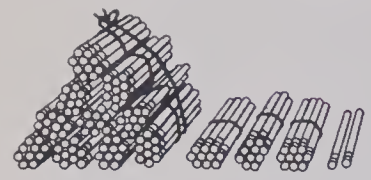
Turn to page 162. Write the work for problems 1–7.

What 3-Place Numerals Show

Expanded notation [O]

1. Joan said, "It is easy to count pencils in bundles of 10." Tell how Joan counted: 10, 20, 30, 40 ?, 50 ?, 60 ?, 70 ?, 80 ?, 90 ?, 100 ?, 110, 120, 130, 131, ?.

2. Jack said, "It is even easier to count them if you put 10 ten-bundles together in a hundred-bundle." How many hundred-bundles are there at the left? 1 How many ten-bundles? 3 How many ones? 2



3. 1 hundred and 3 tens and 2 ones are 100 + 30 + 2 in all. Study the work in box A.

A

Hundreds	Tens	Ones
1	0	0
	3	0
+		2
1	3	2

* In 132, we think "2 ones" for the 2 in one's place, "3 tens" for the 3 in ten's place, and "1 hundred" for the 1 in hundred's place.

The numeral 132 has three places. It is read, "One hundred thirty-two."

Read Ex. 4–8. Ex. 4 is read, "Two hundred nine." Because 0 is in ten's place, we do not say any tens.

Three hundred fifty-one Three hundred four

4. 209 5. 351 ∧ 6. 470 ∨ 7. 304 ∧ 8. 564 ∨

Four hundred seventy Five hundred sixty-four

Three-place numerals show numbers of hundreds, tens, and ones.

* Developmentally, the pupils have already had experience through hundreds. Now the idea of place value becomes the focus of attention.

Teaching Pages 166 and 167

Pupil's Objectives

(a) To learn what 3-place numerals show; and (b) to rename numbers shown with 3-place numerals by using expanded notation.

New Word

(page 166) *easy*

Background

Pupils have had practice in reading 3-place numerals. Fuller understanding of 10 as the *base* and the *positional nature* of our numeration system is essential to building computational skills with hundreds, tens, and ones.

Up to this time, children have learned to express numbers in terms of tens and ones. They have learned that 10 ones are equal to 1 ten. However, they have not generalized about the use of 10 as the *base* of a system for denoting number ideas.

In this lesson, the renaming of 10 tens as 1 hundred is an extension of the idea of 10 as the base of our numeration system. In order to foster as complete and flexible an understanding of hundreds tens, and ones as is possible, *expanded notation* must be explored in many different forms. Consider the numeral 584. At this time children should learn to think of it as

5 hundreds 8 tens 4 ones, or
58 tens 4 ones, or
5 hundreds 84 ones, or
584 ones.

As your pupils progress through school, they will learn to write 584 in more abstract expanded-notation forms such as:

$$(5 \times 100) + (8 \times 10) + (4 \times 1)$$
$$(5 \times 10 \times 10) + (8 \times 10) + (4 \times 1)$$
$$(5 \times 10^2) + (8 \times 10^1) + (4 \times 1)$$

Teacher's Preparation

Have available Arithme-Sticks and materials like splints, tongue depressors, or milk straws that can be bundled together to show 10 tens.

Pre-Book Lesson

• To set the stage for this lesson, write the numerals 425, 524, and 254 on the chalkboard. Ask questions such as these:

a. "In what ways are these numerals alike?"

b. "In what ways are they different?"

c. "Is the 4 in 425 read in the same way as the 4 in 254?"

Bring out in the discussion that the *place* in which the digit is written partly determines its value; that 4 always means 4, but 4 in one's place tells us that there are 4 ones, 4 in ten's place tells us that there are 4 tens, and so on.

• Help children to understand 100 by assembling 10 bundles of ten splints into 1 hundred-bundle. Show that 10 tens = 1 hundred; 20 tens = 2 hundreds, and so on.

• Use a pocket chart with bundles of splints or tickets. Have the children illustrate 3-place numerals. Have them identify 3-place numerals that others illustrate.

Bring out the following understandings in using the pocket chart:

a. The chart illustrates the numeral 286.

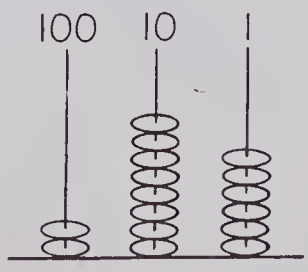
b. It shows 2 hundreds and 8 tens and 6 ones.

c. 286 is the *sum* of 2 hundreds (200) + 8 tens (80) + 6 ones (6).

Hundreds	Tens	Ones
2	0	0
	8	0
+		6
2	8	6, or 286

• You may use Arithme-Sticks or an abacus picture as a more abstract means of demonstrating 3-place numerals. Each child should have his own abacus picture on which he illustrates numerals as directed.

286



Other 3-place numerals suggested for demonstration are 419, 342, 199, 211.

After demonstrating a numeral like 199, ask

If we add 1 to 199 what will the sum be?

Then, have the children show 1 more bead on the abacus and go through the experience of exchanging 10 ones for 1 ten. There will then be 10 tens, which must be exchanged for 1 hundred.

• Using the procedures suggested above, have pupils illustrate 3-place numerals containing 0, such as

200 [no tens and no ones]
401 [no tens]
570 [no ones]

Record some of these types in each of the following ways:

a. 401

b. 4 hundreds and 0 tens and 1 one

c. four hundred ones.

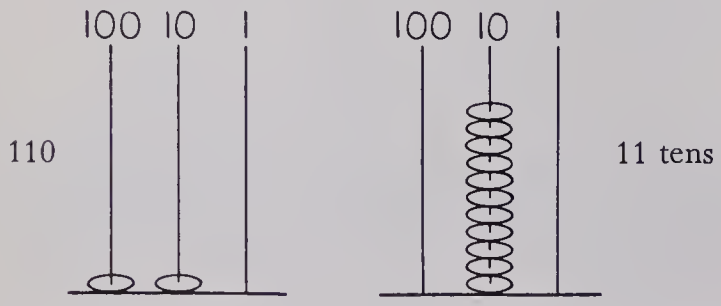
d.

Hundreds	Tens	Ones	
4	0	0	
	0	0	
+		1	
4	0	1	1, or 401

e. 400 + 1

• Use bundles of tens to show children that 110 can be renamed 11 tens. This can also be demonstrated very effectively on the Arithme-Sticks or an abacus picture because it requires the changing of 1 hundred to 10 tens.

a. b.



Show and rename other numbers:

210 may be renamed 21 tens.

510 may be renamed 51 tens.

120 may be renamed 12 tens.

220 may be renamed 22 tens.

400 may be renamed 40 tens.

124 may be renamed 12 tens and 4 ones (120 + 4).

526 may be renamed 52 tens and 6 ones (520 + 6).

• Summarize by illustrating, thinking, reading, and writing 286 in two different ways:

a. 2 hundreds and 8 tens and 6 ones

200 + 80 + 6

b. 28 tens and 6 ones

280 + 6

Using the Text Pages

• Discussion of Ex. 1–12 will show you which pupils are ready to proceed with the written work.

• While they are doing the written work permit children to use a pocket chart or Arithme-Sticks or an abacus picture to help them "see" the number, if they wish.

Individualizing Instruction

• *All pupils* may draw abacus pictures on which they will make rings on the appropriate vertical marks to illustrate 3-place numerals. Below each abacus have the pupil write the 3-place numeral he has illustrated.

• Write on the chalkboard or duplicate on paper some exercises for *all pupils* like the following:

645 = _ ? _ hundreds and _ ? _ tens and _ ? _ ones
408 = _ ? _ tens and _ ? _ ones
530 = _ ? _ tens and _ ? _ ones
_ ? _ = 7 hundreds and no tens and 4 ones
_ ? _ = 42 tens
_ ? _ = 48 tens and 4 ones
_ ? _ = 11 tens and 1 one

• *Slower learners* may need to do more oral and visual work. Use the teaching aid that appears to be most useful to foster further understanding of what 3-place numerals show. Do more oral work of the type developed in Ex. 11 and 12 in the text.

• *More capable children* may use Extra Activities Set 95.

* This expanded form should be used often if pupils have difficulty.

Read each numeral in rows 9 and 10, and tell how many hundreds, tens, and ones it shows.

	a	b	c	d	e	f	g
9.	720	756	523	304	126	843	587
10.	266	364	340	483	806	679	807

B

Hundreds	Tens	Ones
5	2	8

C

Tens	Ones
5	2 8

We can rename in more than one way the number shown by a 3-place numeral. 528 =

a. 5 hundreds and 2 tens and 8 ones,
* or 500 + 20 + 8. See Ex. B.

b. 52 tens and 8 ones, or 520 + 8. See Ex. C.

Say the missing words or numerals in Ex. 11 and 12.

11. 624 = **a.** 6 hundreds and 2 tens and 4 ones,
or 600 + 20 + 4.

b. 62 tens and 4 ones, or 620 + 4.

12. 506 = **a.** 5 hundreds and 6 ones, or 500 + 6.

b. 50 tens and 6 ones, or 500 + 6.

[W]

For each of Ex. 13–24, write two names as in Ex. 11 or Ex. 12.

13. 970	**14.** 934	**15.** 730	**16.** 824	**17.** 247	**18.** 509
19. 608	**20.** 510	**21.** 462	**22.** 400	**23.** 200	**24.** 236

Write 3-place numerals for these:

25. 43 tens and 8 ones — 438

26. 8 hundreds and 4 ones — 804

27. 1 hundred and 18 ones — 118

28. 6 hundreds and 3 ones — 603

29. 70 tens and 0 ones — 700

30. 5 hundreds and 38 ones — 538

31. 33 tens — 330

32. 50 tens — 500

33. 45 tens — 450

●**Extra Activity.** Work Set 95.

* Note the reference to earlier work with ones. This ties in with knowledge of basic addition facts.

Adding and Subtracting Hundreds

[o]

1. The truck driver left 300 copies of one paper and 200 copies of another. That was how many papers?

a. Count by 100's. 100, 200, 300, 400, 500 hundreds

b. Finish: 3 hundreds + 2 hundreds = 5 hundreds, or 500.

c. You can add 300 + 200 these two ways:

$$300 + 200 = 500 \qquad \begin{array}{r} 300 \\ +\ 200 \\ \hline 500 \end{array}$$

Say the sums for Ex. 2–7.

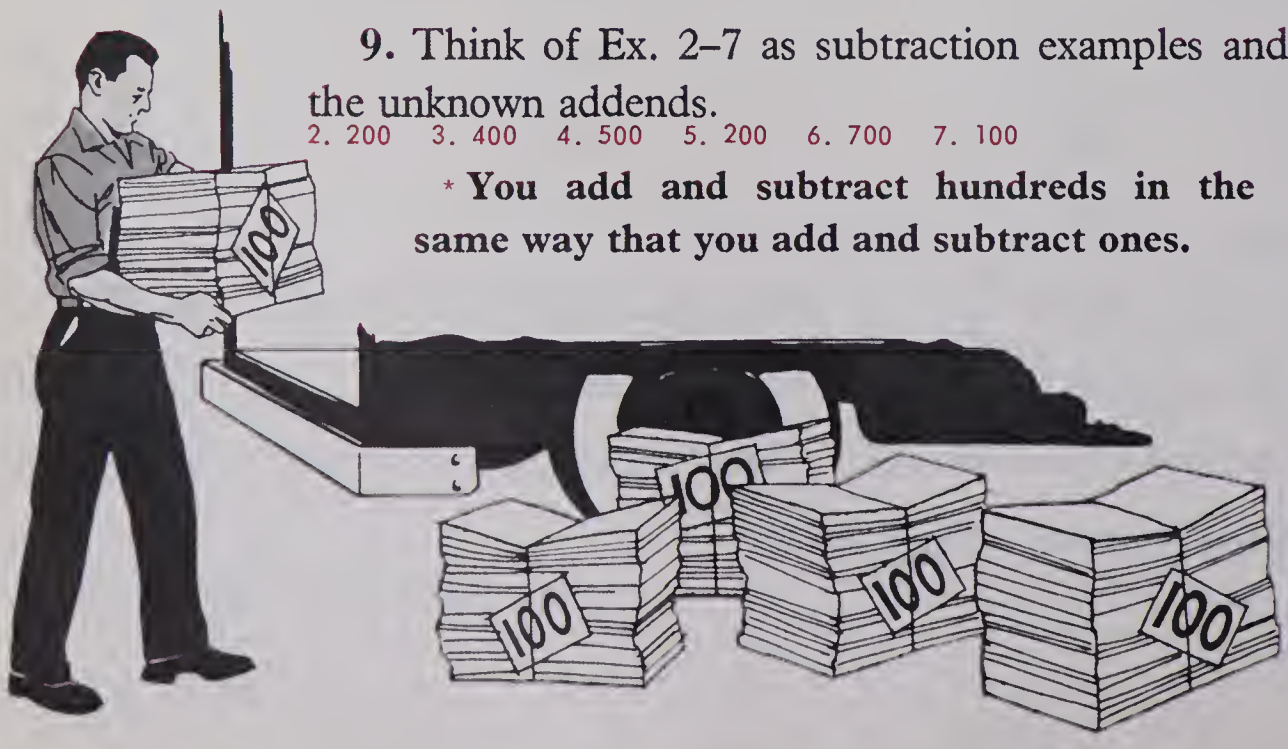
2.	3.	4.	5.	6.	7.
300	600	700	500	800	400
+100	+200	+200	+300	+100	+300
400	800	900	800	900	700

8. If the paper boy sells 400 papers, how many of the 500 papers will be left?

a. Finish: 5 hundreds − 4 hundreds = 1 hundred, or 100.

b. Subtract 500 − 400 these two ways:

$$500 - 400 = 100 \qquad \begin{array}{r} 500 \\ -\ 400 \\ \hline 100 \end{array}$$



9. Think of Ex. 2–7 as subtraction examples and say the unknown addends.

2. 200 3. 400 4. 500 5. 200 6. 700 7. 100

* **You add and subtract hundreds in the same way that you add and subtract ones.**

Teaching Page 168

Pupil's Objective

To learn to add and subtract hundreds.

Background

This lesson is the first of a series of six pages which deal with adding and subtracting with 3-place numerals without and with renaming.

The pupils have been adding and subtracting with 2-place numerals, with and without renaming, and they should know the algorism and the general procedures. Also, they have just completed work with 3-place numerals from the standpoint of positional value.

Page 168 of the pupil's book begins with 3-place numerals which name hundreds. The pupils have already learned to count by 100's. Addition of hundreds is introduced on the pupil's page by making use of this ability to count by 100's. Children should arrive at the generalization that numbers of hundreds are added and subtracted just as are numbers of ones.

Teacher's Preparation

If possible, have available Arithme-Sticks and obtain several sets of objects packaged in sets of 100 each. You may be able to obtain some of the following from the school storeroom or cafeteria: milk straws, tongue depressors, paper clips, construction paper, chalk.

Pre-Book Lesson

• If a problem can be devised so that it involves local data and numbers in the even hundreds, begin with it. Otherwise, devise a problem using the packaged materials that have been obtained for this purpose. A lesson using boxes of paper clips may develop as follows:

a. Show the box of paper clips to the children. Tell them it contains 100 paper clips (or, you may have a child read from the box the numeral telling the number of clips).

b. Place four boxes in a pile. Have a child tell how many clips there are. Have another child write *400* on the chalkboard.

c. Place two boxes of clips in another pile. Repeat the discussion as above.

d. Have a child place the two sets of paper clips with the four sets. Have someone else add the two numbers on the chalkboard.

• You may wish to illustrate other similar examples using the Arithme-Sticks or an abacus picture or the pocket chart. After the first number has been shown, have a child write the numeral on the chalkboard. Then, place "beads" on the Arithme-Sticks or the abacus picture to show another number (such that the sum of the hundreds is less than 10). Have him write the second numeral on the chalkboard and then add.

• Be sure to emphasize that adding the numbers of hundreds is like adding the numbers of ones.

• Use the packaged objects to demonstrate subtraction of hundreds, tens, and ones in much the same manner as that suggested for addition. Stress the different subtraction ideas as follows:

a. Show 8 hundreds and take away 5 hundreds.

b. Show 8 hundreds and 5 hundreds. Find the difference between 8 hundreds and 5 hundreds. Find how many more 8 hundreds are than 5 hundreds. Find how many fewer 5 hundreds are than 8 hundreds. When we work the example for this situation, we find out how much greater 800 is than 500, and how much less 500 is than 800.

c. Show 8 hundreds and with all eyes blindfolded, remove 5 hundreds. Have children determine how many are gone.

Be sure to write each example on the chalkboard as it is being developed.

Using the Text Page

Use the packaged objects to demonstrate Ex. 2–7.

Individualizing Instruction

• *Slower learners* may need more work with visual materials and oral work like that suggested in the Pre-Book Lesson.

• *Slower learners* may need more oral work in fostering greater understanding of what 3-place numerals show in preparation for the work on pages 169–173.

Pupil's Objective

To acquire intelligent skill in adding with two 3-place numerals, without renaming and carrying and with renaming and carrying 1 to ten's place.

Background

No new skills are developed on this page. Your children have already learned to add with two 2-place numerals with and without renaming and carrying and have learned to add hundreds without carrying. Adding with two 3-place numerals is now undertaken and extends some of the skills which were developed previously. Children who have not had difficulty previously should not encounter any difficulties at this time.

Pre-Book Lesson

• On the chalkboard, write "hundreds," "tens," and "ones" as shown below so that different numerals may be written in each blank.

_ ? _ hundreds and _ ? _ tens and _ ? _ ones
\+ _ ? _ hundreds and _ ? _ tens and _ ? _ ones
_ ? _ hundreds and _ ? _ tens and _ ? _ ones

a. First, write 3-place numerals for addition with no renaming involved. Have different children add the numbers shown in each position and explain the result.

b. Next, write 3-place numerals as shown below so that renaming the ones of the sum will result.

257 = 2 hundreds and 5 tens and 7 ones
\+ 318 = 3 hundreds and 1 ten and 8 ones
5 hundreds and 6 tens and 15 ones

At this point, emphasize renaming the 15 ones in the sum to be 1 ten and 5 ones and the carrying of 1 to the ten's column.

c. Examples with 0 in the one's or the ten's place should be presented using the above expanded notation.

• For some examples use partially expanded notation as below:

257 = 200 and 57
\+ 318 = 300 and 18
500 and 75, or 575

Using the Text Page

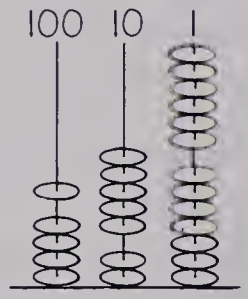
• For Ex. 2 you will probably want to write the example on the chalkboard in order to have discussion. Even though the 1 carried to the ten's column is written above the digits in the ten's column, be sure to stress that the children are not required to write the carried digit. If they can think it, so much the better. Keep in mind, however, that not all children are alike in ability to form mental images and so will need to continue for some time to write the carried digits.

• For Ex. 3 review how to check addition.

Individualizing Instruction

a. For *slower learners*, write on the chalkboard $427 + 156 = n$. Ask a child to read the number sentence. Have the children discuss the example and what it means. What does the sentence ask them to do?

Direct pupils to illustrate 427 on an Arithme-Stick or abacus picture. Be sure they understand that they are to add 156 to the 427. Then, they should illustrate 156 on the abacus picture (see illustration at the right).



Have children determine how many beads are on the one's mark. Someone will know that they should remove 10 beads from the one's mark and place one bead on the ten's mark. Bring out that one bead on the ten's mark stands for (represents) 10 of the beads on the one's mark.

Continue with joining the beads on the ten's mark and on the hundred's mark in the same way. Be sure to record on the chalkboard the complete algorithm for the work illustrated on the Arithme-Sticks or abacus picture.

b. Go through other examples in the same way with children who may need help.

All pupils may work row 5 as follows:

453 = 400 + 50 + 3
\+ 104 = 100 + 0 + 4
500 + 50 + 7 = 557

* This renaming of the sum of the ones and carrying 1 to ten's column is the same skill as pupils had with 2-place numerals.

Adding with Two 3-Place Numerals

Carrying to ten's place [O]

1. One day the paper boy sold 243 copies of one paper and 124 of another, or 367 copies in all. $243 + 124 = n$

a. Tell how to finish the work for Ex. A.

A

2 hundreds and 4 tens and 3 ones
+1 hundred and 2 tens and 4 ones

3 hundreds and 6 tens and 7 ones, or 367

B

Hundreds	Tens	Ones
2	4	3
1	2	4
3	6	7

b. Explain the short way to add 243 and 124 (Ex. B).
Add downward: ones, then tens, then hundreds.
3 + 4 = 7 Write 7 in one's place. 4 + 2 = 6 Write 6 in ten's place.
Tell why 7, 6, and 3 for the sum are placed as they are.
2 + 1 = 3 Write 3 in hundred's place.

2. On still another day the paper boy sold 268 copies of one paper and 129 copies of the other. How many copies did he sell in all? $268 + 129 = n$

C

$$\begin{array}{r} {\scriptstyle 1} \\ 268 \\ 129 \\ \hline 397 \end{array}$$

* Explain the adding for Ex. C. In this example why must you rename and carry 1 to ten's column? The sum of the ones is greater than 9.

3. How can you check the additions in Ex. B and C?
Add upward

In row 4, explain the adding for each example.

	a	b	c	d	e	f
			1	1	1	1
4.	450	$3.75	422	284	453	$3.58
	324	2.03	338	407	218	5.27
	774	$5.78	760	691	671	$8.85

For row 5, tell which sums are shown wrong, and why.

5.	453	$2.09	127	448	139	$7.16
	104	2.57	855	323	214	2.08
	567	$4.66	972	861	343	$9.24
	557		987	771	353	

No 1 to be carried to ten's column.

1 to be carried to ten's column.

1 to be carried to ten's column, but no 1 to be carried to hundred's column.

1 to be carried to ten's column.

Try These

A. with 3-place numerals [W]

Copy the numerals in columns. Add. Check your work. When you add for Ex. 15–21, show the $ and the cent point in your answer.

1. 818 + 102 920
2. 374 + 305 679
3. 736 + 210 946
4. 447 + 127 574
5. 456 + 413 869
6. 355 + 329 684
7. 739 + 229 968
8. 635 + 127 762
9. 514 + 206 720
10. 269 + 218 487
11. 655 + 18 673
12. 609 + 207 816
13. 53 + 118 171
14. 424 + 14 438
15. $4.54 + $2.28 $6.82
16. $3.28 + $0.26 $3.54
17. $0.04 + $3.09 $3.13
18. $5.83 + $4.09 $9.92
19. $4.43 + $0.36 $4.79
20. $4.48 + $3.29 $7.77
21. $2.47 + $0.39 $2.86

◆ **Extra Examples.** Work Set 54.

To Keep in Practice

A. and S. [W]

Rows 1 and 2. Write answers on folded paper. Check.

	a	b	c	d	e	f	g	h	i
1.	412	90	78	91	78	48¢	$1.59	64	50
	+369	−38	+107	−66	−56	+14¢	+5.13	−29	+27
	781	52	185	25	22	62¢	$6.72	35	77
2.	89	526	91	42	562	647	$0.85	36	60
	−42	+38	−44	+57	+8	+136	−0.06	+57	−46
	47	564	47	99	570	783	$0.79	93	14

Ex. 3–11. Copy in columns and work.

3. 700 − 300 400
4. 48 + 609 657
5. 213 + 458 671
6. 83¢ − 37¢ 46¢
7. 807 + 87 894
8. 70¢ − 47¢ 23¢
9. 36¢ + 19¢ + 22¢ + 19¢ 96¢
10. $0.24 + $0.09 + $0.29 $0.62
11. 29 + 12 + 18 + 17 76

Teaching Page 170

Pupil's Objective

To maintain competence in familiar types of A. and S. computation.

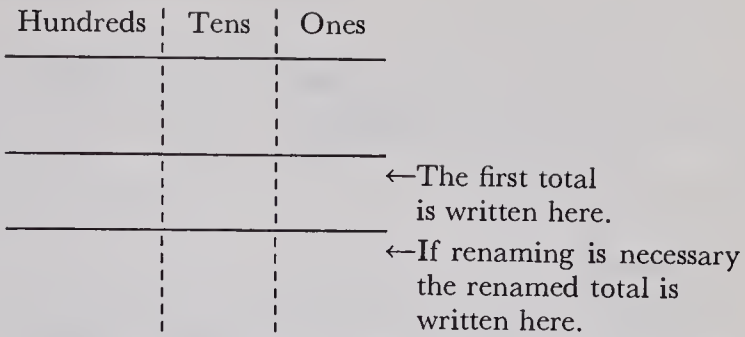
Using the Text Page

• *More capable children* should be able to proceed independently with the written work at the top of the page. *Slower learners* should be permitted to use any written form which gives them confidence. Assign one column at a time to *slower learners*, checking and helping after the work for each column is finished. Before pupils copy Ex. 15–21 in columns show them that the $ is written with the numerals for the first addend and the sum only. Do an example on the board to illustrate the correct way to copy and find the sum with numerals for money numbers.

• Some children will need to copy the examples in rows 1–2 at the bottom of the page before working them. *More capable children* may be able to write answers only on folded paper for Ex. 3–11, as well as for the examples in rows 1–2.

Individualizing Instruction

• Duplicate sheets with the following form for pupils to use if they need to for Ex. 1–21. Place as many of the forms on the page as space permits.



• *All pupils* who are ready may use either of the written forms illustrated below. Full understanding of what each digit and its place in the 3-place numeral tell is essential for the application and use of these forms. They should not be presented to the children as a "trick" we can use in addition.

a. In this example the numbers of ones are added first and their sum recorded below the line as a partial sum. The numbers of tens are added and the numbers of hundreds and recorded in the same way. Be sure to have children show 8 tens as 80 and to think of it as *part of the answer*. The final sum must then be calculated by adding the partial sums.

```
 228
+169
----
  17
  80
 300
----
 397
```

b. The same example may be worked by adding first the numbers of hundreds. The partial sums and the final sum would be shown as illustrated at the right.

```
 228
+169
----
 300
  80
  17
----
 397
```

• Assign Extra Examples Set 54.

NOTES

Pupil's Objective

To learn how to subtract with 3-place numerals when no renaming is required.

New Word: *parents*

Background

This page and the next are developmental lessons concerned with subtracting with 3-place numerals. Readiness for subtracting with 3-place numerals has been provided (a) through lessons on what 3-place numerals show; (b) through experiences in adding with such numerals; (c) through much practice in subtracting with 2-place numerals (which has provided a thorough acquaintance with the algorism and the general procedure used in subtracting); and (d) subtracting with 3-place numerals showing hundreds only.

On page 171 the work is limited to subtraction in which no renaming is involved. Children should be able to do this work with a knowledge of the subtraction facts. Subtracting by using expanded notation of 3-place numerals should be used to foster full understanding.

Perhaps you will want to begin to use the plus symbol in expanded notation as follows:

_ ? _ hundreds + _ ? _ tens + _ ? _ ones.

The use of the + sign emphasizes the additive nature of our decimal (base-10) system.

Pre-Book Lesson

• Consider a problem that involves local data involving hundreds and tens and ones, similar to the following:

A total of 684 children attend Lakeside School. Yesterday 420 children ate in the school cafeteria and the others went home for lunch. How many children went home for lunch?

a. On the chalkboard show hundreds, tens, ones with blanks for a S. example as follows:

$$\begin{array}{r} _\,?\,_ \text{ hundreds} + _\,?\,_ \text{ tens} + _\,?\,_ \text{ ones} \\ -\ _\,?\,_ \text{ hundreds} + _\,?\,_ \text{ tens} + _\,?\,_ \text{ ones} \\ \hline _\,?\,_ \text{ hundreds} + _\,?\,_ \text{ tens} + _\,?\,_ \text{ ones} \end{array}$$

b. Have pupils write the digits that belong in each position for a S. example not requiring renaming of the sum. Be sure to discuss each step in completing the process and have children tell what each digit in the result means.

c. Write examples of each of the following types on the board:

$$\begin{array}{rrrrr} 672 & 847 & 185 & 595 & 449 \\ \underline{-251} & \underline{-\ 37} & \underline{-\ \ 4} & \underline{-205} & \underline{-415} \end{array}$$

• Use partial expanded notation as shown below for illustrating some examples:

$$\begin{array}{rl} 725 = & 700 + 25 \\ \underline{-315} = & \underline{300 + 15} \\ & 400 + 10, \text{ or } 410 \end{array}$$

Using the Text Page

• After discussing Ex. 1–6 orally, have some similar examples done on the board.

• Review the checking of S. as required in Ex. 7.

• Before assigning the written work, discuss how the $ and cent point are used in Ex. 6. Discuss with the class the directions for working Ex. 8–19.

• For the written work, permit *slower learners* to use visual materials if they need them.

Individualizing Instruction

• To locate individual difficulties and to secure a basis for remedial teaching, you may duplicate the set of examples below. Have pupils find the errors and correct them.

$$\begin{array}{rrrrrr} 865 & 958 & 758 & 854 & 629 & 453 \\ \underline{-523} & \underline{-403} & \underline{-\ 43} & \underline{-\ 50} & \underline{-\ \ 7} & \underline{-\ 43} \\ 342 & 545 & 705 & 354 & 612 & 400 \end{array}$$

• *All pupils* may do Ex. 2–6 as follows:

$$\begin{array}{rl} 835 = & 800 + 30 + 5 \\ \underline{-312} = & \underline{300 + 10 + 2} \\ & 500 + 20 + 3 = 523 \end{array}$$

Be sure pupils do not become confused by the plus signs and forget that this is a subtraction example.

• *More capable children* may rewrite Ex. 8–19 as addition number sentences. For example, Ex. 8, 999 − 409, may be written as an addition sentence in two ways: $409 + n = 999$ or $n + 409 = 999$.

Subtracting with 3-Place Numerals

[O]

1. Of the 486 people at our school play, 260 were parents. How many were not parents? $486 - 260 = n$

a. Finish and explain the work for Ex. A.

A

4 hundreds and 8 tens and 6 ones
−2 hundreds and 6 tens and 0 ones
2 _?_ hundreds and _?_ 2 tens and _?_ 6 ones, or _?_ 226

B

Hundreds	Tens	Ones
4	8	6
2	6	0
2	2	6

b. Explain the short way to subtract as shown by Ex. B. Subtract ones, then tens, then hundreds.

6−0=6 Write 6 in one's place. 8−6=2 Write 2 in ten's place.
4−2=2 Write 2 in hundred's place.

Explain the subtracting for each of Ex. 2–6.

2.	3.	4.	5.	6.
835	\$2.39	176	329	\$4.98
312	2.05	41	7	2.04
523	\$0.34	135	322	\$2.94

7. How do you check the subtractions in Ex. 2–6?

Add the two addends.

[W]

Copy in columns and subtract. Check your work. When you subtract for Ex. 16–19, show the \$ and the cent point in your answer.

8. 999 − 409 590
9. 587 − 60 527
10. 554 − 532 22
11. 386 − 275 111
12. 456 − 334 122
13. 608 − 303 305
14. 787 − 26 761
15. 259 − 38 221
16. \$5.00 − \$4.00 \$1.00
17. \$1.66 − \$1.36 \$0.30
18. \$7.17 − \$2.15 \$5.02
19. \$9.89 − \$6.53 \$3.36

◆ **Extra Examples.** Work Set 55.

* Note that this form permits immediate association with former experience in renaming.

Renaming with 3-Place Numerals

To show more ones for subtracting [O]

1. In January, 563 people came to our play. In June, 448 came. How many more came in January than in June?

$$563 - 448 = n$$

No Yes

Ex. A. Can you subtract the ones? Must you rename?

A	B	C
563 448 ?	* 563 = 500 and 63 448 = 400 and 48 100 and 15, or 115	5^{13} $5\not{6}\not{3}$ 448 115 ↑ Check

563 = 500 and 63 448 = 400 and 48

Ex. B. Explain how 563 and 448 are renamed.

Ex. C. Explain the shortest way and the check.

6 tens and 3 ones is renamed 5 tens and 13 ones. 13−8=5 Write 5 in one's column. 5−4=1 Write 1 in ten's column. 5−4=1 Write 1 in hundred's column.

Row 2. Explain each subtraction.

	a	b	c	d	e	f
		7^{10}	7^{13}	4^{10}	3^{15}	6^{12}
2.	578	$6\not{8}\not{0}$	$7\not{8}\not{3}$	$2\not{5}\not{0}$	$\$4.\not{4}\not{5}$	$\$9.\not{7}\not{2}$
	326	373	717	47	0.36	8.43
	252	307	66	203	$4.09	$1.29

[W]

Copy in columns. Subtract. Check your work.

	a	b	c	d
	407	709	105	$4.28
3.	765 − 358	745 − 36	811 − 706	$4.96 − $0.68
	568	711	309	$4.50
4.	992 − 424	839 − 128	918 − 609	$5.70 − $1.20
	405	219	451	$8.38
5.	474 − 69	267 − 48	591 − 140	$8.65 − $0.27
	54	228	608	$2.06
6.	558 − 504	944 − 716	937 − 329	$7.94 − $5.88

Extra Examples. Work Sets 56 and 57.

Pupil's Objective

To learn subtraction with 3-place numerals when renaming to get 10 more ones is required.

Pre-Book Lesson

• Briefly review the renaming of numbers. Write on the chalkboard sentences of the following types:

a. 46 may be renamed _ ? _ tens and 16 ones.

b. 52 may be renamed 4 tens and _ ? _ ones.

c. 6 tens and 14 ones may be renamed _ ? _.

d. 273 may be renamed 2 hundreds and _ ? _ tens and 13 ones.

• Have pupils turn to pages 124 and 132 in the pupil's book where they studied subtracting with 2-place numerals when renaming was required. Review the work on these two pages orally.

• Write the following example on the board:

$$\begin{array}{rcl} 652 & = & 600 + 52 \\ -328 & = & 300 + 28 \end{array}$$

Use the review of pages 132–133 as a basis for finding the answer to this example.

• Write these examples on the board:

a. $\begin{array}{r} {}^{1}\cancel{5}\,{}^{(12)}\cancel{2}\,\cancel{7} \\ -\,1\,0\,6 \\ \hline 4\,1\,6 \end{array}$ b. $\begin{array}{r} 474 \\ -258 \\ \hline 316 \end{array}$

Discuss Ex. a. Have a child explain the subtraction. Be sure he explains the renaming of the tens and ones as shown above the respective columns.

Discuss Ex. b. Ask a child to explain the subtraction. Bring out that we can think the renaming of the tens and ones without writing anything to show it.

Using the Text Page

• Give demonstrations of Ex. 1 and the examples in row 2, if needed, to justify the procedures described in the pupil's book.

• Some of your *more capable children* may be able to write answers on folded paper for rows 3–6. Permit them to do so as long as they do not make mistakes. Remind *all pupils* to check their work.

Individualizing Instruction

• To locate individual difficulties and to secure a basis for remedial teaching, you may duplicate the following set of examples for *all pupils*:

a. Find examples with no renaming and work them.

862	322	241	428	683	359
−337	− 18	−229	−317	−509	− 48

b. In the row above, copy examples where renaming is needed and then work them.

• Your *slower learners* may check some of the above examples with representative materials or by using words and crutches as shown in the two illustrative examples below.

862 = 8 hundreds and 6 tens and 2 ones
−337 = 3 hundreds and 3 tens and 7 ones

or

$$\begin{array}{rcl} 8 \text{ hundreds} + 5 \text{ tens} + 12 \text{ ones} & = & 8\,{}^{5}\cancel{6}\,{}^{(12)}\cancel{2} \\ -3 \text{ hundreds} + 3 \text{ tens} + 7 \text{ ones} & = & -\,3\,3\,7 \\ \hline 5 \text{ hundreds} + 2 \text{ tens} + 5 \text{ ones} & = & 5\,2\,5 \end{array}$$

• *All pupils* may rewrite some of the examples in rows 3–6 as addition sentences (see the third item under Individualizing Instruction on Teacher's Page 171).

• Assign Extra Examples Sets 56 and 57 as needed. These sets should be assigned only when children have demonstrated that they are capable of intelligent independent work.

Pupil's Objective

To have practice in adding and subtracting with 3-place numerals in preparation for the end-of-chapter tests.

New Word

exchange

Background

The extension of two important skills learned in adding and subtracting with 2-place numerals is now being made to work with 3-place numerals—in adding, the renaming of ones and carrying to ten's column, and, in subtracting, the renaming of the sum to show more ones.

The examples at the top of page 173 may be considered as a progress test in preparation for the end-of-chapter testing program. The four types of computation covered and the examples in which they appear are listed below:

a. Adding with 3-place numerals when no renaming is required (Ex. 4, 9, 12)

b. Adding with 3-place numerals when renaming the ones and carrying to the ten's column is required (Ex. 1, 6, 10, 15, 19)

c. Subtracting with 3-place numerals when no renaming is required (Ex. 7, 11, 13, 18, 21, 23, 24)

d. Subtracting with 3-place numerals when renaming to get more ones is required (Ex. 2, 3, 5, 8, 14, 16, 17, 20, 22)

You may wish to use this section for diagnosing individual sources of difficulty. If a child misses two or more examples of any particular type, then his work should be examined carefully to determine the precise source of error. Any remedial work should be in terms of the specific help he needs.

Pre-Book Lesson

• Review the four different types of computation with 3-place numerals described in the Background section. Ask selected youngsters to write examples of each kind on the board and tell why the example is the correct type.

• Tell the children that the written work at the top of page 173 covers examples like these.

Using the Text Page

• *More capable children* may be able to write the answers for Ex. 1–24 on folded paper. Other children, of course, will need to copy the examples in columns before working them.

• The enrichment exercises may be assigned on an individual basis as preparation for class demonstration or they may be discussed with the entire class. If you can have enough cents and dimes coins available, it would be worthwhile to have actual coins used. Otherwise, pupils may use commercially prepared toy money* or toy money the children have made.

The actual exchanging of 1 dime for 10 cents or vice versa should be one more good way of demonstrating meaning for renaming and carrying.

Individualizing Instruction

• *Slower learners* who are having obvious difficulty with the written work on this page may use any form of expanded notation that will help them build further understanding as well as computational skill.

• Assign Extra Examples Sets 58 and 59 as needed.

• *All pupils* may do the following examples for more practice with addition and subtraction with 3-place numerals when finding what n stands for.

1. $n + 215 = 621$
2. $n = 316 + 276$
3. $728 - n = 406$
4. $n + 258 = 793$
5. $n - 263 = 430$
6. $216 + n = 476$
7. $n = 365 - 128$
8. $n - 45 = 748$
9. $547 - 219 = n$
10. $88 + n = 591$
11. $205 + 366 = n$
12. $n + 163 = 492$

* See 16, page xix.

Practice with 3-Place Numerals

A. and S. [W]

Copy in columns and find the answers. Check.

1. 339 + 601 940
2. 791 − 83 708
3. 680 − 674 6
4. 70 + 600 670
5. 791 − 634 157
6. 69 + 825 894
7. 598 − 72 526
8. 673 − 259 414
9. 243 + 725 968
10. 709 + 88 797
11. 188 − 68 120
12. 576 + 202 778
13. 289 − 35 254
14. 790 − 476 314
15. 609 + 57 666
16. 363 − 25 338
17. \$3.91 − \$3.09 \$0.82
18. \$0.89 − \$0.59 \$0.30
19. \$3.29 + \$0.46 \$3.75
20. \$4.63 − \$4.56 \$0.07
21. \$3.67 − \$0.40 \$3.27
22. \$5.81 − \$1.55 \$4.26
23. \$5.75 − \$4.03 \$1.72
24. \$2.96 − \$0.44 \$2.52

◆ **Extra Examples.** Work Sets 58 and 59.

Do You Like to Try New Things?

Enrichment [W]

Get some real money if you can. If you cannot, then make paper bills and coins, for dollars and for dimes and cents.

Use your money to find the answers for Ex. A–F.

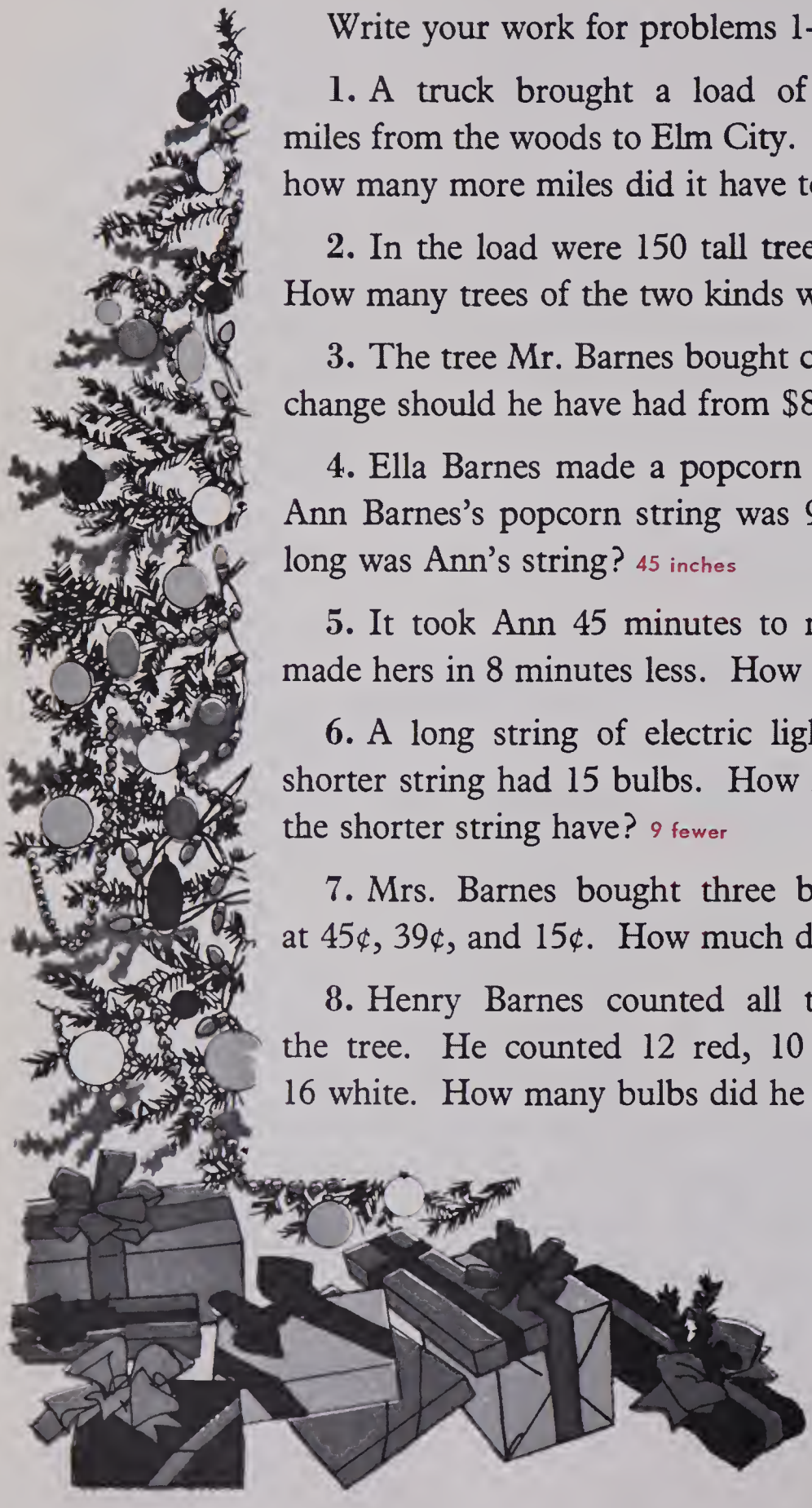
A	B	C	D	E	F
39	147	56	71	180	145
+48	+38	+68	−26	−34	−59
87	185	124	45	146	86

Be ready to explain your work to the class. Use the coins to show why we exchange 10 cents for 1 dime when we would carry to ten's place. Show why we exchange 1 dime for 10 cents when we must show more ones.

◆ **Extra Problems.** Work Set 8.

Can You Solve Problems?

Problem Test 4



Write your work for problems 1–8.

1. A truck brought a load of Christmas trees 730 miles from the woods to Elm City. After going 415 miles, how many more miles did it have to go? 315 miles

2. In the load were 150 tall trees and 107 short trees. How many trees of the two kinds were there? 257 trees

3. The tree Mr. Barnes bought cost $8.60. How much change should he have had from $8.75? $0.15

4. Ella Barnes made a popcorn string 36 inches long. Ann Barnes's popcorn string was 9 inches longer. How long was Ann's string? 45 inches

5. It took Ann 45 minutes to make her string. Ella made hers in 8 minutes less. How long did it take Ella? 37 minutes

6. A long string of electric lights had 24 bulbs. A shorter string had 15 bulbs. How many fewer bulbs did the shorter string have? 9 fewer

7. Mrs. Barnes bought three boxes of colored balls at 45¢, 39¢, and 15¢. How much did she spend? 99¢

8. Henry Barnes counted all the electric bulbs on the tree. He counted 12 red, 10 yellow, 15 blue, and 16 white. How many bulbs did he count? 53 bulbs

Teaching Page 174

Pupil's Objective

To work the end-of-chapter test on problem-solving.

New Words

electric, *string*

Background

This is the first of the group of pages providing the end-of-chapter tests for chapter 4. The problems on this page involve addition and subtraction of the types taught within the chapter or in preceding chapters, so should not be too difficult at this time.

Pre-Book Lesson

Before administering the test, you may wish to review once more some of the situations requiring subtraction and thus provide good background for a test on A. and S. problem-solving.

Using the Text Page

Be prepared to help children who have reading difficulties. You may wish to discuss each item of this test after all children have completed their work.

Individualizing Instruction

- Have *all pupils* write an *n*-example for each problem in the test on page 174. Remind pupils that in these examples *n* should stand for the answer each time.
- For each problem, have the *more capable children* draw an abacus picture and show the adding or subtracting for the problem.
- Assign sets of Extra Problems or Extra Examples as necessary.
- For suggestions about scoring pupils' papers, see Individualizing Instruction on Teaching Pages 50 and 51, and Table of Per Cents on Teaching Pages 176 and 177.

NOTES

Teaching Page 175

Pupil's Objectives

(a) To diagnose difficulties with skills taught in the chapter and to find help in developmental and practice materials on pages listed; and (b) to have practice in estimating answers for A. and S. examples.

Using the Text Page

• Remind pupils about how to use the diagnostic test and explain that, in this way, pupils can find out their weaknesses and review any materials about which they are uncertain. If pupils miss two or more items in any one row they are in need of special help. They may refer to the study pages recommended for each row. Children who miss just one item may need some additional practice. Sets for extra practice are suggested.

• Use the oral practice in estimating answers at the bottom of the page as a "change of pace" between testing sessions. Review rounding to the nearest number of tens before starting. Have pupils tell the number of tens they used to estimate an answer. This is a very good type of practice.

NOTES

Do You Make Mistakes?

Diagnostic Test 4

Write answers on folded paper. Check carefully.

	a	b	c	d	Study Pages	Practice Sets
1.	32 30 +16 78	8 43 +16 67	14 28 +32 74	27 5 +54 86	150	52
2.	17 16 23 +38 94	9 8 37 +45 99	12 14 5 +43 74	11¢ 28¢ 16¢ +28¢ 83¢	152	53
3.	400 +200 600	746 +243 989	457 −324 133	989 −653 336	168, 169, 171	55
4.	716 +209 925	609 +207 816	453 +429 882	739 +128 867	169	54
5.	841 −715 126	680 −463 217	972 −534 438	575 −326 249	172	56–59

Oral Practice in Estimating Answers

Mental procedures

Estimate answers, first by rows, then by columns.

	a	b	c	d	e
1.	42 + 37 80	73 − 39 30	63 + 15 80	58 − 23 40	64 − 32 30
2.	65 − 28 40	18 + 47 70	92 − 49 40	87 − 46 40	57 − 26 30
3.	39 + 21 60	82 − 54 30	78 − 27 50	17 + 18 40	26 + 44 70

Do You Understand?

Test of Information and Meaning 4

1. When the number for a set is 14, the numbers for three parts are 6, 3, and _?_. 5

2. Copy each of these that is less than 1 pound:
18 ounces 11 ounces 21 ounces 15 ounces
11 ounces; 15 ounces

A

Write the letter of the picture that shows

3. sixths. C 4. eighths. B 5. fifths. D

B

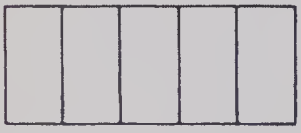
Write what is missing in each of Ex. 6–12.

6. In 1 dozen plums there are _?_ plums. 12

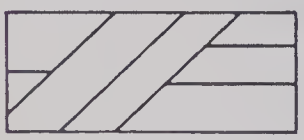
7. If you get $\frac{1}{10}$ of a cake, there can be _?_ other pieces of the same size. 9

C

8. If a 2-place numeral ends in 5, you round the number to the next _?_ ten. greater

D 

9. A box is marked $\frac{1}{2}$ doz. If nothing has been taken from the box there should be _?_ things in it now. 6

E 

10. Picture *E* does not show sevenths because the parts are not the _?_ _?_. same size

11. 806 = _?_ hundreds and _?_ ones 8 6
= 80 _?_ and 6 _?_, or _?_ + _?_ tens ones 800 6

12. 759 = _?_ hundreds and _?_ tens and _?_ ones 7 5 9
= _?_ tens and _?_ ones, or _?_ + _?_ 75 9 750 9

Show the number *n* stands for in each of Ex. 13–17.

13. $n - 40 = 30$ 70 14. $90 - n = 50$ 40 15. $30 + n = 60$ 30

16. $6 + 9 = n$ 15 17. $(3 + 2 + 8) - n = 0$ 13

Teaching Pages 176 and 177

Pupil's Objective

To evaluate the learning of the content of chapter 4.

New Word

(page 176) *plums*

Using the Text Pages (pages 176–177)

It would be a good idea to discuss each of the items in this test after pupils have written their answers. Ask questions like, "Why?" "Is there another way to get the answer?"

Using the Text Page (page 177)

- From the results of this Computation Test, you should get a very good idea of the abilities of your pupils to add and subtract with 2- and 3-place numerals.
- Use the tables below to find the per-cent scores for the individual record cards (see Teaching Pages 50 and 51).

Table of Per Cents for Chapter 4 Scores

Problem Test		Computation Test					
Score	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*
1	13	1	4	9	38	17	71
2	25	2	8	10	42	18	75
3	38	3	13	11	46	19	79
4	50	4	17	12	50	20	83
5	63	5	21	13	54	21	88
6	75	6	25	14	58	22	92
7	88	7	29	15	63	23	96
8	100	8	33	16	67	24	100

Suggestions for Material to Accompany End-of-Chapter 4 Tests

After administering the tests on pages 174–177, in all probability you will find that some pupils need additional help or review before proceeding to the work of the next chapter. Meanwhile, you may find one or more of the following suggestions valuable for use with those children who have finished their work and are ready to go on to the work in chapter 5.

Alternate Uses of Pages in Chapter 4

Page 143. Children may draw models of rectangles (some may be squares) and then separate the inside of each into any of two to ten parts of the same size. Then, in each rectangle, they may shade or color one of the parts and label it with the correct fraction.

Page 144 (bottom). Children may write and complete *if-then* sentences of their own. Then, they may exchange papers with others and check.

Page 149 (bottom). Pupils may write sentences showing a number (between 20 and 30) as equal to 3 of its addends. They then may try to write sentences showing a number (between 15 and 25) as equal to 4 of its addends. Examples:

$$28 = 12 + 13 + 3$$
$$19 = 4 + 5 + 7 + 3$$

Page 153. From the table on page 153, pupils may make up five problems and then solve them.

Page 165. Children may make a chart like the one on this page in which they fill in the appropriate information in each column for other items.

Supplementary Activities

• *More capable children* may try these examples:

3,257	80,495	12,345	2,468
+4,612	+ 2,503	+54,321	+ 7,531
4,627	27,058	65,806	77,664
+1,154	+61,927	+ 2,079	+12,018

• In the following exercises, the child is to indicate the appropriate (A. or S.) operation sign.

3 ○ 24 = 27	n ○ 4 = 1
8 ○ 5 = 3	32 ○ 16 = 48
8 ○ 5 = 13	38 ○ 16 = 22

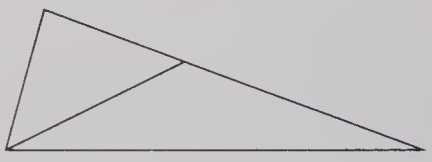
• Pupils may write 3-place numerals in expanded form, as shown below and continue to write others.

$$427 = 400 + 20 + 7$$
$$309 =$$

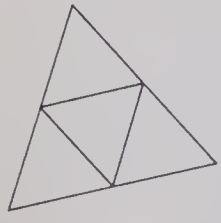
• *More capable learners* should be able to find the number for n in each of the following sentences:

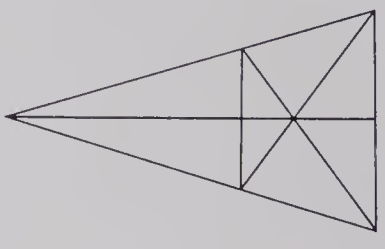
$(42 + n) - 15 = 34$	$87 = (52 + n) + 10$
$12 + (59 - n) = 68$	$n = 25 - (13 - 8)$
$n - (27 - 22) = 14$	$0 = 47 - (25 + n)$

• Do you see three triangles in this picture?



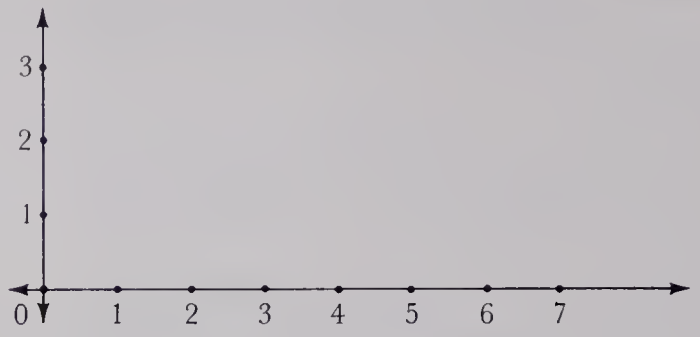
How many triangles do you see in these pictures?



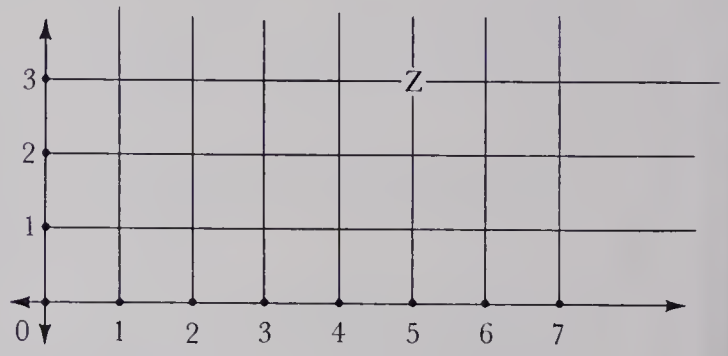


• Two activities in coordinate geometry (see Teaching Pages 92–93 and 138–139) have been suggested to help children identify or describe a location—using numerals in an array chart and using numerals naming points on a number-line picture. The number line, more often than not, is drawn horizontally. However, it may be drawn vertically, diagonally, or in any way one wishes.

For the activity suggested below, children must think of a picture of two number lines, one horizontal and one vertical, which intersect at 0.

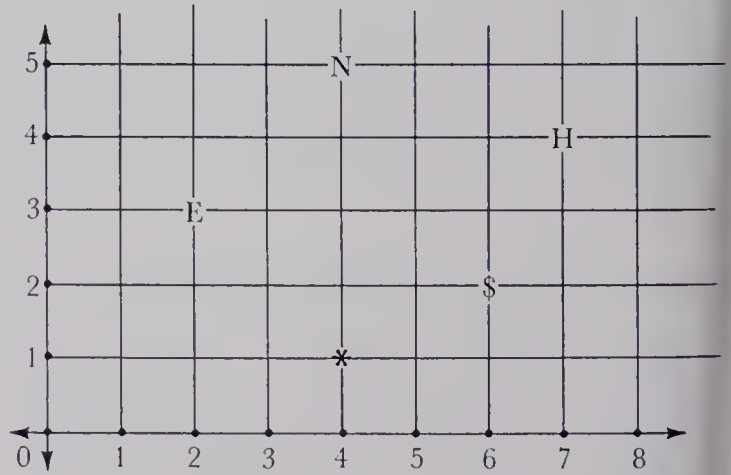


The idea is to describe the location of a position on a flat surface by using the picture of two number lines. In addition to the two number lines pictured, we need *coordinate* lines to help us. In the drawing below, the location of Z is over 5 and up 3.



It is customary always to list the horizontal coordinate first. Eventually children may describe the location of Z by merely writing the ordered pair (5,3).

Ditto a drawing similar to the one below. Children may tell and/or write the locations of the various symbols on the picture.



The E (or $ or N or * or H) is over ____ and up ____.

Write each example with its answer a shorter way.

18. $5 + 5 + 5 = 15$ three 5's = 15

19. $2 + 2 + 2 + 2 = 8$ four 2's = 8

20. Draw a picture of a rectangle and show the inside of it in fourths.

Which answer must be wrong in Ex. 21 and Ex. 22?

21. $25 + 47$ **72** **14** **62** 14

22. $41 - 27$ **52** **24** **14** 52

Copy and write $>$ for is greater than or $<$ for is less than or $=$ for is equal to where you see _?_.

23. $(35 - 5)$ $\overset{=}{?}$ $(20 + 10)$

24. $(14 + 50)$ $\overset{<}{?}$ $(90 - 10)$

Write an *n*-example for each of these problems on page 174:

25. Problem 1 730 − 415 = n

26. Problem 2 150 + 107 = n

27. Problem 4 36 + 9 = n

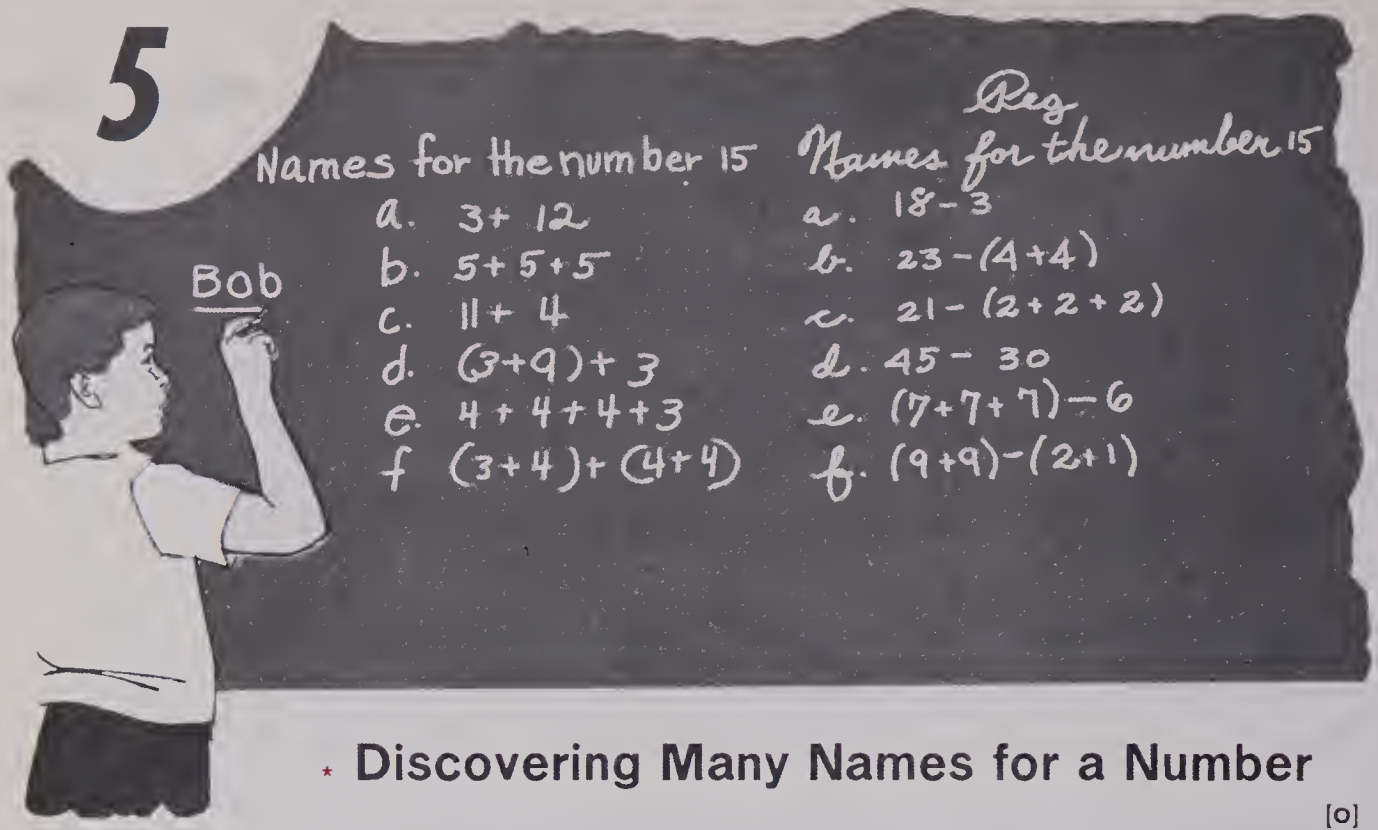
How Well Can You Compute?

Computation Test 4

Copy in columns, and add or subtract. Check.

1. $480 - 79$ 401
2. $408 + 287$ 695
3. $91 - 56$ 35
4. $345 + 227$ 572
5. $87¢ - 22¢$ 65¢
6. $207 + 636$ 843
7. $245 + 34$ 279
8. $992 - 638$ 354
9. $49 + 38$ 87
10. $587 - 567$ 20
11. $816 - 707$ 109
12. $655 - 546$ 109
13. $\$5.06 - \$3.06$ $2.00
14. $\$6.87 - \$4.59$ $2.28
15. $\$8.02 + \$0.69$ $8.71
16. $\$1.88 - \$1.39$ $0.49
17. $\$6.06 + \$2.53$ $8.59
18. $\$9.44 - \$5.28$ $4.16
19. 2, 40, 28, 24 94
20. 47, 9, 7, 17 80
21. 9¢, 29¢, 46¢, 10¢ 94¢
22. 19¢, 37¢, 8¢, 7¢ 71¢
23. $0.18, $0.59, $0.03, $0.18 $0.98
24. $0.39, $0.06, $0.18, $0.15 $0.78

5



* Discovering Many Names for a Number

[O]

As you know, a number may have more than one name. The number 15 has the names $6 + 9$, $8 + 7$, and $10 + 5$ but it has other names, too.

With the help of the class, Bob and Peg wrote on the board other whole-number names for 15.

Look at the picture, and study Bob's list of names.

1. In his list, addition was used. Explain why each set of numerals in Bob's list names 15.

2. On the board write four other whole-number names for 15. Use any of the ways you see in Bob's list.

Answers will vary.

Now study Peg's list of names for 15.

3. Subtraction was used this time. Explain why each set of numerals in Peg's list names 15.

4. On the board write four other whole-number names for 15. Use any of the ways you see in Peg's list.

Answers will vary.

* Pupils have been working casually with other names for numbers, as (4 + 3) for 7, and renaming numbers, as 4 tens and 7 ones = 3 tens and 17 ones. Now the topic is pointed up to show pupils that, by using A. and S., they may give very many names to a number.

Overview — Chapter 5

• With chapter 5, your pupils are beginning the second half of their arithmetic work in grade 3. Now the work is extended in addition to renaming the sums of both the ones and the tens and carrying the necessary digits. In subtraction, the extension is made to renaming to show more tens and more ones when subtraction was not possible.

In this chapter the work with multiplication really gets under way. Emphasis is put on the idea that, to find the number for a total, we may always add but when the sets of things for which we want to find the number for the total are equivalent—that is, when the numbers for the sets are equal—we can multiply. In this chapter the meaning and the tables of multiplication facts for 2 and for 2's are developed. Immediately now and with the development of other multiplication tables, and hence pairs of facts, pupils are putting into use the Commutative Property of Multiplication—i.e., $2 \times 4 = 4 \times 2$, $3 \times 5 = 5 \times 3$, and so on.

In order to emphasize the ease with which we work when we use our base-ten numeration system and its place values, some work with Roman Numerals—a numeration system which does not use place value—is introduced.

• **Problem-Solving.** In this chapter, while maintaining the choosing and writing of *n*-sentences for problems, making problems, and working problems orally, work is extended now to problems with data missing and to those with extra data.

• **Maintenance.** Topics begun in earlier chapters are extended now to make use of 3-place numerals—inequalities, work with *n* to stand for a number, and *if-then* sentences for work with logic.

In addition to incidental references to geometric forms used in work with fraction numbers, geometry is maintained and extended in this chapter to include work on the naming of polygons and their sides in the several ways.

• **Enrichment.** The enrichment provided in this chapter for the more capable children, in addition to the reservoir of suggestions at the end of the pupil's text, includes extended work with parentheses—the making of examples for comparing two numbers—, an exercise in finding answers for multiplication examples by using a number line and adding, and also some work with magic squares.

• **Testing.** There appear as usual the maintenance, mid-chapter review, and the end-of-chapter tests—Problem-Solving Test, Diagnostic Test, Test of Information and Meaning, and Computation Test.

Teaching Pages 178 and 179

Pupil's Objectives

(a) To learn more about the many different ways of renaming a number; and (b) to learn to read problems carefully, in order to detect the absence of relevant material.

New Word

list

Background

The idea of *renaming* a number has been developed extensively in the MWN program. For the most part, the renaming of a number has been limited to two numerals and one operation. The material on these two pages extends renaming to the use of three or more numerals and two operations.

A number may be named in many different ways. We use numerals to *name* numbers. For example, $(7 - 5)$ is said to be a *numeral* that names the number two. Young children often become confused when the number-numeral distinction is belabored. You will find it possible to discuss and write *names* for numbers and to *rename* numbers without always having to resort to using the term "numeral."

We use parentheses to identify numbers that have been renamed. In $(3 + 4) - (2 + 1)$ two numbers have been named; 7 and 3. As shown, operations indicated within parentheses should be done before proceeding with other operations.

The work with problems at the bottom of page 179 is new. Ex. 1–7 are samples of incomplete problem situations. Experience in telling what is missing in these exercises is intended to alert your pupils to the advisability of reading problems all the way through and of making sure they have at hand all the information needed for problem solutions.

Pre-Book Lesson (pages 178–179, top)

• During class discussion, ask pupils to tell different ways to *name* eight. As each way is mentioned, write it on the chalkboard.

You may find that if the first child uses addition in naming eight, all subsequent suggestions will be in terms of addition. Encourage children to be good thinkers and try to name eight in as many different ways as possible. Make certain that both addition and subtraction are used.

You must also be prepared for suggestions which have never been studied formally in class. For example, some child may name eight as (100 − 92).

• If eight has not been named with the use of three or more addends, you may introduce the possibility as follows:

Perhaps 4 + 4 has been suggested. Ask if 4 can be renamed in any other way. Indicate the suggestions for renaming 4 by using parentheses. You may have examples like the following written on the board:

$$(3 + 1) + 4$$
$$(6 - 2) + 4$$

Be sure to emphasize the use of parentheses as a means of indicating part of a name.

• After eight has been named using two and three numerals, ask pupils if they think there are any other names for the number eight. Guide pupils to understand that a number may have so many different names that we could not even give all of them.

Using the Text Pages (pages 178–179, top)

You may find it useful to have different children write Ex. 5–7 on the board during the accompanying discussion. The work with the parentheses may require rather careful development.

Pre-Book Lesson (page 179, bottom)

Before doing the problem work, you may discuss briefly some familiar instances where people must anticipate needs or consider the total situation before taking action. Common experiences would include making plans for a trip or having materials ready before undertaking a task.

Using the Text Page (page 179, bottom)

• Note that Ex. 3 gives all the information necessary to find a solution. See if any youngster can see this.

• You may wish to consider translating the information given in a problem into a mathematical sentence. In doing so, children will be able to see that the sentence would show two unknown numbers.

• Notice that numbers less than 50 are to be used in working the problems. (This means that 50 cannot be used.)

Individualizing Instruction

• You may expand upon the problem-solving lesson in an effort to develop foresight as a general characteristic among your pupils. If you are not already doing so, try utilizing your daily program with *all pupils* participating in suggesting solutions for actual problems that may arise.

• *Slower learners* may need renaming practice with numbers no greater than the teens or the twenties. Each pupil may work with a partner. Have each partner write names for the same number. Then they may check each other's work to see if each name is correct.

• *All pupils* would extend their understanding of the decimal system and expanded notation by showing tens and ones to name a number shown by a 2-place numeral. For 45 they may show (20 + 20 + 5) or (10 + 10 + 10 + 10 + 5) or (50 − 5), and so on.

Write 2-place numerals on the board or duplicate them on paper and have children name the number shown in terms of tens and ones only.

• *More capable children* may name the number shown by a 2-place or a 3-place numeral in any way they wish. Permit them the opportunity to be as creative as possible.

• Use Extra Activity Set 96 as needed.

Finish Ex. 5–7 by saying what is missing.

5. Names for 17

a. $22 - _?_$ 5

b. $(3 + 3 + 3) + _?_$ 8

c. $(10 + 10) - (2 + _?_)$ 1

6. Names for 28

a. $20 + _?_$ 8

b. $7 + 7 + 7 + _?_$ 7

c. $(12 + 4 + 8) + _?_$ 4

7. Names for 32

a. $(8 + 7 + 9) + _?_$ 8

b. $(60 - 20) - _?_$ 8

c. $40 - (2 + _?_)$ 6

[W]

Write four other names for the number

8. 21. **9.** 14. **10.** 27. **11.** 18. **12.** 30.

Answers will vary.

● **Extra Activity.** Work Set 96.

Problems for Careful Readers

Providing missing data; solving problems [O]

In some of these problems something is missing. What is it?

1. Sue bought eggs. How much of her 75¢ was left?

Price of the eggs.

2. Two of Ed's words were wrong, so _?_ were not.

How many words in all.

3. From head to tail our cat measures 28 inches. Our dog measures 17 inches more. How long is our dog?

Nothing is missing.

4. Mike has 33¢. How much more does he need to buy a ball? Price of the ball.

5. Tom is 8 years older than Fred. How old is Fred?

Age of Tom.

6. Jean spent 27¢ for a book. She spent what was left for some paper. How much did the paper cost?

Amount of money Jean had at first.

7. Tom has 18 jacks now. How many has he lost?

How many jacks he had in all.

[W]

Where you need numbers, use any that are less than 50 and work Ex. 1–7. Answers will vary.

*Since so few Roman numerals involve subtraction, strive for immediate recognition of these.

Roman Numerals

I through XII [O]

1. The Roman I means 1. Why can you remember this? What is our numeral for II? 2 for III? 3

2. Writing I after a numeral means that the new number is greater by _?_. 1

3. The letter V was used by the Romans to mean 5. Some say that V was used to mean all the fingers on one hand. Do you see a V in the picture at the left? Yes

4. Write on the board our numeral for VI; VII; VIII. 6 7 8

The letter X was used by the Romans to mean 10.

5. In X, do you see two V's (two 5's)? Yes Explain. Two V's which have been joined.

*6. The Roman numeral for 4 is IV and for 9 is IX.

a. Writing I before V means 1 less than _?_. 5

b. Writing I before X means 1 _?_ than _?_. less 10

Tell what these Roman numerals mean, and why:

7. XI 11; 1 more than 10

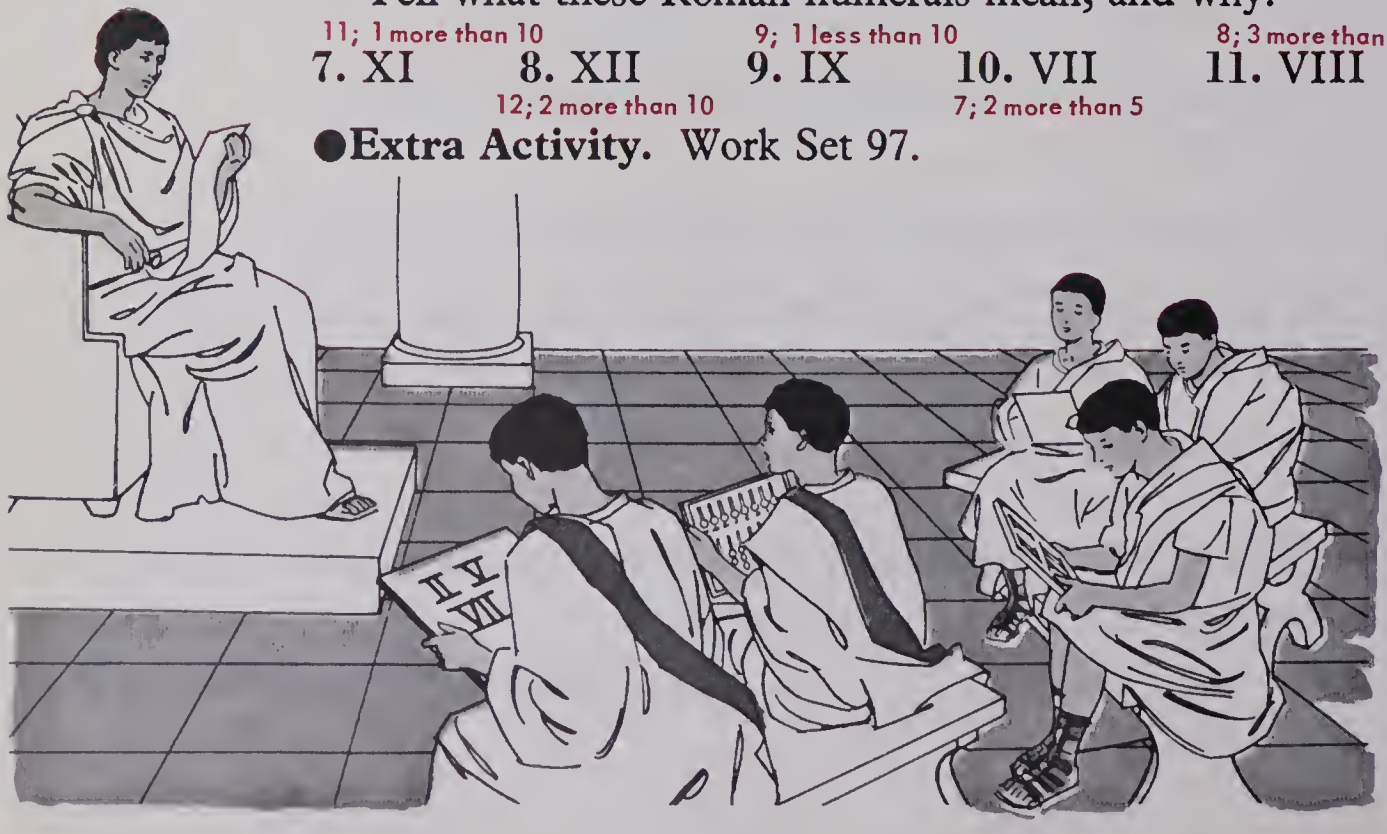
8. XII 12; 2 more than 10

9. IX 9; 1 less than 10

10. VII 7; 2 more than 5

11. VIII 8; 3 more than 5

●**Extra Activity.** Work Set 97.



Teaching Pages 180 and 181

Pupil's Objectives

(a) To learn the rationale of the Roman numerals through XII and to learn how to read and write them; and (b) to practice finding the number for n in addition and subtraction sentences.

Background

Today Roman numerals are seldom used. Yet an important reason for learning about the Roman system of numeration is to foster greater understanding of the system we are now using. Children should understand that the Hindu-Arabic decimal system they are learning is not the only means of recording number ideas that has been used or is now being used.

The story about the "fisherman's" system suggested for page 4 of the pupil's text is but one of the many stages in the historical development of the system of notation presently in use. The fisherman's system was similar to the abacus with a base of 10 and was positional in nature. At another time and in another type of work, sheepherders developed a system with a base of 10 which was not positional in nature.

The Romans developed a means of recording number ideas which had neither a base of 10 nor was positional in nature. They developed a set of symbols resembling some of the capital letters as we now know them: I, V, X, L, C, M. Where our present system is positional and additive in nature, the Roman system is not positional (L stands for 50 independent of a position) and it is subtractive as well as additive. Anytime that a symbol precedes one showing a greater number, the lesser number is subtracted from the greater one to identify the number. (IX stands for ten minus one.) When a lesser number is shown directly after a greater one, the two are added to identify the number. (XI stands for ten plus one.)

In his book, *Mathematics for the Million*, Hogben commented that in the day of Charlemagne the learned scholar, comparable to today's college graduate, studied mathematics which is now mastered by most ten-year-old children. This happened because of the difficulty encountered in trying to perform operations with the Roman system.

Teacher's Preparation

If possible, obtain a clockface with Roman numerals. Collect or prepare materials which utilize Roman numerals. Perhaps the cornerstone or plaque in the school by use of Roman numerals indicates the date it was built. Chapters in textbooks are frequently identified with Roman numerals. Of course, old movies shown on television use Roman numerals to show the year of production.

You may wish to have some pictures of Roman attire and living as well as maps as you discuss who the Romans were and where they lived.

Pre-Book Lesson (pages 180–181)

- Point to the III on the clockface (or in a table of contents) and ask the pupils if they know what it means. Discuss the meaning of I and II at the same time. The Romans actually used tally marks / , // , /// to record the numbers one, two, and three. There are as many tally marks as there are objects in the set being recorded.
- Point to the V. Ask, "What does it mean? How do *we* record the idea of five?" After the pupils know the use of V as a symbol for five, discuss the way the Romans recorded four and six. Four is actually thought of as five minus one, while six is thought of as five plus one. Be sure to emphasize this feature of the Roman system.
- Write a V on the board. Ask the class how much two fives are. Tell them you are going to show ten with two fives the way the Romans did. Then, make an inverted V below, joining the one on the board. The result is X, the Roman symbol for ten.

Using the Text Pages (pages 180–181)

- Supplement the oral work with pictures, materials, and stories you have prepared or gathered to show use of Roman numerals.
- Erase all that has been written on the board in connection with Ex. 1–14 before assigning Ex. 15 and 16.

More Problems for Careful Readers

Discovering unnecessary data; writing n-examples [O]

In some of these problems there are numbers you do not need to use in finding the answers. Which are they?

1. Tom's father weighs 178 pounds. His mother weighs 115 pounds. Tom weighs 69 pounds. How many pounds fewer does Tom weigh than his father? 115
109 pounds 178−69=n

2. A book costs $1.25. A doll costs $1.65. A game costs $0.80. Ella bought the book and the doll. How much did she pay? $0.80
$2.90 $1.25+$1.65=$n

3. The school bus carried 235 children the first week and 242 children the second week. How many children did the bus carry in the two weeks? 477 children
235+242=n

4. Ella read one book of 93 pages and another of 68 pages. How many more pages did one book have than the other? 25 pages 93−68=n

5. The Smith family rode 346 miles Monday and 432 miles Tuesday. The whole trip was 970 miles. By Tuesday night how many miles had they gone? 970
778 miles 346+432=n

6. A truck can carry 500 folding chairs. In one load it carried 144 chairs of one kind and 132 chairs of another kind. How many chairs were in that load? 500
276 chairs 144+132=n

[W]

Work problems 1–6.

Write an *n*-example for each of the problems.

◆**Extra Problems.** Work Set 9.

●**Extra Activity.** Work Set 98.

* This plan of analyzing a variety of problems via the addends-sum analysis and then gradually via number sentences develops both approaches thoroughly.

Pupil's Objectives

(a) To learn to distinguish between relevant and irrelevant information given in a problem; and (b) to practice writing an *n*-sentence for a problem.

Background

An important step toward maturity in solving problems is the realization that not all information given or known about a particular situation is necessary in solving a particular problem. In previous lessons pupils have provided information to complete problems. In this lesson unnecessary information is included in some of the problems. By careful reading of the question posed, pupils should be able to determine which information is pertinent to the question and which is not.

In the written part of this lesson pupils will have their first experience in writing *n*-sentences showing numbers representing money. They should be taught to write the sentence using the \$ symbol just as it appears with each numeral in the problem.

Teacher's Preparation

Duplicate or write on the board a problem similar to the following in which extra information is given:

Saturday afternoon at 3 o'clock Mrs. Blue went to the store. She bought a 10-pound bag of potatoes for \$0.55. How much change did she receive from the \$0.75 she gave the lady clerk?

Pre-Book Lesson

• Refer to the problem which has been prepared in advance. Focus all the discussion on the question that is asked and the information that is given.

• Make the point that in most problem settings a great deal of information is available that is not essential to finding the solution to a particular problem. Ask questions about the problem like the following:

a. Does it make any difference what time Mrs. Blue went to the store? (It is conceivable the store was running a special at that particular time on Saturday.)

b. Can we find the answer without knowing how much the potatoes weighed?

c. Do we have to know that it was potatoes that Mrs. Blue bought?

d. Would she get the same change if the clerk was a man instead of a lady?

e. Suppose the clerk had red hair (yellow dress, wooden leg, and so on). Would the answer be different? (Exaggeration or stressing the ridiculous is often useful to help make a point with children.)

• Ask a child to write an *n*-sentence on the chalkboard for the problem they have been discussing. Be sure to include the \$ symbol in the *n*-sentence. ($\$0.75 - \$0.55 = \$n$).

Using the Text Page

• The Pre-Book Lesson should lead very naturally into a similar discussion about the problems in the pupil's book. Be sure to emphasize the information given rather than finding the solution as such. Have children tell why some information given is not necessary. Help them to see that in problems 3 and 4, all the information must be used to find solutions.

• For the written work at the bottom of the page, your pupils should first write a solution to each problem without being required to write *n*-sentences. If pupils want to write sentences first, they should, of course, be permitted to do so.

Individualizing Instruction

• Extra Problem Set 9 may be used as follows:

a. *Slower learners* merely find a solution to each problem.

b. *More capable children* may solve the problems and then write an *n*-example for each one.

• Children with reading difficulties may need special help in problem analysis. Individual or small-group reading with attention to word analysis and unknown vocabulary may be useful.

• *More capable children* may do Extra Activity Set 98.

Teaching Page 183

Pupil's Objectives

(a) To review and use skills in rounding to the nearest ten for estimating the answers to addition and subtraction examples; and (b) to learn to rename 10 tens as 1 hundred.

Background

The role of estimation as a useful tool in problem-solving cannot be overemphasized for it is one important basis for determining whether an answer is *reasonable*. Estimating in A. and S. examples at the top of page 183 gives practice which will help in estimating answers for problems. In this set, you will note that the estimated answer may vary considerably from the exact answer, as is so often the case in a social situation.

Each estimation should be made mentally, writing only the answer. Pupils having difficulty in forming or retaining mental images may be permitted to show each of the rounded numbers as well as the answer. In an example like 48 + 24, they would write, "50 + 20 = 70."

The child may copy and work each example immediately after he has estimated an answer, or he may make all estimates first before copying the examples. In either case, pupils should be encouraged to compare their actual answers with their estimated answers.

The exercises at the bottom of page 183 are in preparation for addition and subtraction with 3-place numerals in which renaming to show hundreds and tens and carrying occur. This is merely an extension of the decimal feature of our numeration system. Children have already learned that 10 ones may be renamed 1 ten. By extending this idea to renaming 10 tens as 1 hundred, children are ready to begin to generalize that ten of the value of any place is enough to make one of the value of the next place to the left.

Pre-Book Lesson (page 183, top)

• Use your permanent number line* to assist in reviewing the principles involved in rounding to the nearest ten. (A number shown with 5 in the one's place is rounded to the next greater ten). For review you may wish to look at pages 158–160.

• Write 37 + 42 = *n* on the board. Ask a child to round 37 to the nearest ten. Do the same with 42. Write *4 tens* + *4 tens* on the board as the children respond to your questioning.

Ask, "Four tens plus four tens is how many tens?" Encourage pupils to think "tens" when they round.

• Repeat the same kind of discussion with a subtraction example.

Using the Text Page (page 183, top)

Have children write the estimated answers only unless some need to show the rounded numbers as well.

Pre-Book Lesson (page 183, bottom)

• Use a place-value teaching aid** which has three positions (1's, 10's, 100's). Show 6 ones. Ask a pupil to tell how many ones there are. Show 3 more ones. Then ask for the number of ones.

Have another child show three more ones with the others. At this time the pupils should determine there are 12 ones. Review with the children the renaming of 12 ones as 1 ten and 2 ones. Show the renaming on the place-value aid.

• Repeat the procedure just completed using the ten's position. Be sure to bring out the generalization that we rename 10 tens in the same way we rename 10 ones.

• Extend the renaming of tens by asking: "If 10 tens can be renamed 1 hundred, how can 20 tens be renamed?"

• Write the following examples on the board and have pupils complete the work to show the renaming.

a. 20 tens = _ ? _ hundreds and _ ? _ tens
b. 18 tens = 1 _ ? _ and _ ? _ tens
c. 32 tens = _ ? _ hundreds and 2 _ ? _
d. 38 tens = 3 _ ? _ and _ ? _ tens

Using the Text Page (page 183, bottom)

Discuss Ex. 1–3 orally. If any pupils have difficulty, do more work using a teaching aid.

* See 6, page xix.

** See 21, page xix.

To Keep in Practice

Estimating and computing; A. and S. [W]

Write the numerals 1–20 on your paper. Do not work Ex. 1–20 now, but show your estimates of the answers. Estimates shown above the sign of operation.

1. $41 \overset{80}{+} 39$ 80	5. $65 \overset{40}{-} 29$ 36	9. $28 \overset{60}{+} 26$ 54	13. $91 \overset{30}{-} 56$ 35	17. $74 \overset{20}{-} 48$ 26
2. $83 \overset{50}{-} 52$ 31	6. $72 \overset{90}{-} 37$ 35	10. $89 \overset{0}{-} 45$ 44	14. $45 \overset{70}{+} 31$ 76	18. $37 \overset{50}{+} 46$ 83
3. $86 \overset{40}{-} 35$ 51	7. $44 \overset{90}{+} 48$ 92	11. $63 \overset{40}{-} 59$ 4	15. $27 \overset{30}{+} 38$ 65	19. $76 \overset{90}{-} 32$ 44
4. $75 - 38$ 37	8. $62 + 29$ 91	12. $81 - 35$ 46	16. $55 - 34$ 21	20. $42 + 47$ 89

Now copy and work Ex. 1–20. Check your work.

* Renaming to Show Hundreds and Tens

Readiness for carrying to hundred's place [O]

1. 5 tens and 7 tens = 12 tens
   = 1 hundred and $\overset{2}{_?_}$ tens
2. 10 tens = $\overset{1}{_?_}$ hundred and $\overset{0}{_?_}$ tens
3. 21 tens = 2 $\overset{\text{hundreds}}{_?_}$ and $\overset{1}{_?_}$ ten

[W]

Copy and finish each of Ex. 4–6.

4. 15 tens = 1 hundred and $\overset{5}{_?_}$ tens
5. 25 tens = 2 $\overset{\text{hundreds}}{_?_}$ and $\overset{5}{_?_}$ tens
6. 22 tens = $\overset{2}{_?_}$ hundreds and $\overset{2}{_?_}$ tens

When the sum of the tens is greater than 9 tens, we rename that sum to show hundreds and tens. Rename these sums to show hundreds and tens:

7. 24 tens ∧ 2 hundreds and 4 tens	8. 17 tens ∧ 1 hundred and 7 tens	9. 11 tens ∨ 2 hundreds and 0 tens	10. 27 tens ∧ 2 hundreds and 7 tens
11. 13 tens ∨ 1 hundred and 3 tens	12. 20 tens ∧ 2 hundreds and 0 tens	13. 19 tens ∨ 1 hundred and 9 tens	14. 35 tens ∨ 3 hundreds and 5 tens

* This is an extension of earlier work in renaming and prepares for the renaming of the sum of the tens in addition and the carrying of a digit to hundred's column.

Carrying to Hundred's Column

[o]

A

Hundreds	Tens	Ones
	4	5
+	8	2
1	2	7

B

$$\begin{array}{r} {\scriptstyle 1} \\ 164 \\ +\ 50 \\ \hline 214 \end{array}$$

1. Mike has 45 marbles in his pockets and 82 marbles in a bag. Find how many marbles in all. $45 + 82 = n$

Ex. A: Add ones. Is there anything new here? No

Add tens: $4 + 8 = 12$. *Think,* "12 tens = 1 hundred and 2 tens." Write "2" in ten's place and "1" in hundred's place for the sum.

2. Find how many in all: 164 marbles and 50 marbles.

Ex. B: Must you carry to hundred's column? Yes Explain. 6 tens + 5 tens = 11 tens, or 1 hundred and 1 ten.

Explain the addition for each of Ex. 3–7.

3.	4.	5.	6.	7.
95	72	$\overset{1}{}427$	$\overset{1}{}\$0.34$	$\overset{1}{}\$3.62$
33	36	180	0.85	0.95
128	108	607	$1.19	$4.57

Ex. 11 & 12

In Ex. 8–12, which sums are shown wrong? Tell why.

Ex. 11. 0 + 1 = 1 Ex. 12. 4 + 8 = 12; (carry 1 to hundred's column) 1 + 1 + 6 = 8

8.	9.	10.	11.	12.
42	350	86	$0.65	$1.43
67	379	93	1.23	6.85
109	729	179	$2.88	$7.28

Teaching Pages 184 and 185

Pupil's Objectives

(a) To learn to rename tens and carry a digit to hundred's column; and (b) to have practice in column addition.

Background

Renaming 10 tens and carrying a digit to the hundred's column (no similar renaming of ones at this time) is developed first in simple cases like 56 + 72 and 174 + 240. In such examples the sum of the tens is 10 or greater and, obviously, something must be done. The idea that 10 tens may be renamed 1 hundred has just been introduced. In this lesson, the idea is applied in working addition examples such as the two examples written above.

The practice material at the bottom of page 185 affords the opportunity to review and apply the Commutative and Associative Properties of Addition. As children copy the information into column form, the order in which they show the addends will not affect the sum. The Associative Property may be stressed in adding four addends.

Pre-Book Lesson

• Suggest a classroom or local problem (based on totals of books, children, tickets, and so on) and have your pupils use objects in a pocket chart (or a similar place-value teaching aid) to find answers for examples like 47 + 82, 93 + 35, 156 + 72, and 240 + 92.

• The use of *hundreds*, *tens*, and *ones* may help to review previous learnings and to establish new learnings.

$$\begin{array}{rcl} 156 & = & 1 \text{ hundred } 5 \text{ tens } 6 \text{ ones} \\ \underline{+73} & = & \underline{\qquad\qquad 7 \text{ tens } 3 \text{ ones}} \\ & & 1 \text{ hundred } 12 \text{ tens } 9 \text{ ones} \end{array}$$

Since 12 tens = 1 hundred and 2 tens, the 1 hundred should be added to the 1 hundred already shown, so the sum is renamed 2 hundreds 2 tens 9 ones.

You may wish to write *hundreds*, *tens*, and *ones* on a chart with a pocket for each place value. Numerals on cards may be inserted in these pockets. Or, you may write the words on the chalkboard for each place value. Numerals may be written before each of the place-value names.

$$\begin{array}{l} _?_ \text{ hundreds } _?_ \text{ tens } _?_ \text{ ones} \\ \underline{_?_ \text{ hundreds } _?_ \text{ tens } _?_ \text{ ones}} \\ _?_ \text{ hundreds } _?_ \text{ tens } _?_ \text{ ones} \end{array}$$

These plans may be used for both addition and subtraction.

• Use a variety of examples involving this particular skill. Have the children explain the work each time.

Using the Text Pages

• The discussion and explanations in connection with Ex. 1–12 are very important. Be sure to ask probing questions about each explanation of Ex. 3–12. For example:

a. In Ex. 5 what does the little 1 above the 4 mean?

b. Where does it come from?

c. What does the 0 in the answer mean?

Ask questions about the renaming of 10 tens so that pupils will state the generalization at the top of page 185 in their own words.

• Ex. 13–23 may be checked in one of two ways: (a) the numbers may be added in a different order (Commutative Property of Addition) to show that the sum remains unchanged or (b) one addend may be subtracted from the sum which should result in the other addend (inverses).

• If some pupils have difficulty in writing the answers to rows 22 and 23 on folded paper, they may copy the examples on paper.

• In Ex. 1–16 at the bottom of page 185, there are some examples in which 1 is carried to ten's place, others in which 1 is carried to hundred's place, and some in which there is no carrying at all. Point this out to the children and caution them to work carefully.

You may wish to review the Commutative Property of Addition with the class before making the assignment. Suggest that they may copy the numerals in Ex. 1–16 in any order. Checking may be done as mentioned above.

Individualizing Instruction

• *All pupils* may do Ex. 8–12 as follows:

$$\begin{array}{rl} 350 = & 300 + 50 + 0 \\ \underline{+379} = & \underline{300 + 70 + 9} \\ & 600 + 120 + 9 = 729 \end{array}$$

• Mimeograph or write on the board the following sets of examples, and use them with your *slower learners* as directed:

a. In which examples must you rename the sum of the tens? Why?

56	28	434	107	349	33
+36	+127	+85	+504	+560	+86

b. Tell the missing digits.

62	172	515	197	361	483
+126	+435	+243	+80	+383	+351
?88	?07	?58	?77	?44	??4

• Ask *more capable children* to devise ways they can demonstrate renaming 10 tens. They may use a variety of materials. Some of the ways suggested may present difficulties. Let them work out a solution or obtain help from each other, but do not let them present an idea to the class until they can give a clear, reasonable demonstration and explanation.

• *Slower learners* may need review and practice with the more difficult addition facts. You may wish to use one of the following activities for this purpose:

a. Match answer cards with number-sentence cards:

Answer cards					Sentence cards	
1	3	5	9	4	n+5=8	6+7=n

b. Play the *Show the Answer* game. Write *n*-sentences on the board or hold up number-sentence cards. Have children select the appropriate answer to hold up on signal.

• Use Extra Examples Set 60 as needed.

Reminder

Continue to maintain the following:

a. The geometric concepts: point, line segment, and polygons

b. Ability to understand and use ordinals as far as fiftieth

c. Ability to tell time

NOTES

When the sum of the tens is greater than 9 tens, rename the tens as hundreds and tens.

Write the digit for the tens in ten's place and carry the digit for the hundreds to the hundred's column.

[W]

Copy each of Ex. 13–21 in a column and add. For some examples you do not rename. Check your work.

13. 42 + 372 414
14. 73 + 95 168
15. 116 + 172 288
16. 632 + 94 726
17. 43 + 56 99
18. 371 + 167 538
19. \$0.64 + \$3.73 \$4.37
20. \$0.94 + \$1.95 \$2.89
21. \$6.85 + \$2.13 \$8.98

For rows 22 and 23, show the sums on folded paper.

	a	b	c	d	e	f	g
22.	30	48	460	381	85	\$6.74	\$0.82
	477	581	180	175	52	0.80	2.63
	507	629	640	556	137	\$7.54	\$3.45
23.	80	93	587	590	95	\$0.41	\$1.52
	90	352	40	240	374	0.98	1.90
	170	445	627	830	469	\$1.39	\$3.42

◆ **Extra Examples.** Work Set 60.

Try These

[W]

Copy each set of numerals in a column and add. Check.

1. 208, 649 857
2. 35, 336 371
3. 300, 400 700
4. 215, 481 696
5. 65, 164 229
6. 194, 234 428
7. 59, 14 73
8. 162, 27 189
9. 312, 591 903
10. \$0.79, \$4.15 \$4.94
11. \$1.41, \$0.96 \$2.37
12. \$1.25, \$1.09 \$2.34
13. 28, 17, 13, 16 74
14. 16, 9, 8, 2 35
15. \$0.08, \$0.29, \$0.02, \$0.37 \$0.76
16. \$0.40, \$0.19, \$0.12, \$0.27 \$0.98

Carrying to Ten's and Hundred's Columns

[o]

	Hundreds	Tens	Ones
	1	1	
	2	8	3
+	2	5	9
	5	4	2

One day Sue's father drove his bus 283 miles. The next day he drove 259 miles. How far did he drive in the two days? $283 + 259 = n$

Study the work in the box. Why do you rename the sum of the ones? It is greater than 9 ones. the sum of the tens? It is greater than 9 tens.

Explain the addition for each example in row 1.

	a	b	c	d	e	f
	1 1	1 1	1 1	1 1	1 1	1 1
1.	425	279	241	87	345	$2.47
	276	552	179	513	198	0.56
	701	831	420	600	543	$3.03

Tell the missing digit for each addition in row 2.

2.	264	174	408	233	259	$0.78
	367	129	193	279	87	1.54
	6?31	3?3	6?1	5?12	3?6	$2.?2
		0	0		4	3

Tell which sums shown for row 3 are wrong.

3.	653	238	419	59	197	$4.08
	57	167	84	395	108	1.98
	700	295	503	344	295	$6.06
	710	405		454	305	

Sometimes you carry more than once.

The only new skill here is to combine renaming the sum of the ones with carrying of a digit to ten's column and renaming the sum of the tens with carrying of a digit to hundred's column.

Teaching Pages 186 and 187

Pupil's Objectives

(a) To learn to carry to both ten's and hundred's columns in adding with 3-place numerals; and (b) to have practice in addition and subtraction computation.

New Word

drove

Background

Your pupils have had experience with renaming 10 ones as 1 ten, and 10 tens as 1 hundred. Adding with 3-place numerals with either carrying to ten's place or to hundred's place has been taught. This should be adequate readiness for examples in which these two skills are combined.

Pre-Book Lesson

• Write these examples on the board:

451	258	147
+273	+310	+628

Ask questions like the following:

a. In which of the examples is it necessary to rename 10 ones? Tell why.

b. Is there an example in which no renaming is necessary? Which one? Tell why.

c. Is there an example that requires renaming 10 tens? Which one? Tell why.

• Ask, "Do you think it is possible to write an example in which both ones and tens must be renamed?" Ask a child to write an example of this type on the board.

Ask, "Is it necessary to rename ones in this example?" Have a child tell why.

"Must tens be renamed in this example?" Have another child tell why.

• Use the *hundreds*, *tens*, and *ones* chart to demonstrate working the example 465 + 279 and write the example on the board in the following form:

465 = 4 hundreds 6 tens 5 ones
+279 = 2 hundreds 7 tens 9 ones

Go through the example with the children somewhat as follows:

a. How many ones are there? (fourteen)

b. How should we rename the ones? (1 ten and 4 ones)

c. With the 1 ten we got by renaming the ones, how many tens are there in all? (fourteen)

d. How should we rename the tens? (1 hundred and 4 tens)

e. How many hundreds are there including the 1 hundred we got by renaming the tens? (seven)

• Then write the example in this form:

100's	10's	1's
4	6	5
+2	7	9

Using this form, lead pupils to a shorter thought process, as follows:

5 ones plus 9 ones = 14 ones. Write "4," carry 1 to ten's column.

(6 + 1) 7 tens plus 7 tens = 14 tens. Write "4," carry 1 to hundred's column.

(4 + 1) 5 hundreds plus 2 hundreds = 7 hundreds.

• Finally, guide pupils to think:

465	5 + 9 = 14.	Write "4," carry 1.
+279	7 + 7 = 14.	Write "4," carry 1.
?	5 + 2 = 7	

Using the Text Pages

You may wish to work the examples in rows 1–3 on the board in connection with the discussion and explanations. Extensive board work would be useful in preparation for the written work.

• The *more capable children* who have complete mastery of the skills and the associated understandings should be permitted to write answers on folded paper for rows 4 and 5 on page 187.

• Emphasize the Commutative Property of Addition as you instruct children to copy Ex. 6–14.

• Children should copy each example at the bottom of page 187 as directed for each set of examples. All of the *carrying* skills which have been taught to date are included in these exercises. Some of the *more capable children* may be permitted to write answers on folded paper.

Individualizing Instruction

• For your *slower learners*, the following activities may be helpful.

a. Copy the examples in which there is no carrying.

527	365	43	450	638	49
+292	+224	+86	+88	+79	+270

b. Copy the examples in which you carry to hundred's column only. (10 tens are renamed as 1 hundred.)

c. Copy the examples in which you carry to both ten's and hundred's columns.

d. Copy the examples for which the sums are shown wrong. Then do the examples correctly.

85	69	543	316	283	481
+78	+122	+276	+285	+174	+437
155	191	719	591	457	818

e. On the board, write exercises of the following types for renaming practice.

4 hundreds 12 tens 8 ones may be renamed:
2 hundreds 5 tens 16 ones may be renamed:
5 hundreds 17 tens 14 ones may be renamed:
3 hundreds 0 tens 18 ones may be renamed:
6 hundreds 16 tens 16 ones may be renamed:

• *All pupils* may do the examples in row 4 by using expanded form. For Ex. a:

$$\begin{array}{rcl} 736 & = & 700 + 30 + 6 \\ +46 & = & \underline{\qquad\quad 40 + 6} \end{array}$$

Find the sum by using the following steps:

a. Add 6 and 6.

b. Add 30 and 40.

c. Show 700.

d. 700 + 70 + 12 = ?

• Ask *more capable children* to write two examples with 2-place, 3-place, or mixed 2- and 3-place numerals in which they must

a. carry not at all.

b. carry to ten's column only.

c. carry to hundred's column only.

d. carry both to ten's and hundred's columns.

• You may wish to have selected *more capable children* extend all addition skills developed thus far to adding with 4-place and 5-place numerals.

• Use Extra Examples Set 61 with *all pupils* as needed.

• For the work at the bottom of page 187, *more capable children* may do the following:

For Ex. 1–12, show a sum 60 greater than the sum you found for each example. For Ex. 13–24, show an addend 80 less than the addend you found for each example.

NOTES

[W]

Copy each example in rows 4 and 5 and add. You may have to rename and carry more than once.

	a	b	c	d	e	f	g
4.	736	567	408	509	78	129	$0.27
	46	134	392	238	596	585	4.77
	782	701	800	747	674	714	$5.04
5.	190	444	89	567	636	663	$3.93
	428	228	709	280	57	294	1.59
	618	672	798	847	693	957	$5.52

Copy each of Ex. 6–14 in a column and add.

6. 726 + 53 779 **9.** 260 + 640 900 **12.** $2.16 + $3.62 $5.78

7. 78 + 739 817 **10.** 175 + 824 999 **13.** $2.69 + $2.53 $5.22

8. 126 + 386 512 **11.** 103 + 587 690 **14.** $7.43 + $0.35 $7.78

◆ **Extra Examples.** Work Set 61.

To Keep in Practice

A. and S. [W]

1–6. Add 346 to each of these numbers:

407 753 273 619 469 815 298 644 175 521 340 686

7–12. Add 239 to each number shown above.

646 512 708 537 414 579

13–18. From each of these numbers, subtract 57:

480 692 379 586 997 870

423 635 322 529 940 813

19–24. From each number shown above, subtract 266.

214 426 113 320 731 604

Copy each set of numerals in a column. Add and check.

25. 20, 16, 8, 19 63 **28.** 41¢, 6¢, 8¢ 55¢ **31.** 58, 0, 20, 4 82

26. 12, 30, 42, 5 89 **29.** 22¢, 7¢, 4¢ 33¢ **32.** 26, 32, 28, 13 99

27. 37, 9, 18, 24 88 **30.** 42¢, 12¢, 9¢ 63¢ **33.** 48, 9, 9, 28 94

If . . ., Then . . .

Logic [W]

Work each example, using the number for n. Then write the whole example with the answer.

For Ex. 1a write, "427 − 219 = 208."

For Ex. 1b write, "118 + 309 = 427."

1. If $n = 427$,
   a. $n - 219 = ?$ 208
   b. $118 + _?_ = n$ 309

2. If $n = 246$,
   a. $471 - n = ?$ 225
   b. $n = 123 + _?_$ 123

3. If $n = 558$,
   a. $n - 39 = ?$ 519
   b. $977 - n = ?$ 419

4. If $n = 347$,
   a. $234 + n = ?$ 581
   b. $750 - n = ?$ 403

5. If $n = 755$,
   a. $n - 706 = ?$ 49
   b. $137 + _?_ = n$ 618

6. If $n = 671$,
   a. $n - 509 = ?$ 162
   b. $890 - n = ?$ 219

7. If $n = 386$,
   a. $n = 149 + _?_$ 237
   b. $n - 218 = ?$ 168

8. If $n = 149$,
   a. $39 + _?_ = n$ 110
   b. $n + 258 = ?$ 407

9. If $n = 860$,
   a. $n - 159 = ?$ 701
   b. $_?_ + 133 = n$ 727

Oral Work with Problems

Discovering missing data and extra data [O]

In each of Ex. 1–4 tell which number you do not need to use in solving the problem.

1. Jim is 8 years old and Sam is 17 years old. How much younger than Sam will Jim be in 10 years? 10 9 years

2. Jane is 9 years old and Sue is 11. How old will Jane be in 10 years? 11 19 years

3. Jack is 48 inches tall. Bob is 6 inches taller and Mike is 2 inches shorter than Jack. How tall is Mike? 6 46 inches

4. Sam had 75¢. He spent 30¢ in one store and 15¢ in another store. How much did he spend? 75¢ 45¢

Teaching Pages 188 and 189

Pupil's Objectives

(a) To have addition and subtraction practice in *if-then* exercises; (b) to have practice in reading problem information carefully to find missing data or unnecessary data; and (c) to have practice in renaming sums shown with 3-place numerals in preparation for subtraction with renaming the sum.

Background

The use of logic as a means of establishing greater precision and clarity in mathematical thought has been developed throughout the MWN program in the form of *if-then* exercises. In the exercises at the top of page 188, "then" has been omitted; however, the word "then" should be used when reading the exercises orally and children should be encouraged to say "then" when they are writing the work.

The problems at the bottom of page 188 contain information which is not essential to finding the solution to the problem. The initial purpose in reading these problems is to find the irrelevant data. At the top of page 189 are some problems in which essential information is missing. Each of these skills has been developed in previous lessons.

Renaming hundreds, tens, and ones is essential in performing all operations. This renaming is fundamental to building full understanding of the decimal system of notation. The renaming exercises at the bottom of page 189 should prove useful in reviewing the decimal nature of our numeration system as well as preparing the children for renaming in subtraction.

Pre-Book Lesson (page 188, top)

Review *if-then* thinking. Begin with an example such as: If $n = 7$, then $n - 4 = ?$ This would be an appropriate opportunity to review the addends-sum relationship.

(a) When the addends are known, we find the sum by adding.

(b) When the sum and one addend are known, we find the other addend by subtracting.

Using the Text Page (page 188, top)

You may wish to have the pupils write the n-sentence ($118 + 427 = n$); then, show their work; and, finally write the complete sentence ($118 + 427 = 545$).

$$\begin{array}{r} 118 \\ +427 \\ \hline 545 \end{array}$$

Pre-Book Lesson (page 188, bottom; page 189, top)

• Consider a problem using local information similar to the following:

In Room 18 there are 15 boys and 16 girls. How many children are there in all?

Discuss the information that is given in the problem. Have selected pupils tell which information is necessary for solving the problem. Then tell which information is unnecessary for solving the problem.

• Consider problems in which essential information is missing. Conduct guided discussion about the missing data.

Using the Text Page (page 188, bottom; page 189, top)

• In problems 1–4, children are to tell what information given is not essential for solving the problem. Ask pupils to tell why.

• Problems 5–7 cannot be solved until additional information is given. Have different children tell what information is needed. Ask several children to tell how they would complete the problem so that it can be solved.

Pre-Book Lesson (page 189, bottom)

• Write hundreds, tens, ones on the chalkboard as shown below:

hundreds tens ones

a. Ask a child to write a 3-place numeral with a digit written before each of the three words on the board.

b. Ask another child to read the numeral.

c. Ask if anyone can rename the number shown to get more tens. For example, if 475 is written on the board, and renamed as follows: 4 hundreds 7 tens 5 ones = 3 hundreds 17 tens 5 ones.

• Each time a number is renamed, ask questions of the children to determine their understanding.

• Have several different 3-place numerals written and then rename the numbers shown. Show possible renamings for each number.

Using the Text Page (page 189, bottom)

Stress the importance of this type of renaming for subtraction.

Individualizing Instruction

• Some *slower learners* may need easier examples for the *if-then* work. Perhaps some children need more practice with addition and subtraction facts. You may wish to ask a *more capable child* to write or dictate *if-then* sentences limited to the basic facts.

• *More capable children* may be permitted to write *if-then* number sentences and questions in any way they wish. Encourage creativity in thought and expression.

• Extend practice in using problems similar to those at the bottom of page 188 and at the top of page 189. *All pupils* may write problems with data missing or irrelevant information present.

• The renaming exercises at the bottom of page 189 may be performed orally with the *slower learners*. Perhaps it would be more effective with these children to use a hundreds, tens, and ones chart which will permit the manipulation of *numeral* cards to show what happens when renaming occurs.

NOTES

You cannot solve problems 5–7. In each problem tell what is missing.

5. Ruth is 18. How much older is she than Helen? Helen's age

6. Mr. Good is 69 inches tall. How much taller is Mr. Good than Mrs. Good? Mrs. Good's height

7. John earned 20¢ more than Joe earned last week. How much did John earn? Amount Joe earned

Say the answers for problems 1–7. Where numbers are missing, make up some of your own. Answers will vary.

Renaming to Show More Tens

[O]

1. 423 = 4 hundreds and 2 tens and 3 ones, or
3 hundreds and 12 tens and 3 _?_ ones.

2. 602 = 6 _?_ hundreds and 0 tens and 2 _?_ ones, or
5 hundreds and 10 tens _?_ and 2 _?_ ones.

3. 596 = 5 hundreds _?_ and 9 _?_ tens and 6 ones _?_, or
4 hundreds and 19 _?_ tens and 6 _?_ ones.

[W]

In examples when we cannot subtract the tens, we must rename the sum to show 10 more tens. Copy and finish each of Ex. 4–6 to show 10 more tens.

4. 208 = 1 hundred and 10 _?_ tens and 8 _?_ ones

5. 623 = 5 hundreds and 12 _?_ tens and 3 _?_ ones

6. 982 = 8 _?_ hundreds and 18 tens _?_ and 2 _?_ ones

For Ex. 7–11, work as in Ex. 4–6.

7. 618 **8.** 752 **9.** 350 **10.** 506 **11.** 184

618 = 5 hundreds and 11 tens and 8 ones
752 = 6 hundreds and 15 tens and 2 ones
350 = 2 hundreds and 15 tens and 0 ones
506 = 4 hundreds and 10 tens and 6 ones
184 = 18 tens and 4 ones

More Renaming of Sums for Subtracting

To show more tens [O]

A

Hundreds	Tens	Ones
4	2	5
−	8	5
?	?	0

B

$$\begin{array}{r} {}^{3}\,{}^{(12)} \\ \not{4}\,\not{2}\,5 \\ -\;\;8\;5 \\ \hline 3\;4\;0 \end{array}$$

C

$$\begin{array}{r} {}^{3}\,{}^{(12)} \\ \not{4}\,\not{2}\,5 \\ 1\;3\;0 \\ \hline 2\;9\;5 \end{array}$$

1. Mr. Brown had an order for 425 bags of potatoes. He had 85 bags ready to sell. How many bags did he still have to get ready? $425 - 85 = n$

Ex. A. Can you subtract the ones? Yes the tens? No Why or why not? Not enough tens shown.

Ex. B. Explain the renaming. 425 = 3 hundreds and 12 tens and 5 ones

Subtract ones: $5 - 5 = ?$ 0

Subtract tens: $12 - 8 = ?$ 4 *Think,* "12 − 8 = 4." Write "4" in ? ten's place.

Subtract hundreds: Write "3" in hundred's place. 3

2. After two days, only 130 of the 425 bags still had to be filled with potatoes. How many bags of potatoes were ready to sell? $425 - 130 = n$

Explain the subtracting in Ex. C. Why did we rename?

Tell how to check Ex. C. 5 − 0 = 5 Write 5 in one's place. 12 − 3 = 9 Write 9 in ten's place. 3 − 1 = 2. Write 2 in hundred's place.

295 + 130 = 425 (addend plus addend equals sum)

When you cannot subtract the tens, you rename the sum to show more tens.

Teaching Pages 190 and 191

Pupil's Objective

To learn that 1 hundred can be renamed 10 tens to make it possible to subtract.

Background

Your pupils have had enough practice to assure readiness for the new skill of renaming a hundred. They have renamed ones, tens, and hundreds. They have *carried* to ten's place and to hundred's place. They have renamed a ten. They are familiar with the notion that sometimes numbers must be renamed before an operation can be completed.

Pre-Book Lesson

• Consider a problem similar to the following in which only 1 hundred must be renamed:

Teddy wants to find 250 match covers. He now has 170. How many more does he need to find?

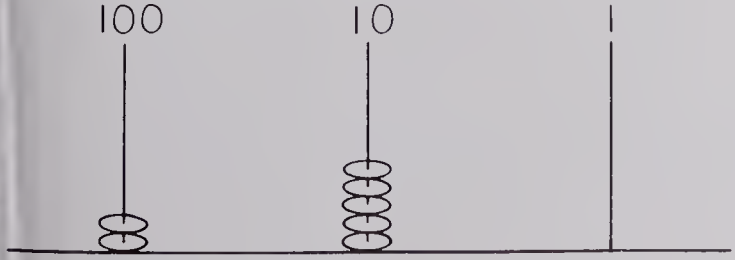
CAUTION: Some children will be able to solve problems of this type mentally using many different patterns of solution. Be prepared to accept thinking of this type: "He needs 30 more to make 200, and 50 more to make 250. 30 plus 50 is 80."

• Engage in a discussion about appropriate number sentences to express the problem. Ultimately, a subtraction example should be written with 3-place numerals:

$$\begin{array}{r} 250 \\ -170 \\ \hline \end{array}$$

• Use an appropriate place-value teaching aid* to demonstrate the renaming that is necessary when subtraction is involved. For example, the example under consideration should be written on the board and demonstrated on the abacus as follows:

a. Show the sum 250 on the abacus.



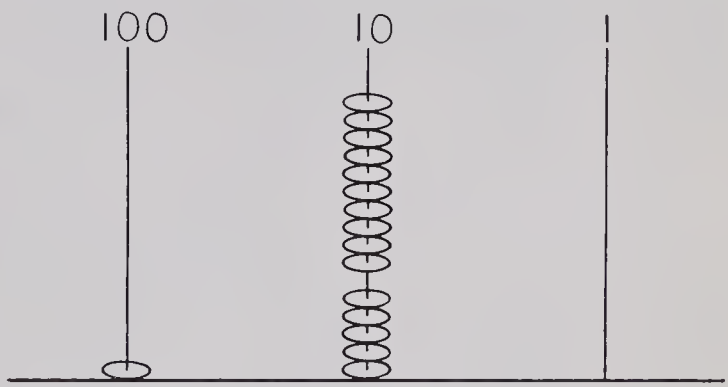
* See 10 and 21, page xix.

CAUTION: For a subtraction example, only the sum should be shown on the teaching aid. The known addend should *not* be shown. The known addend is kept in mind for the demonstration in order to tell how many of each position are to be removed. By showing the known addend the situation becomes confusing to the child when the removing takes place.

b. Ask,

How many ones are we to subtract?
How many tens are we to subtract?
Do we have enough tens to subtract 7 tens?
Where can we get some more tens?

c. Upon a child's suggestion that one of the hundreds may be renamed 10 tens, have a child demonstrate the renaming on the abacus.

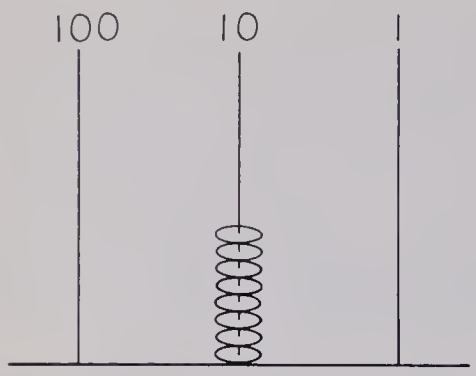


d. Illustrate the renaming of 1 hundred with the example written on the chalkboard. Use the same questioning as used with the abacus. Cross out the 2 to indicate that the hundreds have been renamed. Write 1 above the 2 to show that there is now 1 hundred. The renamed hundred now becomes 10 tens. Add 10 tens to the 5 tens and write (15) above the 5.

$$\begin{array}{r} 1\,(15) \\ \not{2}\,\not{5}0 \\ -\,1\,7\,0 \\ \hline \end{array}$$

e. Remove 7 tens from the abacus. Show the subtracting of 7 tens from 15 tens in the example on the board. Be sure to point out the answer as shown both on the board and by the 8 tens left on the abacus.

f. Remove the 1 hundred on the abacus. Subtract hundreds in the example on the board.



$$\begin{array}{r} {}^{1}\cancel{2}\,{}^{(15)}\cancel{5}0 \\ -\,1\,7\,0 \\ \hline 8\,0 \end{array}$$

• If necessary, use the above procedure with other examples before going to the book.

Using the Text Pages

• Guide youngsters to state in their own words the generalization at the bottom of page 190.

• Permit all children to write the *crutches* which show the result of renaming. However, pupils should be encouraged to write the subtraction result without using the *crutches* if they are able to do so. Do not be so anxious to eliminate the use of the *crutches* that understanding and mastery of the skill is jeopardized.

Individualizing Instruction

• *Slower learners* may profit from recording words and computations similar to the following:

$$\begin{array}{rcl} 250 & = & 2 \text{ hundreds } 5 \text{ tens } 0 \text{ ones} = 200 + 50 + 0 \\ \underline{-170} & = & \underline{1 \text{ hundred } 7 \text{ tens } 0 \text{ ones}} = \underline{100 + 70 + 0} \end{array}$$

after renaming

$$\begin{array}{rcl} 1 \text{ hundred } 15 \text{ tens } 0 \text{ ones} & = & 100 + 150 + 0 = {}^{1}\cancel{2}\,{}^{(15)}\cancel{5}0 \\ \underline{1 \text{ hundred } 7 \text{ tens } 0 \text{ ones}} & = & \underline{100 + 70 + 0} = \underline{1\,7\,0} \\ & & 8\,0 \end{array}$$

• *Slower learners* may need more practice in renaming of the type provided at the bottom of pages 183 and 189.

• Use Extra Examples Set 62 with *all pupils* as needed.

NOTES

Ex. 3–7. Explain the renaming of the sums.

3. $\overset{8}{\cancel{9}}\overset{\textcircled{10}}{\cancel{0}}7$	**4.** $\overset{4}{\cancel{5}}\overset{\textcircled{11}}{\cancel{1}}3$	**5.** $\overset{3}{\cancel{4}}\overset{\textcircled{13}}{\cancel{3}}8$	**6.** $\overset{6}{\$\cancel{7}}.\overset{\textcircled{16}}{\cancel{6}}0$	**7.** $\overset{2}{\$\cancel{3}}.\overset{\textcircled{17}}{\cancel{7}}7$
694	483	86	4.80	0.96
213	30	352	$2.80	$2.81

Row 8. Tell each missing hundred's digit.

	a	b	c	d	e	f	g
8.	348	180	765	387	566	625	$4.29
	192	70	259	196	423	273	3.73
	?56	?10	?06	?91	?43	?52	$?.56
	1	1	5	1	1	3	0

Row 9. Which subtractions are wrong? Why?

Sums in Ex. a, b, e, and f were not correctly renamed.

	a	b	c	d	e	f	g
9.	648	347	859	517	726	438	$9.43
	60	184	722	395	84	182	2.72
	688	263	137	122	742	356	$6.71
	588	163			642	256	

[W]

Rows 10 and 11. Copy and subtract. Check.

	a	b	c	d	e	f	g
10.	847	235	419	778	535	936	$4.19
	57	173	238	293	164	445	0.84
	790	62	181	485	371	491	$3.35
11.	468	256	538	359	728	329	$3.75
	396	186	251	265	95	257	1.82
	72	70	287	94	633	72	$1.93

Copy each of Ex. 12–20 and subtract. Be careful! For some examples you do not rename the sum.

12. 529 − 77 452	**15.** 427 − 257 170	**18.** $4.53 − $1.47 $3.06
13. 796 − 156 640	**16.** 764 − 48 716	**19.** $6.09 − $4.09 $2.00
14. 915 − 23 892	**17.** 513 − 451 62	**20.** $3.68 − $3.59 $0.09

◆ **Extra Examples.** Work Set 62.

Renaming More than Once for Subtracting

To show more ones and more tens [O]

A

Hundreds	Tens	Ones
3	4	2
−1	7	5
?	?	?

B

$$\begin{array}{r} {}^{3}\,{}^{(12)} \\ 3\,\not{4}\,2 \\ 1\,7\,5 \\ \hline 7 \end{array}$$

C

$$\begin{array}{r} {}^{2}\,{}^{(13)}\,{}^{(12)} \\ \not{3}\,\not{4}\,\not{2} \\ 1\,7\,5 \\ \hline 1\,6\,7 \end{array}$$

1. Our school gets 342 bottles of milk a day. One milkman brings 175 bottles. How many does the other milkman bring? $342 - 175 = n$

Ex. A. Can you subtract the ones? No ∧ the tens? No ∧ Why or why not? There are neither enough ones nor enough tens.

* Ex. B. Can you subtract the ones now? Yes ∧ Why? There are enough ones. ∧ Can you also subtract the tens? No ∧ Why or why not? There are not enough tens.

* Ex. C. What has been done here? Hundreds have been renamed hundreds and tens. ∨ Can you subtract the tens now? Yes ∨ Explain. There are enough tens.

Ex. 2–5. Explain the renaming and the subtracting.

2. $\begin{array}{r} {}^{7}\,{}^{(12)}\,{}^{(16)} \\ \not{8}\,\not{3}\,\not{6} \\ 5\,8\,9 \\ \hline 2\,4\,7 \end{array}$

3. $\begin{array}{r} {}^{8}\,{}^{(12)}\,{}^{(10)} \\ \not{9}\,\not{3}\,\not{0} \\ 6\,8\,6 \\ \hline 2\,4\,4 \end{array}$

4. $\begin{array}{r} {}^{3}\,{}^{(16)}\,{}^{(18)} \\ \not{4}\,\not{7}\,\not{8} \\ 9\,9 \\ \hline 3\,7\,9 \end{array}$

5. $\begin{array}{r} {}^{4}\,{}^{(14)}\,{}^{(17)} \\ \$\not{5}.\not{3}\,\not{7} \\ 0.5\,8 \\ \hline \$4.9\,9 \end{array}$

Sometimes when you subtract you rename more than once.

Ex. 6–10. Which subtractions are wrong? Why?

Sums for Ex. 6, 8 and 9 were not correctly renamed.

6. $\begin{array}{r} 614 \\ 77 \\ \hline 647 \\ 537 \end{array}$

7. $\begin{array}{r} 543 \\ 454 \\ \hline 89 \end{array}$

8. $\begin{array}{r} 739 \\ 205 \\ \hline 524 \\ 534 \end{array}$

9. $\begin{array}{r} 912 \\ 436 \\ \hline 486 \\ 476 \end{array}$

10. $\begin{array}{r} \$8.14 \\ 1.99 \\ \hline \$6.15 \end{array}$

Sum for Ex. 8 should not have been renamed.

Ex. 11–15. Tell the missing digits in each answer.

11. $\begin{array}{r} 880 \\ 674 \\ \hline ??6 \\ 20 \end{array}$

12. $\begin{array}{r} 721 \\ 24 \\ \hline ??7 \\ 69 \end{array}$

13. $\begin{array}{r} 840 \\ 588 \\ \hline ?5? \\ 2\ \ 2 \end{array}$

14. $\begin{array}{r} 572 \\ 299 \\ \hline ??3 \\ 27 \end{array}$

15. $\begin{array}{r} \$6.25 \\ 1.97 \\ \hline \$?.?? \\ 4\ \ 28 \end{array}$

16. In which of Ex. 11–15 did you rename the sum to show more ones and more tens? Ex. 12-15

* Renaming the sum to show both enough ones and enough tens is considered difficult, so pupils must think of the renaming one step at a time. For some time, most children may want to use the crutch of showing the renaming.

Teaching Pages 192 and 193

Pupil's Objectives

(a) To learn to subtract in examples when both tens and hundreds must be renamed; and (b) to have oral practice in working addition and subtraction examples.

Pre-Book Lesson

• On the board, write 463 − 248 (in vertical form). Ask a pupil to work the example at the board while other pupils work it on paper. If any pupils have difficulty with the example, give individual help while the others proceed with the next type of subtraction example using 3-place numerals.

• On the board, write 516 − 268 (in vertical form). Have one child work the example at the board while the others do the work on paper at their desks.

• Compare the two examples to see how they are different.

• If there is difficulty at any point, ask questions to help children identify or locate the trouble spot. Guide them in such a way that they will suggest what is to be done.

• Responses to the second example and to Ex. 2–5 on page 192 in the pupil's book should follow this model.

Ones: There are not enough ones to subtract 7. Rename tens as tens and ones. Show the tens by crossing out the 4 in ten's column and placing a small 3 (for 3 tens) above it. Add the 10 ones and the 5 ones shown in one's column. Write 15 in one's column and ring it to show that it means "15" ones. Subtract 7 ones from 15 ones. Write 8 in one's column in the answer.

$$\begin{array}{r} 5\,\overset{13}{\cancel{4}}\,\overset{15}{5} \\ -\,2\,8\,7 \\ \hline 3\,5\,8 \end{array}$$

Tens: There are not enough tens to subtract 8 tens. Rename hundreds as hundreds and tens. Show the hundreds by crossing out the 6 in hundred's column and writing a small 5 (for 5 hundreds) above it. Add the 10 tens and the 3 tens shown in ten's column and ring the 13 to show that it means "13" tens. Subtract 8 tens from 13 tens. Write 5 in ten's place in the answer.

Hundreds: Subtract 2 hundreds from 5 hundreds. Write 3 in hundred's place in the answer.

Using the Text Pages

• If *slower learners* have difficulty with Ex. 1–5 they should be given help as suggested below in Individualizing Instruction before they try Ex. 6–16.

• Some children may not be ready to proceed with the written work. Circulate among the pupils to determine individual sources of difficulty. Frequently, just a word or a demonstration for the individual is adequate provision for his specific needs.

• Some children will find the Oral Practice too difficult to do mentally. *More capable children* may listen to each other say the answers while the others write the examples showing their work and the answers on paper.

Individualizing Instruction

• *Slower learners* may need more practice in renaming through the use of objects and place-value teaching aids. Use pocket charts, line abacus, Ginn Arithmé-Sticks, and so on, to show the renaming that is necessary and how the physical model looks.

• *Slower learners* may need more work recording words and computations similar to the following:

$$\begin{array}{rcl} 325 & = & 3 \text{ hundreds } 2 \text{ tens } 5 \text{ ones} = 300 + 20 + 5 \\ -136 & = & 1 \text{ hundred } 3 \text{ tens } 6 \text{ ones} = 100 + 30 + 6 \end{array}$$

after renaming

$$\begin{array}{rcl} 2 \text{ hundreds } 11 \text{ tens } 15 \text{ ones} & = & 200 + 110 + 15 \\ 1 \text{ hundred } \quad 3 \text{ tens } \quad 6 \text{ ones} & = & 100 + \quad 30 + \quad 6 \\ \hline 1 \text{ hundred } \quad 8 \text{ tens } \quad 9 \text{ ones} & & 100 + \quad 80 + \quad 9 = 189 \end{array}$$

• It may help some of the children to make use of a teaching aid to explain the work in Ex. 2–5 on page 192.

• If it helps, permit children to use the place-value device to help in working rows 17 and 18 on page 193.

• *All pupils* may use the following exercises as needed.

a. Tell the missing digits.

571	620	734	453	826	338
−378	−546	−247	−66	−557	−289
1?3	?74	4?7	??7	?6?	??9

b. Which subtractions are wrong? Why?

353	527	423	840	932	645
−264	−348	−345	−356	−745	−246
189	189	78	594	187	409

• Use Extra Examples Set 63 with *all pupils* as needed.

NOTES

[W]

Rows 17 and 18. Copy and subtract. Check.

	a	b	c	d	e	f	g
17.	577	453	828	532	741	930	$6.43
	479	189	39	278	356	445	5.87
	98	264	789	254	385	485	$0.56
18.	781	631	162	720	656	513	$4.70
	485	358	93	246	98	326	0.98
	296	273	69	474	558	187	$3.72

Ex. 19–30. Copy and subtract. For some examples you do not rename. For others you rename once or more.

19. 955 − 778 177 **23.** 889 − 326 563 **27.** $3.29 − $0.83 $2.46

20. 864 − 566 298 **24.** 760 − 49 711 **28.** $9.88 − $8.99 $0.89

21. 692 − 59 633 **25.** 783 − 474 309 **29.** $9.99 − $1.45 $8.54

22. 820 − 677 143 **26.** 699 − 387 312 **30.** $8.80 − $2.83 $5.97

Extra Examples. Work Set 63.

Oral Practice

A. and S.; mental procedures [O]

Work these examples in your head and say only the answers. For Ex. 1a *think*, "117 − 8 = 109." Say, "109."

	a	b	c	d
1.	117 − 8 109	93 − 7 86	526 − 7 519	981 + 9 990
2.	789 − 8 781	7 + 42 49	800 − 300 500	426 − 9 417
3.	9 + 110 119	30 + 40 70	215 − 6 209	8 + 345 353
4.	88 − 8 80	611 − 7 604	6 + 73 79	782 − 8 774
5.	400 − 8 392	214 − 9 205	324 + 6 330	76 − 7 69
6.	564 − 4 560	215 + 9 224	70 − 50 20	8 + 94 102

To Keep in Practice

A. and S. [W]

Ex. 1–6. Copy each set of numerals in a column. Add.

1. 39, 26, 28 93
2. 19, 37, 38 94
3. 12, 44, 8, 19 83
4. 3, 55, 4, 17 79
5. 23¢, 4¢, 8¢, 55¢ 90¢
6. 48¢, 10¢, 4¢, 20¢ 82¢

Rows 7–9. Write answers on folded paper. Check.

	a	b	c	d	e	f	g
7.	410	716	89	342	925	\$8.47	\$0.80
	−74	−386	+79	+575	−78	−3.57	−0.29
	336	330	168	917	847	\$4.90	\$0.51
8.	457	324	625	83	82	\$1.39	\$6.53
	−59	+183	−270	+69	+476	−0.46	−5.78
	398	507	355	152	558	\$0.93	\$0.75
9.	515	93	827	78	475	\$0.74	\$1.41
	+364	+780	−38	−57	−187	+0.89	−0.95
	879	873	789	21	288	\$1.63	\$0.46

Extra Examples. Work Set 64.

Many Things

Maintenance [W]

Ex. 1–3 show numbers for equivalent sets. Copy and finish. Then show another way to find each answer.

1. 2 + 2 + 2 + 2 = ? 8 (four 2's = 8)
2. two 5's = ? 10 (5 + 5 = 10)
3. 6 + 6 + 6 = ? 18 (three 6's = 18)

Write four other names for the number

4. 35.
5. 60.
6. 27.
7. 46.
8. 59.

Answers will vary.

Write our numeral for 9. IV. 4 10. VIII. 8 11. XI. 11

Copy and finish Ex. 12 and 13.

12. 17 tens + 9 tens = 2 _?_ (hundreds) and _?_ (6) tens
13. 867 = 7 _?_ (hundreds) and _?_ (16) tens and 7 _?_ (ones)

Teaching Pages 194 and 195

Pupil's Objectives

(a) To have practice in all the skills in addition and subtraction that have been taught to date; (b) to check understandings of ideas studied to this point; (c) to have practice with units of measurement that have been learned.

Background

Practice must be provided to maintain skills already developed. Maintenance practice also affords the opportunity to diagnose sources of difficulty of individual children.

The enrichment material at the bottom of page 195 presents a magic square. Encourage the more capable children to look for all the different ways of showing four addends whose sum is 34. Children should be encouraged to try things, be creative in seeking patterns, relationships, or similarities that exist. Facility with mathematics permits one to become more flexible rather than restrictive in thought.

Using the Text Page (page 194, top)

With *slower learners*, you may wish to check their work after they have completed each row or set of exercises. If certain errors are persistent, give the appropriate remedial assistance before assigning more work.

Using the Text Pages (pages 194–195)

• Before assigning the written work for Many Things, you may wish to review the following:

1 ft. = 12 in.	1 hr. = 60 min.
1 lb. = 16 oz.	1 doz. = 12 things
1 yd. = 3 ft.	1 wk. = 7 days

• As pupils are doing this written work, circulate among the class to observe which pupils are having difficulty with any of this maintenance work. Give individual help as needed.

• The enrichment material appearing at the bottom of page 195 presents the idea of many names for a number in a different setting—namely, the magic square. *More capable children* will work rapidly with this material and will find many different combinations whose sum is 34. Commuting the addends such as 7 + 14 + 9 + 4 to 4 + 9 + 7 + 14 should not count as a different combination.

Individualizing Instruction

• *Slower learners* may need renaming practice similar to the following:

4 hundreds 12 tens may be renamed _ ? _ hundreds 2 tens.

5 hundreds 4 tens may be renamed 4 hundreds _ ? _ tens.

6 hundreds 3 tens may be renamed _ ? _ hundreds 13 tens.

3 tens 6 ones may be renamed 2 tens _ ? _ ones.

• You may wish to duplicate more exercises of the type presented in Ex. 1–3 at the bottom of page 194. This is good readiness material for multiplication.

a. 3 + 3 + 3 = ?	f. 9 + 9 + 9 = ?
b. three 4's = ?	g. 4 + 4 + 4 = ?
c. two 8's = ?	h. 5 + 5 + 5 + 5 = ?
d. 7 + 7 + 7 = ?	i. two 6's = ?
e. 2 + 2 + 2 + 2 + 2 = ?	j. four 3's = ?

• You may wish to duplicate more exercises of the type dealing with measurement at the top of page 195.

a. 1 wk. and 5 da. = _ ? _ da.
b. 1 ft. and 7 in. = _ ? _ in.
c. 1 lb. and 6 oz. = _ ? _ oz.
d. 1 ft. and 3 in. = _ ? _ in.
e. 1 hr. and 15 min. = _ ? _ min.
f. 1 yd. and 1 ft. = _ ? _ ft.
g. 1 doz. and 6 things = _ ? _ things
h. 1 wk. and 8 da. = _ ? _ da.
i. 1 lb. and 15 oz. = _ ? _ oz.
j. 1 hr. and 25 min. = _ ? _ min.

• Assign Extra Examples Set 64 to *all pupils* as needed.

• You may wish to use the enrichment material at the bottom of page 195 with *all pupils*. Direct the *more capable children* to do the following exercises:

11	16	5	18
6	17	12	15
20	7	14	9
13	10	19	8

a. Find the sums for the numbers shown in each row; in each column; along each diagonal.

b. Compare the number shown in the upper left corner with the number shown in the corresponding corner of the first magic square on the text page. Make other comparisons between corresponding numbers of these two magic squares.

c. Have pupils construct their own magic squares by adding 10 to each number; adding 100; and so on.

d. Have pupils discuss each of the magic squares they construct.

• You may use the following exercise with the *more capable children.*

For each example in rows 7–9 on page 194, write a new answer. To find the new answer, subtract 95 from the answer you found and then add 200 to that number.

• Use Extra Activity Set 99 with the *more capable children.*

NOTES

Copy and finish Ex. 14–21.

14. $\frac{1}{2}$ doz. = 6 things

15. 1 doz. and 3 things = 15 things

16. 1 lb. and 10 oz. = 26 oz.

17. 1 hr. and 45 min. = 105 min.

18. 1 ft. and 8 in. = 20 in.

19. 1 yd. and 2 ft. = 5 ft.

20. 1 wk. and 3 da. = 10 da.

21. 1 lb. and 5 oz. = 21 oz.

22. Copy the examples in which you carry two times.

467	375	618	485	872	563	983	758
+249	+205	−468	+69	−604	+227	−254	+156

467 + 249, 485 + 69, 758 + 156

23. Copy the examples above in which you rename to show more ones. 872 - 604, 983 - 254

Do You Like to Try New Things?

Enrichment [W]

The figure at the right is a magic square.

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

1. Copy the numerals in the first row. Add the numbers. $7 + 12 + 1 + 14 = ?$ 34

2. Copy the numerals in the first column. Add the numbers. $7 + 2 + 16 + 9 = ?$ 34

3. Write all the other four-addend A. examples with sums of 34 in the magic square. For example, use the four numerals in the corners.

Answers will vary. Pupils may find more than 20 different combinations.

4. Does each of your A. examples give a different name to 34? Yes

5. Now copy and finish the magic square at the right.

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

● **Extra Activity.** Work Set 99.

More capable children will find work with magic squares quite intriguing.

* Renaming can be done in one step when the 3-place numeral has 0 in ten's place since the pupil can think of the hundreds, tens, and ones simply as tens and ones.

* Renaming When the Ten's Digit Is 0

To show 10 more ones in the sum [O]

1. John has read 158 of the 306 pages in his book. How many more pages has he to read? $306 - 158 = n$

A	B
306 158 ?	2 9 (16) ~~3 0 6~~ 1 5 8 1 4 8

Ex. A. Can you subtract the ones? Why must you rename the sum? No To show more ones

Ex. B. Explain the renaming and the work.

306 = 30 tens and 6 ones, or
29 tens and 16 ones.

C	D
205 47 ?	1 9 (15) ~~2 0 5~~ 4 7 1 5 8

2. After reading 47 of the 205 pages in his book, Joe has how many pages still to read?

$205 - 47 = n$

Ex. C. Can you subtract the ones? Why? No ∨ There are not enough ones.

Ex. D. Explain the renaming and the work.
205 = 19 tens and 15 ones; 15 − 7 = 8, 9 − 4 = 5. Write 1 in hundred's place.

E	F
7 9 (14) ~~8 0 4~~ 1 7 8 6 2 6	5 9 (13) ~~6 0 3~~ 3 6 5 2 3 8

3. Explain the subtractions in Ex. E and F.
E 804 = 79 tens and 14 ones; 14 − 8 = 6, 9 − 7 = 2, 7 − 1 = 6
F 603 = 59 tens and 13 ones; 13 − 5 = 8, 9 − 6 = 3, 5 − 3 = 2

In Ex. 4–7, which subtractions are wrong?

4.	5.	6.	7.
801 83 728 718	407 299 108	608 369 249 239	$3.04 0.74 $3.30 $2.30

[W]

Ex. 8–19. Copy and find the answers. Check.

8. 906 − 859 47	12. 705 − 447 258	16. $8.08 − $6.04 $2.04 ∧
9. 503 − 134 369	13. 402 − 225 177	17. $7.80 − $7.63 $0.17 ∧
10. 904 − 877 27	14. 551 − 443 108	18. $2.03 − $0.89 $1.14 ∧
11. 602 − 588 14	15. 902 − 73 829	19. $3.60 − $1.38 $2.22 ∧

◆ **Extra Examples.** Work Set 65.

Teaching Page 196

Pupil's Objective

To learn a special way of renaming when the ten's digit of the numeral for the sum is 0.

Background

On pages 166 and 167 of the pupil's book, children were introduced to the idea of various renamings of numbers shown with 3-place numerals. For example, the number 482 may be renamed:

4 hundreds and 8 tens and 2 ones; or
48 tens and 2 ones; or
4 hundreds and 82 ones; or
482 ones.

Now, consider the number 506. It may be renamed:

5 hundreds and 0 tens and 6 ones; or
50 tens and 6 ones; or
506 ones.

When renaming is necessary to get more ones in subtracting from a sum like 506, it is more convenient to think of this sum as 50 tens and 6 ones. In this way, when 1 ten is renamed 10 ones, 49 tens remain and no more renaming is necessary to get more tens.

$$\begin{array}{r} 49\,(16) \\ \cancel{50}\,6 \\ -\,2\,3\,8 \\ \hline \end{array}$$

The thought process should be something as follows:

Ones: There are not enough ones to subtract 8. Show more ones by renaming 50 tens as 49 tens and 10 ones. Show this by crossing out the 50 and writing 49 above it. Add the 10 ones and the 6 ones. Show this sum by writing 16 in one's column. Ring it to show that it means "*16*" ones. Subtract 8 ones from 16 ones. Write 8 in one's place in the answer.

Tens: Subtract 3 tens from 9 tens. Write 6 in ten's place in the answer.

Hundreds: Subtract 2 hundreds from 4 hundreds. Write 2 in hundred's place in the answer.

Pre-Book Lesson

• Write the example 407 − 158 on the board (in vertical form). Go through each step in working the example, asking pupils questions to foster as much understanding as possible. Be sure to stress the value of thinking of 40 tens, as noted in the Background section.

• Write the following numerals on the board:

a. 602 b. 305 c. 608 d. 406

Have children rename each number shown in the following way: 602 may be renamed 60 tens and 2 ones; and 60 tens and 2 ones may be renamed 59 tens and 12 ones.

In working an example such as 602 − 351, renaming to get more ones isn't necessary.

Using the Text Page

• During oral work, be sure to stress all types of renaming suggested in the previous sections.

• Have children write the *crutches* in Ex. 8–11 to show that they are using the recommended thinking when renaming a ten as 10 ones. *More capable children* may write answers only for Ex. 12–19.

Individualizing Instruction

• While pupils are doing the written work, circulate among them to check on the thinking of individuals when they rename the sum. Later, when 4-place numerals are involved in subtraction, the computation will be much easier if they develop the correct thought patterns now.

• Use bundles of hundreds, tens, and ones to help *slower learners see* the relationship between 20 tens and 2 hundreds and the renaming of 20 tens.

• Use the following examples for *slower learners* if needed:

a. Tell the missing digits.

501	605	704	403	806	308
−378	−546	−247	−66	−557	−289
1?3	?9	4?7	??7	?4?	?9

b. Which subtractions are wrong? Why?

303	507	403	804	902	605
−264	−348	−345	−356	−745	−246
139	169	58	558	157	469

• Use Extra Examples Set 65 with *all pupils* as needed.

Pupil's Objectives

(a) To have practice in choosing the correct *n*-sentence for a problem; and (b) to maintain skill in problem-solving.

New Words

materials, *model*

Background

In these problems, all necessary information is given. The main emphasis in this lesson should be on translating the problem into a number sentence. Children should read the problem carefully so as to understand the situation. (Often it is necessary to state the question in one's own words in order to understand it fully.) When the pupil decides whether he knows two addends and must find a sum or he knows a sum and an addend and must find the other addend, he is ready to choose the correct *n*-sentence for the problem.

Teacher's Preparation

You may wish to use a flannel board* during the Pre-Book Lesson. It is usually motivating to use a variety of materials with this device. Have numerals, operation symbols, *n*, and equals sign** available which will adhere to the flannel board. They may be used for showing different ways of writing a mathematical sentence for a problem.

Pre-Book Lesson

• Write a problem appropriate to the local situation, similar to the one below:

There are 525 children who go to Park School. At Hillside School there are 678 children. How many more children go to Hillside School than Park School?

• Ask a child to show the number of children that go to Park School on the flannel board. Do the same for Hillside School.

• First engage in a general discussion about finding numbers for a total set and its parts. Then point out that it is more difficult to find these numbers when working with a difference situation, such as comparing the numbers of pupils in the two schools. When we try to match members of the larger set with members of the smaller set, we see that some of the members of the larger set cannot be matched. The number for the set of unmatched members becomes the unknown addend, which can be found by subtracting the number for the set of matched members from the number for the total set.

• Ask a child to show a mathematical sentence (*n*-example) for the problem on the flannel board. Discuss the sentence. Is it appropriate?

• Ask if anyone can show a different sentence for the problem. Discuss it for appropriateness. *Slower learners* may need to consider the writing of number sentences with 1-place numerals before attempting to write them with 3-place numerals. Frequently, a relatively difficult problem can be simplified by substituting lesser numbers for the greater numbers given in the problem.

• You may wish to consider other problem situations before proceeding to Using the Text Page.

Using the Text Page

• Ask the children to read each problem and then to copy the correct *n*-example for it.

• Remind your pupils that when writing *n*-examples using symbols for dollars and cents, the *n* is written with the dollar sign ($*n*).

• After the written work has been completed, have pupils explain their choices of *n*= examples.

Individualizing Instruction

• *Slower learners* may be asked to substitute lesser numbers in each problem to help them arrive at an appropriate mathematical sentence. In Ex. 1, for example, one could substitute 5¢ for the $3.25. The child can then think 5¢ + 5¢ as an appropriate solution. After writing "5¢ + 5¢ = *n*¢," he then looks at the two alternatives given to see if one is the same as the model he used.

• Use Problem Set 10 as needed with *all pupils*.

• *More capable children* may write *n*-examples for Problem Set 10.

* See 2, page xix. ** See 9, page xix.

Which *n*-Example?

Problem-solving [W]

Copy the correct *n*-example, or number sentence, for each problem.

1. Ned and Sam each had \$3.25 to buy materials for model airplanes. Together they had _?_. $6.50

$\$3.25 - \$3.25 = \$n$ $\boxed{\$3.25 + \$3.25 = \$n}$

2. The materials for a large model plane cost \$1.89 and those for a small model plane cost \$0.85 less, or \$_?_. $1.04

$\$n = \$1.89 + \$0.85$ $\boxed{\$1.89 - \$0.85 = \$n}$

3. When they had bought everything they needed, Sam found he had 27¢ of his \$3.25 left. He had spent \$_?_. $2.98

$\boxed{\$3.25 - \$0.27 = \$n}$ $\$3.25 + \$0.27 = \$n$

4. Ned spent \$2.98 of his \$3.25. He had \$_?_ left. $0.27

$\boxed{\$3.25 - \$2.98 = \$n}$ $\$3.25 + \$2.98 = \$n$

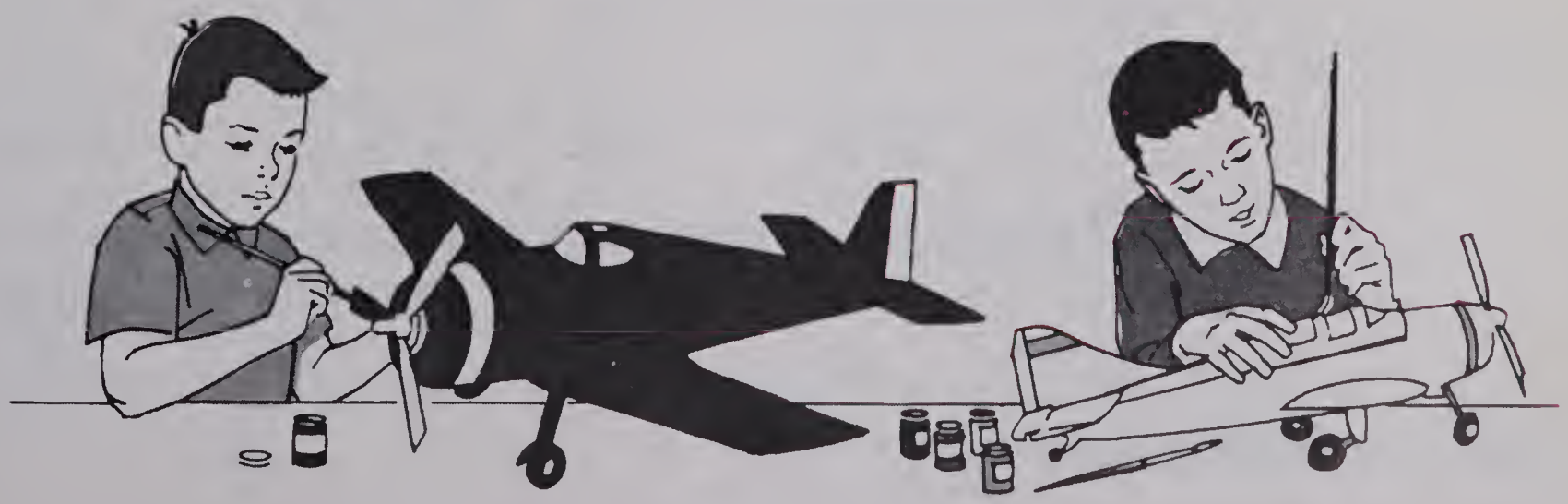
5. One day the small plane flew 235 feet. The big plane flew 306 feet, or _?_ feet more than the small one. 71

$n = 306 + 235$ $\boxed{306 - 235 = n}$

6. Another day the big plane flew 240 feet. The small plane flew 65 feet more, or _?_ feet in all. 305

$240 - 65 = n$ $\boxed{240 + 65 = n}$

♦ **Extra Problems.** Work Set 10.



*0 in Ten's and One's Places

Renaming the sum to show 10 more ones [O]

1. Betsy's father has 300 sheep. He has sheared 174 of the sheep. How many sheep does he still have to shear?

$$300 - 174 = n$$

Ex. A. Can you subtract the ones? No Why must you rename the sum? To show more ones

Ex. B. If 30 tens and 0 ones is one name for 300, why do we rename that number to be 29 tens and 10 ones? So we can subtract ones

Explain the work in Ex. B.
300=29 tens and 10 ones; 10−4=6, 9−7=2, 2−1=1

A	B	C	D	E
300	2 9 (10) ~~3 0 0~~	5 9 (10) ~~6 0 0~~	6 9 (10) ~~7 0 0~~	4 9 (10) ~~$5.0 0~~
174	1 7 4	4 6 8	8 9	2.6 8
?	1 2 6	1 3 2	?	?

2. 600 − 468 = ? Can you subtract or must you rename the sum? You must rename. Explain what has been done in Ex. C.
600=59 tens and 10 ones; 10−8=2, 9−6=3, 5−4=1

3–4. Ex. D and E. Explain what has been done, and why. Now say the answers. D. 611 E. $2.32

Which subtractions in Ex. 5–9 are wrong? Why?
Sums for Ex. 5, 6, 8, and 9 not correctly renamed.

5.	6.	7.	8.	9.
700	$5.00	600	$9.00	$7.00
354	4.36	87	3.52	0.64
356	$0.74	513	$6.58	$6.46
346	$0.64		$5.48	$6.36

10. For which examples must you rename two times? Ex. a-e

a.	b.	c.	d.	e.
652	700	808	520	500
98	78	79	73	146

* Another hard type of renaming unless pupils use the plan shown by thinking of hundreds, tens, and ones simply as tens and ones!

Teaching Pages 198 and 199

Pupil's Objectives

(a) To learn to subtract in examples when 0 occurs in both the one's and ten's places in the numeral for the sum; (b) to practice computation in subtraction with many types of 3-place numerals; and (c) to write and work specific addition and subtraction examples as directed.

New Word

sheared

Background

The method taught in a previous lesson for renaming numbers to make it possible to subtract is equally useful with sums which are hundreds (no tens or ones). In an example like 400 − 157, the child should first think of 400 as 40 tens and 0 ones. To show some ones, 1 of the tens is renamed 10 ones. Consequently, only 39 tens are shown.

Having completed all types of addition and subtraction examples using 2- and 3-place numerals, the children are better able to make applications of their learnings to specific needs. They will not meet any examples within the limits of 3-place numerals for which they have not had necessary preparation.

Pre-Book Lesson

• Write a numeral like 500 on the board (or show it on the flannel board). Ask children to tell all the ways they can of *thinking* the number 500. Elicit the following responses:

5 hundreds and 0 tens and 0 ones
50 tens and 0 ones
500 ones

• Emphasize the importance of thinking of 500 as 50 tens and 0 ones in order to subtract from it. Ask, "If we rename one of the 50 tens, how many tens will be left?" Ask the same question about other numbers like 20 tens, 30 tens, 70 tens.

• Have a child demonstrate on the chalkboard how to show the renaming of the number 500.

In general, the thinking should be something like the following: There are no ones, so we must rename one of the tens. When I take one of the tens there are 49 tens left. This is shown by crossing out the 50 and writing 49 above. The 10 ones are shown in the one's place by writing 10 and placing a ring around it.

• Demonstrate the renaming involved in an example like 500 − 264.

Using the Text Pages

• During the oral lesson, you may wish to have certain pupils work Ex. 1–10 on the board.

• Most children should be required to copy each example in rows 11–12. However, it is possible that some *more capable children* will be able to write the answers only on folded paper.

• Ex. 13–30 may be used as a progress test on skills in subtraction. Before assigning the work, you may ask your pupils to inspect the examples to see if they can find subtraction:

a. with no renaming (Ex. 20, 22)

b. with renaming a ten only (Ex. 13, 21, 23, 29 and possibly include Ex. 17, 19, 24, 25, 27)

c. with renaming a hundred only (none)

d. with renaming a ten and a hundred (Ex. 14, 15, 16, 17, 18, 19, 24, 25, 26, 27, 28, 30). Accept omission of Ex. 17, 19, 24, 25, 27 if listed under *b* above.

• Before assigning the written work at the bottom of page 199, you may wish to write one example of each type on the board to serve as a model for pupils who are uncertain.

Individualizing Instruction

• *Slower learners* may need further renaming practice of the following types:

a. 400 = 40 tens, or
39 tens and _ ? _ ones.

b. 600 = 60 tens, or
_ ? _ tens and 10 ones.

c. _ ? _ = 50 tens, or
49 tens and 10 ones.

• *All pupils* may have practice in renaming of the following type:

Write four other names for the numbers named below: (See Teaching Pages 178–179.)

a. 17 b. 24 c. 45 d. 21

• The results of the Progress Test may be the basis for assigning Extra Examples Set 66 for additional work.

• *More capable children* may use the examples in rows 11 and 12 on page 199 and write another example with the same answer. Ex. 11a could be written 700 − 417 = 699 − 416. This activity checks to see how well these pupils understand the use of compensation in subtraction.

NOTES

[W]

Rows 11 and 12. Copy and subtract. Check.

	a	b	c	d	e	f	g
11.	700	600	800	300	500	$9.00	$7.00
	417	39	748	154	234	0.83	2.26
	283	561	52	146	266	$8.17	$4.74
12.	600	500	700	400	300	$9.00	$2.00
	377	455	98	329	86	6.62	0.11
	223	45	602	71	214	$2.38	$1.89

Ex. 13–30. Copy each example in a column and subtract. Watch 0's, and watch renaming!

13. 570 – 169 401	**19.** 203 – 169 34	**25.** $8.00 – $3.42 $4.58
14. 114 – 38 76	**20.** 607 – 305 302	**26.** $1.14 – $0.47 $0.67
15. 547 – 278 269	**21.** 680 – 579 101	**27.** $7.00 – $1.11 $5.89
16. 722 – 478 244	**22.** 178 – 175 3	**28.** $8.06 – $1.18 $6.88
17. 400 – 174 226	**23.** 940 – 635 305	**29.** $4.80 – $0.43 $4.37
18. 521 – 438 83	**24.** 200 – 27 173	**30.** $7.12 – $2.95 $4.17

◆**Extra Examples.** Work Set 66.

* Making and Working Your Own Examples

A. and S. [W]

Make up and work two examples of each of these kinds. Use two 3-place numerals, sometimes with one or two 0's.

1–2. Addition, carrying to ten's column

3–4. Addition, carrying more than once

5–6. Subtraction, renaming to show more ones

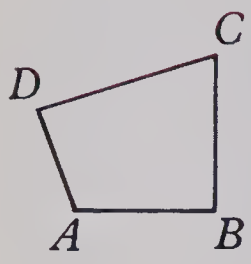
7–8. Subtraction, renaming more than once

* These exercises provide an excellent check on each pupil's level of understanding.

* The geometric ideas taught on pages 120 and 121 are here extended to include refinements. Again, remind pupils that they see pictures of geometric forms, not the forms themselves.

Naming Polygons and Their Sides

Geometry [O]



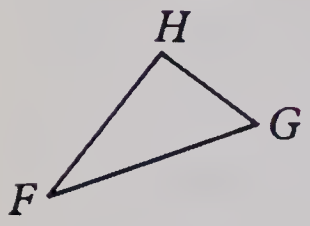
The polygon pictured at the left may be named polygon *ABCD*. Another name for it is polygon *CBAD*.

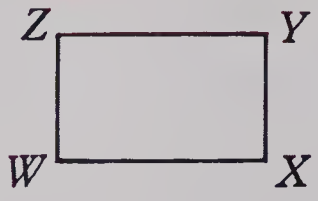
1. What other names can be used for this polygon?
DABC, BCDA, CDAB, BADC, DCBA, ADCB

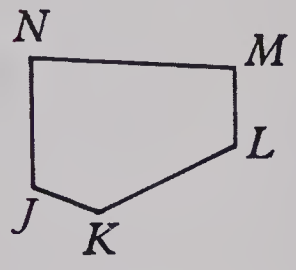
2. We can name each side of the polygon. One side may be named *AB* or *BA*. Name the three other sides.
BC or CB, CD or DC, DA or AD

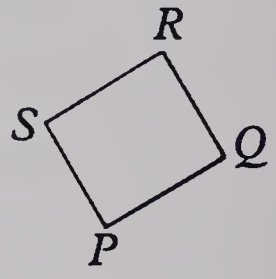
3. Name in two ways each polygon pictured below.
Answers will vary. See Ex. 1.

4. Name in two ways each side of each polygon.
Answers will vary. See Ex. 2.









Greater Than? Less Than? Equal To?

Equalities and inequalities [W]

First do the work in the ( )'s. Then write the answer. In Ex. 1a, 300 − 149 = 151, and 102 + 49 = 151. So write, "(300 − 149) is equal to (102 + 49)."

1. (300 − 149) is equal to (102 + 49)
2. (456 − 406) is greater than (798 − 770)
3. (99 + 412) is less than (600 − 38)
4. (256 + 139) is less than (87 + 603)
5. (27 + 89) is greater than (600 − 572)
6. (460 − 267) is less than (312 − 47)

** For Ex. 7–10, use sides of polygons above. Because these are not numbers, instead of "is greater than" or "is less than" use "is longer than" or "is shorter than."

7. *HG* is shorter than *FG*
8. *ZY* is longer than *WZ*
9. *KJ* is shorter than *MN*
10. *LM* is shorter than *FH*

Now turn back to page 197 and solve the problems.

** Note the (tie-up) between geometric ideas and inequalities.

Teaching Page 200

Pupil's Objectives

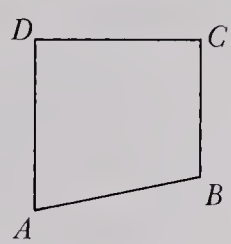
(a) To learn about naming polygons; and (b) to use *is greater than* and *is less than* to make true sentences.

Background

Refer back to the Background section for pages 120 and 121 in which points, lines, line segments, and closed curves were defined.

Polygons are created by the intersection of line segments in the same plane. The simplest polygon is the triangle which is made up of three line segments. There is no limit to the number of line segments (sides) of which a polygon may be made.

The naming of a polygon is accomplished by using the end points of the line segments. For example, in the polygon shown at the right there are four line segments: *AB*, *BC*, *CD*, and *DA*. (A line segment with the end points *A* and *B* is designated by the symbol *AB*.) The polygon may be named by using the points *A*, *B*, *C*, and *D*. When naming the polygon, you may begin at any point and proceed either clockwise or counterclockwise using the other points in the order in which they occur. The polygon above may be named *DABC* or *BADC*. However, it may not be named in any manner other than using the letters in consecutive order (i.e., *ACDB* is not a name for the polygon above).



For the work with polygons, you may wish to guide *more capable children* to the realization that one polygon has many different names.

Teacher's Preparation

Measuring square corners was introduced on pages 120 and 121 of the pupil's book. For that lesson it was recommended that you use the corners of any standard paper you have available as *square-corner measures*. It would be appropriate to make the same preparations for use in this lesson.

Pre-Book Lesson

• Review the geometric concepts which were introduced previously, such as point, path between two points, line segment, and closed path.

• Discuss the following polygons: square, triangle, rectangle. Identify each polygon by pointing out models of it in the classroom. (Window pane, door, table top, and so on.)

• Draw a model of a polygon on the board and label the end points of the line segments with capital letters. Ask children to use the labels to name the line segments shown on the board. Ask how many line segments make this polygon. Ask the class to name two polygons having four line segments.

Using the Text Page

• Identify line segments in the various polygons. Bring out the idea that each line segment is a side of a polygon.

• Discuss the written work at the bottom of the page before making the assignment. Elicit the idea that the numeral in each ( ) names a number. In each exercise pupils are to compare the two numbers named and then write a *true* sentence. It would be appropriate to write Ex. 1 on the board as it is presented in the directions.

Individualizing Instruction

• You may wish to modify the work at the bottom of the page in one or more of the following ways:

a. *Slower learners* may write the work for each number named in the following way.

$$\begin{array}{rr} 99 & 600 \\ +412 & -38 \\ \hline 511 & 562 \end{array}$$

(99 + 412) is less than (600 − 38)
or 511 is less than 562.

b. *More capable children* may write only the words *is greater than* or *is less than* as an answer.

• *More capable children* (perhaps *all pupils*) may write true mathematical sentences of their own creation using symbols of inequality (> or <).

• The written work suggested at the bottom of page 200, will give *all pupils* a chance to solve the problems on page 197 for which *n*-sentences had been written.

• *More capable children* may rewrite each of Ex. 1–6 so that *is equal to* may be used.

Teaching Page 201

Pupil's Objective

To check understandings and skills recently learned.

Background

The exercises on this page may be used as a progress test of important ideas which have been learned. More important, perhaps, is to use the material for maintaining the ideas and skills which have been developed. Drill in a mathematics program serves two important purposes. First, it is used immediately after understanding has been developed in order to establish the concept or skill. Then, it must be provided periodically to maintain the understandings and skills.

Using the Text Page

• After *all pupils* have completed the written work at the top of the page, discuss each item and have pupils explain their work. Do remedial work with groups or individuals who had difficulty with any of the written assignment.

• *Slower learners* may be assigned just four or five examples in each of rows 1–3 at the bottom of the page. You may wish to have these pupils copy the examples.

Individualizing Instruction

• *All pupils* may draw models of polygons. Have them label the end points of each line segment with letters of their choice and name the polygon.

• *More capable children* may draw models of rectangles and show the inside of each rectangle in parts of the same size.

Reminder

Continue to maintain

a. understandings and knowledge about fraction numbers and fractions.

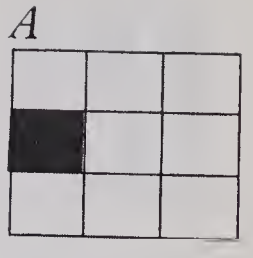
b. understandings and knowledge of geometric concepts.

NOTES

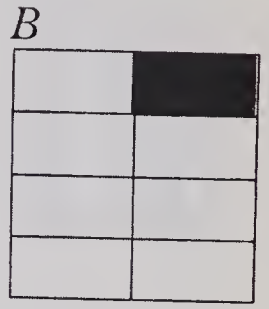
* Another mid-unit review of information and meanings!

* Many Things

Maintenance [W]



1. The inside of each of rectangle A, square B, and circle C is shown in parts of the same size. What is the fraction number for the red part of A? $\frac{1}{9}$ of B? $\frac{1}{8}$ of C? $\frac{1}{6}$

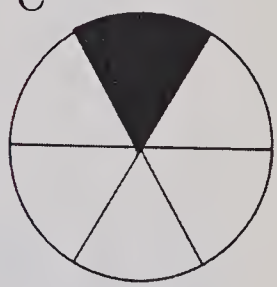


Copy and finish Ex. 2–5.

2. $(89 + 237)$ _?_ $(400 - 42)$. (is less than)

3. To find $\frac{1}{7}$ of a pie, you can cut the pie into _?_ (7) parts of the same size.

4. $572 = 4$ _?_ (hundreds) and _?_ (17) tens and 2 _?_ (ones), or _?_ (57) tens and 2 _?_ (ones). Are these other names for the number 572? Yes



5. $420 - 117 = n$, (303) so $n + 117 = 420$. (303)

6. Write four names to show the same number as 32. (Answers will vary.)

7. Copy each example you can write in a short way. (Ex. b and d)

a. $6 + 2 + 9$ **b.** $4 + 4 + 4$ **c.** $3 + 5$ **d.** $6 + 6 + 6$

To Keep in Practice

A. and S [W]

Write answers on folded paper. Check your work.

	a	b	c	d	e	f	g	h	i
1.	498	278	97	60	338	74	90	352	$7.60
	−479	+369	−92	+40	−306	−36	−37	−158	−3.59
	19	647	5	100	32	38	53	194	$4.01
2.	83	357	450	83	832	49	94	565	$3.00
	−9	+506	−80	−47	−653	+77	−58	−347	+1.68
	74	863	370	36	179	126	36	218	$4.68
3.	22	44	10	31	15	24	26	27	$0.13
	16	8	57	46	26	33	7	20	0.08
	37	7	4	8	9	5	19	33	0.48
	+19	+26	+28	+9	+24	+18	+20	+8	+0.05
	94	85	99	94	74	80	72	88	$0.74

* Note the association of two numbers with a unique third number.

When Sets Are Equivalent

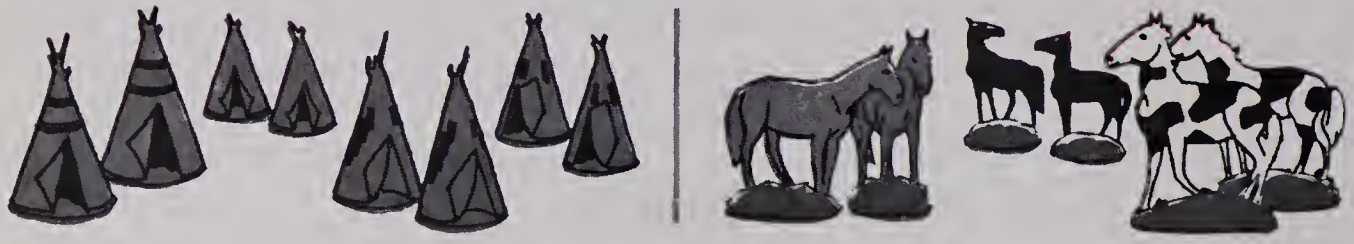
Meaning of multiplication [O]

1. How many sets of toy Indians has Tom? 4 How many are in each set? 3 Are the sets equivalent? Yes Explain. They each have the same number of members.

2. What is the number for the total set? To find out, you can
   a. count by 3's: 3, 6, _?_, 9 _?_ 12
   b. add 3's: 3 + 3 + 3 + 3 = ? 12
   c. say, "Four 3's equal _?_." 12

* When you know that four 3's = 12, you are multiplying. The answer is called the product.

3. Tom has 4 _?_ sets of 2 tepees. Find the number for the total set three ways, as in Ex. 2. 2, 4, 6, 8; 2 + 2 + 2 + 2 = 8; Four 2's = 8



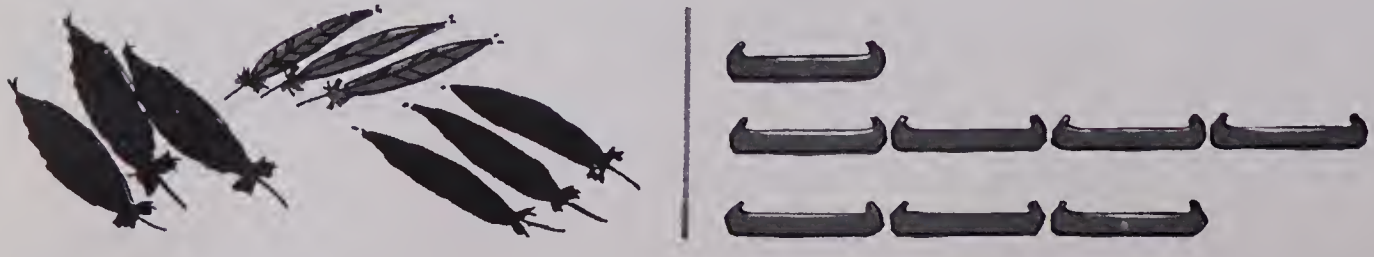
4. Tom's Indian ponies are in 3 _?_ sets of 2 _?_. Find the number for the total three ways. 2, 4, 6; 2 + 2 + 2 = 6; Three 2's = 6

5. For the pictures below, can you multiply to find the number for all the feathers? Yes all the canoes? No Explain. You can multiply only when sets are equivalent.

6. If sets are not equivalent, can you multiply to find the number for the total set? No Why? See answer for Ex. 5.

On the board show each of Ex. 7–10 with its product.

7. four 2's 8 8. two 5's 10 9. five 3's 15 10. four 4's 16



Teaching Pages 202 and 203

Pupil's Objectives

(a) To extend understanding of the relationship that exists between addition and the new way of finding the number of members in the total set when the sets being joined are equivalent (three 2's are 6); (b) to find the number for the total set; and (c) to learn the terms *multiply* and *product*.

New Words

(page 202) *multiplying*, *product*

Background

On pages 164 and 165 the pupils were introduced to the joining of equivalent sets as well as to the writing of a mathematical sentence to show the joining.

Now, pupils are to develop understanding of the operation used when working with equivalent sets. Multiplication, a binary operation, is the operation on two numbers called *factors* which results in a unique third number called the *product*. One of the factors tells the number of members in each of the equivalent sets. The other factor tells how many equivalent sets there are. Thus, if there are four groups of children with three in each group, the statement "Four 3's equal 12" is one of multiplication.

Note that the term *factor* is not used in this lesson. The symbol $\times$ is not introduced either. Both will be used in the very near future.

Frequently, one hears that multiplication is a *short cut* for addition or a quick way to add. Each of these impressions should be avoided. Instead, as the relationship between addition and multiplication is established, children should begin to see that under certain circumstances multiplication is a shorter way of dealing with a quantitative situation which could have been handled by using addition. When the quantitative situation is one in which there is a prescribed number of equivalent sets and the number of the set is given, such as 5 sets of children with 4 children in each set, it is possible to use the multiplication 5×4 rather than the addition $4 + 4 + 4 + 4 + 4$.

Teacher's Preparation

Be prepared to use and show groups of objects in the classroom. Flannel shapes—squares, circles, or triangles of different colors—may be used on the flannel board to show several equivalent sets.

Pre-Book Lesson

• Arrange objects in the room (children, books, chairs, and so on) so as to show three sets of four each. Ask selected pupils to describe what they see. Ask someone to write a number sentence to describe the situation. (Be sure to ask a question that would suggest joining the sets. The number sentence children have learned for the joining of disjoint sets is an addition sentence.)

As a result of the discussion, a sentence like the following should be written on the board: $4 + 4 + 4 = 12$.

• Have different children tell ways in which the sentence may be read. (Some will say, "four plus four plus four equals twelve." Others will say, "three fours equal twelve.") Be sure to encourage both responses. Urge children to begin to use the latter as the lesson progresses.

• You may wish to illustrate on the flannel board another example of the type described above. There is value in making a transition from the concrete stage using objects in the classroom to the semi-concrete presentation using geometric materials on a flannel board. The flannel-board designs are more abstract and less distracting than children or chairs.

Using the Text Pages

• The oral work sets the stage for the child's development in moving toward maturity in learning multiplication. Be sure you discuss each of Ex. 1–11 carefully. Each child's future success in multiplication will be determined by this initial approach.

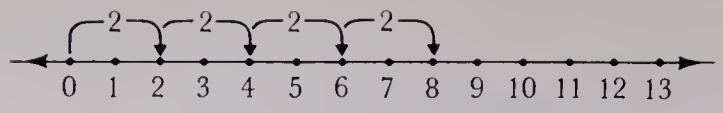
• In Ex. 7–10, pupils will write each of the three ways shown in Ex. 2. As a child writes on the board, have him tell what he is writing and what it means.

• You may wish to complete the work for Ex. 11 on the board so it can be used as a model for the work in Ex. 12–15.

• Before writing the work for Ex. 12–15, emphasize the importance of writing each of the four ways shown in the box. The first way involves making dot pictures. The box shows dot pictures for 6. Ask pupils to show different dot pictures for 2, 3, 4, and 5.

Individualizing Instruction

• Duplicate a page of number-line pictures and have *all pupils* illustrate Ex. 7–15 using a number-line picture. For Ex. 7, the following number-line picture shows how to find the product.



• Partners may work together so that one shows a number of equivalent sets and the other tells the total. The Ginn Arithme-Sticks* are very useful for this purpose.

• *More capable children* may engage in the *Show the Answer* game. You or a child may write "four 2's" on the board. Pupils select a numeral for the correct product and insert it in a holder. Upon a signal, pupils raise their answers for you to see.

* See 3, page xix.

NOTES

11. Jean's class made a picture of an Indian village. They made 3 sets of trees with 6 in each set. The number for the total set of trees in the picture was _?_. 18

The box shows four ways to find the number for the total set. Explain each way.
(1) Count the dots. (2) Count by 6's.
(3) Add. (4) Multiply.

[W]

Solve each of Ex. 12–15. Write your work four ways, as in the box.

(1) ••• ••• •••
•• •• ••
•   •

(2) 6, 12, _?_

(3) 6 + 6 + 6 = ?

(4) Three 6's = ?

12. Jean pasted 3 sets of 4 Indians on the picture. That was how many Indians? 12 Indians

13. Mike made 3 sets of 3 tepees for the picture. How many tepees did he make for the picture? 9 tepees

14. Two Indians were placed in each of 4 canoes. How many Indians were in the canoes? 8 Indians

15. Each of 4 Indians had 5 feathers in his headband. How many feathers was that all together? 20 feathers

* **When sets are equivalent, you can multiply instead of adding to find the number for the total set.**

* On pages 164-165, preparation was made for this meaningful approach to the operation of multiplying. Even at this early stage, pupils should be able to tell in their own words the substance of the generalization stated here.

* Emphasize the separateness of the addition operation and the multiplication operation.

Ways to Show Multiplication Facts

Multiplication a binary operation [W]

If May sews 3 buttons on each doll dress, how many buttons will she sew on 4 doll dresses?

You can add four 3's. The sets are equivalent, so it is quicker to multiply. *Think*, "Four 3's = 12."

We can show a multiplication (M.) fact three ways, as in boxes A, B, and C. We can read the fact like this for all three ways shown: "Four 3's equal 12."

A	C
four 3's = 12	3 ×4 12 product
B	
$4 \times 3 = 12$	

We use × as the sign for multiplication and read it, "times." It tells us to multiply, just as + tells us to add.

Copy Ex. 1–9. Find the sums. Then beside each show the M. fact three ways, as in the boxes.

1. $5 + 5$ 10
2. $8 + 8$ 16
3. $9 + 9$ 18
4. $4 + 4 + 4$ 12
5. $3 + 3 + 3$ 9
6. $6 + 6 + 6$ 18
7. $2 + 2 + 2 + 2$ 8
8. $5 + 5 + 5 + 5$ 20
9. $4 + 4 + 4 + 4$ 16

Multiplication Facts for 2

[W]

M. Facts for 2
$2 \times 1 = 2$
$2 \times 2 = ?$ 4
$2 \times 3 = ?$ 6
and so on to
$2 \times 9 = ?$ 18

Does $3 + 3 = 6$ mean the same as the M. fact, $2 \times 3 = 6$? Yes In the number sentence $2 \times 3 = 6$, the first numeral, 2, tells the number of equivalent sets. The second numeral, 3, tells how many in each set.

The M. facts for 2 are easy. The products are the same as the sums of A. facts having two equal addends.

Copy and finish the M. Facts for 2 table that is started for you. Use A. facts if you need help.

$2 \times 4 = 8$ $2 \times 6 = 12$
$2 \times 5 = 10$ $2 \times 7 = 14$
$2 \times 8 = 16$

Pupil's Objectives

(a) To learn two new ways to show M. facts; and (b) to identify and start learning the multiplication (M.) facts for 2.

New Words

multiplication, *sews*

Background

The two ways of showing M. facts using the symbol × are developed at the top of page 204 without any regard for a particular set of M. facts. The purpose is to teach pupils two new ways of showing multiplication facts.

You will note that the word *times* is not used yet in reading the multiplication sentences. Multiplication of 3 by 4 is read "four 3's." The use of *times* in reading sentences of this type first appears on page 246.

The table of M. facts for 2 started at the bottom of the page does not mean that you should *give* the table to your pupils, or that instruction on the table should merely be aimed at getting pupils to learn the facts. At this time, they should be completing the table independently. Withhold the use of the M. table as such until needed for comparison with pupil-made tables and for reference later.

The M. facts in this table have *2* as the number by which you multiply. These facts would be read "Two 1's," "Two 2's," "Two 3's," and so on.

Pre-Book Lesson

• You may test pupils to determine how many of the M. facts for 2 they already know. The written test may be given in two steps:

a. Have the children show the product for each M. example that you read. Pause only long enough for them to show the product, but not long enough for them to *find* it by counting or adding. Explain that you will give another chance later, so no child need feel embarrassed. Dictate them in mixed order, somewhat as follows:

(1) two 3's	(4) two 8's	(7) two 9's
(2) two 5's	(5) two 1's	(8) two 4's
(3) two 2's	(6) two 6's	(9) two 7's

b. Next, have pupils write each example, on their papers, as you say them again, pausing after each one this time. Then, have them show how to find products in any way they want (dot pictures, adding, and so on). Pupils' work on these exercises will give you considerable information concerning the teaching you will have to do in working with the next four pages of the book.

• In preparation for the work at the top of page 204, show four sets of three objects on the flannel board (or in some other way) and have pupils write the different ways they know to describe the sets of objects.

• Review the meanings of the abbreviations A. and S. Ask what letter we should use to mean *multiplication*.

Using the Text Page

• Go through the material at the top of the page orally before assigning the written work. Work with the objects on the flannel board referred to in the Pre-Book Lesson. Use the term "product" to mean the *answer* in multiplication.

• Go over the material at the bottom of the page orally before assigning the written work. Be sure children understand what it is they are to do. Permit them to use any means they wish to help them determine the product for each of the M. facts for 2.

Individualizing Instruction

• *Slower learners* may develop the table of M. Facts for 2 as begun below:

Two 1's = 2
Two 2's = 4
Two 3's = 6

• Help *all pupils* use the completed table to bring out new relationships and understandings such as,

a. the products increase by 2;

b. each product is shown by a numeral that ends with 0, 2, 4, 6, or 8. Numbers whose standard names end with such digits are *even* numbers. (The standard name for twelve is 12, for twenty-five is 25, and so on.)

Pupil's Objectives

(a) To learn about the M. facts for 2's; and (b) to learn to distinguish between M. facts for 2 and M. facts for 2's.

Background

The M. facts at the bottom of page 204 show 2 to mean the number of equivalent sets. In the M. facts for 2's, 2 tells how many in each set. Each fact identifies another number of 2's. In the lesson, the multiplication examples are presented in mixed order. Pictures are used to show most of the relationships and to assist in finding answers. These relationships and answers, when found, are stated in different ways as is shown on page 205. The lesson ends at the bottom of the page in a written exercise of completing the table of M. Facts for 2's. This table, in contrast to the one on the previous page, shows 2 to mean the size of each equivalent set.

Pre-Book Lesson

• Direct the children to use objects, representative materials, and/or dots in groups of two. Have them show a different number of 2's each time. Then have them show the numbers of 2's in mixed order. For example, seven 2's may be shown before four 2's. Each child should discover what is meant by *five 2's*, *eight 2's*, and so on, without relying upon a table as such.

• You may wish to give an inventory-type test as suggested for the preceding lesson.

Using the Text Page

• Use objects on the flannel board* to accompany the discussion of each of Ex. 1–3. The use of objects will help pupils to see what each numeral means in a multiplication sentence—either how many in each equivalent set or the number of sets.

• In Ex. 2, discuss why it is easier to find the answer by multiplication rather than by addition. This is in preparation for Ex. 5–8.

• Whether or not pupils understand the difference between M. facts for 2 and M. facts for 2's will be shown by their performance on Ex. 5–8.

Individualizing Instruction

• *Slower learners* may develop the multiplication table for Ex. 4 as begun below:

One 2	= 2
Two 2's	= 4
Three 2's	= 6

• After *all pupils* have completed the multiplication table in Ex. 4, compare it with the one completed for the preceding lesson. They should note that the corresponding facts in each table have the same product. (This idea will be developed more completely in the next lesson.)

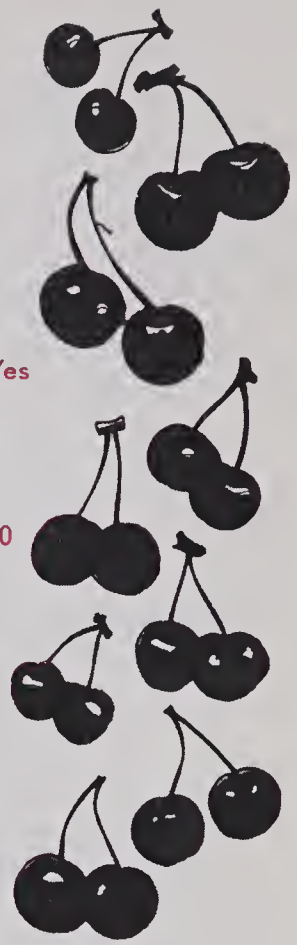
• Review *even numbers* recently discussed and then discuss *odd* numbers. (Odd numbers are *one* more than a product with 2 as one of the factors.) Have children show counting numbers from 1 through 20 in two columns under headings of *Odd* and *Even*. Encourage discussion as to why odd numbers are never the product when 2 is a factor. See Teaching Pages 308–309 for explanation of this idea.

• *More capable children* may make a list of things that we think of in pairs and list the M. facts for 2's.

• From the following set of numbers, have *all pupils* select the even numbers and then tell the M. fact for which the number is a product: 6, 3, 5, 10, 4, 1, 14, 17, 18, 11, 12, 16, 13, 7, 19, 8, 9.

* See 2, page xix.

Multiplication Facts for 2's



[O]

1. The number for each set of cherries is 2. What is the number for 3 sets of cherries?

a. Count by 2's: 2, _?_[4], _?_[6] 2+2+2=6

b. On the board show the A. example with the sum.

c. To find the number for the total, can you multiply? Yes Explain. The sets are equivalent.

2. How many cherries are there in 5 sets?

a. Count by 2's. b. Add 2's: 2 + 2 + 2 + 2 + 2 = ?[10]

c. The M. fact is 5 × 2 = 10. Which numeral tells how many sets? 5 Which tells how many in each set? 2

3. How many cherries are there in 7 sets?

a. To find the number for the total, count by 2's. 14

b. Which is the M. fact, 2 × 7 = 14 or 7 × 2 = 14? 7×2=14

[W]

4. Copy and finish the M. Facts for 2's table that is started for you. To find products, count by 2's if you need help.

M. Facts for 2's
1 × 2 = 2
2 × 2 = ?
[3]? × 2 = 6
and so on to
9 × 2 = ? 18

4 × 2 = 8
5 × 2 = 10
6 × 2 = 12
7 × 2 = 14
8 × 2 = 16

Show the correct M. fact for each addition.

5. 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 18

a. 9 × 2 = 18 b. 2 × 9 = 18

6. 2 + 2 + 2 + 2 + 2 + 2 = 12

a. 2 × 6 = 12 b. 6 × 2 = 12

7. 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 16

a. 2 × 8 = 16 b. 8 × 2 = 16

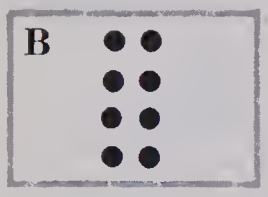
8. 2 + 2 + 2 + 2 + 2 + 2 + 2 = 14

a. 7 × 2 = 14 b. 2 × 7 = 14

* In an array, as in A-D, all rows have the same number of members and all columns have the same number of members.

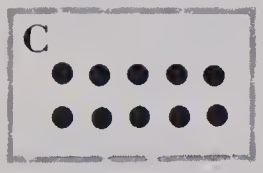
Discovering Pairs of M. Facts about 2

* *Use of arrays; using the Commutative Property of M.* [O]



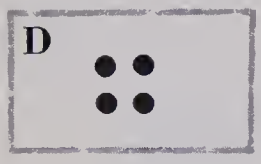
1. Box A has [2] _?_ rows of dots with [3] _?_ dots in each row. There are [6] _?_ dots in all. $2 \times 3 = n$ 6

2. Box A also has [3] _?_ columns of dots, with [2] _?_ dots in each column. $3 \times 2 = n$



** 3. Do 2×3 and 3×2 have the same product? Yes Then do they go together in a pair of M. facts? Yes

4. Box B has [2] _?_ columns of [4] _?_ dots each. $2 \times$ [4] _?_ $= n$ 8
Box B has [4] _?_ rows of [2] _?_ dots each. [4] _?_ $\times$ [2] _?_ $= n$ 8
$2 \times 4 = 8$ and $4 \times 2 = 8$ are a pair of M. facts.



5. What pair of M. facts can you make from box C? $2 \times 5 = 10$; $5 \times 2 = 10$

6. Why does box D show that no different M. fact goes with $2 \times 2 = 4$? The number of rows and the number of columns are the same.

Most multiplication facts go in pairs.

[W]

Finish each example and show the M. fact that goes with it. For Ex. 7 write, "$5 \times 2 = 10$ and $2 \times 5 = 10$."

7. 5×2 10 ($2 \times 5 = 10$)
8. 2×3 6 ($3 \times 2 = 6$)
9. 1×2 2 ($2 \times 1 = 2$)
10. 2×7 14 ($7 \times 2 = 14$)
11. 6×2 12 ($2 \times 6 = 12$)
12. 2×8 16 ($8 \times 2 = 16$)
13. 4×2 8 ($2 \times 4 = 8$)
14. 2×9 18 ($9 \times 2 = 18$)
15. 2×5 10 ($5 \times 2 = 10$)
16. 9×2 18 ($2 \times 9 = 18$)
17. 2×6 12 ($6 \times 2 = 12$)
18. 7×2 14 ($2 \times 7 = 14$)
19. 2×4 8 ($4 \times 2 = 8$)
20. 8×2 16 ($2 \times 8 = 16$)
21. 3×2 6 ($2 \times 3 = 6$)
22. 2×1 2 ($1 \times 2 = 2$)
23. two 6's 12
24. five 2's 10
25. two 8's 16
26. eight 2's 16 (eight 2's = 16; two 8's = 16)

Copy only the examples in which you can multiply.

27. 5 + 5
28. 4 + 3
29. 3 + 7
30. 8 + 5
31. 7 + 7
32. 4 + 4
33. 2 + 2 + 2
34. 3 + 4 + 7
35. 3 + 3 + 3
36. 4¢ + 4¢ + 4¢
37. 2¢ + 8¢ + 5¢
38. 5¢ + 5¢ + 5¢

** There is a Commutative Property of Multiplication just as there is a Commutative Property of Addition. Here is a good chance to point out that when they learn one fact of a pair of M. facts, pupils know the other fact of the pair.

Pupil's Objectives

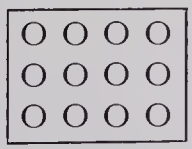
(a) To discover that most M. facts go in pairs; and (b) to extend the idea that a M. fact can be associated with equivalent sets of objects.

Background

Your pupils have been introduced to some sets of numbers (*counting* numbers and *whole* numbers). They have learned two *operations* (addition and subtraction) which may be performed on these sets of numbers, and are now being introduced to another operation (multiplication). Certain key properties which govern the operations and the sets of numbers have been developed intuitively (Commutative, Associative, Closure, and Identity Element for Addition).

Learning the structure of mathematics involves exploring the properties governing each of the operations. Now that multiplication is being developed formally, the properties which have been learned in connection with addition and subtraction must be explored again.

The commutative property is concerned with the order in which numbers are presented when performing addition. We learned that the order of two numbers may be changed when adding without affecting the sum. However, in subtraction, changing the order of the numbers will affect the result so the commutative property does not hold for subtraction. When multiplying two whole numbers, the order of the factors may be changed without affecting the product. This is the Commutative Property of Multiplication. A rectangular *array* of objects in which there are rows and columns is very useful for building an understanding of this property. All rows contain the same number of objects and all columns contain the same number of objects. The numbers of rows and columns are easily seen in the array. The product of the number of columns (4) and the number of rows (3) is the number of objects (12) in the array; the product of the number of rows (3) and the number of columns (4) is also the number of objects (12) in the array.



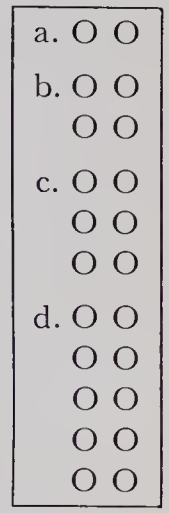
Pre-Book Lesson

• Build an array of five 2's with objects on the flannel board. Have pupils construct the array with one-inch circle shapes. They should construct this array on a piece of paper. First, show, and direct the pupils to show a set of two. Ask, "How many two's are there?"

Next, place another *two* below the first one. Again, ask, "Now, how many two's are there?"

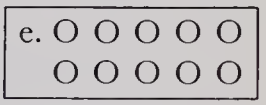
Show another *two* below the others. Repeat the question about the number of two's.

Continue the construction until an array of five 2's is completed, as in d.



• Show the M. fact on the board for the array in d. ($5 \times 2 = 10$)

• Turn your flannel board and direct the pupils to carefully turn their papers so that the array now shows two 5's as in e.



• On the board, write the multiplication sentence for the array in e. ($2 \times 5 = 10$)

• Ask: "In what ways are the two facts alike?" "In what way are the two facts not alike?"

Be sure to emphasize that both involve multiplication; both have the same products; and both have the same factors. They differ in the order in which the factors are shown.

Using the Text Page

• You may want to have the pupils construct the arrays for each of Ex. 1–6 and use them during the oral work.

• Perhaps you will assign only part of the written work for *slower learners*.

Individualizing Instruction

• *Slower learners* (perhaps *all pupils*) may draw arrays to show six 2's and two 6's, and so on. They will get a *pair* of facts in every instance with the exception of two 2's.

• *More capable children* may have timed contests in the folded-paper exercise of showing only products for Ex. 7–26.

Teaching Page 207

Pupil's Objective

To further discover the factors-product relationship using the facts for 2 and for 2's tables.

New Word

factors

Background

The two tables of M. facts in which 2 is one of the factors should be studied together in order to build some important generalizations about multiplication. In one table of facts, *2* tells the number of members of the equivalent sets being joined. In increasing the number of these sets by *1*, you are adding *2* more members and the product increases by *2*. In the other table of facts, *2* tells the number of equivalent sets being joined. In increasing the number of members of each of these sets by *1*, you are adding *2* more members, and the product increases by *2*.

When two whole numbers are multiplied, we always get another whole number. This means the property of *closure* holds for multiplication as well as for addition.

The term "factors" is introduced for the first time. *Factors* and *products* will be used regularly throughout the *Mathematics We Need* program.

Teacher's Preparation

You may wish to write on the board each example from the tables of facts on pupil's page 207. Do not show the products. Pupils can do this as part of their lesson.

Pre-Book Lesson

Before working Ex. 5 of page 207, have different pupils go to the board and show a product for the examples you've shown on the board. Then ask them to explain the increasing products in their tables of facts.

Using the Text Page

Use Ex. 1–8 and the tables at the top of the page for building relationships and understandings.

Individualizing Instruction

You may have an oral discussion with *more capable children* in connection with the products shown in the tables of M. facts on the board. Ask, "What is the product of four 2's?" "What is the product of two 2's?" Add the two products. "What other product do you get?" Through this questioning, pupils should begin to see that four 2's plus two 2's equals six 2's. Try other similar combinations.

NOTES

* The tables provide one of the variety of approaches used in clinching learning of facts.

*

Table of Multiplication Facts for 2

1	2	3	4	5	6	7	8	9
$\frac{2}{2}$	$\frac{2}{4}$	$\frac{2}{6}$	$\frac{2}{8}$	$\frac{2}{10}$	$\frac{2}{12}$	$\frac{2}{14}$	$\frac{2}{16}$	$\frac{2}{18}$

*

Table of Multiplication Facts for 2's

2	2	2	2	2	2	2	2	2
$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{5}{10}$	$\frac{6}{12}$	$\frac{7}{14}$	$\frac{8}{16}$	$\frac{9}{18}$

Factors-product relationship [O]

1. Cover the products shown in both tables. Try to say the facts. Do not take them in order.

** 2. Any two numbers you multiply to get a product are its factors. For $2 \times 5 = 10$, the factors are 2 and 5. In which fact shown above are the factors equal? $2 \times 2 = 4$

3. In M. facts for 2, by what factor do you multiply each time? 2 What is the change in the other factor? 1 greater

4. In M. facts for 2's, is there a change in the factor by which you multiply each time? Yes What is the change? 1 greater Is there a change in the other factor? No

5. Study the products shown in the tables. How much greater is each as you go from fact to fact? 2 Why?

Table of M. Facts for 2 — the number in each set is 1 greater.
Table of M. Facts for 2's — the number of sets is 1 greater.

6. Are these products all odd or all even numbers? even

7. Do most of the M. facts go in pairs? Yes Give examples of pairs of M. facts using one fact from each table. Answers will vary.

8. Which M. fact has no other fact to make a pair? $2 \times 2 = 4$

** Help pupils to begin at once to use correctly the new terms factor and product. It is a good time, at this point, to have pupils generalize that the product is always an even number when one factor is 2.

Try These

Oral practice on M facts having 2 as a factor [O]

Say the unknown factor for each example in rows 1–7.

	a	b	c	d
1.	$\overset{2}{_?_} \times 3 = 6$	$5 \times \overset{2}{_?_} = 10$	$2 \times \overset{7}{_?_} = 14$	$\overset{8}{_?_} \times 2 = 16$
2.	$2 \times \overset{2}{_?_} = 4$	$\overset{9}{_?_} \times 2 = 18$	$\overset{4}{_?_} \times 2 = 8$	$2 \times \overset{5}{_?_} = 10$
3.	$2 \times \overset{3}{_?_} = 6$	$2 \times \overset{4}{_?_} = 8$	$\overset{6}{_?_} \times 2 = 12$	$\overset{2}{\square} \times 4 = 8$
4.	$\overset{2}{_?_} \times 6 = 12$	$\overset{2}{_?_} \times 2 = 4$	$9 \times \overset{2}{_?_} = 18$	$\overset{5}{_?_} \times 2 = 10$
5.	$4 \times \overset{2}{_?_} = 8$	$8 \times \overset{2}{\square} = 16$	$\overset{2}{_?_} \times 5 = 10$	$2 \times \overset{9}{_?_} = 18$
6.	$\overset{2}{_?_} \times 7 = 14$	$6 \times \overset{2}{_?_} = 12$	$2 \times \overset{1}{_?_} = 2$	$7 \times \overset{2}{\triangle} = 14$
7.	$\overset{3}{_?_} \times 2 = 6$	$\overset{7}{_?_} \times 2 = 14$	$3 \times \overset{2}{_?_} = 6$	$2 \times \overset{6}{_?_} = 12$

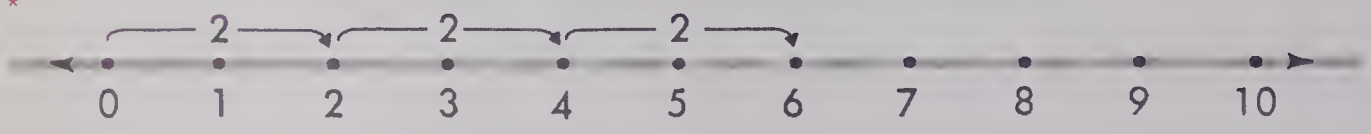
Do You Like to Try New Things?

Enrichment [W]

You have been using a number line for finding answers to A. and S. examples, like $4 + 6 = n$ and $12 - 7 = n$. Be ready to show the class how you can use a number line to find the product for a M. example.

1. $3 \times 2 = n$. What does n stand for? 6 You know that three 2's = $2 + 2 + 2$. So, by using a number line to add $2 + 2 + 2$, you can find the product n stands for in $3 \times 2 = n$.

*



2. Draw a number-line picture and find each product.

a. $5 \times 2 = n$ 10	d. $6 \times 3 = n$ 18	g. $4 \times 2 = n$ 8	j. $4 \times 6 = n$ 24
b. $8 \times 2 = n$ 16	e. $4 \times 3 = n$ 12	h. $2 \times 6 = n$ 12	k. $5 \times 3 = n$ 15
c. $3 \times 3 = n$ 9	f. $3 \times 5 = n$ 15	i. $7 \times 3 = n$ 21	l. $3 \times 7 = n$ 21

* Make sure that pupils realize that they used a number line to find $2 + 2 + 2$, but that the sum 6 is also the product for 3×2.

Teaching Page 208

Pupil's Objective

To have oral practice in the finding of unknown factors.

Background

Though pupils have not been introduced to division, the oral practice material is an intuitive development of the relationship between multiplication and division. In the exercises at the top of page 208, one factor and a product are known. The pupil is to determine the unknown factor, which is precisely what is involved in using division. However, the purpose of this material is to help children build now complete mastery of multiplication.

Using the Text Page

• Review the terms *factors* and *product*. Read the sentences on the top of page 208 in different ways. Ex. 1a may be read:

a. "How many 3's equal 6?"

b. "The product is 6; one of two factors is 3; what is the other factor?"

• Have pupils tell how they could prove that the answers they have given are true. They may rely upon using the commutative property, adding two groups of a given size, or adding a given number of two's.

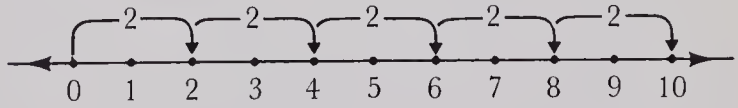
Individualizing Instruction

• You may work with the *slower learners* with material at the top of page 208 while *more capable children* do the work with partners.

• After working with the exercises at the top of page 208 as an oral lesson, have *slower learners* do the work as a written lesson. Tell pupils to omit examples they find difficult. Examine the papers for incorrect and omitted answers. Then have these children make cards showing the examples and cards showing the answers for them. In this book, we see the horizontal example and its answer as forming a mathematical sentence. The sentence shows a fact. If the example is a M. example as is the case here, a M. fact is shown.

Allow frequent opportunities for matching cards. When a fact has been mastered, that set of cards may be eliminated.

• *More capable children* (perhaps *all pupils*) may do the enrichment material at the bottom of page 208. You may wish to duplicate a page of number-line pictures to minimize the time spent on drawing them instead of minimizing the time doing arithmetic. Ex. 2a should be illustrated on a number-line picture as shown below:



NOTES

Pupil's Objectives

(a) To have practice in giving answers for the M. facts for 2's and for 2; (b) to find and write answers for addition and subtraction examples; and (c) to use = (is equal to) and ≠ (is not equal to) to make true sentences.

Background

The exercises on this page are in preparation for the end-of-chapter tests which follow. By this time your pupils should have command of the multiplication facts in which 2 is a factor. They should be able to say the unknown factor in the exercises at the top of page 208.

Addition and subtraction with 3-place numerals with and without renaming should be reasonably well mastered. The written work on page 209 affords the opportunity for work in addition and subtraction with 2- and 3-place numerals in a useful mathematical setting. The need to complete mathematical sentences to make true sentences is more motivating than merely writing the answers for examples.

Pre-Book Lesson

Review the terms *factors* and *product*. Write some multiplication sentences on the board for which either a factor is unknown or the product is unknown.

Using the Text Page

- First, do the oral work at the top of the page with the class as a whole. Then, have pupils work in pairs helping each other work the examples.
- Before the written work is assigned, remind pupils to add numbers when addition is indicated, and to subtract numbers when subtraction is indicated, and to be careful when renaming is necessary. Be sure the directions for Ex. 1–5 are understood. Review how to copy a set of numerals in a column for the addition work at the bottom of the page.

Individualizing Instruction

- *Slower learners* may write the answers for the exercises on page 208 and compare their results with the work from the previous day.
- *Slower learners* may do the written work for Ex. 1–5 as follows:

$$\begin{array}{rrr} \text{1a.} & 68 & 527 \\ & +32 & -405 \\ \hline & 100 & 122 \end{array} \qquad 100 \neq 122$$

- *All pupils* may write the addition examples in Ex. 6–14 at the bottom of page 209 as follows:

$$\begin{array}{rrrr} (53 + 10) & + & (19 + 8) & = n \\ 63 & + & 27 & = n \\ & & 90 & = n \end{array}$$

- *More capable children* may change the sentences in rows 1–5 so that using the equals sign will make true sentences. The sentences which are already true for the *is-equal-to* relation need not be rewritten.

Oral Practice on Multiplication Facts

Facts having 2 as a factor [O]

Use the examples in rows 1–7 on page 208.

1. Say each fact. This time work by columns.

2. For column a, turn each example around. For Ex. 1a, say, "Two 3's equal 6, so 6 equals two 3's."

3. Say the M. fact that goes with each fact for columns b, c, and d, if there is one.

4. Make Study Cards for facts that you do not know. Study these facts until you know them.

To Keep in Practice

Comparisons and column A. [W]

Is not equal to or is equal to?

In Ex. 1a, $68 + 32 = 100$ and $527 - 405 = 122$. So, write, "$(68 + 32) \neq (527 - 405)$." Work this way in the other examples in rows 1–5, using $=$ or $\neq$.

	a	b
1.	$(68 + 32) \overset{\neq}{?} (527 - 405)$	$(586 - 577) \overset{=}{?} (800 - 791)$
2.	$(743 - 96) \overset{\neq}{?} (232 + 567)$	$(490 + 87) \overset{\neq}{?} (272 + 168)$
3.	$(139 + 136) \overset{\neq}{?} (288 - 48)$	$(359 - 139) \overset{\neq}{?} (27 + 155)$
4.	$(911 - 399) \overset{\neq}{?} (700 - 83)$	$(98 + 122) \overset{=}{?} (300 - 80)$
5.	$(440 - 80) \overset{\neq}{?} (208 + 193)$	$(587 - 99) \overset{\neq}{?} (223 + 280)$

Copy each set of numerals in a column. Add and check.

6. 53, 10, 19, 8 90
7. 8, 19, 4, 49 80
8. 7, 22, 37, 17 83
9. 16, 13, 15, 19 63
10. 8, 47, 18, 20 93
11. 16, 9, 30, 17 72
12. \$0.13, \$0.42, \$0.27 \$0.82
13. 5¢, 46¢, 6¢, 39¢ 96¢
14. \$0.28, \$0.26, \$0.06 \$0.60

Can You Solve Problems?

Problem Test 5

Write your work for problems 1–8.

1. In Jane's skating party there were 15 boys, 16 girls, and 3 fathers. How many were in the party? 34 people

2. They came to the skating pond in small buses. One bus had 7 seats, and each seat could hold 2 people. How many people could ride in that bus? 14 people

3. The pond was 225 feet long and 190 feet wide. How many feet longer was the pond than it was wide?
35 feet

4. Tickets for a group to use the pond cost $3.40. Jane had saved $2.70. How much more did she need?
$0.70

5. For the lunch Jane and her mother had made the sandwiches, 45 with dark bread and 65 with white bread. How many sandwiches had they made? 110 sandwiches

6. Nine girls each ate 2 sandwiches. How many sandwiches did this group of girls eat? 18 sandwiches

7. After lunch, 27 of the 100 cookies Jane's mother had made were left. How many cookies had been eaten?
73 cookies

8. The buses were in use 25 minutes carrying the people to the pond, 25 minutes giving the group a ride after the party, and 30 minutes taking the people home. In all, how many minutes were the buses in use? 80 minutes

Teaching Pages 210, 211, 212, and 213

Pupil's Objectives

(a) To evaluate the learning of the chapter content by taking the four end-of-chapter tests; and (b) to have oral practice in finding the number for n in arithmetic sentences.

Using the Test Pages

- Children who miss two or more examples in any of rows 1–5 of Diagnostic Test 5 would be in need of remedial work (review at least). Use the study pages and practice sets as they seem appropriate.
- In discussing each item of the Test of Information and Meaning, be sure to have children explain their thinking.

Individualizing Instruction

- Use the oral work at the bottom of page 211 to minimize fatigue in taking the tests. It should provide welcome relief between tests.
- *All pupils* may write n-sentences for the problems on page 210.
- Use the tables below to find the per-cent scores for the individual record cards.

Table of Per Cents for Chapter 5 Scores

Problem Test		Computation Test					
Score	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*
1	13	1	4	9	38	17	71
2	25	2	8	10	42	18	75
3	38	3	13	11	46	19	79
4	50	4	17	12	50	20	83
5	63	5	21	13	54	21	88
6	75	6	25	14	58	22	92
7	88	7	29	15	63	23	96
8	100	8	33	16	67	24	100

Suggestions for Material to Accompany End-of-Chapter 5 Tests

Alternate Uses of Pages in Chapter 5

Page 180. *More capable children* may look up the ancient Egyptian system of numerals. Perhaps they will want to make a chart to show the other members of the class. They may show the way a number may be recorded using the Roman system, the ancient Egyptian system, and the base-10 system we use today.

Page 182. First, have the children write any five A. or S. n-sentences. Then, have them create a problem to go with each n-sentence.

Page 187. Write the solution to the examples in row 4 in the following form:

$$\begin{array}{rl} 736 = & 700 + 30 + 6 \\ \underline{+46} = & \underline{\qquad\quad 40 + 6} \\ & 700 + 70 + 12 = 700 + 80 + 2 = 782 \end{array}$$

Page 189. Rename the numbers shown in Ex. 7–11 at the bottom of the page in this way:

$$549 = 500 + 40 + 9 \text{ or } 400 + 140 + 9 \text{ or } 500 + 30 + 19$$

Page 193. Write the solution to the examples in row 17 as follows:

$$\begin{array}{rcl} 577 & = & 500 + 70 + 7 = 400 + 160 + 17 \\ \underline{-479} & = & \underline{400 + 70 + 9} = \underline{400 + \ 70 + \ 9} \\ & & 90 + \ 8 \end{array}$$

Page 200. Rewrite the sentences in Ex. 1–6 so that "is equal to" will make the sentence true. The sentences which are already true for the *is-equal-to* relation need not be rewritten.

Page 207. Refer to the Table of M. Facts for 2 at the top of the page. Prepare pupils for a M. chart which they shall encounter in the near future. You may duplicate the outline or the children may draw a grid design like the one below. On the outside of the grid at the left show the factor 2.

	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18

Above each square across the top show the factors 1 through 9. In each square is shown the product of the factor 2 (at the left end) and the factor shown immediately above the square. The children should be able to show the factors and products.

Page 208. Rewrite the sentences in column a at the top of the page using n instead of _ ? _. Then, write each sentence so that n appears in a different place in the sentence. For Ex. 1a,

$$n \times 3 = 6; \quad 3 \times n = 6.$$

Page 209. Rewrite Ex. 1–5 at the bottom so that each will be a true sentence by using the equals sign. The sentences which are already true for the *is-equal-to relation* need not be rewritten.

Supplementary Activities

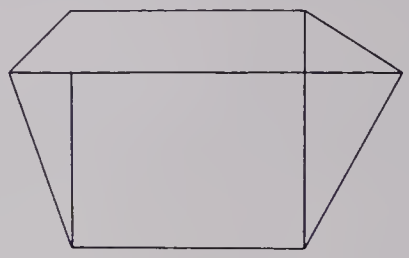
• *More capable children* may find the following examples challenging:

50,487	80,029	66,548	34,400
+61,254	+5,748	+28,496	+74,859
84,029	59,017	70,000	38,005
−31,762	−46,354	−35,267	−4,621

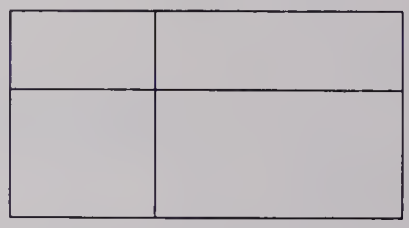
• In the following *if-then* sentences, the children are to write the operation symbol which will make each a true sentence.

If	$n = 7$,	If $n = 16$,
then	$n \bigcirc 7 = 14$;	then $13 \bigcirc n = 29$;
	$11 \bigcirc n = 4$;	$9 \bigcirc 7 = n$;
	$2 \bigcirc n = 14$;	$2 \bigcirc 8 = n$;
	$n \bigcirc n = 0$.	$n \bigcirc 5 = 11$.

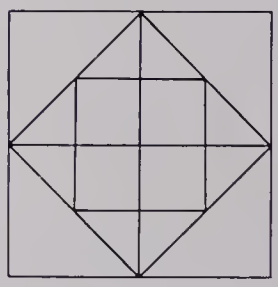
• How many polygons are shown in this drawing?



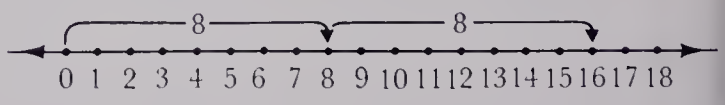
• How many rectangles do you see pictured?

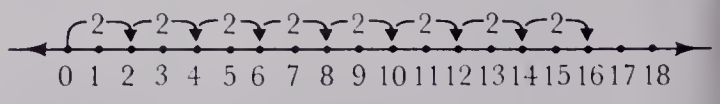


• How many squares do you see pictured?



• Draw two number-line pictures and illustrate in two ways the M. fact when the factors are 2 and 8.





Do the same when the factors are these:

6 and 3	9 and 2
4 and 5	4 and 6
5 and 3	3 and 8

* Are any of your pupils having difficulties? The Study Pages will provide remedial help.

* Do You Make Mistakes?

Diagnostic Test 5

Write answers on folded paper.

	a	b	c	d	Study Pages	Practice Sets
1.	5 ×2 = 10	2 ×7 = 14	9 ×2 = 18	2 ×8 = 16	204–207	
2.	199 +650 = 849	384 +355 = 739	605 +299 = 904	366 +295 = 661	184–186	60, 61
3.	628 −258 = 370	376 −95 = 281	862 −370 = 492	946 −364 = 582	190	62
4.	533 −69 = 464	521 −378 = 143	824 −59 = 765	864 −694 = 170	192	63, 64
5.	802 −68 = 734	603 −444 = 159	500 −235 = 265	900 −257 = 643	196, 198	65, 66

Oral Practice with *n*

Say the numbers for *n*, first by rows, then by columns.

	a	b	c	d
1.	$26 - \overset{10}{n} = 16$	$2 \times 7 = \overset{14}{n}$	$34 - \overset{10}{n} = 24$	$2 \times 8 = \overset{16}{n}$
2.	$3 \times 2 = \overset{6}{n}$	$28 - \overset{8}{n} = 20$	$\overset{40}{n} + 8 = 48$	$6 + \overset{16}{n} = 22$
3.	$30 + \overset{20}{n} = 50$	$15 + 10 = \overset{25}{n}$	$9 \times 2 = \overset{18}{n}$	$\overset{80}{n} - 20 = 60$
4.	$\overset{4}{n} + 12 = 16$	$6 \times 2 = \overset{12}{n}$	$\overset{30}{n} = 15 + 15$	$\overset{17}{n} + 7 = 24$
5.	$17 = \overset{9}{n} + 8$	$\overset{18}{n} - 12 = 6$	$\overset{36}{n} - 8 = 28$	$5 \times 2 = \overset{10}{n}$

Do You Understand?

Test of Information and Meaning 5

Copy and finish showing these names for 27:

1. $(7 + 6 + 7) + _?_$ 7 2. $(70 - 40) - _?_$ 3

3. Write the Roman numeral a. for 4. IV b. for 9. IX

4. What numbers do you not need for solving? $3.80; 65

Mrs. White spent $3.80 for things that weighed 65 pounds. The sugar weighed 10 pounds and the oranges 6 pounds. How much did these two things weigh?

Copy the letter of the example in the box in which you carry

A	B	C	D
216	588	425	42
+203	+79	+80	+39

5. to ten's place only. D

6. to hundred's place only. C

7. to both ten's and hundred's places. B

8. Copy each example in which you can multiply. b; c

a. $2 + 5 + 9 + 0$ b. $3 + 3 + 3 + 3$

c. $2 + 2 + 2 + 2$ d. $3 + 3 + 3 + 4$

9. Draw a picture of polygon *ABCD* and give three other names for it. BCDA, CDAB, DABC

Write an *n*-example for each of these on page 210:

10. Problem 1 $15 + 16 + 3 = n$ 11. Problem 3 $225 - 190 = n$ 12. Problem 7 $100 - 27 = n$

E	F	G	H
72	186	211	846
−36	−35	−185	−275

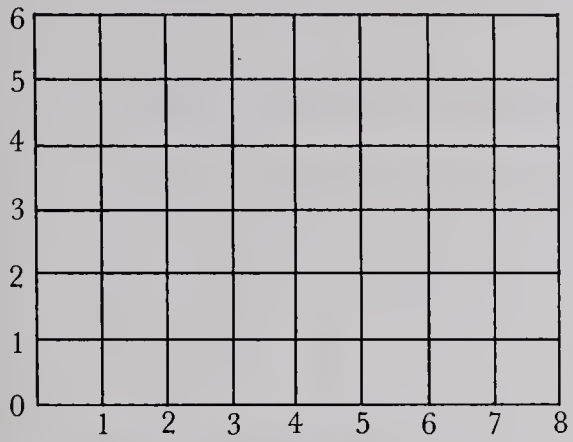
Copy the letter of the example in which you rename the sum to show

13. more tens only. H 14. more ones only. E

15. more ones and more tens. G

• Coordinate Geometry

Perhaps you can obtain some $\frac{1}{2}$-inch graph paper on which children can draw the horizontal and vertical axes. Or, you may prefer to duplicate some coordinate systems similar to the one shown below.



1. Make dots at these points:

a. Over 6, up 4
b. Over 2, up 5
c. Over 7, up 1
d. Over 4, up 6

2. Make dots at these points:

a. Over 8, up 2
b. Over 7, up 3
c. Over 6, up 4
d. Over 5, up 5
e. Over 4, up 6

Join the dots. At this time, try not to talk formally about lines as not until Chapter 7 will pupils be introduced to the concepts of line segment and line.

3. Make a coordinate system and show a portion of a line. Then, have children tell the location (the number of units over and up) of some points in the line indicated. Lines may be vertical, horizontal, or diagonal. See if children can draw any generalizations about the points in a particular line.

NOTES

See Teacher's Page 210 for suggestions about the end-of-chapter tests.

NOTES

Copy and finish Ex. 16–17.

16. 24 tens = $\overset{2}{_?_}$ hundreds and $\overset{4}{_?_}$ tens

17. 316 = 30 $\overset{\text{tens}}{_?_}$ and $\overset{16}{_?_}$ ones

Write "Yes" or "No" for an answer to each of these:

18. Is 73 the next odd number after 71? Yes

19. Does n equal 30 in the example $43 - n = 13$? Yes

20. Has $2 \times 2 = 4$ another M. fact to make a pair? No

21. Does this picture show fifths? No

22. Is $(8 + 6)$ equal to (2×7)? Yes

23. If you have $\frac{1}{4}$ of a candy stick, could 3 others have pieces of the same size from that stick? Yes

How Well Can You Compute?

Computation Test 5

Copy in columns and add or subtract. Check your work.

1. 415 − 87 328

2. 257 + 500 757

3. 389 − 89 300

4. 410 − 373 37

5. 187 + 98 285

6. 700 − 489 211

7. 353 + 49 402

8. 968 − 359 609

9. 100 − 74 26

10. 42 + 738 780

11. 439 − 76 363

12. 923 − 738 185

13. \$7.98 − \$7.25 \$0.73

14. \$4.06 + \$3.08 \$7.14

15. \$2.22 − \$1.97 \$0.25

16. \$6.14 − \$3.08 \$3.06

17. \$5.28 + \$2.79 \$8.07

18. \$0.85 − \$0.48 \$0.37

19. 40, 6, 39, 7 92

20. 9, 5, 6, 8 28

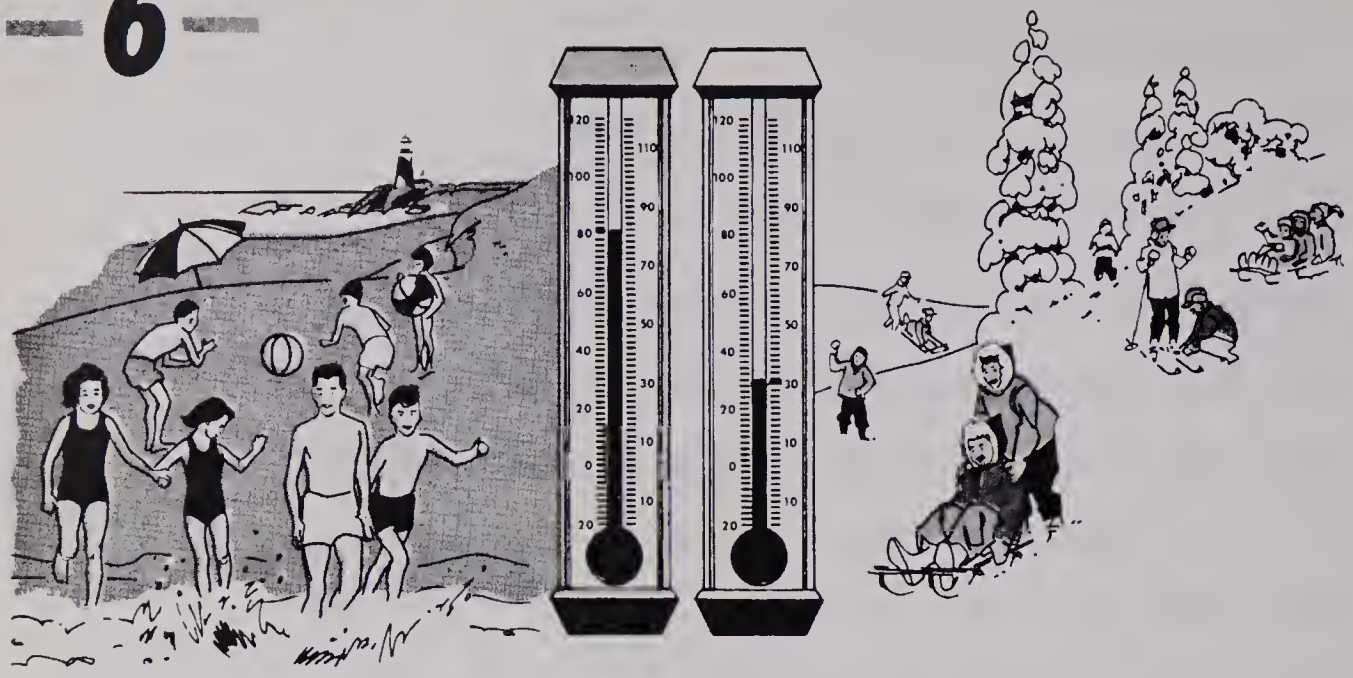
21. 38¢, 18¢, 7¢, 18¢ 81¢

22. 19¢, 16¢, 34¢, 18¢ 87¢

23. \$0.51, \$0.07, \$0.17, \$0.20 \$0.95

24. \$0.07, \$0.35, \$0.27, \$0.27 \$0.96

6



Measuring Temperature

Using a number line [O]

1. In Ted's town it is hot in summer and cold in winter. In what months might he play in the snow? December through March In what months might he go swimming? June through September

2. Look at the pictures above. For each, how warm or cold was it? very warm; very cold How could you tell? Actions and clothing of children

3. What do we use to measure time? clock to measure weight? weight scale to measure lengths? ruler

4. Do we have something for measuring temperature? Yes

5. In the pictures above, do you see what we use for measuring temperature? Yes What do we call it? Thermometer

A thermometer measures temperature in degrees.

The little marks between the numerals showing tens on the thermometer help us to count by 2's in reading the number of degrees. We do the counting only when the temperature reading is between the numerals.

Answers will vary.

Children probably have had experiences with vertical, horizontal, and round thermometers. It may be well to demonstrate all kinds. Most pupils will quickly see the parallel between the scale on a thermometer and number-line pictures.

Overview—Chapter 6

• In Chapter 6 the new work in multiplication involves facts having a factor 3 or 4, and one factor a number containing tens and ones. In multiplying tens and ones, the pupil now uses another of the properties of arithmetic—the Distributive Property of Multiplication over Addition.

• **Problem-Solving.** Work in problem-solving also is extended to use of facts having a factor 3 or 4 and maintains work begun earlier in differentiating among the needs for addition, subtraction, and multiplication. Estimating answers for problems is still confined to problems requiring addition or subtraction, as is the writing of an *n*-sentence to show how to work the problem.

• **Maintenance.** Work with the set of fraction numbers is maintained and makes a definite advance. The pupil learns how to write fractions for fraction numbers involving several parts of the same size. He learns too that the numeral below the fraction bar shows the number of parts of the same size in the particular whole and so helps to name the fraction number; also that the numeral above the fraction bar shows the number of those parts being named. Furthermore, the pupil learns to compare fraction numbers for parts of the same whole—that one part is larger or smaller than another and, correspondingly, that one fraction number is greater or less than another; he learns to count in sequences of fraction numbers; and he learns to use fractions with measurements.

Measurements are maintained and now extended to simple work with temperatures and liquids.

Work is maintained and extended with inequalities, logic, *n*, and with other number names.

• **Enrichment.** In this chapter the more capable children will learn how to subtract by the equal-additions method, to add by obtaining partial sums, to show multiplication facts within parentheses as other names for numbers to be added or subtracted. There is also for these children a section of unreasonable statements to justify.

• **Testing.** With the usual maintenance, mid-chapter review, and end-of-chapter tests, the work of this chapter is evaluated.

Teaching Pages 214 and 215

Pupil's Objectives

(a) To learn more about the measurement of temperature; (b) to begin to learn about the set of *integers*; and (c) to learn the term *degree* and the symbol ° for degree.

New Words

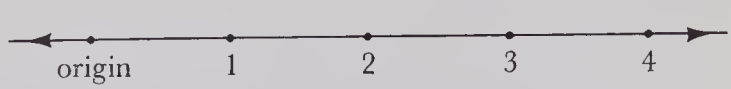
(page 214) *degrees*, *temperature*, *thermometer*
(page 215) *boil*, *freeze*

Background

The thermometers pictured show a portion of a number line. Usually, a number-line picture is drawn horizontally but children should feel comfortable in seeing and in drawing a number-line picture in any position.

In many parts of the country, children are very familiar with the terminology "below zero." By listening to weather reports and experiencing extreme cold, children become familiar with a special set of numbers intuitively. These numbers are technically known as the *negative numbers*. Though these numbers should not be developed formally at this time, children should have many experiences with them in their mathematical development.

Up to this time the set of counting numbers, or positive integers, and the set of whole numbers have been formally explored and developed.



The numbers shown below for points to the right of the origin (zero) are positive integers and the numbers for points to the left of the origin are negative integers. The number for the origin (zero) is neither positive nor negative.

The points to the left of zero for *negative integers* are shown the same distance apart as those for counting numbers (or positive integers).

The set of integers is that set of numbers which includes all the negative integers, zero, and the positive integers. Since this set is infinite, only a portion of it can be shown here. When there is no chance for confusion, the plus symbol is not written in the numeral for the positive integers. For the most part, only zero and the positive integers are formally developed in the first five or six grades in school. Note, all whole numbers are integers, but not all integers are whole numbers.

The symbol for showing negative integers is like the symbol indicating subtraction but is placed to the left of the numeral and raised, such as in $^{-}4$ or $^{-}6$. The position of the symbol is chosen to avoid confusion with the symbol for subtraction. Similarly, a small raised plus symbol is used for positive integers. The symbols for negative and positive numbers are not symbols of operation; however, the ideas of addition and subtraction are closely related. In more advanced mathematics, $^{+}4$ indicates either the number *positive four* or the operation *add 4*; $^{-}6$ indicates either the number *negative six* or the operation *subtract 6*.

Teacher's Preparation

It would be a good idea to have two or three actual thermometers available. If you do not already have a large demonstration-model thermometer,* perhaps you will wish to make one.

Pre-Book Lesson

• Start a discussion about temperature by asking a question such as: "How cold (or hot) is it?" "How do you know?"

At this point, be sure to have an outside thermometer available which the children can see. Discuss the reading of it and compare it with an inside thermometer. Be sure to emphasize that the thermometer is an instrument for measuring temperature. Temperature is measured in degrees just as distance may be measured in inches.

• Measure the temperature of other materials or objects in the room. (Near a cold window; over a radiator or heater; near a burning light bulb.) Each time, have children read the thermometer and tell why the temperature has changed.

• Use a large demonstration thermometer. Ask a child to use a red ribbon to show any temperature he wishes. Have another child read the temperature. Continue this for a while.

• Set the ribbon at $^{-}10$ (10 below zero). Discuss 0 (zero) as a reference point on the thermometer. Some numbers are shown "above zero" and some numbers are shown "below zero." Have children use these terms in reading temperatures shown on the demonstration model.

Using the Text Pages

Refer to the Pre-Book Lesson for Ex. 1–5.

More capable children may use an encyclopedia to obtain additional information about Ex. 6–12. In Ex. 9, for instance, it is *contraction* as a result of cooling, and *expansion* as a result of heating that causes the rise and fall of the red liquid. In Ex. 12, 212° is considered the boiling point of water at sea level. The boiling point decreases 1° F. for every 550 feet of increase in altitude due to the decrease in pressure. At 10,000 feet, water boils at 194° F.

Individualizing Instruction

• If possible, obtain the temperatures of the coldest day of the year and the warmest day of the year. Use this information as a guide for deciding temperatures that indicate a very hot and a very cold day for your community.

• *All pupils* may ask at home about different types of thermometers. Have them describe and draw a picture of the thermometer and tell its use.

• *More capable children* may try the enrichment activity at the bottom of page 215.

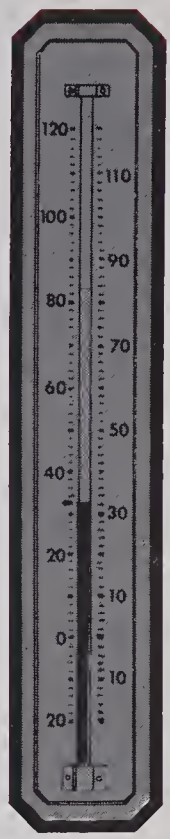
The rationale for this subtracting is based on *compensation*. (The same number is added to the sum and one of two addends without changing the other.) For example, $8 - 2 = 6$. If 1 is added to both 8 and 2 the other addend will still be 6.

In box B at the bottom of child's page 215, 10 has been added to both numbers. Ten was added to 82—ten more ones are shown. Ten was added to 47 in the ten's place—one more ten is shown. The difference, 35, is the same as in the example in box A.

• You may wish to use Extra Activities Set 100.

* See 15, page xix.

* The answer for "at 200?" is "No" in most localities. At very high altitudes, the answer would be "Yes." See the comments under Using the Text Pages on Teacher's Page 215.



6. On the winter thermometer, the red liquid pictured ends at the mark for what number? 30 Then, the temperature that day was 30? degrees.

7. Tell the temperature on the summer thermometer. 80 degrees

Just as we write "62 ft." for "62 feet," we write "35°" for "35 degrees." The little ° means degrees.

8. On the board show the room temperature now.

9. When it gets warmer, the top of the red liquid in the thermometer moves upward. When it gets colder, it moves downward. Can you tell why? Heat causes it to expand.

10. At 32°, water starts to freeze. What do we mean, then, by temperature above freezing? Warmer than 32° below freezing? Colder than 32°

11. Will water freeze at 10°? Yes at 50°? No Why? 10° is colder than 32°, 50° is warmer than 32°

At 212° water almost always begins to boil. So we call 212° the boiling point of water.

* 12. Can you boil water at 99°? No at 200°? Not usually. Why? Both temperatures are colder than 212°.

● **Extra Activity.** Work Set 100.

Do You Like to Try New Things?

Enrichment [W]

You have learned to subtract 47 from 82 by renaming the sum, as shown in Ex. A.

** Many people work this example as shown in Ex. B. They *think*, "7 from 12, 5; 5 from 8, 3." Try to figure out what they do to get the same answer you do.

Turn back to page 133. Do the subtraction examples in rows 6–9 the new way, as in Ex. B.

A

$$\begin{array}{r} {\scriptstyle 7}\,{\scriptstyle (12)} \\ \not{8}\,\not{2} \\ 4\,7 \\ \hline 3\,5 \end{array}$$

B

$$\begin{array}{r} {\scriptstyle (12)} \\ 8\,\not{2} \\ {\scriptstyle (5)} \\ \not{4}\,7 \\ \hline 3\,5 \end{array}$$

** Since ten is added to both the sum and the known addend, this is often called the equal-additions method.

A

B

Making Change

From 25¢, 50¢, $1; mental procedures [O]

1. Mrs. Long gave 25¢ for a 13¢ cake of soap. Use the coins shown in picture A to count her change.

a. Before giving any change you say, "13¢," the cost of the soap.

b. Point to one cent shown in box A and say, "14."

c. What would you say for the other cent? 15

d. Why do you then say, "25," for the dime? 15¢ + 10¢ = 25¢

2. Al's stamps cost 18¢. Point to the coins shown in picture B and tell how the clerk made change from 25¢. 18¢, 19, 20, 25

C

3. Mrs. Gray gave the clerk a half dollar for things that cost 22¢. The clerk said, "22¢," the cost, and then "23, 24, 25, 50." Explain, using picture C. 22¢ + 1¢ = 23¢; 23¢ + 1¢ = 24¢; 24¢ + 1¢ = 25¢; 25¢ + 25¢ = 50¢

4. Jean gave the clerk 50¢ for a 31¢ cake. He said, "31¢, 32, 33, 34, 35, 40, 50." What coin did he count when he said, "40"? nickel when he said, "50"? dime

D

5. Jack bought a 35¢ cowboy rope. Count his change from a half dollar. 35¢, 40, 50

6. Picture D shows Clara's change after she bought a 35¢ book. How much did she have at first? $1.00 (Answers will vary.)

Teaching Pages 216 and 217

Pupil's Objectives

(a) To acquire skill in making change from a quarter, a half dollar, and a dollar; and (b) to have practice in all of the addition and subtraction skills learned so far.

New Words

(page 216) *clerk*, *soap*
(page 217) *amount*

Teacher's Preparation

Have actual money to perform some of the transactions described in the pupil's book. Many toy coins and dollar bills will help make group work practicable.

You may wish to devise a *store* situation of some type in which children may simulate money transactions.

Pre-Book Lesson

• Review the names and the values of the different coins that have been taught so far. You may want to make use of lists or charts showing equivalent relationships.

• Introduce the idea of making change by stating a *spending* situation in which the child gave the clerk a quarter for an item that cost 10¢. Have the class determine how much change he would get. Discuss the various ways the change could be found.

• If no child has mentioned it, introduce the method of *counting up* to the value of the coin given the clerk.

• State a spending situation in which the item cost 14¢. See if the children can count up to 25; first with 1¢, then 10¢ (either a dime or two nickels).

Correct responses may be:

14 and 1 makes 15; and 5 is 20; and 5 more is 25.
14 and 1 makes 15; and 10 more is 25.

Using the Text Pages

• If possible, dramatize in small groups many of the transactions in Ex. 8–35. Some children may take the part of storekeepers and others the part of customers. Encourage pupils to check change as it is given.

• For Ex. 1–14 at the bottom of page 217, *more capable children* may be permitted to use the new method of subtraction from the enrichment section of page 215. They should not use it if they have not learned to do so.

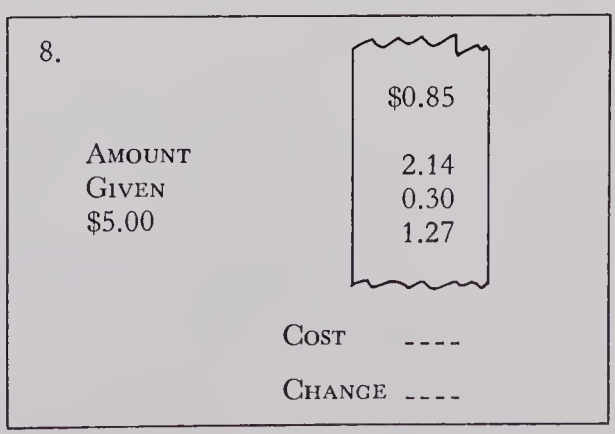
Individualizing Instruction

• A classroom store may be devised in which merchandise is for sale. Children may make price tags for the various items available for sale. The type of store could possibly fit in with a unit on social science, science, and so on under consideration at the moment. Children may take turns being storekeepers and customers in which making change for the purchases is the main activity.

• For *all pupils*, write on the chalkboard or duplicate examples like the following, having pupils complete the last column by drawing pictures of coins and labeling them:

Amount Given	Amount Spent	Change
$1.00	71¢	?
$3.50	$3.32	?

• Ask children to bring sales tapes from home and make cards like those suggested for pages 150 and 151. This time, the sales tapes selected may be more difficult than previously. The column may be longer; 2- and 3-place numerals may be used; renaming may be involved in both addition and subtraction. The drawing below is representative of what all cards may be like:



a. Use heavy cardboard or tagboard for making the cards.

b. Number each card in the upper left-hand corner.

c. Indicate on each card the amount given the clerk.

d. Be sure to prepare an answer card.

e. Cards may be arranged by sets according to the level of difficulty of addition and subtraction for each sales tape.

Reminder

Continue to maintain the following:

a. ability to recognize and use common geometric terms like polygon, line, line segment, triangle, square, rectangle, and so on

b. ability to recognize common measuring instruments and occasions for their use

c. increasing ability to tell time

d. use of symbols of inequality for "is greater than," "is less than," and "is not equal to"

NOTES

E

7. Mrs. Hill paid 7¢ for cabbage. She gave the clerk a dollar. Why was her change not correct? (picture E)

7¢ + 3¢ = 10¢; 10¢ + 5¢ = 15¢; 15¢ + 5¢ = 20¢; 20¢ + 25¢ = 45¢; 45¢ + 25¢ = 70¢; 70¢ + 25¢ = 95¢

Tell how to make change with the fewest coins:

You give 25¢ and the amount to pay is

8. 8¢. ∧ 2 cents, 1 dime, 1 nickel
9. 17¢. ∨ 1 dime
10. 15¢. ∧ 3 cents
11. 22¢. ∧ 2 cents, 1 dime
12. 13¢. ∧ 1 dime, 1 nickel
13. 10¢. ∧ 3 cents, 1 nickel
14. 21¢. ∨ 4 cents
15. 18¢. ∨ 2 cents, 1 nickel
16. 12¢. ∧ 3 cents, 1 dime
17. 9¢. ∨ 1 cent, 1 dime, 1 nickel

You give 50¢ and the amount to pay is

18. 29¢. ∧ 1 cent, 2 dimes
19. 38¢. ∨ 2 cents, 1 dime
20. 11¢. ∧ 4 cents, 1 dime, 1 quarter
21. 46¢. ∨ 4 cents
22. 27¢. ∧ 3 cents, 2 dimes
23. 25¢. ∧ 1 quarter
24. 42¢. ∨ 3 cents, 1 nickel
25. 15¢. ∧ 1 dime, 1 quarter
26. 28¢. ∨ 2 cents, 2 dimes
27. 35¢. ∧ 1 dime, 1 nickel

You give $1.00 and the amount to pay is

28. 28¢. ∧ 2 cents, 2 dimes, 1 half dollar
29. 65¢. ∨ 1 dime, 1 quarter
30. $0.17. ∧ 3 cents, 1 nickel, 1 quarter, 1 half dollar
31. $0.30. ∨ 2 dimes, 1 half dollar
32. 45¢. ∧ 1 nickel, 1 half dollar
33. 78¢. ∨ 2 cents, 2 dimes
34. $0.37. ∨ 3 cents, 1 dime, 1 half dollar
35. 55¢. ∨ 2 dimes, 1 quarter

Practice in Subtraction

[W]

Copy and subtract. Check your work.

1–7. Subtract 279 from each number shown below.

500 221 287 8 609 330 380 101 799 520 900 621 810 531

8–14. Subtract 186 from each number shown above.

314 101 423 194 613 714 624

A. or S. — Which?

Problems without numbers [W]

Write only "A." or "S." to show which you would use.

1. Joe knows the number of pages in his stamp book and the number of pages he has filled. How can he tell the number of pages to be filled? S.

2. He knows how many old stamps and how many new stamps he has. How can he tell how many stamps he has? A.

3. Joe knows how many stamps he has and how many Tim has. How can he tell how many more or fewer stamps he has than Tim? S.

4. How can Joe tell how many stamps he and Tim have together? A.

5. Joe knows how many stamps he bought and how many of them are in his book. How can he tell how many are not in the book? S.

6. He knows how many red stamps he has just bought and how many red stamps he had. How can he tell how many red stamps he has in all? A.

7. Joe knows how many stamps he has now and how many he had Monday. How can he tell how many of his stamps are lost? S.

8. Joe has more brown stamps than red stamps. How can he tell how many more red stamps he needs to have the same number of red stamps as brown stamps? S.

Teaching Page 218

Pupil's Objective

To have practice in selecting the appropriate operation for problems which do not include numbers.

Background

The emphasis in this activity should be placed on reading the material and then determining the most appropriate operation to use to find the solution.

All too often, too much emphasis is placed on the answer as such and little attention is given to procedures used in finding the answer. Of course, the mere act of reading is important in the solving of problems. However, the role of *translating* the written material to an *n*-example involving the appropriate operation is equally important.

Teacher's Preparation

You may wish to have some simple materials ready for use in demonstration and illustration of problem situations if it is deemed necessary.

Pre-Book Lesson

• Set the stage for the idea of problems with no numbers by stating orally a problem similar to the following:

You know how tall you are and how tall your father is. How can you tell how much taller your father is than you?

• As you discuss the problem with the pupils, ask them to tell why they would use the operation they have chosen.

• Use children in the class to demonstrate the same type of problem. Suggest to pupils that they try to form *mental images* of the problem setting or pictures in their minds.

Using the Text Page

Most children should be able to proceed independently with this material. They should read the problem and then write either "A." or "S." to indicate the appropriate operation for the problem.

Individualizing Instruction

• With *slower learners* or pupils with reading difficulties, it may be appropriate to direct this activity orally in small-group instruction. Ask different children to read the problems. Where there is lack of understanding, dramatize the situations with representative materials. After you have covered each problem orally, ask the children to read each problem independently and show the appropriate operation.

• Extend practice for *all pupils* by using other problems similar to those below:

a. You know how much a basket weighs and how much the things in it weigh. How can you tell how much the whole load weighs?

b. You know how long your kite string is and how long another piece of string is. How can you tell about how long the kite string will be if you tie the extra piece of string to it?

c. You know how many children a bus will hold and that there will be empty seats when it is used to take the children to the museum. How can you tell how many empty seats there will be?

d. You know how many minutes it took you to walk to school and how many minutes it took to walk home. How can you tell how many minutes you spent in walking to and from school?

• Either orally or as a written activity, *all pupils* may suggest numbers that could be used in the problems.

Pupil's Objective

To learn how the facts which have been developed in multiplication may be used in finding the solution to problems.

Background

The idea of multiplication has been developed in connection with the joining of sets which have equal numbers of members. One known factor tells the number for each set (the number of members) and the other known factor tells the number of sets. The multiplication of two known factors yields a third number called the product.

Each of the problems in this lesson involves sets having the same number of members, i.e., sets which are equivalent. The pupils are to determine what the two factors are and then are to find the product of these factors.

Pre-Book Lesson

• Use blocks, chairs, children, and so on to arrange several sets of 2. Have the children tell how many 2's there are in each instance. Then have a child show on the chalkboard the multiplication fact for the demonstration.

• Guide pupils to state in their own words that we multiply to find the number for the total when the sets being joined are equivalent.

Using the Text Page

• Before assigning the written work, discuss each problem with the entire class, referring constantly to the generalization stated in the Pre-Book Lesson. Have children specify the number for each group, the number of groups, and the possibility of finding the total by addition or multiplication. Have them state why they prefer one or the other operation. Use representative objects and dramatization to clarify situations as needed. Remind pupils to rewrite each answer with a label as shown in the box for problem 1.

• When the written lesson is completed, as oral work have pupils explain why $7 \times 2 = n$ is an appropriate number sentence for problem 1. Then, have different pupils tell number sentences for the remaining problems. Translating problem situations into number sentences is a vital part of problem solving.

Individualizing Instruction

• *All pupils* may use the problems below to practice telling whether they can multiply instead of add; to decide on appropriate number sentences.

a. I saw 3 birds on each of 2 telephone wires. How many birds did I see?

b. I saw 3 birds on one telephone wire, 2 on another, and 4 on another. How many birds did I see?

c. Two cars were taking 4 children each. That was how many children in all?

d. Two children rode in one car, 3 in another, and five in another. How many children were in the three cars?

e. Four red cows, 3 brown cows, and 2 black cows are how many cows in all?

f. How many cows in all are there in 5 groups of 2 black cows each?

• *All pupils* may prepare a list or make a chart of things that frequently occur in 2's (gloves, shoes, and so on) and in 2 groups (2 teams in a game). The lists or charts may suggest subjects for original problems which may be written and presented to the class.

* Previously, pupils have thought of problem-solving activities in terms of A. or S. Now they add a third operation, M., to the operations by which they may arrive at an answer.

Multiplication for Problems

Using M. facts for 2's and for 2 [W]

Write the multiplication work and labeled answer for each problem. The work for problem 1 is in the box.

$$\begin{array}{r} 2 \\ \times 7 \\ \hline 14 \end{array}$$

14 eggs

1. Mrs. Tell decided to hide eggs for the egg hunt in groups of 2. How many did she need for 7 groups?

2. Ned found 3 eggs. Tom found the same number. How many eggs did these 2 boys find in all? 6 eggs $2 \times 3 = n$

3. Sue found 8 eggs. Lois found 2 times as many. How many eggs did Lois find? 16 eggs $2 \times 8 = n$

4. Bill found 2 eggs in each of 6 places. How many eggs did he have then? 12 eggs $6 \times 2 = n$

5. Five children each found 2 eggs. Together how many eggs did they find? 10 eggs $5 \times 2 = n$

6. Ann had 2 eggs in her basket. If the boys all together had 9 times as many, how many eggs did they have? 18 eggs $9 \times 2 = n$

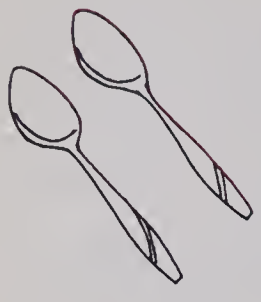
[O]

The *n*-example, or number sentence, for problem 1 is $7 \times 2 = n$ or $n = 7 \times 2$. Explain. Tell the *n*-example for each of problems 2–6. See above for n-examples.

* Previously, differentiating operations to use has been between A. and S. Now pupils must differentiate between using A. and M.

Multiply or Add — Which?

Differentiating A. and M. problems [O]



* To find the number for a total set, you add if the sets to be put together are not equivalent; you multiply if the sets are equivalent.

Should you add or should you multiply for each of Ex. 1–10? Tell why.

1. Jean put 2 small spoons at each of 9 places on her party table. How many small spoons were on the table? Multiply because the sets are equivalent. 18 spoons. $9 \times 2 = n$

2. In all, Jean used 18 small spoons and 6 larger ones. All together, how many spoons did she use? Add because the sets are not equivalent. 24 spoons. $18 + 6 = n$

3. Two boys and 3 times as many girls were at the party. How many girls were at the party? Multiply because the sets are equivalent. 6 girls. $3 \times 2 = n$

4. For 5 boys, how many shoes are needed? Multiply because the sets are equivalent. 10 shoes. $5 \times 2 = n$

5. Jack has 2 nickels. How much money has he? Multiply because the sets are equivalent. 10¢. $2 \times 5¢ = n¢$

6. Ann missed 4 examples Monday and 2 examples Tuesday. In the two days she missed how many examples? Add because the sets are not equivalent. 6 examples. $4 + 2 = n$

7. Two plates each have 6 oranges. How many oranges are on the two plates? Multiply because the sets are equivalent. 12 oranges. $2 \times 6 = n$

8. One bag has 9 oranges. Another bag has 10 oranges. How many oranges are in the two bags? Add because the sets are not equivalent. 19 oranges. $9 + 10 = n$

9. At 2¢ each, how much will 6 candy drops cost? Multiply because the sets are equivalent. 12¢. $6 \times 2¢ = n¢$

10. Find the cost of two pears selling for 8¢ and 9¢. Add because the sets are not equivalent. 17¢. $8¢ + 9¢ = n¢$

[W]

Now write the answers for problems 1–10.

Write an *n*-example for each problem.

♦ **Extra Problems.** Work Set 11.

Pupil's Objectives

(a) To learn more about differentiating between addition and multiplication as operations to use in solving problems; and (b) to gain more experience in writing the correct *n*-example for a problem.

Background

You will want your pupils to grasp and apply the following ideas in solving problems: (a) Finding the number for a total is involved when either addition or multiplication is used; and (b) Multiplication is the more appropriate operation when the problem deals with several equivalent sets. In solving problems based on classroom situations, distinctions between the two operations should be made easily.

Pre-Book Lesson

• Use sets of objects in the classroom to demonstrate the joining of sets not having the same number of members. Then, through guided discussion, lead children to the conclusion that addition is the appropriate operation to be shown in the *n*-example.

• By the same techniques, arrange equivalent sets which are joined. If the sets are joined carefully, an array can be formed which will show the size of each set and the number of sets. The orderly arrangement of the array lends very nicely to *seeing* and *stating* the number sentence involving multiplication.

Using the Text Page

• Select children to read each problem. Have pupils tell whether they would add or multiply to find the solution. Be sure to have them tell why they would use the particular operation selected.

• After the discussion about each problem and the differences in situations where addition and multiplication are appropriate, have pupils write the answers for problems 1–10.

Then, after they have written answers, have the children write a number sentence for each problem.

Individualizing Instruction

• From information like that which follows, *all pupils* may write some problems which require addition and some problems which require multiplication:

a. Telephone calls, 5¢ each

b. Parking meter, 1¢ for 12 minutes

c. 3 dolls in a set: each doll has 6 dresses, 2 hats, 3 coats, 4 bonnets

d. Cost of items at the school cafeteria: apples, 3¢; milk, 6¢; cookies, 2¢; sandwiches, 9¢

• *Slower learners* may profit from making drawings to illustrate the information given in the problems.

• Use Extra Problem Set 11 as deemed necessary for *all pupils*.

Pupil's Objectives

(a) To begin to learn the multiplication facts in which 3 is one factor; (b) to extend understanding of the relationship between addition and multiplication as operations for finding the result of joining sets; and (c) to extend understanding of the Commutative Property of Multiplication.

Background

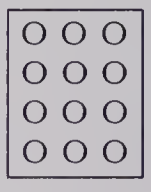
Multiplication has been developed as a special operation for describing the joining of several equivalent sets. One of the factors tells the size of the sets being joined and the other factor tells how many sets are being joined. The product tells how many objects are in the new set created as a result of joining the several sets. Technically, multiplication is defined as the operation on two numbers called *factors* which yields a third number called the *product*.

On page 206 of the pupil's book, the idea was introduced that most multiplication facts go in pairs. This was the first lesson dealing with the Commutative Property of Multiplication. Understanding this property is a very useful tool in learning the multiplication facts.

Teacher's Preparation

Have materials available which pupils may use to form arrays. The materials should be such that the rows and columns of the array can be seen distinctly. Also, they should be movable so the array may be placed in different positions. Felt designs* placed on a flannel board* are very useful for this purpose. Blocks, spools, discs, and so on placed on a piece of construction paper or tagboard work very nicely, also.

Pre-Book Lesson

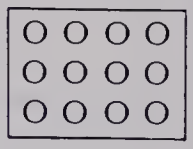


• Direct pupils to make an array in which there are 4 sets of 3 members. (3 columns and 4 rows)

Refer to the array as a picture of 4 sets of 3, or merely as four 3's. Have a child write a number sentence on the chalkboard.

* See 17 and 2, page xix.

• With $4 \times 3 = 12$ written on the board, review the terms *factors* and *product*. The factor 3 tells the number of members in the equivalent sets and the factor 4 tells how many sets are being joined. The product tells the number of members in the newly formed set.



• Have pupils turn the array so there are 4 columns and 3 rows. Now, the sentence is $3 \times 4 = 12$.

• Analyze the two sentences on the board. Have children state in their own words a generalization about the effect of changing the order of the factors.

Using the Text Page

• Be sure to focus on the commutative property in discussing Ex. 1 and 2. Pupils may find it helpful to turn the book to *see* the facts in Ex. 1.

• The discussion in connection with Ex. 3–16 is based upon written work which should be done on the board by different children.

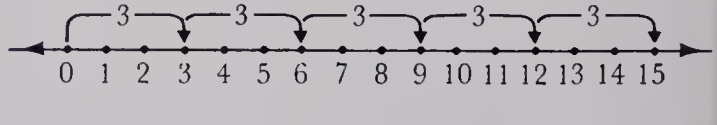
While a child is writing or drawing the appropriate material on the board for each example, the children at their seats should be doing the same thing with pencil and paper.

Throughout all the discussion, stress the commutative property.

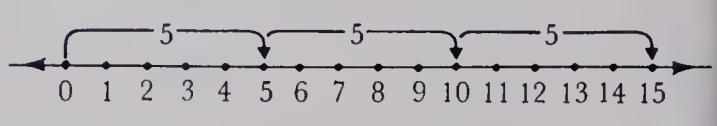
Individualizing Instruction

• *All pupils* may construct number-line pictures to illustrate pairs of M. facts as follows:

$5 \times 3 = 15$



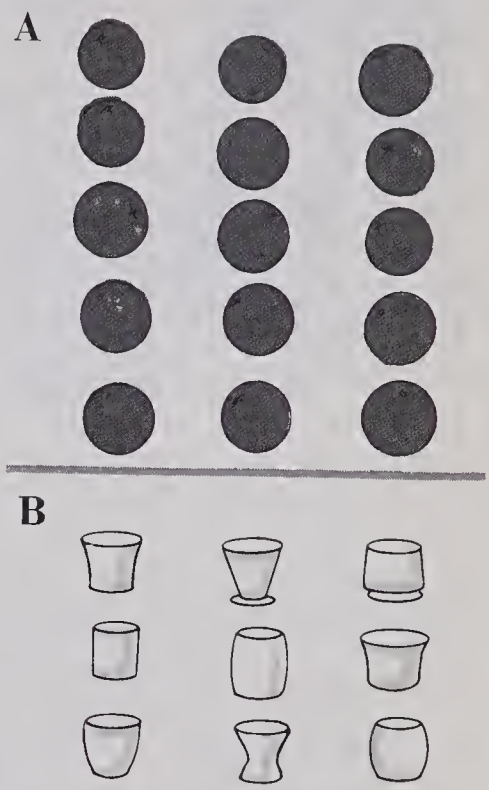
$3 \times 5 = 15$



• *Slower learners* may make dot pictures (arrays) for each of the M. facts in Ex. 9–15.

Discovering M. Facts with a Factor 3

Using Commutative Property of M.; arrays [O]



1. Picture A. Use the rows of oranges to finish the M. fact $5 \times 3 = n$. Use the columns to finish the M. fact $3 \times 5 = n$.

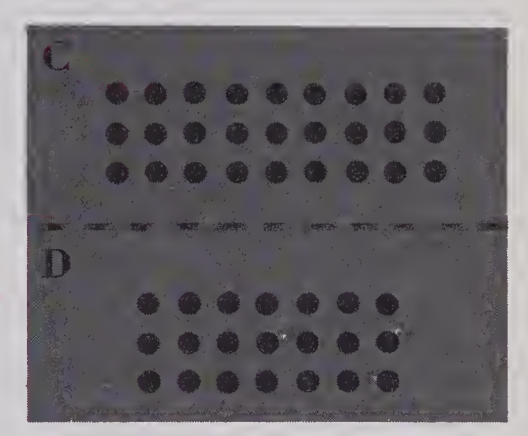
2. Picture B. Use both the rows and the columns to finish the M. fact $3 \times 3 = n$. Has this M. fact another to make a pair? No Why? The numbers for rows and for columns are equal.

Remember: Most M. facts go in pairs. If you know one, you should know the other.

For Ex. 3–16, work on the board.

3. Picture C. If we use the rows, we have the example $9 + 9 + 9 = n$.

Show the pair of M. facts. $3 \times 9 = 27$, $9 \times 3 = 27$



4. Picture D. Use the columns and count by 3's. Then show the pair of M. facts. $7 \times 3 = 21$, $3 \times 7 = 21$

Draw a dot picture for

5. 6×3. $6 \times 3 = 18$, $3 \times 6 = 18$

6. 3×4. $3 \times 4 = 12$, $4 \times 3 = 12$

7. 8×3. $8 \times 3 = 24$, $3 \times 8 = 24$

8. Under each of the pictures for Ex. 5–7, show the pair of M. facts.

Copy Ex. 9–15 and show the products. For help, add numbers or count or use a number line.

9. $3 \times 9 = 27$, $9 \times 3 = 27$

10. $3 \times 6 = 18$, $6 \times 3 = 18$

11. $7 \times 3 = 21$, $3 \times 7 = 21$

12. $3 \times 5 = 15$, $5 \times 3 = 15$

13. $3 \times 8 = 24$, $8 \times 3 = 24$

14. $4 \times 3 = 12$, $3 \times 4 = 12$

15. $9 \times 3 = 27$, $3 \times 9 = 27$

16. Under each M. fact you showed for Ex. 9–15, show the M. fact that goes with it to make a pair.

Multiplication Facts for 3's

Terms of multiplication [O]

A	
3	addend
3	addend
3	addend
3	addend
?	sum

B	
3	factor
4	factor
?	product

1. The children in Jane's room are working in sets of 3. How many children are there in 4 sets?

a. You can count by 3's: 3, 6, [9]?, [12]?.

b. You can add 3's, using a number line.

c. You can add 3's as in Ex. A. The sum is ?. [12]

d. What M. fact do you know now (Ex. B)? $4 \times 3 = n$ [12]

Count by 3's to find how many children in

2. 5 sets. (3, 6, 9, [12]?, [15]?)

3. 7 sets. [21]

4. 2 sets. [6]

5. 3 sets. [9]

6. 9 sets. [27]

7. 8 sets. [24]

8. 6 sets. [18]

[W]

Remember: You can show a M. fact in three ways.

Copy and finish these to show the M. fact for Ex. 1:

Four 3's = n [12]

$4 \times 3 = n$ [12]

$$\begin{array}{r} 3 \\ \times 4 \\ \hline n \end{array}$$ [12]

Show the M. fact for each of Ex. 2–8 in three ways.

Answers can be found in several ways, so again pupils explore various ways of finding the answer.

Teaching Pages 222 and 223

Pupil's Objectives

(a) To discover the multiplication facts for 3's; and (b) to continue to establish the relationship between addition and multiplication.

Background

The factor 3 can have two meanings as shown in the two arrays below. In array E, 3 denotes the number of columns or tells the number of members in each of the equivalent sets shown. We could find the product by adding the 3's as well as using 3 as a factor for a multiplication. Array G, on the other hand, shows 3 to be the number of rows or the number of equivalent sets. Here, 3 tells us how many 7's to add to find the total for the array. It also is a factor for a multiplication.

E

X X X
X X X
X X X
X X X
X X X

G

X X X X X X X
X X X X X X X
X X X X X X X

In this lesson we are focusing upon 3 as the number for several equivalent sets which are being joined. The attention should be on 3's and the result of joining 3's.

Teacher's Preparation

Have objects available for demonstrating and illustrating the joining of 3's. Use a variety of materials such as chairs, children, blocks, discs, and so on. Using geometric designs on a flannel board is very effective.

It is recommended that you have some 1-inch square shapes of construction paper or tagboard for children to use at their seats. Each child should have about 20 shapes.

On the board, draw a number-line picture numbered from 0 to 30.

Pre-Book Lesson

• Arrange 2 sets of chairs with 3 chairs in each set. Have children analyze the physical setting and describe it in their own words. Ask a child to join the two sets of chairs.

Guide the discussion so that children understand the *joining* of two equivalent sets. Then, have a child write a sentence on the board to describe the physical demonstration just completed. You should have an addition sentence and a multiplication sentence written. Discuss with the class that both sentences are appropriate.

Use other objects on the flannel board to demonstrate three 3's and four 3's. Be sure to write an addition and multiplication sentence for each of these demonstrations.

• Illustrate the same demonstrations with number-line pictures. In all instances emphasize the adding of 3's.

• Direct *all pupils* in the following activity. Each child should have 20 one-inch square shapes.

a. Have each child show a row of three on the left side of his desk. (You should do the same thing on a flannel board to guide the children.) Ask, "How many 3's are there?"

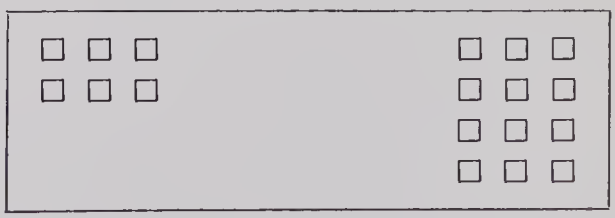
Have them show another three immediately under the first one as shown in A. Ask, "Now, how many 3's are there?" Be sure the pupils are thinking in terms of 3's.

A

□ □ □
□ □ □

b. Have the children show a three on the right side of their desk. (Your flannel board should show something similar to that below.)

c. Show one more three under the one on the right. Continue this until four 3's are shown on the right. The child should have two arrays on his desk like those shown below.



d. Ask, "How many 3's do you have on the left?" How many 3's do you have on the right?" "How many 3's are there in all?"

e. In the ensuing discussion, be sure that pupils understand that six 3's is equal to two 3's plus

four 3's. In fact, they should see that six 3's can be distributed in several ways. They may use what they already know about 3's to determine a greater number of 3's.

The purpose of this activity is to begin to establish a very important property of multiplication. We may use addition to help us find the product in working more difficult multiplication examples. It is referred to as the Distributive Property of Multiplication.

Using the Text Pages

- As you discuss these exercises, use a number-line picture and objects to illustrate and demonstrate each example. Begin to write the result of joining 3's by using each of the three forms of sentences at the bottom of page 222.
- Be sure children understand that they are to show the facts for Ex. 1–8 in three ways.
- Refer to Ex. 9–17 as *sentences*. Some are true sentences; some are false sentences. After checking each sentence, they are to rewrite the false sentences to make them true sentences. Emphasize that they are to change only the product to make true sentences.
- You may wish to read Ex. 19–23 for children who have reading difficulties.
- Encourage children to make use of the distributive property to help find the unknown products for Ex. 24–38. For example, two 8's may be thought of as two 4's plus two 4's—i.e., the multiplication may be distributed. Rather than rely upon a table, they should be encouraged to *think* the answer in this way.

Individualizing Instruction

Duplicate number-line pictures (perhaps 10 on a page) for *slower learners* to illustrate Ex. 24–38.

NOTES

Some of the products shown in Ex. 9–17 are wrong. Check each product by adding 3's or by counting by 3's or by using a number line. Then show Ex. 9–17 with the correct products.

9. $6 \times 3 = 17$ 18
10. $8 \times 3 = 24$
11. $4 \times 3 = 9$ 12
12. $7 \times 3 = 21$
13. $3 \times 3 = 9$
14. $9 \times 3 = 18$ 27
15. $6 \times 3 = 12$ 18
16. $5 \times 3 = 15$
17. $7 \times 3 = 24$ 21

18. Copy and finish the M. Facts for 3's table.

M. Facts for 3's

$1 \times 3 = 3$
$2 \times 3 = ?$ (6)
$? \times 3 = ?$ (3, 9)
4 × 3 = 12
5 × 3 = 15
6 × 3 = 18
7 × 3 = 21
8 × 3 = 24
and so on to
$9 \times 3 = 27$

Write your work for Ex. 19–23 like that for Ex. 19 in box C. We multiply for Ex. 19–23 in the same way that we multiply other numbers.

*19. Five 3¢ candies cost how much? 15¢

20. Four 3¢ pencils cost how much? 12¢

21. John saves 3¢ a day. In 6 days how much does he save? 18¢

22. How much must you pay for 9 balloons at 3¢ each? 27¢

23. What is the cost of 7 buttons at 3¢ each? 21¢

C

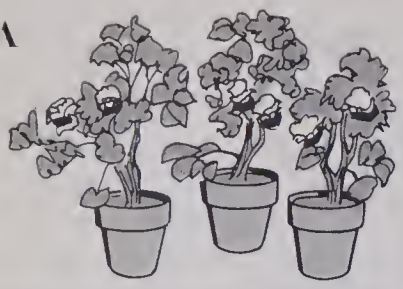
$$\begin{array}{r} 3¢ \\ \times 5 \\ \hline 15¢ \end{array}$$

Copy Ex. 24–38, writing numerals in place of *n*. Try to show the products without looking at the table.

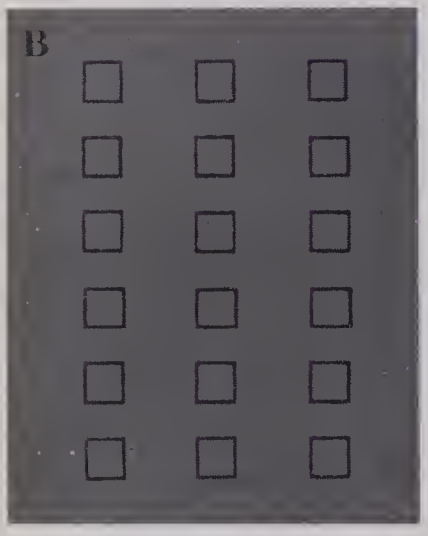
24. $2 \times 9 = n$ 18
25. $6 \times 3 = n$ 18
26. $7 \times 2 = n$ 14
27. $9 \times 2 = n$ 18
28. $8 \times 3 = n$ 24
29. $5 \times 2 = n$ 10
30. $6 \times 2 = n$ 12
31. $2 \times 8 = n$ 16
32. $9 \times 3 = n$ 27
33. $2 \times 6 = n$ 12
34. $7 \times 3 = n$ 21
35. $8 \times 2 = n$ 16
36. $4 \times 3 = n$ 12
37. $2 \times 7 = n$ 14
38. $5 \times 3 = n$ 15

*Refer to box C and remind pupils that we use ¢ with numerals to help us remember that we are showing numbers of cents. The multiplying involves the numbers.

A

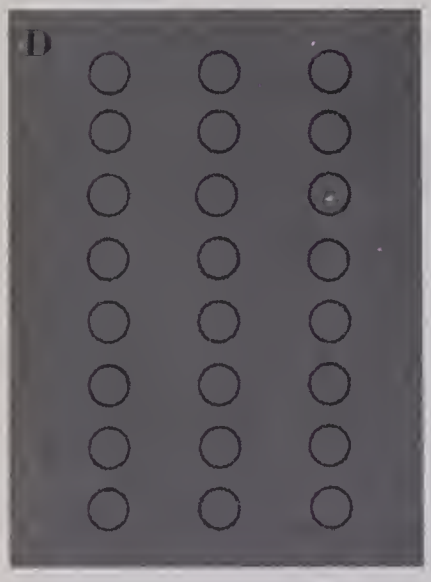


B



C

D



Multiplication Facts for 3

[O]

1. Picture A. Joe saw 3 plants each with 2 blossoms. How many blossoms was that in all?

a. To find the number for the total, finish the addition example $2 + 2 + 2 = n$. 6

b. Can you use a M. fact instead of adding? Yes

c. Are there 2 sets of 3 or 3 sets of 2? 3 sets of 2

d. Say the multiplication fact you can use. $3 \times 2 = 6$

2. Picture B. To find the number for the total, use the columns of squares.

a. Say the addition example with the sum. $6 + 6 + 6 = 18$

b. On the board show the multiplication fact three ways. three 6's = 18 $3 \times 6 = 18$ $\begin{array}{r} 6 \\ \times 3 \\ \hline 18 \end{array}$

3. Picture C. To find how many triangles in all, say the addition and the multiplication examples with their answers. $4 + 4 + 4 = 12$ three 4's = 12 $3 \times 4 = 12$ $\begin{array}{r} 4 \\ \times 3 \\ \hline 12 \end{array}$

4. Picture D. To find how many circles in all, use the columns. Say the addition and the multiplication examples with the answers. $8 + 8 + 8 = 24$ three 8's = 24 $3 \times 8 = 24$ $\begin{array}{r} 8 \\ \times 3 \\ \hline 24 \end{array}$

[W]

Copy and add for each of Ex. 5–12.

5. $3 + 3 + 3$ 9
6. $5 + 5 + 5$ 15
7. $8 + 8 + 8$ 24
8. $2 + 2 + 2$ 6
9. $7 + 7 + 7$ 21
10. $9 + 9 + 9$ 27
11. $4 + 4 + 4$ 12
12. $6 + 6 + 6$ 18

13–20. For each of Ex. 5–12 show a M. fact in three ways.

Teaching Pages 224 and 225

Pupil's Objectives

(a) To discover the multiplication facts for 3; and (b) to begin to see the relationship between the M. facts for 3's and the M. facts for 3.

New Word

(page 224) *blossoms*

Background

In the previous lesson we considered 3 as indicating the number for several equivalent sets. Now, we shall consider 3 as indicating the number of sets which are equivalent.

It should be noted that in this program a distinction is made between the M. facts for 3's and the M. facts for 3. It is especially important for the child to realize that, although the product is the same, the situation denoted by 3×6 is not the same as the situation denoted by 6×3. As the M. facts for 3 are being developed, a M. fact for 4's, a M. fact for 5's, a M. fact for 6's, and so on is involved. Furthermore, because of the Commutative Property of Multiplication, the child is learning two M. facts simultaneously so that, when the 9's are encountered, there will be only one fact (9×9) which the child has not previously learned.

Teacher's Preparation

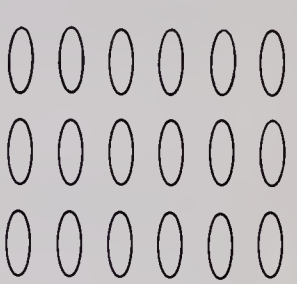
Have materials ready for children to use in demonstrating the M. facts for 3. A peg-board, flannel board, Ginn Arithme-Sticks*, and so on, should be available for your use.

Pre-Book Lesson

• Perhaps your children are seated at tables with 4, 5, or 6 children at each table. If so, use 3 sets of children at tables as a basis for discussion and demonstration. It is necessary that there be the same number of children in each set.

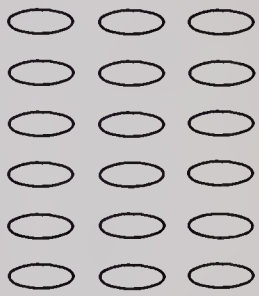
For example, have the children at three tables stand. Analyze the situation in terms of equivalent sets. How many equivalent sets are there? Write an addition sentence ($5 + 5 + 5 = 15$) on the board. Then, write a multiplication sentence ($3 \times 5 = 15$) on the board as another way to write about three 5's.

• Use Ginn Arithme-Sticks to show three 6's. (A peg-board, a flannel board or some other teaching aid may be used just as well.) You may want children to show the same array at their desks with materials of their own.



three 6's

After you discuss the idea of three 6's and write a multiplication sentence to denote the idea, direct the pupils to turn the array very carefully so that six 3's are seen.



six 3's

Write a multiplication sentence for six 3's. Ask questions about the array. "Did we change the number of objects in the array when we turned it?" "Are there any more or any fewer objects in the array after turning it?" "Are the products for the two M. facts the same?"

Be sure to emphasize the relationship between three 6's and six 3's.

• You may wish to demonstrate another example as suggested above in which 3 is a factor.

Using the Text Pages

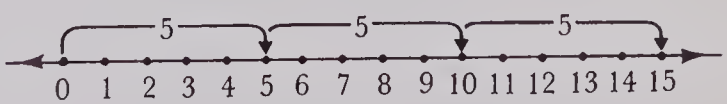
• In Ex. 1–4, the primary purpose is to discover and extend the idea of using the factor 3 to denote three equivalent sets.

It is important to stress the different sentences which may show this idea.

• See the suggestions in Individualizing Instruction for different ways of using Ex. 5–35.

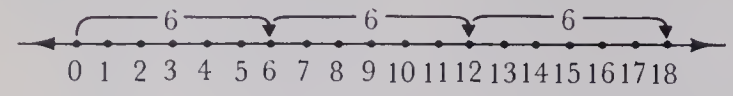
Individualizing Instruction

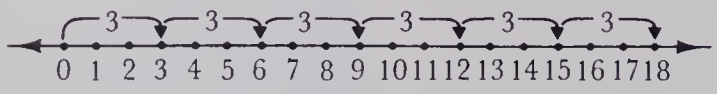
• *All pupils* may illustrate Ex. 5–12 in number-line pictures.



* See 3, page xix.

• *More capable children* may illustrate each of Ex. 5–12 in a number-line picture and also illustrate the related M. fact.

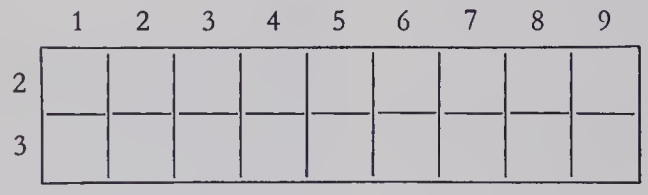




• *Slower learners* may make arrays with dots, circles, x's, squares, and so on, to illustrate Ex. 5–12.

• *All pupils* may make drawings or arrays to illustrate the problem situations in Ex. 22–25.

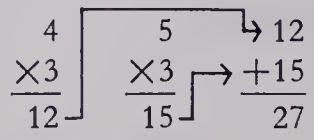
• *All pupils* may make a grid like the one below, or you may prefer to duplicate the empty grid for children to use. Sometimes so much time is spent by the child in making the grid that its use for mathematical learning is slight.



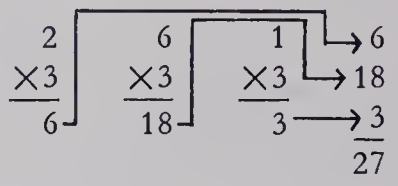
This activity may be used for review as well as practice. The children should show the factors 2 and 3 at the left end of the grid and the factors 1 through 9 across the top. Practice with facts comes in showing products in appropriate squares in the grid. (See Teacher's Page 211.)

You may wish to make this a directed activity in which you or some child says, "I am thinking of a product whose factors are 3 and 7." Then, the pupils show the product in the correct square. "The product I am thinking of is 18; one of its factors is 2. What is the other factor? Where is the product shown?"

• Have *all pupils* refer to Ex. 21 in which they have completed the table of M. facts for 3. From the table they may select as many different ways as possible to confirm that $3 \times 9 = 27$. For example, they may select:



Some of the more creative children may even do something like the example below.



Both examples illustrate distributing multiplication over the factor which tells the size of each equivalent set. This is good preparation for work which is to follow involving the distribution of multiplication over hundreds, tens, and ones.

• Use the enrichment material at the bottom of page 225 with *more capable children*. It is possible that you may wish to use the material as oral work instead of written work with all of your class.

This material is extremely important in helping youngsters use their mathematical learning effectively. If children learn mathematical facts but are unable to use them in problem situations, then the teaching and learning has not been complete. Do not minimize the importance of this enrichment material.

21. Copy and finish the M. Facts for 3 table. 3×4=12, 3×5=15, 3×6=18, 3×7=21, 3×8=24

Write your work for Ex. 22–25.

22. Three sets of 7 boys are how many boys in all? 21 boys

23. How much do 3 apples cost at 8¢ each? 24¢

24. It takes 9 children to make a baseball team. How many children are needed for 3 teams? 27 children

25. If each boat holds 5 Scouts, how many Scouts can ride in 3 boats? 15 scouts

M. Facts for 3

$3 \times 1 = 3$

$3 \times 2 = ?$ (6)

$? \times 3 = ?$ (9)

and so on to

$3 \times 9 = ?$ (27)

Copy Ex. 26–35 and show the products.

26. 3×5 15 **28.** 3×9 27 **30.** 3×8 24 **32.** 5×2 10 **34.** 2×6 12

27. 7×2 14 **29.** 6×2 12 **31.** 3×6 18 **33.** 3×4 12 **35.** 3×3 9

Do You Like to Try New Things?

Enrichment [W]

Right or wrong? Explain why.

1. Dick's estimate 50 for 39 + 26 was a good one. Wrong 40+30=70

2. Three 2-pound bags hold more than a 5-pound bag. Right 3×2=6

3. Ella is 11 years old. Her sister, Ida, is 2. Ella said that she is just five times as old as Ida. Wrong 5×2=10

4. Ruth and Fran cut a candy bar carefully into halves. Ruth took the larger piece. Wrong. Halves of the same whole are the same size.

5. Sam said that, to find the sum of 2 + 1 + 3, you can use the multiplication fact $3 \times 2 = 6$. Wrong. The sets are not equivalent.

6. Joe has two boards 1 foot long and 9 inches long. He said that the boards together are 22 inches long. Wrong. 12+9=21

Table of Multiplication Facts for 3's

3	3	3	3	3	3	3	3	3
1	2	3	4	5	6	7	8	9
3	6	9	12	15	18	21	24	27

Table of Multiplication Facts for 3

1	2	3	4	5	6	7	8	9
3	3	3	3	3	3	3	3	3
3	6	9	12	15	18	21	24	27

Factors-product relationship [O]

1. Cover the products shown in both tables and try to say them. Do not take the facts in order.

2. In $2 \times 3 = 6$, what do we call 2 and 3? Factors

3. Which factor is always the same for all facts
   a. in the table for 3's? 3 b. in the table for 3? 3

4. For each table say the facts with products that
   2×3=6 4×3=12 6×3=18 8×3=24 3×2=6 3×4=12 3×6=18 3×8=24
   a. are even numbers. ∧ b. are odd numbers. ∨
   1×3=3 3×3=9 5×3=15 7×3=21 9×3=27 3×1=3 3×3=9 3×5=15 3×7=21 3×9=27

5. After 1×3 and 3×1, each product is how much greater than the product just before it? 3 Why? See below.

6. Explain what the product would be for 10×3.
   9×3=27, so 10×3=27+3, or 30.

For rows 7–8, say products by rows and by columns.

	a	b	c	d	e	f	g
7.	5×3 15	3×4 12	4×3 12	8×3 24	3×8 24	7×3 21	9×3 27
8.	8×2 16	6×3 18	3×9 27	3×6 18	3×3 9	9×2 18	3×7 21

Make Study Cards for help and use them for practice.

♦ **Extra Examples.** Work Set 67.

5. For 3's, there is one more set of 3 members each time.
   For 3, each set increases by 1 member each time.

Pupil's Objectives

(a) To learn more about the factors-product relationship; (b) to see the table of M. facts for 3's and the table of M. facts for 3; and (c) to have oral practice with M. facts in which 2 or 3 is a factor.

Background

The M. facts for 3's and the M. facts for 3 have been presented separately in previous lessons. In this lesson, the two are presented together. Be sure to emphasize two important learnings in this lesson: (1) the Commutative Property of Multiplication; and (2) the terms *factor* and *product*.

Pre-Book Lesson

On the board, show the first three facts of each of the two tables as indicated below:

one 3 = 3	three 1's = 3
two 3's = 6	three 2's = 6
three 3's = n	three 3's = n

Have pupils complete each table by themselves. If they need to use objects, pictures, or drawings they should be permitted to do so. When they are finished, they may compare their work with the tables in the book to study relationships that are important for full understanding of M. facts.

Using the Text Page

• Use the pupil-organized tables, the tables at the top of the pupil's page, and the questions in Ex. 1–6 to guide the children to see and state in their own words generalizations like the following:

(a) The products increase by 3.

(b) Saying the products in order is like counting by 3's.

(c) When multiplying an even number by 3, the product is an even number.

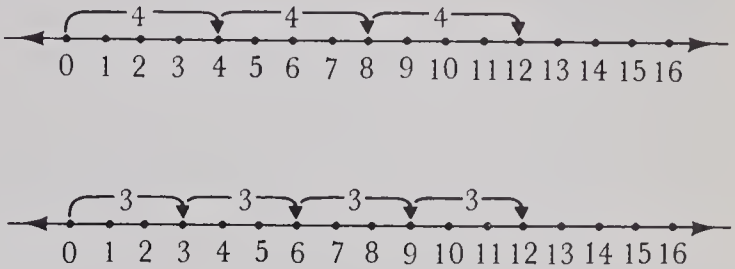
(d) When multiplying an odd number by 3, the product is an odd number.

(e) The product of four 3's plus the product of five 3's equals the product of nine 3's (12 + 15 = 27). This is true for the sum of other products which may be seen in the tables.

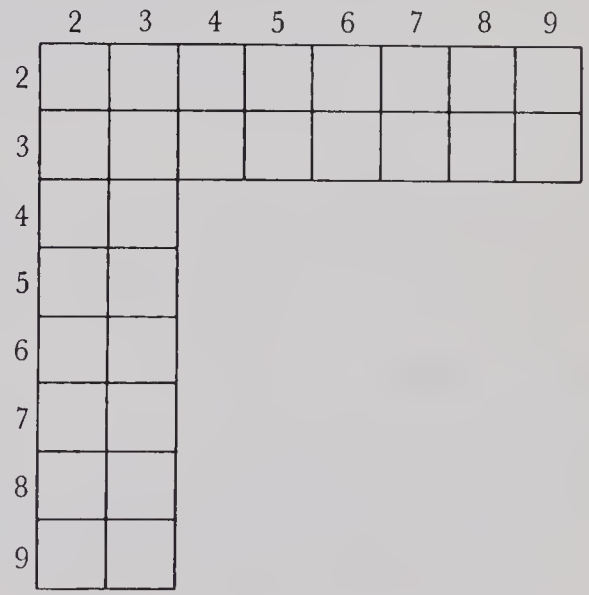
• You may have the pupils say the products only or you may have them read each sentence as: "Five 3's equal 15." or "Three 9's equal 27."

Individualizing Instruction

• *All pupils* may illustrate on a number line each M. fact and its related M. fact in Ex. 7–8.



• Duplicate on paper (or children may draw their own) a grid like the one below.



The factors 2 through 9 should be shown across the top and the factors 2 through 9 should be shown down the left side. In the table, the M. facts for 2's and 3's as well as the M. facts for 2 and 3 are to be shown. Children may show products as an independent activity or you may prefer to direct the use of the table for part of the work.

• Have pupils make Study Cards as they seem to be needed.

• Assign Extra Examples Set 67 as deemed appropriate.

Pupil's Objectives

(a) To learn the principle of using 1 in multiplication; and (b) to further understand the idea that M. facts go in pairs.

New Word

peg

Background

• Multiplication has been developed to show the joining of several equivalent sets. In every instance, we know how many sets and the number for each set. The problem is to determine the number for the total set. This is done by multiplication. We say that multiplication is a binary operation on two numbers (factors) which yields a unique third number called a product.

• All understandings about M. facts developed thus far have involved M. facts in which 2 or 3 is one factor. When 1 is a factor, the product is equal to the other factor. As a result, 1 is said to be the Identity Element for Multiplication.

Teacher's Preparation

Have a flannel board and objects (or a similar teaching device) ready for demonstration. Pupils should have objects to use at their desks.

Pre-Book Lesson

• Demonstrate an array of four 3's, two 5's, seven 2's and write a sentence for each. Review the commutative property by turning each array. As pupils compare this new array with the one associated with it, they will be able to write another sentence to go with the first sentence written. Both sentences, of course, will show the same product.

• You may wish to demonstrate the same ideas with objects in the room. (Show 4 stacks of 2 books each and 2 stacks of 4 books each.)

• Demonstrate and direct pupils to show just 1 set of 4, 1 set of 7, 1 set of 5. Each time, have a pupil write an appropriate sentence on the chalkboard to describe the physical model.

Ask, "What is the product when we multiply a number by 1?"

• Direct pupils to show just 1 object. Then 1 more, one more, and so on, until five ones have been shown. Focus on the idea of five 1's. Ask a pupil to write a sentence for this situation.

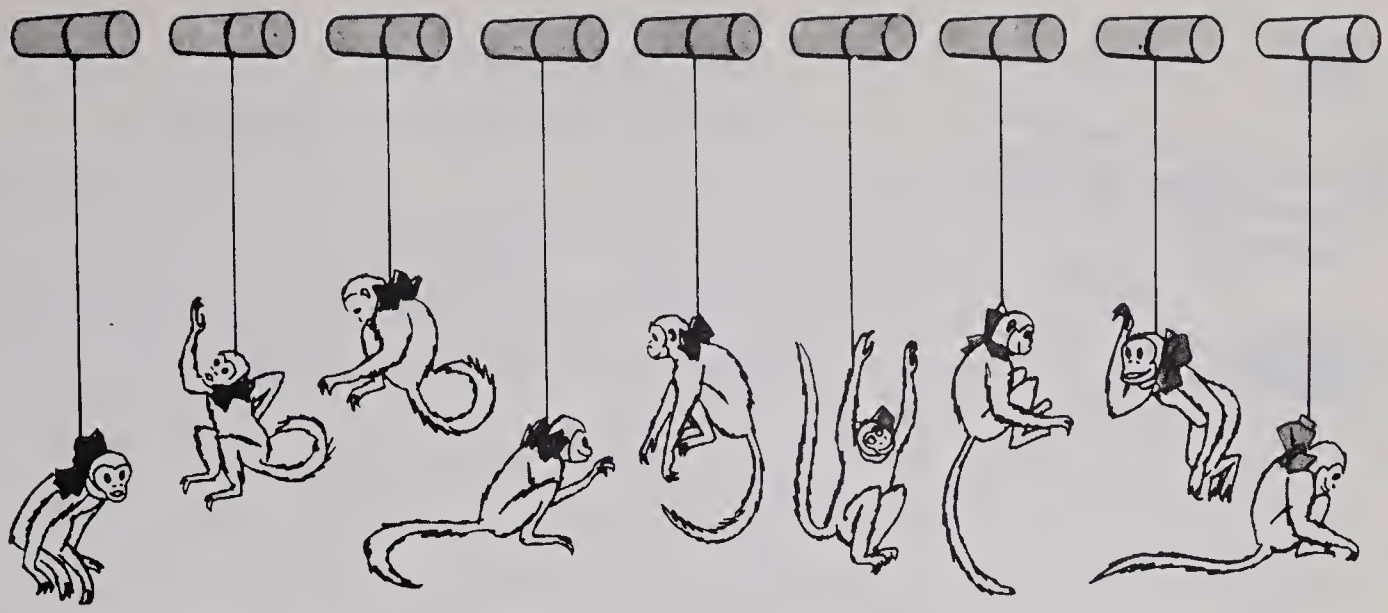
Ask, "What is the product when we multiply 1 by a number?"

• Through your guided questions and directions, pupils should be able to state in their own words something to the effect that when 1 is a factor, the product is equal to the other factor.

Using the Text Page

• As this material is being discussed with the children, ask many different pupils to state in their own words what the product is when we multiply 1 by some number or when a number is multiplied by 1.

• In Ex. 5 and 6, you may have a pupil say the product for a fact and then ask another child to say the related fact and its product.



When One of Two Factors Is 1

*Identity for M.; using the Commutative Property of M. [O]

1. To hang 1 monkey on each peg, how many monkeys did Mr. Mack use for 3 pegs? $3 \times 1 = n$ 3

2. How many monkeys will Mr. Mack need for

a. 6 pegs? $6 \times 1 = n$ 6 b. 9 pegs? $9 \times 1 = n$ 9 c. 5 pegs? $5 \times 1 = n$ 5 d. 8 pegs? $8 \times 1 = n$ 8

3. Because $2 \times 1 = 2$, then $1 \times 2 = 2$. Why? We may change the order of the two factors without changing the product.

4. Say the M. fact that goes with each of Ex. 2a–d.

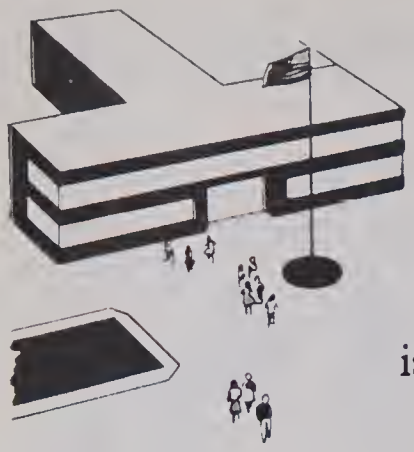
When 1 is a factor, the product equals the other factor.

Say the products for the examples in rows 5 and 6.

	a	b	c	d	e	f	g	h
5.	1	6	9	1	5	8	1	1
	$\frac{8}{8}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{7}{7}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{6}{6}$	$\frac{9}{9}$
6.	2	1	4	1	1	3	7	1
	$\frac{1}{2}$	$\frac{5}{5}$	$\frac{1}{4}$	$\frac{2}{2}$	$\frac{4}{4}$	$\frac{1}{3}$	$\frac{1}{7}$	$\frac{3}{3}$

7. Say the M. fact to go with each example in rows 5–6.

*When 1 is a factor, the product is identical to the other factor. This understanding makes products easy when 1 is a factor.



Add or Subtract — Which?

Estimating; writing n-examples [O]

Tell why you should add or subtract. See foot of page.

1. Our schoolhouse is 32 feet high. The flagpole is 18 feet higher. How tall is the flagpole? 50; 50 ft.; 32 + 18 = n

2. Of the 88 children in Grade 3, 42 walk to school. How many come to school other ways? 50; 46 children; 88 − 42 = n

3. One week 19 of the 88 children were absent one day or more. How many children were not absent? 70; 69 children; 88 − 19 = n

4. Of the 88 children in Grade 3, 27 are in Miss Lee's class. How many children have other teachers? 60; 61 children; 88 − 27 = n

5. Ruth rides on the bus 48 miles a week. Sue rides 19 more miles than Ruth. How many miles a week does Sue ride on the bus? 70; 67 miles; 48 + 19 = n

6. Miss Dunn's pupils brought money to buy her a present. Monday they brought $2.65 and Tuesday they brought $1.68. How much did they have then? $4.33; $2.65 + $1.68 = $n

7. At last the pupils had $4.50 for the present. They wanted to buy a pen set that cost $1.75 more than they had. How much did the pen set cost? $6.25; $4.50 + $1.75 = $n

8. Instead, they bought Miss Dunn a book for $3.90. How much of the $4.50 did they have left? $0.60; $4.50 − $3.90 = $n

[W]

For each of problems 1–5, show an estimate of the answer to the nearest ten and work the problem.

Then work problems 6–8.

Write an *n*-example for each of problems 1–8.

Ex. 1, 5, 6, 7: Add; the numbers for the parts are given.
Ex. 2, 3, 4, 8: Subtract; the numbers for the total and one part are given.

Teaching Page 228

Pupil's Objectives

(a) To have practice in deciding whether to add or subtract on the basis of information given in a problem; (b) to have practice in estimating the answers for both A. and S. problems; and (c) to write *n*-sentences for problems.

New Words

absent, *pupils*

Background

As you know, children frequently have difficulty solving problems because they fail to understand the problem situation. Often, it is due to carelessness; perhaps due to haste. Sometimes it is because of reading a word incorrectly. And, of course, faulty reading habits contribute to incorrect solutions to problems.

The exercises on page 228 are to give pupils additional experience in analyzing the problem and deciding on the appropriate operation. It is recommended as an oral activity to give you the advantage of hearing children tell why they would use a particular operation.

Using the Text Page

- It may be necessary to review the work on rounding to the nearest ten. (See Teaching Pages 158–160.) Have children use this method of estimating the answer for a problem. After a child has given his estimate of the answer, have him tell why he thinks that it is a good answer.
- To continue the emphasis on estimating, it is suggested that children write their estimates to Ex. 1–5 before writing the complete answer for each problem.

After pupils have written the solutions to all problems, have them write an appropriate *n*-sentence for each problem. They should be permitted to solve each problem before they are asked to write the *n*-sentences.

Individualizing Instruction

Slower learners may need further practice in rounding to the nearest ten.

NOTES

Pupil's Objectives

(a) To have practice in *naming* a number in many ways; and (b) to have practice in finding the number for *n* in addition and subtraction sentences.

Pre-Book Lesson

• Write (14 − 6) on the board. Ask, "What number has been named?" "Can someone write another name for the same number?" As different children write different names for *8* be sure to bring out the generalization that a number may be named in many different ways.

• Review with the entire class the generalizations regarding the addends-sum relationship:

a. When we know two addends, we add to find the sum.

b. When we know the sum and one addend, we subtract to find the other addend.

• On the board, write number sentences like the following:

$$582 + 216 = n$$
$$n + 418 = 822$$
$$964 - n = 550$$

Have selected children tell whether the number *n* represents is an addend or a sum. Be sure to have pupils tell why the number is an addend or a sum.

Using the Text Page

• Have children continue to explore and express names for a number in many different ways. Encourage creativity and ingenuity. In Ex. 2, another way of expressing 12 could be $2 \times (3 \times 2)$. Some children will find it exciting to write twenty or thirty different names for the same number.

• Encourage ingenuity as pupils work Ex. 9–14.

• The Pre-Book Lesson reviewing the addends-sum relationship should be adequate background for doing the work at the bottom of the page independently.

Individualizing Instruction

• *Slower learners* may use the practice cards and the *Show the Answer* game for practice on M. facts.

• Write the following sentence on the board: Let △ represent one of two factors and □ represent the other factor of the product represented by ○. Children should draw △, □, and ○ as column headings on their papers. They should show a number below each heading that will make a true sentence. Be sure they understand that a shape may represent a different number in each sentence.

• *All pupils* may try examples like the following:

WHAT NUMBER IS NAMED?

a. $4 + (2 \times 4)$
b. $(6 - 4) \times 3$
c. $3 \times (12 - 5)$

• Use Extra Activities Set 101 as you deem necessary.

* Pupils have been using names for numbers which involved the operations of A. and S. Now they will use M. in giving other names to numbers.

* Discovering More Names for Numbers

Using A., S., and M. [O]

You have been using addition and subtraction to give many names to whole numbers.

1. For 17 you may have used these names: $8 + 9$; $(4 + 6) + 7$; $18 - 1$; $12 + 5$; $3 + 6 + 6 + 2$; $(19 - 9) + 7$. Explain each. 8+9=17; 4+6=10; 10+7=17; 18−1=17; 12+5=17; 3+6=9, 9+6=15; 15+2=17; 19−9=10; 10+7=17

You can use multiplication, too, to give more names to numbers.

2. For example, 2×6 and 3×4 are names for the number 12. So are $(3 \times 3) + 3$, and $(2 \times 8) - 4$, and $2 + (5 \times 2)$. Explain. 3×3=9; 9+3=12; 2×8=16; 16−4=12; 5×2=10; 2+10=12

As in Ex. 2, write on the board two names for

3. 10. 4. 14. 5. 19. 6. 9. 7. 13. 8. 20. Answers will vary.

[W]

As in Ex. 2, write three names for

9. 8. 10. 18. 11. 16. 12. 15. 13. 22. 14. 11. Answers will vary.

● **Extra Activity.** Work Set 101.

Finding the Unknown Addend or Sum

[W]

Work each example to find the unknown number.

1. n = 507: $n + 300 = 807$
2. $\triangle$ = 545: $\triangle = 456 + 89$
3. n = 620: $867 - n = 247$
4. $\square$ = 605: $\square - 28 = 577$
5. n = 82: $700 - n = 618$
6. n = 907: $n - 300 = 607$
7. n = 402: $710 - n = 308$
8. $\square$ = 416: $570 + \square = 986$
9. n = 899: $770 + 129 = n$
10. $\triangle$ = 419: $506 - \triangle = 87$
11. $n¢$ = 19¢: $98¢ - n¢ = 79¢$
12. $n¢$ = 24¢: $72¢ - n¢ = 48¢$
13. $n¢$ = 62¢: $28¢ + n¢ = 90¢$
14. $n¢$ = 65¢: $n¢ - 56¢ = 9¢$
15. $\square¢$ = 95¢: $\square¢ = 27¢ + 68¢$

* This Try These exercises may serve as a quick test of your pupils' knowledge of the M. facts having one factor no greater than 3.

* Try These

One factor no greater than 3 [W]

Show the products on folded paper.

	a	b	c	d	e	f	g	h	i	j
1.	5	3	9	7	2	3	2	3	3	2
	$\frac{3}{15}$	$\frac{8}{24}$	$\frac{2}{18}$	$\frac{3}{21}$	$\frac{6}{12}$	$\frac{9}{27}$	$\frac{8}{16}$	$\frac{3}{9}$	$\frac{6}{18}$	$\frac{4}{8}$
2.	2	6	2	3	2	3	3	8	2	9
	$\frac{7}{14}$	$\frac{3}{18}$	$\frac{9}{18}$	$\frac{7}{21}$	$\frac{5}{10}$	$\frac{1}{3}$	$\frac{4}{12}$	$\frac{2}{16}$	$\frac{2}{4}$	$\frac{3}{27}$
3.	3	2	5	7	4	4	2	6	8	3
	$\frac{5}{15}$	$\frac{3}{6}$	$\frac{2}{10}$	$\frac{2}{14}$	$\frac{2}{8}$	$\frac{3}{12}$	$\frac{1}{2}$	$\frac{2}{12}$	$\frac{3}{24}$	$\frac{2}{6}$

The Numeral below the Fraction Bar

Relative sizes of parts [O]

A

1. The inside of square *A* is shown in _?_ parts.

a. Are the parts the same size? Yes

b. The colored part is 1 of the _?_ parts.

c. What is the fraction number for 1 of the 4 parts? $\frac{1}{4}$

2. Is only $\frac{1}{5}$ of the inside of square *B* colored? No

B

3. Is $\frac{1}{6}$ of the inside of square *C* colored? Yes Explain. 1 of the 6 parts of the same size is colored.

4. In the fraction $\frac{1}{4}$ the 4 shows that there are _?_ parts of the same size in the whole. The 1 means _?_ part.

5. What does the 6 in $\frac{1}{6}$ show? There are 6 parts of the same size.

C

The numeral below the bar in a fraction shows how many parts of the same size there are in the whole. It helps to name the fraction number.

6. The 7 in $\frac{1}{7}$ means _?_ parts of the same size.

In this grade, pupils learn the meaning of the denominator but are not taught to use the term. If you wish to have your pupils use the term denominator, make sure that they understand that the denominator is a number and not the numeral below the line.

Teaching Pages 230 and 231

Pupil's Objectives

(a) To have practice in showing products for M. facts; (b) to learn the meaning of the numeral below the bar of a fraction; and (c) to see and understand that halves, fourths, and so on of different-sized wholes are not the same size.

New Word

(page 231) *juice*

Background

Some discussion of the technical terminology used in the *Mathematics We Need* program is appropriate at this time. The result of dividing a counting number by a counting number is a member of the set of positive *rational numbers*. However, at this particular level of the child's progress toward mathematical maturity, the following terminology and distinctions are made:

a. *Parts*. The word *parts* is used in connection with the objects, segments, elements, and so on, of the same size which constitute one whole thing or one whole collection of things.

b. *Fraction number*. This term is used in much the same way the term *rational number* is used. It is used when referring to a number idea. One half, seventeen thirds, and twelve fourths are all number ideas. We call them fraction numbers.

c. *Fraction*. Numeral and symbol are other terms that may be used in place of *fraction*. In this program, *fraction* is used when referring to the actual symbol which names a fraction number.

Teacher's Preparation

You should have materials available for pupils to use for showing, seeing, and comparing various fractional parts. One way of preparing materials of this type is to use standard sized construction paper. For example, let one full sheet of yellow construction paper represent one whole. Cut red paper in halves, blue paper into thirds, orange paper into fourths, and so on, until you have as many different sizes of parts as you need. It is rather easy and fast to cut the various sizes on a paper cutter. Enough parts should be cut so each child has a full set.

It would be a good idea to have a similar set of materials for your own use in demonstrating.

Pre-Book Lesson

• Distribute the sets of construction-paper kits to pupils.* Direct pupils to use the materials to find the answer to questions like the following:

How many of the red (one half) pieces are needed to make a piece the same size as the yellow piece?
Are the red pieces the same size?
Are the orange pieces the same size?
How many of the orange ones are the same size as one yellow one?

Be sure to establish the idea that all of the pieces of the same color are the same size.

• Say something like this, "When two pieces are the same size (like the red ones), there is a special name we can use to tell how they compare with the yellow piece."

At this point most children will know you are referring to the idea of one half. Let them express it in their own words.

"We say that the size of the red piece is one half of the size of the yellow piece. Is the size of the red piece one half of the size of your desk? of this book?" and so on.

It is important to build firm understanding that the size of the red piece is one half of the size of the yellow piece or anything else having the size of the yellow piece.

Continue this type of discussion, giving the special names *thirds*, *fourths*, and so on, that you plan to develop.

• "Numbers like one half, one third, and one fourth are called fraction numbers. We show a fraction number in a special way. We use two of the numerals that we know, and write a fraction."

a. Explain the fraction $\frac{1}{3}$. Point out that two numerals have been used to write the fraction; one above the bar and one below the bar.

b. The numeral below the bar names the number of parts of the same size in the whole.

* See 1, page xix.

c. Since this lesson is limited to showing the numerator 1, point out that the *1* shows just one of the thirds. Ask different children to go to the chalkboard and write the fraction for one fourth. Say, "Let's see if Sue can write the fraction that shows us one of the fourths."

Write other fractions for one part, such as $\frac{1}{3}$, and so on and explain what the numeral above the bar and the numeral below the bar mean.

Using the Text Pages

• Ex. 1–3 at the top of the page may be used for drill at any time. You may use the practice for timed work. The time may be determined for rows or for the entire section. However, do not stress *time* to the detriment of *accuracy*.

• As Ex. 1–5 at the bottom of page 230 are discussed, have pupils use their construction-paper parts to help clarify the material. Lead children to state in their own words the generalization at the bottom of the page.

• In addition to referring to the pictures on page 231, it may be more revealing to the children to use appropriate objects and materials while discussing Ex. 6–15. Use as many different examples and illustrations as you find necessary to aid children in building complete understanding of the generalization following Ex. 12.

Individualizing Instruction

• *All pupils* may show the M. fact that goes with each example in rows 1–3 to make a pair of M. facts.

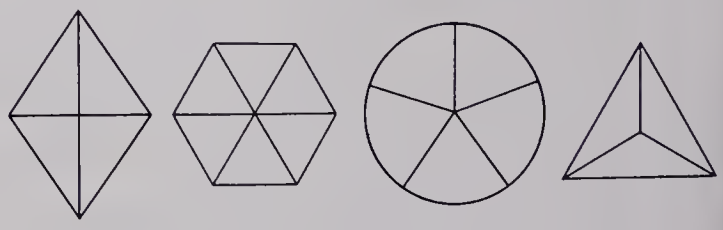
• Many kindergarten teachers have sets of blocks of different shapes and sizes. If such blocks are available, perhaps you may borrow them. *All pupils* may use the blocks to further understanding of one half, one third, one fourth, and so on. One important idea which can evolve from this type of independent experimental activity is that one particular block may be a whole, one half, one fourth, and so on. It takes on its name and meaning in terms of some other block to which it is being compared.

• Children may write descriptions or make charts which depict some fraction relationships through the use of measures (inch, foot, yard, minute, hour, ounce, pound, block, mile, and so on).

• You may ask questions such as, "Which is more, $\frac{1}{2}$ of a minute or $\frac{1}{2}$ of an hour?"

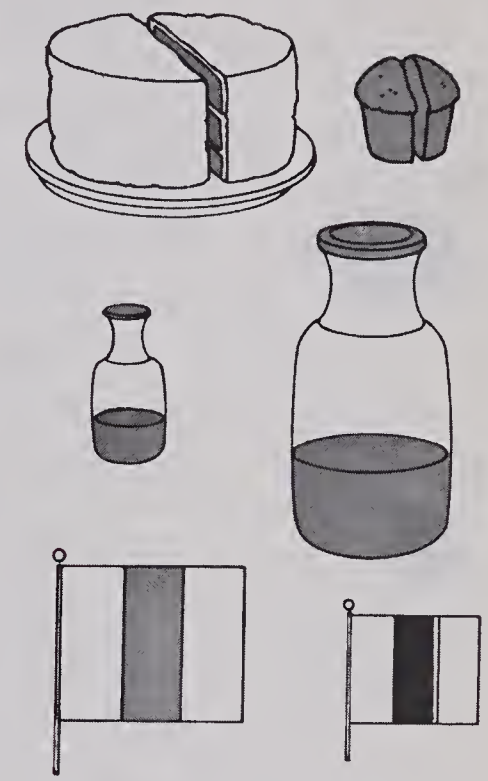
• Some children may wish to decrease the time taken to complete the written work at the top of page 230. Children may time each other. Be sure to build the attitude that speed is only important as long as accuracy is improved or maintained. Do not let the pupils sacrifice accuracy for speed.

• Duplicate some drawings as below and have pupils label the parts of each drawing.



7. There are 9 parts of the same size in a whole. On the board, write the fraction for one part. $\frac{1}{9}$ What is written below the fraction bar? 9

* 8. Each cake in the picture is cut into halves. Why is $\frac{1}{2}$ of the cupcake not so large as $\frac{1}{2}$ of the big cake? The cakes are not the same size.



9. Is $\frac{1}{4}$ of the little bottle of juice as much as $\frac{1}{4}$ of the big bottle of juice? No Why? The bottles are not the same size.

10. If the colored part of each flag is $\frac{1}{3}$ of the flag, why are the colored parts not the same size? The flags are not the same size.

11. Is $\frac{1}{8}$ of a cherry as large as $\frac{1}{8}$ of an apple? No Why? The cherry and the apple are not the same size.

12. Is $\frac{1}{10}$ of a yard as long as $\frac{1}{10}$ of a mile? Why? A yard is not as long as a mile.

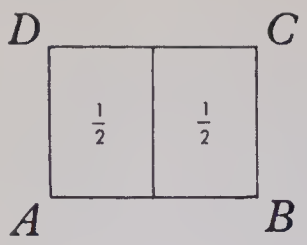
For wholes of the same size, halves of those wholes are the same size, thirds of those wholes are the same size, and so on.

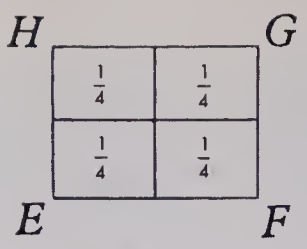
13. Are the halves of all oranges the same size? No Why? All oranges are not the same size.

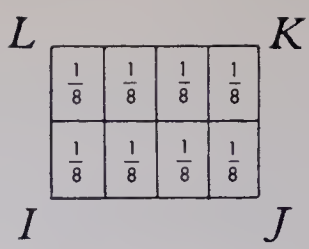
14. On the board, draw pictures of a large circle and a small circle. Show the inside of each circle in halves. Are all the halves the same size? No Why? The whole circles are not the same size.

15. Draw pictures of two line segments, one much longer than the other. Show each line segment in thirds. Are all the thirds of the two line segments the same size? No Why is this so? The whole line segments are not the same size.

* The idea of relative sizes of halves or thirds or fourths and so on may be new to your pupils. They may have been thinking that all halves are the same size, all thirds are the same size, and so on. Make sure that the correct concept is clearly defined now.







Discovering More about Parts of Wholes

Comparing parts of wholes of the same size [O]

1. Rectangles *ABCD*, *EFGH*, and *IJKL* are all the same size. The inside of each rectangle is shown in parts of the same size.

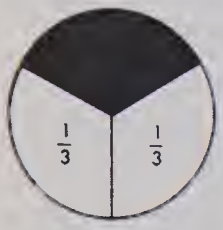
a. Which part is largest, $\frac{1}{2}$ or $\frac{1}{4}$ or $\frac{1}{8}$? $\frac{1}{2}$

b. What fraction is shown on the smallest part? $\frac{1}{8}$

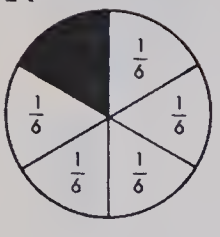
* **c.** The part labeled $\frac{1}{2}$ is larger than the part labeled $\frac{1}{8}$, but the 2 of $\frac{1}{2}$ is less than the 8 of $\frac{1}{8}$. How can this be?

The lesser the number of parts in a whole, the larger is each part.

M



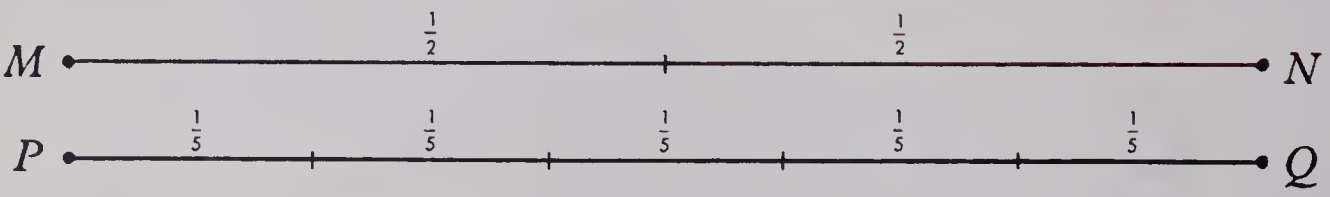
N



2. The insides of circles *M* and *N* are the same size.

a. Which part is larger, $\frac{1}{3}$ or $\frac{1}{6}$? $\frac{1}{3}$

b. In which fraction is the greater number shown below the fraction bar? $\frac{1}{6}$ Explain. The greater the number of parts in a whole, the smaller is each part.



3. If line segments *MN* and *PQ* are the same length,

a. which segment has longer parts of the same size? MN

b. in which fraction is the greater number shown below the fraction bar? $\frac{1}{5}$ Why is this so? Line segment PQ is shown separated into a greater number of parts than line segment MN.

[W]

For each of Ex. 4–6 write "larger" or "smaller."

4. If you are given $\frac{1}{4}$ of a stick of candy, you get a larger piece than if you get $\frac{1}{6}$ of it.

5. Each part of a rope will be larger if you cut the rope into 3 pieces of the same length instead of 8 pieces.

* Constantly stress that if a whole has a greater number of parts, each part will be smaller.

Teaching Pages 232 and 233

Pupil's Objectives

(a) To learn about the relationship between different sized parts of the same whole; (b) to learn that when the number shown below the bar of a fraction is greater, the part is smaller; and (c) to have practice in solving *if-then* sentences.

New Word

(page 233) *melon*

Teacher's Preparation

Have the kits of fractional parts* ready for use by all pupils.

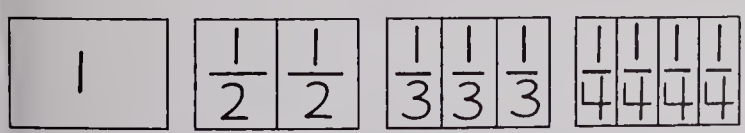
Pre-Book Lesson

Under your direction, each pupil should use the materials in his fraction-parts kit to discover, see, and finally state the generalization that one half is greater than one fourth, and so on.

a. Have children show and state the number of red pieces needed to be the same size as a yellow piece. Do the same with the orange and blue pieces of paper. Review briefly the special names we use to describe the orange, red, and blue pieces.

b. Now compare the orange pieces with the red and blue pieces. Which pieces are larger?

c. You may wish to place some pieces of construction paper in a conspicuous spot in the room. Write on each piece of paper the fraction for that particular piece. They could be clipped on a wire across the top of the board in the front of the room somewhat as shown below.



Direct children to write the fraction on each piece of construction paper showing how its size compares with the size of the yellow piece.

Have the children tell you: "On each red piece of paper we write $\frac{1}{2}$," and so on.

d. As the writing of the fraction is being completed, direct the pupils' attention to the number shown below the bar. Once again, develop the generalization about the size of each piece of paper and the name and number we use to express it.

* See 1, page xix.

Using the Text Pages

- Use the materials you have prepared for pupils to help clarify or extend the ideas developed in Ex. 1–3. Showing line segments on the board similar to those in the pupil's book in Ex. 3 should be especially valuable.
- Permit pupils to use the fraction cutouts if they are needed to find the solutions to Ex. 4–14.
- You may wish to discuss Ex. 4–9 orally.
- Be sure pupils understand the directions for Ex. 4–14. You may wish to have similar examples done at the chalkboard to be sure the pupils are ready for this written work.
- The work at the bottom of page 233 may be assigned any time for practice in addition and subtraction with 3-place numerals. Before making this assignment, review the nature of *if-then* sentences. Give an example of an *if-then* statement on the board. It may be profitable to review the addends-sum relationship before asking pupils to work these examples.

Individualizing Instruction

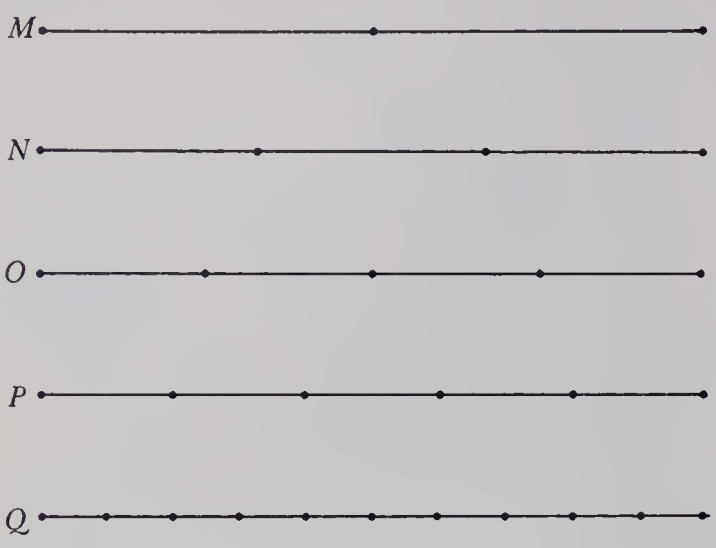
- *Slower learners* may need additional work with fractional parts. An activity having a gamelike quality could be organized in which you or a *more capable child* would serve as a leader.

Each child should have his fractional parts on the desk before him. Ask the pupils to hold up one of the red pieces. Then ask a child to tell what fraction number tells the size of a red piece as compared with a blue piece. Have the child tell why. Do this kind of questioning with pieces of other colors.

Variety may be added to the activity by having different children tell the others what color to hold up.

Then, you may change the directions by asking them to hold up a piece that is $\frac{1}{4}$ of the yellow one. At all times, ask questions and give directions which require pupils to reveal understandings.

• Duplicate models of line segments which are separated into parts from one half to one tenth. *All pupils* may label each part with the appropriate fraction.



• Duplicate or write on the board, examples like those below. *More capable children* should write *is greater than* or *is less than* to make each a true sentence.

a. $\frac{1}{5}$ ------ $\frac{1}{2}$
b. $\frac{1}{8}$ ------ $\frac{1}{4}$
c. $\frac{1}{3}$ ------ $\frac{1}{9}$
d. $\frac{1}{6}$ ------ $\frac{1}{7}$

• *More capable children* should find the following activity interesting and enriching.

Arrange the following in order from greatest to least.

a
$\frac{1}{9}, \frac{1}{2}, \frac{1}{15}, \frac{1}{10}, \frac{1}{5}, \frac{1}{20}$

b
$\frac{1}{30}, \frac{1}{4}, \frac{1}{7}, \frac{1}{12}, \frac{1}{8}$

Arrange the following in order from least to greatest.

a
$\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{8}, \frac{1}{25}$

b
$\frac{1}{50}, \frac{1}{5}, \frac{1}{25}, \frac{1}{15}, \frac{1}{10}$

• *Slower learners* may use their fraction cutouts to show fraction numbers from greatest to least and from least to greatest. Have some children show the same thing using the flannel board.

Reminder

Continue to maintain and extend understanding and use of the following:

a. reading the thermometer

b. recognizing coins of different values and the ability to make change

c. recognition of different polygons

d. relationships of different measures of length, weight, and time

NOTES

6. Sam has $\frac{1}{3}$ of a melon. Joe has $\frac{1}{4}$ of the melon. Joe's part is smaller than Sam's part.

Copy Ex. 7–9. Write "greater" or "less" for _?_

7. When there is a greater number of parts in a whole, the number shown below the fraction bar is greater

8. When the parts are smaller, the number shown below the fraction bar is greater.

9. When the parts are larger, the number shown below the fraction bar is less.

Each fraction number shown tells about a part of the same whole. Copy the fractions in each set to list the fraction numbers in order from least to greatest.

10. $\frac{1}{3}, \frac{1}{8}, \frac{1}{5}$ $\frac{1}{8}, \frac{1}{5}, \frac{1}{3}$ **11.** $\frac{1}{9}, \frac{1}{6}, \frac{1}{3}$ $\frac{1}{9}, \frac{1}{6}, \frac{1}{3}$ **12.** $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}$ $\frac{1}{8}, \frac{1}{5}, \frac{1}{2}$ **13.** $\frac{1}{4}, \frac{1}{10}, \frac{1}{7}$ $\frac{1}{10}, \frac{1}{7}, \frac{1}{4}$ **14.** $\frac{1}{3}, \frac{1}{7}, \frac{1}{9}$ $\frac{1}{9}, \frac{1}{7}, \frac{1}{3}$

If . . ., Then . . .

Logic [W]

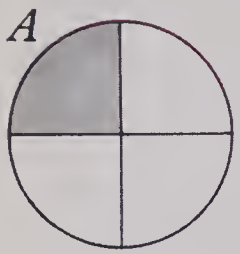
In each of Ex. 1–6, find the answers. Then copy Ex. a, b, and c, using numerals in place of n and the ?.

	a	b	c
1. If $n = 285$, then	$n + 166 = ?$ 451	$600 - n = ?$ 315	$n + n = ?$ 570
2. If $n = 189$, then	$n - 93 = ?$ 96	$217 + n = ?$ 406	$207 - n = ?$ 18
3. If $n = 507$, then	$711 - n = ?$ 204	$n - 88 = ?$ 419	$n + 85 = ?$ 592
4. If $n = 67¢$, then	$88¢ - n¢ = ?$ 21¢	$\$1.50 - \$n = ?$ $0.83	$\$1.16 = \$n +$ _?_ $0.49
5. If $n = 400$, then	$n - 316 = ?$ 84	$n + 300 = ?$ 700	$750 - n = ?$ 350
6. If $n = 89¢$, then	$\$1.00 - \$n = ?$ 11¢	$n¢ - n¢ = ?$ 0	$\$n = \$6.05 -$ _?_ $5.16

*Guide pupils to see that the white parts have threeness just as earlier sets had threeness.

Fraction Numbers for Several Parts

[O]

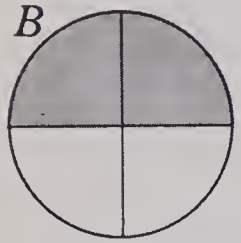


1. The inside of circle *A* is shown in 4 parts of the same size. So, each of the parts is 1 of the 4 parts.

a. The colored part is $\frac{1}{4}$ of the whole.

b. How many of the parts are white? 3

* **c.** We write the fraction for the number telling about the white parts like this, $\frac{3}{4}$. What does the 3 in $\frac{3}{4}$ show? 3 of the 4 parts



2. The inside of circle *B* is shown in 4 parts of the same size.

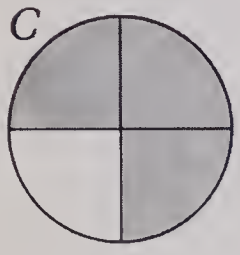
a. How many of the parts are colored? 2

b. What fraction number tells about 2 of the 4 parts? $\frac{2}{4}$

c. In the fraction $\frac{2}{4}$, what does the 4 show? What does the 2 show? 4 parts of the same size; 2 of the 4 parts

d. On the board write the fraction for the number telling about the white parts. $\frac{2}{4}$

e. Write the fraction for one of the 4 parts. $\frac{1}{4}$



3. The inside of circle *C* is shown in fourths. On the board write the fraction for the number telling about

a. the parts that are colored. $\frac{3}{4}$

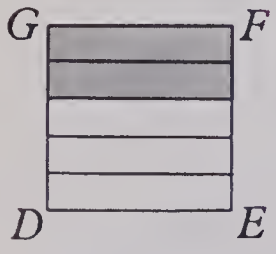
b. the part that is white. $\frac{1}{4}$

4. The inside of square *DEFG* is shown in 5 parts of the same size.

a. Each part is $\frac{1}{5}$ of the inside of the square.

b. The colored parts are $\frac{2}{5}$ of the whole.

c. The white parts are $\frac{3}{5}$ of the whole.



d. On the board write the fractions for the numbers telling about the colored parts and the white parts. $\frac{2}{5}$, $\frac{3}{5}$

e. For each fraction you wrote for **d**, explain what the numerals above and below the fraction bar show. The 5 in each fraction shows 5 parts of the same size in the whole. The 2 in $\frac{2}{5}$ shows 2 of the 5 parts. The 3 in $\frac{3}{5}$ shows 3 of the 5 parts.

Teaching Pages 234 and 235

Pupil's Objectives

(a) To learn how to name two or more parts of the same size; and (b) to begin to apply to various units of measurement the knowledge and understanding of fraction numbers.

Background

Pupils need many experiences with real objects as they move to higher levels of maturity in mathematical abstractions. They also need experiences with a variety of materials. Cutout parts have been recommended in previous lessons. Lest children tend to think that fraction-number ideas are always associated with rectangles, it would be a good idea to prepare cutouts in the shape of triangles, squares, and circles.

It may be that you find adequate experiences and materials outside of the developmental lessons so that providing other shapes as a part of the developmental activities is unnecessary.

Teacher's Preparation

You may wish to provide a different set of parts for circle or square shapes. Perhaps you have access to commercially-prepared materials of this type.* Otherwise, the materials cut from construction paper, recommended for the two previous lessons, should be adequate for this lesson.

You may wish to prepare in advance some square or circle shapes (either on the board or on paper) which have been separated into 3, 5, 6, 9 parts of the same size. Shade some of the parts.

Some teachers use the overhead projector for this type of activity. The use of a series of overlays on the overhead projector makes it possible to show one whole object which is then separated into parts of the same size some of which are then shaded. The sequence would be:

(1) a square shape representing one whole thing

(2) the shape separated into five parts of the same size

(3) three of the parts shaded which serves as a basis for discussion ($\frac{3}{5}$ shaded; $\frac{2}{5}$ not shaded)

* See 1, page xix.

If you have access to an overhead projector, you may wish to use it in connection with the Pre-Book Lesson suggested below.

Pre-Book Lesson

• Direct children to place the blue pieces (thirds) on the yellow piece (one whole). By this time, they should know that the size of each of the blue pieces is one third the size of the yellow one. Ask, "How many of the thirds have the same size as one whole? Who can write the fraction that names one of the thirds?"

At this time, review what the two numerals in the fraction $\frac{1}{3}$ mean. "The 3 below the bar tells us that there are three parts of the same size in the whole. The 1 above the bar shows us that one of the thirds is being considered."

Ask a child if he would try to write the fraction that names two of the thirds. Say, "We are still writing about thirds. Now, we want to show that we are considering two of the thirds instead of just one of them."

After a child has successfully written $\frac{2}{3}$, discuss what each of the numerals in the fraction means.

• Continue using materials and writing fractions naming fourths, fifths, sixths, and so on to tenths.

Using the Text Pages

• You may wish to use the cutout parts in connection with Ex. 1–4. *Slower learners* in particular need many experiences with a variety of materials to help them build adequate understandings.

• Tell the children they now have a chance to show what they have learned about fraction numbers. After Ex. 5–17 are completed, have class discussion about each item. Use diagrams on the board to help clarify any misunderstandings.

Individualizing Instruction

• *Slower learners* may use their cutout parts to help find the answers to the written work.

• *More capable children* may label points in a number-line picture with appropriate fractions. Duplicate number-line pictures which show labeled points for 0 and 1 and also show fraction numbers

between 0 and 1 identified by points. The child then labels with fractions the points for the fraction numbers between 0 and 1.

• *More capable children* may be able to do one or more of the following activities:

a. Provide them with a number-line picture which is separated into 12 parts of the same size, as shown below.

By referring to the number line (if they need to), they should write $>$ or $<$ to make each a true sentence.

$\frac{1}{2}$ ------ $\frac{2}{3}$
$\frac{3}{4}$ ------ $\frac{2}{3}$
$\frac{7}{8}$ ------ $\frac{1}{2}$
$\frac{3}{4}$ ------ $\frac{5}{6}$

b. Arrange the following fractions so as to show the fraction numbers in order from greatest to least:

a
$\frac{1}{12}, \frac{1}{4}, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}$

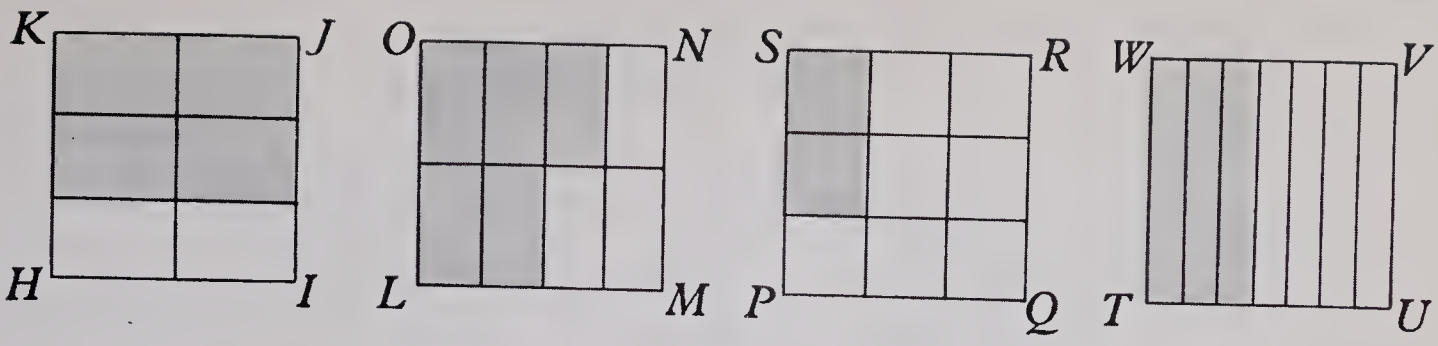
b
$\frac{2}{4}, \frac{2}{8}, \frac{2}{3}, \frac{2}{6}$

c. Arrange the following fractions so as to show the fraction numbers in order from least to greatest:

a
$\frac{3}{6}, \frac{3}{8}, \frac{3}{4}, \frac{2}{3}, \frac{1}{3}$

b
$\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{5}{6}, \frac{1}{3}$

NOTES



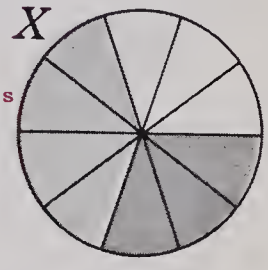
[W]

Study the inside of each square shown above. Then write the letters of the square for which the number for

5. the colored parts is PQRS **a.** $\frac{2}{9}$. HIJK **b.** $\frac{4}{6}$. TUVW **c.** $\frac{3}{7}$. LMNO **d.** $\frac{5}{8}$.

6. the white parts is HIJK **a.** $\frac{2}{6}$. TUVW **b.** $\frac{4}{7}$. LMNO **c.** $\frac{3}{8}$. PQRS **d.** $\frac{7}{9}$.

7. The parts of the inside of circle X are the same size. Do all parts together make the whole? Yes Is this $\frac{10}{10}$? Yes



8. What fraction number tells about these parts in X?

a. red $\frac{4}{10}$ **b.** yellow $\frac{3}{10}$ **c.** white $\frac{2}{10}$ **d.** blue $\frac{1}{10}$

9. Copy and finish the list of fraction numbers to use in counting from $\frac{1}{5}$ through $\frac{5}{5}$: $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}?, \frac{4}{5}?, \frac{5}{5}$

Show the fraction numbers to use in counting

10. from $\frac{1}{7}$ through $\frac{6}{7}$.

11. from $\frac{3}{9}$ through $\frac{9}{9}$.

12. from $\frac{1}{8}$ through $\frac{7}{8}$.

13. from $\frac{4}{10}$ through $\frac{10}{10}$.

10. $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{7}{7}$

11. $\frac{3}{9}, \frac{4}{9}, \frac{5}{9}, \frac{6}{9}, \frac{7}{9}, \frac{8}{9}, \frac{9}{9}$

12. $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}$

13. $\frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \frac{10}{10}$

Try to write the missing fractions.

14. 1 yard = 3 feet. So, 2 feet = $\frac{2}{3}$?_ of 1 yard.

15. 1 week = 7 days. So, 3 days = $\frac{3}{7}$?_ of 1 week.

16. May has walked 3 of the 8 blocks from her home to school. She has walked $\frac{3}{8}$?_ of the whole way.

17. The distance from a to b on the line shown below is $\frac{2}{3}$?_ of the distance from a to c.

* Another mid-chapter review of information and meanings!

* Many Things

Maintenance [W]

1. Write as Roman numerals: a. 9 IX b. 12 XII c. 4 IV

Estimate each answer to the nearest ten.

2. 47 − 29 20 3. 23 + 51 70 4. 19 + 67 90 5. 92 − 68 20

6. Copy and finish: 1 ft. and 8 in. = 20 _?_ in.

Write "Yes" or "No" to answer Ex. 7–12.

7. Is $\frac{1}{5}$ of a cake larger than $\frac{1}{3}$ of the same cake? No

8. When we write about $\frac{5}{6}$ of a cake, does the 6 show that there were 6 parts of the same size in the whole? Yes

9. If you pay 12¢ for something, should your change from a half dollar be 3 cents, 1 dime, and 1 quarter? Yes

10. Is a triangle always a polygon? Yes

11. Is it very cold at 78°? No

12. Does the 3 in $\frac{3}{5}$ mean 3 of 5 parts of the same size? Yes

Greater Than? Less Than? Equal To?

Equalities and inequalities [W]

First do the work in ( )'s. In Ex. 1, $3 \times 6 = 18$ and $9 + 8 = 17$, so you would write, "$(3 \times 6) > (9 + 8)$."

Copy Ex. 2–10, using $=$ or $>$ or $<$ for _?_.

1. (3×6) > _?_ $(9 + 8)$
2. $(5 + 5)$ < _?_ (3×4)
3. (9×2) > _?_ $(15 - 4)$
4. $(19 - 9)$ < _?_ (5×3)
5. (9×2) > _?_ (2×6)
6. (8×2) > _?_ $(3 + 10)$
7. $(7 + 7)$ > _?_ (6×2)
8. $(5 + 10)$ = _?_ (3×5)
9. (8×3) = _?_ (3×8)
10. (7×3) = _?_ $(21 - 0)$

Pupil's Objectives

(a) To review some of the main ideas taught in other chapters; (b) to have practice with the M. facts with 2 or 3 as a factor; and (c) to use = or > or < symbols to make true sentences.

Teacher's Preparation

You may wish to use the exercises at the top of the page as a practice test. If so, use your usual procedure for a testing situation.

The material at the bottom of the page serves two important purposes. First, it provides drill in an interesting setting on the M. facts with 2 or 3 as a factor. Second, it requires thinking in terms of "is equal to," "is greater than," and "is less than" in order to make a true sentence. Be sure to emphasize the idea of writing true sentences.

Pre-Book Lesson

• If you use the work at the top of the page as a practice test, you will provide the necessary paper and ask the pupils to proceed with the exercises.

• On the other hand, you may use the material as a formal directed review of important ideas developed throughout the book thus far. If so, it would be appropriate to engage in discussion activities like the following:

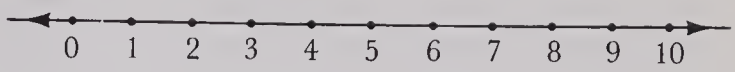
a. What are Roman numerals? Have pupils show numbers with Roman numerals.

b. Discuss principles of rounding numbers to the nearest ten. Use a number-line picture to help illustrate the idea if needed.

c. Use objects or a number-line picture to demonstrate the meaning of the numeral above the bar and the numeral below the bar in a fraction.

d. What is a polygon? What is a line segment? Discuss paths and the special kinds of paths. What are some of the special names we call some polygons? (square, rectangle, triangle)

• Before assigning the written work at the bottom of the page, review the ideas of *greater than* and *less than* by using a number-line picture. You may need to review the symbols > and <.



Is 9 *greater than* or *less than* 5?

Is 4 *greater than* or *less than* 7?

Then, discuss the idea of *naming* numbers in many ways. Ask, "What number is named by 4×2?"

Using the Text Page

• After the written work at the top of the page has been completed by all pupils, be sure to discuss each item. Have children tell or illustrate their answers with an explanation.

• Be sure each pupil understands the directions at the bottom of the page.

Individualizing Instruction

• *All pupils* may draw models of line segments and label end points with capital letters. Then, they may show any polygons they wish. Some children may want to show triangles inside of triangles, and so on.

• *More capable children* may like to use the Roman-numeral system. Or, they may like to look up other systems of numeration in the encyclopedia. In particular, the ancient Egyptian symbols would be very interesting.

• *All pupils* may write true number sentences of their own using the symbols =, >, <. Encourage them to name numbers in these number sentences.

a. using the symbol for addition $(3 + 2 + 5)$, $(7 + 5)$, or $(3 + 6)$.

b. using the symbol for subtraction $(12 - 2)$, $(15 - 3)$, or $(17 - 8)$.

c. using the symbol for multiplication (2×5), (4×3), or (3×3).

• *More capable children* may work each of Ex. 1–10 as indicated in the directions, after adding 28 to the first number named and 34 to the second number named.

Pupil's Objective

To learn to differentiate among addition, subtraction, and multiplication as operations to use in solving problems.

New Words

bushes, package, peas

Background

Pupils should understand and apply the following ideas in solving the problems on this page:

a. To find the number for a total we can add or multiply.

b. We multiply to find the number for a total when we know that several groups are of the same size.

c. If we know the number for the total, but must find the number for a part of that total, we subtract.

In solving problems, or finding applications based on situations familiar to the children, distinctions among these operations will be enhanced.

Using the Text Page

• As you proceed from one problem to the next, ask children to tell what operation should be used and why. Then, have a pupil use representative materials* (flannel board, magnet board, Ginn Arithme-Sticks, the Modern Number Line, and so on) to demonstrate the problem. For example, Ex. 2 could be illustrated on a number-line model by moving to 20 as representative of 20 feet and then 15 more for 15 feet, so as to arrive at 35 as representative of 35 feet. Or, Ex. 5 could be demonstrated on a flannel board by showing 3 rows of 9 objects.

* See list on page xix.

• After each problem has been explained and demonstrated, go over them again and have children tell the answer for each one.

• Finally, go over the problems once again and have pupils tell an *n*-sentence for each problem. You may wish to make this last activity written work for all pupils.

Individualizing Instruction

• From situations similar to those below, have *all pupils* make up some addition, subtraction, and multiplication problems. The multiplication problems should be such that one of the factors is not greater than 3.

a. rows of children, desks, chairs, and so on

b. bicycles in bicycle racks

c. parking, 1¢ for 5 minutes

d. days in the week

e. rows and columns which can be seen in the classroom

• Write *n*-sentences on the chalkboard like the following. Then, have pupils write problems to go with the *n*-sentences.

a. $24 - 8 = n$ b. $7 \times 2 = n$

c. $n + 5 = 16$ d. $3 \times 5 = n$

• Use Extra Problem Set 12 as needed.

* This oral lesson will enable you to detect misunderstandings at this stage of crucial differentiating among three operations!

Add or Subtract or Multiply?

Oral problem-solving [O]

Tell why you should add or subtract or multiply.

1. Mr. Ford bought seeds for sweet peas for 65¢. How much change should have been given him from 75¢? Subtract to find number for other part. 10¢ 75¢ − 65¢ = n¢

2. He planted two rows of sweet peas, one 20 feet long and the other 15 feet. How many feet was that in all? Add to find number for the total. 35 ft. 20 + 15 = n

3. At a sale, Sue Ford bought 3 packages of flower seeds for 8¢ a package. How much did the seeds cost? Multiply to find number for total because sets are equivalent. 24¢ 3 × 8¢ = n¢

4. Mr. Ford and Sue set out all but 8 of 78 pansy plants they bought. How many plants did they set out? Subtract to find number for other part. 70 plants. 78 − 8 = n

5. Sue set out 3 rows of pansy plants, 9 in a row. How many plants were there in these rows? Multiply to find number for total because sets are equivalent. 27 plants. 3 × 9 = n

6. In the Ford garden are 4 sets of rose bushes, with 3 bushes in a set. How many rose bushes is that? Multiply to find number for total because sets are equivalent. 12 bushes. 4 × 3 = n

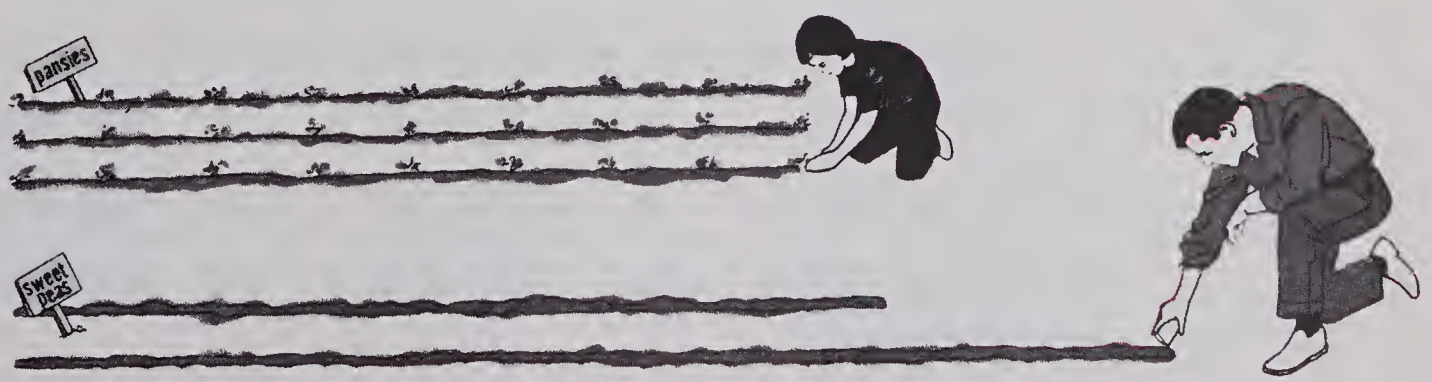
7. A row of white daisies is 38 feet long. A row of yellow daisies is 9 feet shorter, or _?_ feet long. Subtract to find number for other part. 29 ft. 38 − 9 = n

8. Sue planted 3 bulbs. Mr. Ford planted 5 times as many. How many bulbs did he plant? Multiply to find number for total because sets are equivalent. 15 bulbs. 5 × 3 = n

Read each problem again. This time say the answer.

Now say an *n*-example for each problem.

◆ **Extra Problems.** Work Set 12.



To Keep in Practice

A. and S. [W]

Write answers on folded paper. Check your work.

	a	b	c	d	e	f	g
1.	243	819	928	429	409	\$0.73	\$4.30
	+496	−49	−396	+560	−83	+1.58	−0.64
	739	770	532	989	326	\$2.31	\$3.66
2.	82	431	305	703	865	\$5.42	\$7.95
	−78	−76	+208	−48	+96	−4.75	−3.98
	4	355	513	655	961	\$0.67	\$3.97
3.	89	680	819	766	984	\$4.00	\$6.49
	+68	−657	−376	+38	−789	−2.56	−2.68
	157	23	443	804	195	\$1.44	\$3.81
4.	44	2	27	16	40	\$0.09	\$0.36
	9	28	4	19	4	0.17	0.11
	+36	+7	+36	+36	+8	+0.27	+0.47
	89	37	67	71	52	\$0.53	\$0.94

Do You Like to Try New Things?

Enrichment [W]

1. Copy and add: 43, 16, 29. The sum is _?_. 88

2. Ex. A: 18 is the sum of the ones; 70 is the sum of the tens. The sum of those two sums is _?_. 88

3. Copy and add: 457, 348. The sum is _?_. 805

4. Ex. B: See how the numbers are added. If you do not understand what has been done, ask your teacher.

A

43
16
29
18
70
88

B

457
348
15
90
700
805

Copy in columns and add as in Ex. A or Ex. B.

5. 16, 38, 10, 5 69
6. 7, 19, 29, 18 73
7. 54, 9, 7, 28 98
8. 7, 25, 9, 68 109
9. 51, 0, 6, 47 104
10. 3, 46, 28, 16 93
11. 368, 268 636
12. 748, 66 814
13. 255, 376 631

Pupil's Objective

To have practice in writing answers to addition and subtraction examples.

Background

The material in the enrichment activity is one more way of enhancing understanding of the additive nature of our decimal system of numeration—the Hindu-Arabic system. By showing the child the sum of the hundreds, the sum of the tens, and the sum of the ones, he is given another way of *seeing* what actually happens in the working of the example.

Teacher's Preparation

Have paper ready for the folded-paper written work at the top of the page.

Pre-Book Lesson

For *slower learners* only, review the principles involved when renaming numbers for addition or subtraction examples.

a. Have pupils rename 27 as 1 ten and 17 ones.

b. 472 may be renamed 3 hundreds _ ? _ tens and 2 ones.

c. 5 tens and 18 ones may be renamed _ ? _.

Using the Text Page

Most children should be able to proceed with the work at the top of the page independently. Check the work of *slower learners* to determine the extent to which they are able to do the work without additional help.

Individualizing Instruction

• *Slower learners* may need additional practice in using expanded form to show the renaming of numbers. A variety of practice activities have been suggested at different places in this book. Any one of the following may be used:

a. 819 = 800 + 10 + 9
673 = _ ? _ + _ ? _ + _ ? _
_ ? _ = 500 + 40 + 8

b. 492 = 4 hundreds 8 tens _ ? _ ones
492 = _ ? _ hundreds 19 tens 2 ones
492 = 3 hundreds 18 tens _ ? _ ones
_ ? _ = 4 hundreds 8 tens 12 ones

• *Slower learners* and/or *all pupils* may make multiplication charts for facts with 2 or 3 as a factor as suggested for pages 224 and 225.

	1	2	3	4	5	6	7	8	9
3									

• *All pupils* (or only *more capable children*) may work the addition examples in rows 1 and 2 at the top of page 238 as illustrated in box B in the Enrichment section.

NOTES

Pupil's Objectives

(a) To discover pairs of M. facts with 4 as one factor; and (b) to extend what has been learned about the Commutative Property of Multiplication.

Background

The idea of pairs of M. facts is not new. It was developed in introducing the M. facts in which 2 or 3 is a factor. Building an understanding of pairs of M. facts is actually an intuitive development of the commutative property. Using the term *commutative* is not as important as building the idea behind it. This is precisely what we attempt to do in the *Mathematics We Need* program.

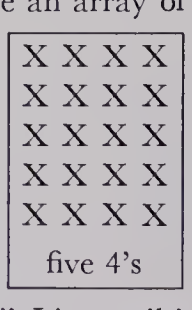
Likewise, it is fully recognized that other interpretations of the multiplication operation exist. However, at this point in the child's progress toward mathematical maturity, we consider the joining of a number of equivalent sets to be the most appropriate means of building an understanding of multiplication.

Teacher's Preparation

Have materials available to distribute to children for use in making arrays. Each child should have some objects and should be involved in making the arrays. Watching someone else make an array is not as valuable to the learner as actually making it for himself.

Pre-Book Lesson

• Each child should have objects and something on which to arrange them. Under your guidance and direction, have the children make an array of five 4's, as was suggested for the lesson on page 221 for showing an array of 3's. It is preferable to have them arrange one 4 at a time.



After all children have the five 4's properly arranged, ask, "How many x's are there in all?" Then ask, "How do you know that five 4's = 20?" List pupils' replies. The following should be among them.

a. "I counted one at a time and got 20."

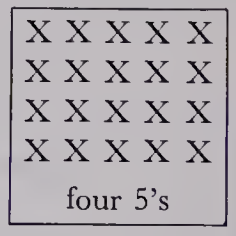
b. "I counted by 4's and got to 20."

c. "I added 4 + 4, then 4 more, and kept adding 4 until they were all gone."

d. "Two 4's = 8; 8 and 8 = 16, and 4 more = 20."

e. "Two 4's = 8; three 4's = 12; 8 + 12 = 20." (This is using the distributive property.)

• Direct *all pupils* to turn their array so that four 5's are shown. (This involves turning the paper on which the array is shown.)



Ask, "How many objects are there in four 5's?"

You should expect the same kinds of responses as described above for five 4's. In each instance, it is desirable to have pupils use the M. facts they have learned already. In learning the 2's and 3's, they learned 2×4, 2×5, 3×4, and 3×5. With the use of these facts and the distributive property, children should be able to find products for 4×4, 4×5, 5×4, and 5×5.

• Have children make arrays for seven 4's and eight 4's and then turn the arrays to help see the related facts.

Using the Text Page

• Have pupils turn the book to see the related facts in each picture. You may have the children make arrays with objects they have at their seats to illustrate each of Ex. 1–3.

• Pupils may draw pictures of buttons, bells, and boxes for Ex. 4–6, or they may draw pictures of circles, triangles, squares, and so on.

Individualizing Instruction

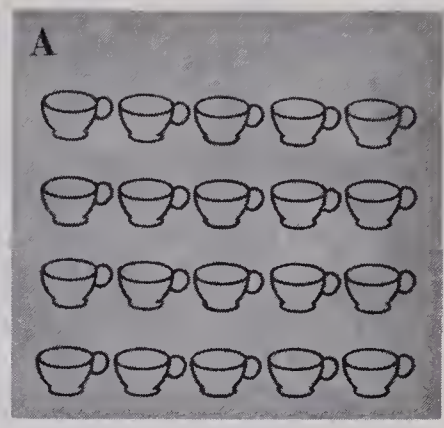
• *Slower learners* may use objects to help with Ex. 7–18.

• *More capable children* may check Ex. 7–18 by using the Distributive Property of Multiplication. For Ex. 18, they may think "$7 = 4 + 3$; $4 \times 4 = 16$, $4 \times 3 = 12$; $16 + 12 = 28$."

• *All pupils* may work in pairs. While one pupil uses objects to make an array, the other pupil says the M. fact shown by the array. To vary the activity, one pupil may give a M. fact and the other pupil may show the fact in an array.

Discovering M. Facts with a Factor 4

Using Commutative Property of M.; arrays [O]

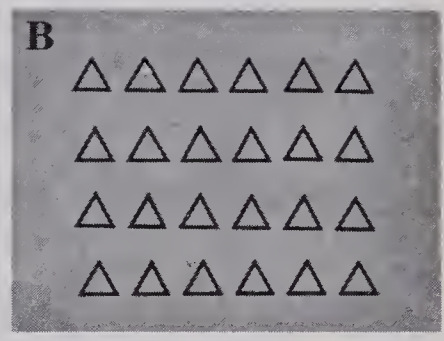


1. You can use one of a pair of M. facts to find how many cups are in picture A.

a. There are [4] ? rows of [5] ? cups. The M. example is $4 \times$ [5] ? $= n$. To find the answer, add four 5's or count by 5's. $n =$? 20

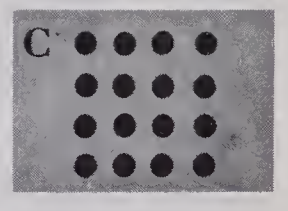
b. Picture A has [5] ? columns of [4] ? cups. The M. example is $5 \times$ [4] ? $= n$. To find the answer, add [5] ? 4's or count by [4] ? 's. $n =$? 20

c. Say the pair of M. facts you have found. 4×5=20 5×4=20



2. Use rows and columns in picture B to help you find and finish this pair of M. facts:

$4 \times 6 = n$ 24 $\qquad$ $6 \times 4 = n$ 24



3. What M. fact can you find from picture C? 4×4=16 Has it another fact to go with it? No Why? The two factors are equal. [W]

Draw pictures for Ex. 4–6. Then beside each picture show the pair of M. facts.

4. 7 rows of buttons, 4 buttons in a row. 7×4=28 4×7=28

5. 4 rows of bells, 9 bells in a row. 4×9=36 9×4=36

6. 8 rows of little boxes, 4 boxes in a row. 8×4=32 4×8=32

Some of the products shown in Ex. 7–18 are wrong. Add to check them. Then write all of Ex. 7–18 to show the correct facts and show the other fact in each pair.

7. 4 × 3 = 12 $3 \times 4 = 12$

8. 4 × 7 = 28 $7 \times 4 = 26$ 28

9. 4 × 5 = 20 $5 \times 4 = 18$ 20

10. 8 × 4 = 32 $4 \times 8 = 32$

11. 9 × 4 = 36 $4 \times 9 = 36$

12. 5 × 4 = 20 $4 \times 5 = 15$ 20

13. 4 × 8 = 32 $8 \times 4 = 30$ 32

14. 4 × 2 = 8 $2 \times 4 = 12$ 8

15. $4 \times 4 = 14$ 16

16. 6 × 4 = 24 $4 \times 6 = 24$

17. 4×9=36 $9 \times 4 = 26$ 36

18. 7 × 4 = 28 $4 \times 7 = 28$

Multiplication Facts for 4's

[W]

Mr. Green has put his flowers in 9 rows of pots with 4 pots in each row.

Copy Ex. 1–9, and find the products. Use the rows of pots and count by 4's. For Ex. 1, use 8 rows.

1. $4 \times 8 = 32$	2. $4 \times 4 = 16$	3. $4 \times 6 = 24$
4. $4 \times 3 = 12$	5. $4 \times 1 = 4$	6. $4 \times 5 = 20$
7. $4 \times 9 = 36$	8. $4 \times 2 = 8$	9. $4 \times 7 = 28$

Some products shown in Ex. 10–21 are wrong. Check by using a number line. Then show each fact correctly.

10. $4 \times 4 = 18$ 16
11. $3 \times 4 = 12$
12. $6 \times 4 = 28$ 24
13. $9 \times 4 = 36$
14. $7 \times 4 = 28$
15. $9 \times 4 = 32$ 36
16. $5 \times 4 = 24$ 20
17. $8 \times 4 = 32$
18. $6 \times 4 = 24$
19. $8 \times 4 = 36$ 32
20. $5 \times 4 = 20$
21. $7 \times 4 = 26$ 28

M. Facts for 4's

$1 \times 4 = 4$
$2 \times 4 = 8$
$3 \times 4 = ?$ 12
and so on to
$9 \times 4 = ?$ 36

22. Copy and finish the M. Facts for 4's table that has been started for you. $4 \times 4 = 16$; $5 \times 4 = 20$; $6 \times 4 = 24$; $7 \times 4 = 28$; $8 \times 4 = 32$

23. Which of these facts are new to you? Show them. $4 \times 4 = 16$; $5 \times 4 = 20$; $6 \times 4 = 24$; $7 \times 4 = 28$; $8 \times 4 = 32$; $9 \times 4 = 36$

24. After each fact shown for Ex. 23, show the M. fact that goes with it to make a pair if there is one. $4 \times 5 = 20$; $4 \times 6 = 24$; $4 \times 7 = 28$; $4 \times 8 = 32$; $4 \times 9 = 36$

25. Does 9×4 mean nine 4's or four 9's? nine 4's

Teaching Page 240

Pupil's Objective

To learn the M. facts for 4's.

Background

The difference between M. facts for 4's and M. facts for 4 should be clear to all pupils who clearly understood the difference between M. facts for 2's and 2 and M. facts for 3's and 3. The M. facts for 4's are those in which 4 denotes the number of members in the equivalent sets. The M. facts for 4 are those in which 4 tells how many equivalent sets there are.

In this lesson we are focusing on 4 as it denotes the number for each set. The number of these sets varies.

The table of M. facts for 4's shows 4 as the second factor. The thinking should be, "one 4," "two 4's," "three 4's," and so on.

$1 \times 4 = 4$
$2 \times 4 = 8$
$3 \times 4 = 12$
and so on

Pre-Book Lesson

• Use objects in the room to demonstrate several different sets of 4 things. For example, you may show 4 chairs, 4 pictures, 4 windows, 4 coats, and so on.

Then, think of joining three sets of 4 things, such as the sets of 4 chairs, 4 pictures, and 4 windows. Ask "What is the number for the total set?"

• Have a child write a sentence on the chalkboard to show the number for the total set.

Using the Text Page

• *Slower learners* may use objects at their desks to assist in finding the products for Ex. 1–9.

• Draw a model of a number line on the chalkboard (or use your commercially-prepared one). Illustrate Ex. 10 on this number-line picture.

You may wish to duplicate a number-line picture with points from 0 through 40 labeled. Pupils may use it to check Ex. 10–21.

Individualizing Instruction

• After *all pupils* have completed the M. facts for 4's table, discuss the table and have pupils state in their own words generalizations like these:

a. Each product is 4 more than the one before it.

b. Saying the products in order is like counting by 4's.

c. The product of three 4's plus the product of five 4's is the same number as the product of eight 4's. In fact, have the pupils tell all the different ways that eight 4's can be found by using the table. This is more work toward the understanding of the Distributive Property of Multiplication over Addition.

Pupil's Objectives

(a) To learn the M. facts for 4; and (b) to extend understanding of the relationship between M. facts for 4's and for 4.

Background

In the last lesson, the emphasis was on sets of 4 things. Now, the stress must be on 4 sets of things. Of course, the 4 sets each have the same number of members.

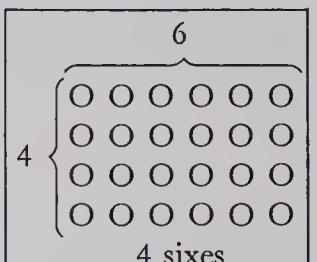
The table of M. facts for 4 shows 4 as the first factor. The thinking should be, "four 1's," "four 2's," and so on.

$4 \times 1 = 4$
$4 \times 2 = 8$
$4 \times 3 = 12$
and so on

Pre-Book Lesson

• Use objects in the room to demonstrate 4 equivalent sets. Perhaps there are 2 tables with 6 children each and 2 windows with 6 panes each. Any four sets of objects, with each set having the same number of members, will serve the purpose.

• You may wish to have children use objects at their desks in which they arrange arrays of four 7's, four 5's, four 8's, and so on. Each time an array is formed by the children, have them tell the number for each set and then how many sets in all.



• Have pupils show on the chalkboard the M. fact for 4 shown by each array.

Using the Text Page

• Before assigning Ex. 1–8, ask children to tell how these examples are alike. Pupils should state something to the effect that all examples show 4 numbers.

Being able to show a M. fact for each example indicates an understanding of the relationship between addition and multiplication. It would be a good idea to discuss this part of the written work, making sure to clearly bring out this relationship to all pupils.

• For variety, encourage pupils to use any way they wish to check the products for Ex. 9–15. They may add, use a number line, or use objects on their desks.

• Have pupils show the M. fact for each of problems 16–20.

Individualizing Instruction

• *Slower learners* may illustrate Ex. 1–8 on a number line. Prepare a sheet of paper showing 10 number-line pictures with points labeled from 0 through 40.

• *All pupils* may complete sentences like the following: *If 6 is one factor and 4 the other, the product is 24.* You may wish to write the sentence on the chalkboard and have pupils label headings on paper as indicated below. Let △ represent one of two factors and □ represent the other factor of the product represented by ○.

△	□	○
6	4	24

Show as many pairs of factors and a product as you can that will make a true sentence. Let one factor be 1, 2, 3, or 4.

Multiplication Facts for 4

[W]

Copy each of Ex. 1–8 and show the sum. Then under each example show the M. fact.

1.	2.	3.	4.	5.	6.	7.	8.
4	6	8	5	7	9	3	2
4	6	8	5	7	9	3	2
4	6	8	5	7	9	3	2
4	6	8	5	7	9	3	2
16	24	32	20	28	36	12	8
$4 \times 4 = 16$	$4 \times 6 = 24$	$4 \times 8 = 32$	$4 \times 5 = 20$	$4 \times 7 = 28$	$4 \times 9 = 36$	$4 \times 3 = 12$	$4 \times 2 = 8$

In Ex. 9–15, some of the products shown are wrong. Check by addition. For Ex. 9 add four 7's, for Ex. 10 add four 6's, and so on. Then write Ex. 9–15 correctly.

9.	10.	11.	12.	13.	14.	15.
7	6	3	8	5	9	4
4	4	4	4	4	4	4
30	24	14	32	22	38	8
28		12		20	36	16

Write answers for each of Ex. 16–20.

16. If 6 girls can sit at a table, 24 girls can sit at 4 tables.

17. Joe has 4 rabbits. Tom has 4 times as many. Tom has 16 rabbits.

18. Four 5¢ stamps cost 20¢.

19. If each box holds 8 pens, in 4 boxes there should be 32 pens.

20. There are 36 boys in all in 4 teams of 9 each.

21. Copy and finish the M. Facts for 4 table.

22. Beside each fact shown in the table show the other M. fact that goes with it to make a pair if there is one.

$4\times4=16$; $4\times5=20$; $4\times6=24$; $4\times7=28$; $4\times8=32$

$1\times4=4$; $2\times4=8$; $3\times4=12$; –; $5\times4=20$; $6\times4=24$; $7\times4=28$; $8\times4=32$; $9\times4=36$

M. Facts for 4

$4 \times 1 = 4$

$4 \times 2 = 8$

$4 \times 3 = 12$

and so on to

$4 \times 9 = 36$

Table of Multiplication Facts for 4's

4	4	4	4	4	4	4	4	4
1	2	3	4	5	6	7	8	9
4	8	12	16	20	24	28	32	36

Table of Multiplication Facts for 4

1	2	3	4	5	6	7	8	9
4	4	4	4	4	4	4	4	4
4	8	12	16	20	24	28	32	36

Factors-product relationship [O]

1. Cover the products shown and try to say them.

2. After 4×1 and 1×4, how much greater is each product than the product just before it?[4] Why? See below.

3. Is one factor always an even number? Yes Are any of the products odd numbers? No

4. What should be the products for these examples? Explain. **a.** 10×4 (40) **b.** 4×10 (40) **c.** 11×4 (44) **d.** 4×11 (44)

$10 \times 4 = (9 \times 4) + 4$, or 40 $4 \times 10 = (4 \times 9) + 4$, or 40 $11 \times 4 = (10 \times 4) + 4$, or 44 $4 \times 11 = (4 \times 10) + 4$, or 44

Say the products by rows and by columns.

	a	b	c	d	e	f	g
5.	7×4 28	4×5 20	8×2 16	4×7 28	8×3 24	9×4 36	3×8 24
6.	4×2 8	9×3 27	4×4 16	4×3 12	5×2 10	6×4 24	3×7 21
7.	6×2 12	4×6 24	9×2 18	5×4 20	7×2 14	3×9 27	4×9 36
8.	3×4 12	4×8 32	6×3 18	3×5 15	8×4 32	5×3 15	7×3 21

Make Study Cards and use them for practice.

♦ **Extra Examples.** Work Set 68.

● **Extra Activity.** Work Set 102.

2. For 4's, there is one more 4 each time.
For 4, there are 4 more 1's each time.

Pupil's Objectives

(a) To learn more about the factors-product relationship; (b) to study the tables of M. facts for 4's and for 4; and (c) to say products for M. facts in which 2, 3, or 4 is one of the factors.

Background

In previous lessons, the M. facts for 4's and the M. facts for 4 have been presented separately. In this lesson, they are presented together. Be sure to stress the Commutative Property of Multiplication and to use the terms *factor* and *product*.

Pre-Book Lesson

Duplicate (or write on the board) the first facts of the two tables as shown below:

one 4 = 4	four 1's = 4
two 4's = 8	four 2's = 8
three 4's = n	four 3's = n

Ask pupils to show the facts to complete the tables for 4 and 4's. If they need to use objects, make drawings, and so on, they should be permitted to do so. After all pupils are finished, have them compare their products with the products shown in the tables on page 242 of the pupil's book.

Using the Text Page

• Use the tables completed by the pupils and the tables at the top of the pupil's page as a basis for answering questions 1–4.

Through your guided questioning, the pupils should state generalizations like these:

a. The products for M. facts about 4 increase by 4 each time.

b. Saying the products in order is like counting by 4's.

c. When 4 is a factor, the product is always an even number.

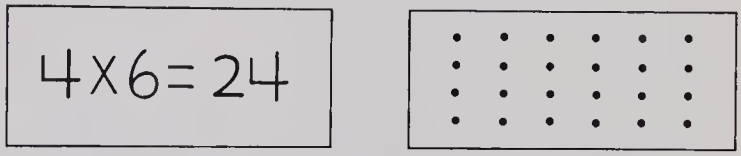
• Set the stage for later work with the distributive property by having pupils show the sum of two products each having 4 as a factor as equal to some other product of which 4 is a factor. For example, five 4's + two 4's = seven 4's.

• Vary the work in Ex. 5–8 by using these alternating procedures. Have pupils say just products for one column and read the entire sentence for the next column (i.e., "seven 4's equal 28"), and so on.

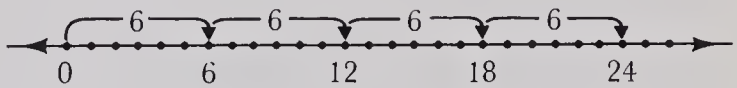
Individualizing Instruction

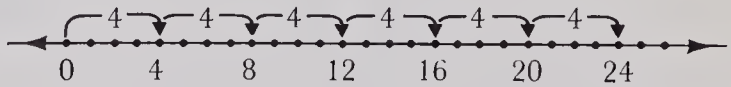
• *All pupils* may make study cards for M. facts in which 2, 3, or 4 is one of the factors. *Slower learners* should make arrays on the back of the study cards to illustrate the facts.

front back



• *All pupils* may illustrate pairs of M. facts on a number line. Use the examples in Ex. 7–8 and their related facts.





• *All pupils* may fill in the squares in a duplicated grid like the one shown on Teaching Page 226.

• *All pupils* may copy and finish each example in rows 5–8. Then they may show the M. fact that goes with the fact shown.

• *More capable children* may do the Extra Activity suggested in Set 102.

• Use Extra Examples Set 68 as needed.

• Write sentences like the following on the board or duplicate them on paper.

$5 \times \square = 20$
$\bigcirc \times 4 = 32$
$7 \times \triangle = 21$
$6 \times 4 = \square$

Teaching Page 243

Pupil's Objectives

(a) To extend skill in column addition to include five addends shown with 1- and 2-place numerals; and (b) to begin column addition of three addends shown with 3-place numerals.

Background

No new understandings are necessary to find the answers to these examples. It is a matter of applying what is already known and understood to working longer examples. Greater concentration and attention span is demanded.

Using the Text Page

Perhaps your pupils will be motivated by the challenge of longer examples. Point out to the youngsters that they possess all the necessary skills to find the answers to these examples.

Ask pupils to describe situations when it may be necessary for them to know how to add a great number of addends.

Before asking the pupils to begin the written work, have them scan the examples to find one like that in box A; then, examples like those in boxes B, C, D, and E. Make the point that the examples in the boxes are models which may be referred to if necessary.

Individualizing Instruction

- *All pupils* may play the *Show the Answer* game with the M. facts in which 2, 3, or 4 is a factor. Hold up a sentence in which either a factor or the product is missing. Pupils hold up (on command) the correct answer. Emphasize the idea of making a true sentence.

Instead of holding up cards with sentences written on them, you may prefer to write sentences on the chalkboard.

- Conduct an individual interview with *slower learners* to determine their sources of difficulties in working column-addition examples. Have them tell you what they are doing as they work an example. Ask specific questions designed to elicit explanations. The results of the interview will give you a basis for assigning practice material to eliminate specific weaknesses.
- *More capable children* may write problems like Ex. 16–20 at the bottom of page 241 and then show the appropriate M. fact to go with the problem they have written.
- Assign Extra Examples Set 69 as needed.

NOTES

* Again, pupils may be reminded that in addition we work with only two numbers at a time.

Adding as Many as Five Numbers

* *Addition a binary operation* [O]

When we add as many as five numbers, we always begin by adding two of them. Then to their sum we add a third number. To that sum we add a fourth number, and to that sum we add the fifth number.

Ex. A. *Think*, "15, 21, 25, 34." Write "34."
6 + 4 = 10, 10 + 8 = 18, 18 + 7 = 25, 25 + 8 = 33.
Ex. B. Explain the work. ∧ Why do you carry? The sum of the ones is greater than 9.

A	B	C	D	E
7	56	67¢	$3.49	$4.27
8	4	19¢	2.56	0.89
6	38	8¢	1.08	0.73
4	77	26¢	$7.13	2.44
9	8	120¢,		$8.33
34	183	or $1.20		

Explain Ex. C–E. How many places do cents use? 2

[W]

Copy each set of numerals in a column. Add and check.

1. 7, 8, 6, 5, 9 35
2. 3, 8, 7, 9, 6 33
3. 4, 6, 9, 9, 8 36
4. 41, 8, 5, 7, 9 70
5. 62, 8, 0, 7, 5 82
6. 26, 5, 2, 7, 8 48
7. 38, 4, 9, 4, 9 64
8. 8, 7, 4, 6, 8 33
9. 9, 7, 7, 9, 8 40
10. 329, 184, 249 762
11. 193, 245, 413 851
12. 248, 256, 67, 396 967
13. 215, 457, 76, 93 841
14. 16¢, 5¢, 87¢, 7¢, 18¢ $1.33
15. $4.07, $3.09, $0.83 $7.99
16. $0.79, $4.12, $3.12 $8.03
17. $1.56, $5.84, $0.03 $7.43

◆ **Extra Examples.** Work Set 69.

* Again a quick test on multiplication facts—this time facts having one factor no greater than 4.

* Try These

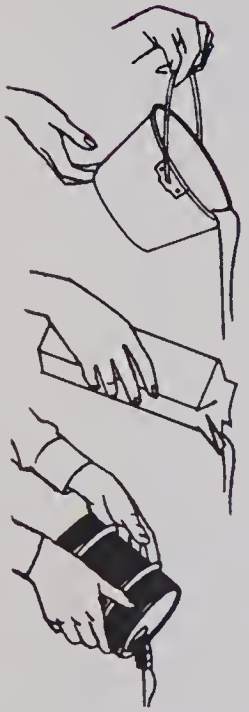
One factor no greater than 4 [W]

Show the products on folded paper.

	a	b	c	d	e	f	g	h	i	j
1.	4	9	2	8	4	3	2	3	6	5
	4	3	8	4	2	9	5	4	3	2
	16	27	16	32	8	27	10	12	18	10
2.	3	2	9	7	2	5	7	9	4	3
	7	4	2	3	6	4	2	4	8	3
	21	8	18	21	12	20	14	36	32	9
3.	7	4	2	1	8	6	4	5	6	1
	4	3	7	4	3	2	9	3	4	3
	28	12	14	4	24	12	36	15	24	3
4.	4	3	4	8	4	3	2	4	3	2
	5	6	7	2	6	5	9	1	8	3
	20	18	28	16	24	15	18	4	24	6

Cups, Pints, Quarts, and Gallons

Measuring liquids [O]



1. What do we call things like water, milk, and oil? liquids

2. When might we want to measure such things? Answers will vary.

3. When we measure milk, we use cups, pints, quarts, and gallons. In the picture on page 245, which thing for measuring is smallest? cup Which is largest? gallon

Not all cups are measuring cups. Some cups hold more than a measuring cup and some hold less. In this book, when you see "cups" you should think of measuring cups.

4. The picture on page 245 shows that it takes 1 cup of liquid to make _?_ $\frac{1}{2}$ pint, _?_ 2 cups to make 1 pint, _?_ 2 pints to make 1 _?_ qt; and 4 qt ?_ to make 1 gallon.

1 cup (c.) = $\frac{1}{2}$ pint (pt.) **2 pints = 1 quart (qt.)**

2 cups = 1 pint **4 quarts = 1 gallon (gal.)**

Teaching Pages 244 and 245

Pupil's Objectives

(a) To practice showing products on folded paper; (b) to learn the units for measuring liquids; and (c) to have practice in giving equivalent liquid measurements.

New Word

(page 244) *gallons*

Background

All units of liquid measurement have been arbitrarily selected and named by man. As was noted earlier, no measurement is exact. We select the unit of measurement that is most appropriate for our needs. The smaller the unit of measurement the more precise the measurement is.

Most children will probably know the names for the units of liquid measurement, but they may not know much about the units themselves. The principal purpose of this lesson is to give these terms fuller meaning.

Teacher's Preparation

Arrange to have some examples of various units of measurement for use in the classroom. Children can probably bring milk cartons for a pint, a quart, and one-half gallon. If it is possible to obtain an example of a cup and a gallon unit it would be helpful.

Pre-Book Lesson

• If water and a sink are nearby, have several children fill various containers with water. Use a pint container and count the number of times it must be filled for the liquid to fill a quart container. (Using a coloring for the water helps you to see the liquid as the container is being filled.) Do the same with a quart and half-gallon container.

• Reverse the filling process. Have a child demonstrate how many pint containers can be filled from the liquid of one quart container. Do the same with a half-gallon and quart container.

• If you have two or more different types of pint containers, have children compare the amount of water each will hold. They should establish the idea that *one pint* is a constant unit of liquid measurement which does not vary even though the pint containers may be many shapes.

• Discuss with children the different kinds of social applications with which they are familiar that require the various units of liquid measurement presented in this lesson. What are some examples or instances in which the cup is the most appropriate unit of measurement? Is the gallon usually used as a unit of measurement in your mother's recipes? Does your father ever buy a cup of gasoline?

Using the Text Pages

• You may wish to use the written work at the top of page 244 as a timed test on the M. facts that have been learned so far. Pupils may want to keep a record of their time and number of errors. Permit them to try to improve in both time and errors. Be careful not to stress time to the point that additional errors are committed. Time may decrease if errors do not increase.

• Use the containers in the classroom when appropriate to help clarify and extend any of the ideas developed in Ex. 1–10. The pictured relationships should be helpful, but experience in using actual containers is necessary.

• Many pupils should be able to use some of the M. facts they have just learned to help find the answers to Ex. 11–14. Encourage pupils to use M. facts if they can. Permit those who need the reassurance to draw pictures, dots, arrays, and so on, to help find the answers.

Individualizing Instruction

• Encourage children to experiment at home with different units of liquid measurement. If possible, they should help their parents mix or make something which requires the use of such units. If there is a school cafeteria, perhaps a visit to the kitchen at a time when units of liquid measurement are being used would be possible.

• Duplicate a list similar to the following. Pupils

are to select the items which may be measured by a unit of liquid measurement. Then, they should indicate the most common unit used for each item selected.

crackers	syrup	salt
fish	tomatoes	gasoline
soda pop	toast	shoes
milk	paint	rolls
ham	nails	water

• Duplicate (or write on the chalkboard) exercises like the following. *All pupils* are to write = or > or < to make each a true sentence.

(4×3) _____ (2×7)
(4×7) _____ (9×3)
(6×4) _____ (3×8)

Pupils may use such objects as buttons or spools to form arrays or they may draw dot arrays to help with these examples.

• *More capable children* may try finding the solutions to sentences like the following:

$(7 \times 3) + 4 = n$ $\quad n = ?$
$(4 \times 2) + n = 11$ $\quad n = ?$
$(n \times 4) + 2 = 18$ $\quad n = ?$
$19 - (5 \times 3) = n$ $\quad n = ?$

There are many different ways to write sentences of this type. The idea is to give additional practice with the M. facts as well as to give practice in using addition and subtraction skills. Drill of this type is interesting and self-motivating.

• For each M. fact completed in Ex. 1–4, *slower learners* may show the M. fact to make a pair.

Reminder

Continue to maintain and extend the following:

a. the construction of charts and tables from data which has been gathered in connection with a science or social-studies unit

b. the solving of problems based on information given in a table or chart

c. the use of measurements of distance and weight which have been taught previously

NOTES

Cups, pints, quarts, or gallons? Tell which.

5. Mrs. Kane buys $\frac{1}{2}$ pint of cream a day.

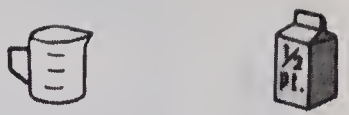
6. Ann's mother used about 1 cup of milk in a small cake.

7. Gasoline for airplanes is measured in gallons.

8. The boys each bought $\frac{1}{2}$ pint of milk for lunch.

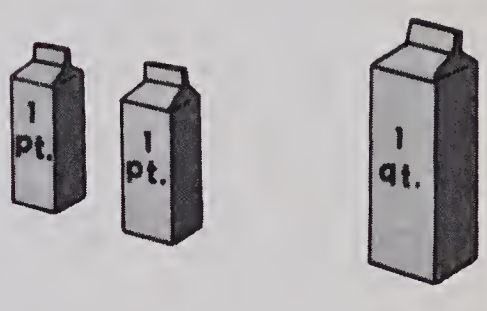
9. 3 quarts is less than 1 gallon.

10. Mr. Bell buys 4 quarts of oil when he changes oil in his car.



1 c. or $\frac{1}{2}$ pt.

2 c. or 1 pt.



2 pt. or 1 qt.

[W]

Write only the missing numerals.

11. 1 pt. = 2 c., so

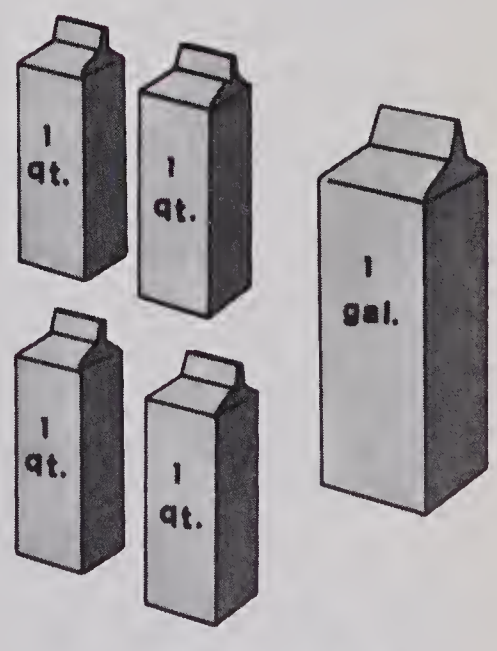
a. 3 pt. = 6 c.
b. 5 pt. = 10 c.
c. 9 pt. = 18 c.
d. 7 pt. = 14 c.
e. 8 pt. = 16 c.
f. 4 pt. = 8 c.

12. 1 qt. = 2 pt., so

a. 2 qt. = 4 pt.
b. 4 qt. = 8 pt.
c. 5 qt. = 10 pt.
d. 7 qt. = 14 pt.
e. 3 qt. = 6 pt.
f. 9 qt. = 18 pt.
g. 6 qt. = 12 pt.
h. 8 qt. = 16 pt.

13. 1 gal. = 4 qt., so

a. 3 gal. = 12 qt.
b. 5 gal. = 20 qt.
c. 6 gal. = 24 qt.
d. 2 gal. = 8 qt.
e. 4 gal. = 16 qt.
f. 8 gal. = 32 qt.
g. 7 gal. = 28 qt.
h. 9 gal. = 36 qt.



4 qt. or 1 gal.

14. Do you know? $\frac{1}{2}$ gal. = 2 qt.

* Multiplying Tens

[o]

A	
10	addend
10	addend
10	addend
n	sum

B	
10	factor
$\times 3$	factor
30	product

Each box in the picture has 10 pieces of candy. How many pieces are there in 3 boxes? $3 \times 10 = n$

1. Count the candies by 10's. How many? 30

2. Ex. A. Add tens. The sum is _?_. 30

3. Why can you multiply instead of adding? The sets are equivalent.

4. Ex. B. *Think*, "3 times 1 ten = 3 tens, or _?_." 30

5. The factors are 10 _?_ and 3 _?_. The product is 30 _?_.

You multiply tens in the same way that you multiply ones.

Say "times" for $\times$, as you read and finish Ex. 6–9.

6. $4 \times 1 = 4$, so
4×1 ten = 4 _?_ tens, or 40 _?_.

7. $3 \times 2 = 6$, so
3×2 tens = 6 _?_ tens, or 60 _?_.

8. $3 \times 3 = 9$, so
3×3 tens = 9 tens _?_, or 90 _?_.

9. $4 \times 2 = 8$, so
4×2 tens = 8 _?_ tens, or 80 _?_.

Explain Ex. 10–12. Tell how to finish Ex. 13–15.

10.	11.	12.	13.	14.	15.
30	20	10	40	20	20
$\times 3$	$\times 2$	$\times 3$	$\times 2$	$\times 3$	$\times 1$
90	40	30	? 80	? 60	? 20

* When pupils find that they multiply tens in the same way that they multiply ones, this new skill will not seem difficult. Using place-value ideas is much help in the more difficult phases of all four operations—A., S., M., and D.

Teaching Pages 246 and 247

Pupil's Objectives

(a) To begin to learn how to multiply tens; and (b) to review and practice some skills in addition.

Background

On page 246 is a first introduction to multiplication using a 2-place numeral and a 1-place numeral. In this lesson, the 2-place numeral shows only a number of tens. Also, you will observe that no carrying is involved when multiplying the number of tens.

In the introductory stage, the total is obtained by counting, then by adding, and finally by multiplying.

The use of objects and teaching aids is deliberately kept at a minimum. By this time, the relationship between addition and multiplication should be well established. Objects have been used extensively in previous lessons when learning basic M. facts. And, they will be recommended in future lessons for discovering and mastering basic M. facts. Now, we are using the M. facts to multiply tens and eventually, of course, to multiply hundreds.

Make sure your pupils understand the writing of numerals for the product and why they are written where they are. An understanding of place value, of course, is the key to the explanation. For example in Ex. 10, 9 in ten's place is used to show the product 9 tens, or 90.

On page 246, the word "times" is used for the first time in reading sentences indicating multiplication. With understanding well established, pupils should be able to say "four times twenty" rather than "four 20's." The advantage of using "times" increases as the numbers being multiplied become greater.

Teacher's Preparation

Have a teaching aid available on which you can show three 10's. You may wish to use a flannel board*, a peg board, an abacus*, and so on.

* See 2 and 10, page xix.

Pre-Book Lesson

• Present a problem similar to the following:

> There are 10 players on a kickball team. How many players would there be on three teams?

Pupils may use objects to represent the players on each team. Direct pupils to arrange objects to show three groups.

You may wish to use a pocket chart** to show *tens* and that the answer will be *tens*. Have children tell different ways they can find the total. Some will count (either by ones or by tens); others will add tens; and some will multiply.

Point out that the problem is concerned with 10 three *times*. We may say "three *times* ten." Write "3 × 10" on the chalkboard and point out that the symbol × may be read "times."

• State a similar problem. This time, omit counting and adding. Have pupils draw ϕ-bundle pictures and then show the multiplication in two ways, as shown below.

ϕ -pictures		
ϕ ϕ	2 tens	20
ϕ ϕ	×3	×3
ϕ ϕ	6 tens	60

• Guide pupils to state in their own words a generalization, such as, "You multiply tens in the same way that you multiply ones."

Using the Text Pages

• Use Ex. 1–15 to extend and strengthen the ideas and understandings just developed in the Pre-Book Lesson. For pupils who need more objective presentation, use objects and teaching aids as appropriate.

NOTE: Multiplication is a *binary* operation. We operate on two numbers and get a third number called the product. Even though we may use addition as a basis for understanding and in finding a product, we eventually want children to perform the *binary* operation of multiplication.

** See 21, page xix.

• When discussing Ex. 10–15, have pupils state that there are no ones in 30, so we must show there are no ones in the product. The 9 tells us there are 9 tens in the product.

$$\begin{array}{r} 30 \\ \times 3 \\ \hline 90 \end{array}$$

• Circulate among the children as they are working Ex. 16–27 to determine if and where they may be having difficulty. Asking a child to tell you how he finds the answer will give you insight as to what kind of reteaching must be done.

Individualizing Instruction

• *All pupils* may do the written work at the bottom of page 247. You may want to ask some pupils to show the sums for each column separately, as suggested on pupil's page 238.

• *Slower learners* may draw ϕ -bundle pictures to illustrate Ex. 16–27 at the top of page 247.

Picture		
$\phi\ \phi\ \phi\ \phi$	4 tens	40
$\phi\ \phi\ \phi\ \phi$	$\times 2$	$\times 2$
	8 tens	80

• *Slower learners* (and perhaps others) may gain greater insight by using dimes to demonstrate the idea presented in this lesson. For example, if an item cost two dimes (20¢), then how much would 4 such items cost? Permit pupils to use coins or representations of coins to illustrate the situation. Other items may cost 3 dimes and still others may cost 4 dimes. Point out the relationship between 3 dimes and 3 tens.

• The enrichment material in the middle of page 247 may be tried by any children who are adequately prepared for it. Bring out the game element in this activity in that matching of items is required. Children are to determine which item in the box matches each of the items in Ex. 2–10. Ask children, "What number is named in Ex. 2?" Then tell them, "Find a name for the same number in the box."

• *Slower learners* may need more practice with the basic M. facts that have been introduced to date. They may match answer cards with multiplication sentences they have prepared. They may play the *Show the Answer* game with the sentences. Or, they may be timed on working the examples at the top of page 244.

NOTES

[W]

Write the work for Ex. 16–27. Box C shows the work for Ex. 16.

C

$$\begin{array}{r} 10 \\ \times\ 3 \\ \hline 30 \end{array}$$

16. three 10's 30

17. two 10's 20

18. 3×20 60

19. two 40's 80

20. 2×20 40

21. 4×20 80

22. two 30's 60

23. 3×10 30

24. 4×10 40

25. three 30's 90

26. 2×40 80

27. three 20's 60

Do You Like to Try New Things?

Enrichment [W]

For Ex. 1, $(4 \times 4) + 4 = 20$. In the box, Ex. b has the same answer. So, write "$(4 \times 4) + 4 = 5 \times 4$."

Follow this plan for Ex. 2–10.

1. $(4 \times 4) + 4 = 5 \times 4$

2. $(9 \times 2) + 6 = 8 \times 3$

3. $(7 \times 4) + 2 = 3 \times 10$

4. $(9 \times 4) - 4 = 4 \times 8$

5. $(3 \times 5) + 6 = 7 \times 3$

6. $(4 \times 10) - 4 = 4 \times 9$

7. $(3 \times 6) + 0 = 2 \times 9$

8. $(6 \times 4) - 9 = 5 \times 3$

9. $(3 \times 7) - 9 = 4 \times 3$

10. $(8 \times 4) - 5 = 9 \times 3$

a. 7×3	**f.** 5×3
b. 5×4	**g.** 9×3
c. 3×10	**h.** 4×9
d. 4×8	**i.** 4×3
e. 8×3	**j.** 2×9

To Keep in Practice

Column A. [W]

Copy each set of numerals in a column. Add and check.

1. 9, 0, 9, 9, 9 36

2. 7, 8, 6, 7, 9 37

3. 58, 1, 5, 9, 7 80

4. 40, 8, 77, 12, 5 142

5. 34, 9, 8, 4, 9 64

6. 47, 4, 8, 6, 9 74

7. 85, 8, 79, 29, 16 217

8. 7, 59, 8, 66, 73 213

9. 452, 24, 300, 126 902

10. \$4.00, \$2.06, \$2.52 \$8.58

11. \$1.56, \$5.04, \$0.83 \$7.43

12. \$2.24, \$3.05, \$0.42 \$5.71

Multiplying with 2-Place Numerals

* *Using the Commutative and Distributive Properties; no renaming* [O]

Each of three mats is made of 32 sticks. How many sticks in all were used to make the three mats?

To find the number for the total, you can add as in Ex. A; but it is better to multiply instead. Why? It is faster.

Ex. B–D show a way to multiply. Think of 32 as $30 + 2$ or $2 + 30$. Then find the product of the ones (Ex. B) and the product of the tens (Ex. C). Then add those products (Ex. D).

A

$$\begin{array}{r} 32 \\ 32 \\ \underline{32} \\ ? \end{array}$$

B **C** **D**

$$\begin{array}{rrr} 2 & 30 & 6 \\ \times 3 & \times 3 & +90 \\ \hline 6 & 90 & 96 \end{array}$$

E

Tens	Ones
3	2
	3
9	6

Ex. E shows the short way to multiply 3×32.

Ones: 3×2. Write "6" in _?_ (one's) place for the product.

Tens: 3×3. Write "9" in _?_ (ten's) place for the product.

The product of 3×32 is _?_. 96

Explain Ex. 1–5 as in Ex. B–D. For Ex. 1 say, "$2 \times 4 = 8$, $2 \times 20 = 40$, $8 + 40 = 48$."

1.	2.	3.	4.	5.
24	31	42	33	43
$\times 2$	$\times 3$	$\times 2$	$\times 3$	$\times 2$
48	93	84	99	86

Now say Ex. 1–5 the short way. For Ex. 1 say, "$2 \times 4 = 8$." Write "8" in one's place for the product. Say, "$2 \times 2 = 4$." Write "4" in ten's place for the product. The product of 2×24 is 48.

* The ideas of expanded notation ($32 = 30 + 2$) and the commutative property, ($30 + 2 = 2 + 30$), have been utilized earlier. Now the idea is utilized of distributing the multiplication over addition, $3 \times (2 + 30)$ expressed as $(3 \times 2) + (3 \times 30)$.

Teaching Pages 248 and 249

Pupil's Objective

To learn to multiply tens and ones by the factor 2, 3, or 4, when there is no carrying to ten's place.

New Word

(page 248) *mats*

Background

The distributive property is expressed in its most general form as $a \times (b + c) = (a \times b) + (a \times c)$. This very important principle is one which brings two operations together. In the child's mind, we use one operation to help find the solution to another. Thus far, we have used the principle in such a way that addition is used to help find the product for a multiplication fact. More technically, we say multiplication is distributed over addition.

The illustration below shows how the distributive property has been used so far to enhance understanding of multiplication and serve as a basis for learning and mastering the M. facts.

$$3 \times 7 = 3 \times (2 + 5) = (3 \times 2) + (3 \times 5)$$
$$a \times (b + c) = (a \times b) + (a \times c)$$

In the lesson on page 248 and in subsequent lessons, the distributive property will be used to give fuller understanding of multiplication with 2- and 3-place numerals.

$$4 \times 52 = (4 \times 50) + (4 \times 2)$$

In this example, 52 has been renamed as (50 + 2). We find the partial product of (4 × 2) and the partial product of (4 × 50) and then add the two partial products to find the product of (4 × 52).

Pre-Book Lesson

• Write the following examples on the chalkboard and ask different pupils to come to the chalkboard, show the products, and explain the work.

40	20	30
×2	×3	×3

• Write the following on the chalkboard and ask pupils to show the products and explain the work. This particular skill has not been developed previously, but it should not be difficult to develop it. It merely involves carrying to hundred's place.

70	40	80	60
×2	×4	×3	×4

• On the chalkboard, write in vertical form the example 2 × 2. Ask a child to show the product and explain what it means.

• Leave the above example on the board and immediately to its left write in vertical form the example 2 × 40. Ask a different child to show the product and explain the work. At this point, you should have written on the board something like the work below.

40	2
×2	×2

• Write and ask, "What number is named by (40 + 2)?" Relate this to the two examples written on the chalkboard. "Suppose we write our example as shown at the right. How was the product found?"

(40 + 2)
× 2
(80 + 4), or 84

Check the product by adding 42 + 42.

You may wish to write the example as

4 tens 2 ones
×2
8 tens 4 ones, or 84

Using the Text Pages

• As Ex. 1–6 are being discussed, refer back to any of the Pre-Book Lesson which may provide additional insight or clarification. Have pupils make ⌀-bundle pictures if it helps in developing understanding.

• Discuss each of Ex. 7–12 separately. Ask a child to read a sentence for each example. Then, have him tell whether the sentence is true or false.

You may wish to have pupils use any of the techniques suggested in the Pre-Book Lesson for checking each answer shown.

• Observe the written work for Ex. 13–16 to note a lack of understanding or incorrect procedures. Ask questions of children if you doubt their comprehension. In these exercises, products are shown with 2- and 3-place numerals.

Work with *slower learners* either individually or in a small group to clarify misunderstandings and to prevent poor procedural habits from becoming fixed.

More capable children may find "timed races" an added bit of excitement in doing the rows at the bottom of page 249. Time them by rows. Pupils with a high degree of accuracy may then improve upon their speed of response.

Individualizing Instruction

• As *slower learners* progress toward correct procedures and understanding, they may be helped by one or more of the following activities:

a. Make Φ -bundle pictures to show the example.

Φ Φ Φ Φ Φ Φ // //
Φ Φ Φ Φ Φ Φ // //

$$\begin{array}{r} 64 \\ \times 2 \\ \hline 128 \end{array}$$

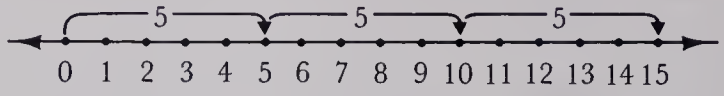
b. Show tens and ones as indicated below.

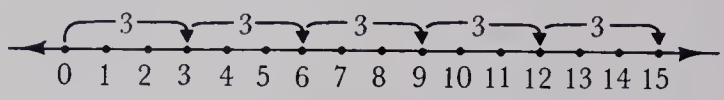
$$\begin{array}{rl} 64 = & 6 \text{ tens and } 4 \text{ ones} \\ \times 2 & \times 2 \\ \hline & 12 \text{ tens and } 8 \text{ ones, or } 128 \end{array}$$

c. Write the 2-place numeral in expanded notation, as shown below.

$$\begin{array}{rl} 64 = & (60 + 4) \\ \times 2 & \times 2 \\ \hline & 120 + 8, \text{ or } 128 \end{array}$$

• Some *slower learners* may need additional work with the basic M. facts. They may use their practice cards or perhaps illustrating M. facts in a number-line picture would be valuable practice. If they use a number-line picture, have them illustrate related facts as indicated below.





• *More capable children* may gain greater insight into the distributive property by engaging in an activity like the following:

A package of balloons cost 31¢. Show in as many different ways as you can the cost of five packages of balloons.

Examples of pupil responses are:

a. 3 packages 93¢
2 packages +62¢
5 packages 155¢, or $1.55

b. 2 packages 62¢
2 packages 62¢
1 package +31¢
5 packages 155¢, or $1.55

c. 1 package 31¢
4 packages +124¢
5 packages 155¢, or $1.55

• Use Extra Examples Sets 70 and 71 for *all pupils* as deemed necessary.

6. Patty and Ann made four mats with 32 sticks in each. For the 4 mats they used 128 ? sticks. $4 \times 32 = n$

Explain the work for Ex. F. What is new in this multiplying? Showing a product having hundreds.

F

$$\begin{array}{r} 32 \\ \times\ 4 \\ \hline 128 \end{array}$$

Which products for Ex. 7–12 are shown wrong? Why? Ex. 8, 10, 12

Ex. 8 $2 \times 3 = 6$ $2 \times 8 = 16$
Ex. 10 $3 \times 1 = 3$
Ex. 12 $3 \times 2 = 6$

7.	8.	9.	10.	11.	12.
92	83	31	71	64¢	92¢
$\times$ 3	$\times$ 2	$\times$ 4	$\times$ 3	$\times$ 2	$\times$ 3
276	85	124	214	128¢, or $1.28	275¢, or $2.75

[W]

Copy and multiply. Check by adding.

	a	b	c	d	e	f	g	h	i	j
13.	72	82	32	93	41	83	44	71	62	73¢
	$\times$ 3	$\times$ 2	$\times$ 3	$\times$ 3	$\times$ 4	$\times$ 3	$\times$ 2	$\times$ 3	$\times$ 4	$\times$ 3
	216	164	96	279	164	249	88	213	248	$2.19
14.	83	31	23	91	72	82	22	62	31	92¢
	$\times$ 2	$\times$ 4	$\times$ 3	$\times$ 2	$\times$ 4	$\times$ 3	$\times$ 4	$\times$ 2	$\times$ 3	$\times$ 2
	166	124	69	182	288	246	88	124	93	$1.84
15.	42	73	52	82	61	53	93	32	62	71¢
	$\times$ 3	$\times$ 2	$\times$ 2	$\times$ 4	$\times$ 2	$\times$ 3	$\times$ 2	$\times$ 4	$\times$ 3	$\times$ 4
	126	146	104	328	122	159	186	128	186	$2.84
16.	43	63	24	92	52	43	91	51	74	51¢
	$\times$ 2	$\times$ 3	$\times$ 2	$\times$ 3	$\times$ 4	$\times$ 3	$\times$ 4	$\times$ 3	$\times$ 2	$\times$ 2
	86	189	48	276	208	129	364	153	148	$1.02

Extra Examples. Work Sets 70 and 71.

To Keep in Practice

A., S., and M. [W]

Copy and work. Watch the signs!

1. 4×91 364
2. $207 + 389$ 596
3. $679 - 452$ 227
4. 7×40 280
5. $863 - 739$ 124
6. 3×73 219
7. $608 - 99$ 509
8. 3×41 123
9. 4×62 248
10. $721 - 98$ 623
11. 2×24 48
12. $502 - 164$ 338
13. $169 - 72$ 97
14. 3×80 240
15. $97 + 732$ 829
16. 4×41 164
17. 4×82 328
18. 3×73 219
19. $2 \times 84¢$ \$1.68
20. $\$6.95 - \$0.78$ \$6.17
21. $3 \times 93¢$ \$2.79
22. $\$0.68 + \$6.74$ \$7.42
23. $\$6.36 - \$5.53$ \$0.83
24. $3 \times 81¢$ \$2.43

Copy each set of numerals in a column, and add.

25. 452, 24, 300, 126 902
26. 29, 389, 293, 95 806
27. \$0.40, \$0.04, \$1.00, \$0.79 \$2.23
28. \$1.69, \$0.84, \$3.08, \$2.64 \$8.25

◆ **Extra Examples.** Work Set 72.

Add? Subtract? Multiply?

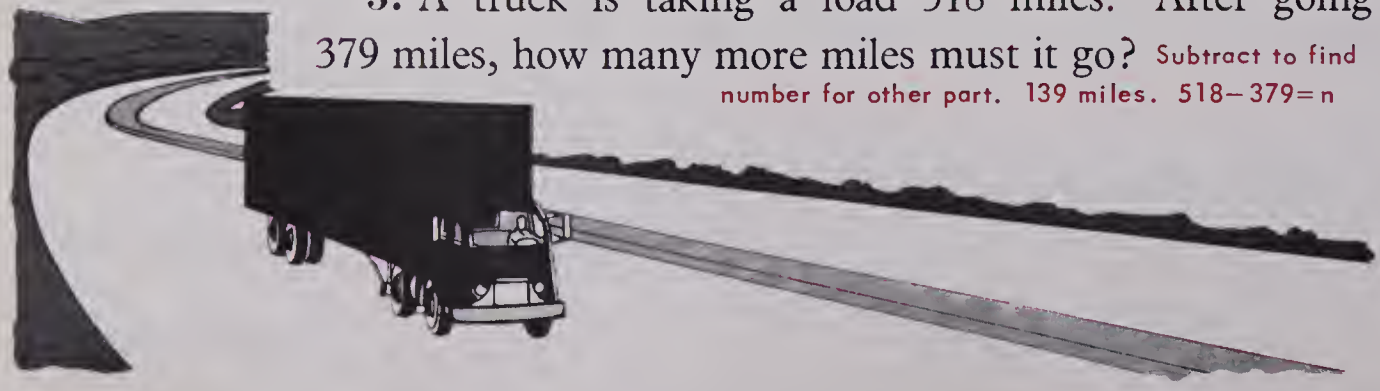
Writing n-examples [O]

Tell why you should add or subtract or multiply in each of problems 1–9.

1. A truck driver worked 152 hours in April and 139 hours in May. How many more hours did he work in April than in May? Subtract to find number for other part. 13 hours. $152 - 139 = n$

2. In the two months, the driver worked how many hours? Use numbers from Ex. 1. Add to find number for total. 291 hours. $152 + 139 = n$

3. A truck is taking a load 518 miles. After going 379 miles, how many more miles must it go? Subtract to find number for other part. 139 miles. $518 - 379 = n$



Teaching Pages 250 and 251

Pupil's Objectives

(a) To review and practice the addition, subtraction, and multiplication skills which have been learned; (b) to write appropriate *n*-sentences for addition, subtraction, and multiplication problems; and (c) to use symbols of equality and inequality in completing addition, subtraction, and multiplication sentences to make true sentences.

New Word

(page 251) *onions*

Background

No new skills are developed in the material on these pages. Perhaps you will want to use each of the three separate sections as a practice test in preparation for the end-of-chapter tests. An analysis and discussion of specific errors made on these practice tests will give pupils a basis for appropriate review or relearning. It will give you a basis for identifying common sources of weaknesses or difficulties in order to conduct small-group remedial lessons.

Pre-Book Lesson

If you use these exercises for practice tests, prepare your pupils as if for a regular testing situation. However, be sure to let them know that each is a practice test and that they are taking it to discover their sources of errors. It will help them to do as well as possible on the regular tests which are coming soon.

Use whatever form of review you consider necessary before having pupils proceed with the written work.

Using the Text Pages (page 250, top)

Slower learners may be assigned just part of Ex. 1–28 if the amount of work seems to be excessive for them.

Using the Text Pages (page 250, bottom; page 251, top)

First, ask different pupils to read each problem and then tell what operation should be used to solve the problem. Be sure to emphasize the *why* of using each particular operation.

Have pupils write the answer to each problem.

Then, have pupils write an *n*-sentence for each problem. It may be necessary for you to read each problem with the *slower learners*.

Using the Text Page (page 251, bottom)

For Ex. 1–12, the pupil is required to work two different examples. Then he must decide about the two answers. Finally, he must write a sentence using the correct symbol to describe the relation between the two numbers. Stress the importance of writing *true* sentences.

Individualizing Instruction

• *More capable children* may find a challenge in estimating the answers in the written work at the bottom of page 251. After they have made their estimates, they should write a sentence using the correct symbol of equality or inequality to make a true sentence. Then, they should check their estimates for accuracy by working each example completely.

• *All pupils* may fill in the frames to make this a true sentence:

One factor is represented by △, the other factor is represented by ○, and the product is represented by □.

△	○	□
6	3	18
2	7	14

• A review of fractions may be appropriate at this time in preparation for the end-of-chapter tests. Use "is greater than" or "is less than" to make each a true sentence.

$\frac{1}{2}$ ------ $\frac{2}{3}$

$\frac{3}{4}$ ------ $\frac{4}{8}$

$\frac{1}{3}$ ------ $\frac{1}{6}$

• You may find some *slower learners* who need review practice on addition and subtraction facts.

Have a *more capable pupil* direct them in the *Show the Answer* drill with facts. Or, you may prefer to have them fill in the frames to make this a *true* sentence. (See the top of the second column):

Let ◇ represent one addend and ○ represent the other addend of the sum represented by □.

◇	○	□
6	7	13

• *More capable children* may rewrite each of Ex. 1–12 on page 251 so that in each parentheses is a name for the same number.

NOTES

4. Jim Troy drove his truck 31 hours a week for 4 weeks. In the 4 weeks how many hours did he drive?

Sets equivalent, so multiply to find number for total. 124 hours. $4 \times 31 = n$

5. Jim had 100 boxes of oranges on his truck, large oranges in 39 boxes and small ones in $\underset{61}{?}$ boxes.

Subtract to find number for other part. $100 - 39 = n$

6. He had 4 bags of onions, each bag weighing 80 pounds. What was the total weight of the onions?

Sets equivalent, so multiply to find number for total. 320 bags. $4 \times 80 = n$

7. Jim Troy and Joe Clark are taking a load 710 miles. If Jim drives 328 miles, how far will Joe drive?

Subtract to find number for other part. 382 miles. $710 - 328 = n$

8. Joe stopped after driving 156 miles. Then he drove 106 miles more. How many miles was that?

Add to find number for total. 262 miles. $156 + 106 = n$

9. If Dick Stone drives his truck about 42 miles an hour, how far can he go in 4 hours?

Sets equivalent, so multiply to find number for total. 168 miles. $4 \times 42 = n$

[W]

Write your work for problems 1–9.

Write an *n*-example for each problem.

Greater Than? Less Than? Equal To?

Equalities and inequalities [W]

In Ex. 1, $4 \times 31 = 124$, and $66 + 66 = 132$. So, you would write, "$(4 \times 31) < (66 + 66)$." Work this way for Ex. 2–12, using $=$ or $>$ or $<$.

1. $(4 \times 31) \underset{<}{?} (66 + 66)$

2. $(800 - 465) \underset{<}{?} (9 \times 40)$

3. $(3 \times 60) \underset{>}{?} (4 \times 42)$

4. $(143 + 136) \underset{=}{?} (3 \times 93)$

5. $(8 \times 40) \underset{>}{?} (846 - 539)$

6. $(511 - 389) \underset{<}{?} (2 \times 94)$

7. $(98 + 107) \underset{>}{?} (5 \times 40)$

8. $(4 \times 72) \underset{<}{?} (510 - 122)$

9. $(3 \times 82) \underset{<}{?} (4 \times 92)$

10. $(505 - 216) \underset{>}{?} (3 \times 72)$

11. $(3 \times 81) \underset{<}{?} (583 - 290)$

12. $(6 \times 30) \underset{>}{?} (2 \times 82)$

Oral Practice

M. facts with factor 2, 3, 4

Say the products by rows and then by columns.

	a	b	c	d	e	f	g	h	i
1.	5	2	7	2	3	6	3	4	5
	2	4	3	8	5	2	9	6	4
	10	8	21	16	15	12	27	24	20
2.	6	2	4	4	9	8	2	3	4
	3	9	8	3	3	4	7	3	2
	18	18	32	12	27	32	14	9	8
3.	3	8	3	4	6	2	9	7	4
	7	2	6	9	4	6	2	2	7
	21	16	18	36	24	12	18	14	28
4.	7	3	5	8	4	2	9	3	4
	4	8	3	3	4	5	4	4	5
	28	24	15	24	16	10	36	12	20

Can You Solve Problems?

Problem Test 6

Write your work for problems 1–8.

1. The first time Jimmy bowled, he learned that he could roll the ball 20 times in a game. In 3 games, how many times could he roll the ball? 60 times

2. In his first game Jimmy's score was 48. In his second game it was 53. How many more points did Jimmy make in the second game than in the first? 5 points

3. The highest score one can make in a game is 300. How much less than this was Jimmy's score of 53? 247

4. It cost 20¢ for Jimmy to bowl a game. He bowled 4 games. How much did he have to pay for them? 80¢

Teaching Pages 252, 253, 254, and 255

Pupil's Objectives

(a) To have practice in saying the products for M. facts in which one factor is 2, 3, or 4; and (b) to take the four end-of-chapter tests.

Using the Text Page (page 252, top)

You may wish to use the oral practice as a "breather" between written tests.

Using the Text Pages (page 252, bottom; page 253, top)

An item analysis of Problem Test 6 will provide you with evidence as to which items are missed most frequently. If a sizable group of children in your class have difficulty with any particular item or items, then you have a responsibility to provide them with additional help in the particular skill involved.

Using the Text Page (page 253, bottom)

As in all previous diagnostic tests, if a pupil misses 2 or more items in any particular row, then there is reason to believe that he needs additional help with the skill measured in that row. Individual interviews with pupils on the skill concerned will give you a better basis for providing the appropriate remedial help.

Using the Text Pages (page 254; page 255, top)

After your pupils have written the answers to the Test of Information and Meaning, be sure to discuss each item with them. Have them tell the *why* of their response.

Using the Text Page (page 255, bottom)

This test, of course, will show whether your pupils can make use of certain skills.

Individualizing Instruction

- *All pupils* may write an *n*-sentence for each problem on pages 252 and 253.
- Use the tables below to find the per-cent scores for the individual record cards.

Table of Per Cents for Chapter 6 Scores

Problem Test		Computation Test					
Score	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*
1	13	1	5	8	38	15	71
2	25	2	10	9	43	16	76
3	38	3	14	10	48	17	81
4	50	4	19	11	52	18	86
5	63	5	24	12	57	19	91
6	75	6	29	13	62	20	96
7	88	7	33	14	67	21	100
8	100						

Note: For suggestions to accompany the end-of-chapter tests, see Teacher's Pages 253–255.

Suggestions for Material to Accompany End-of-Chapter 6 Tests

Alternate Uses of Pages in Chapter 6

Page 229. For each of Ex. 1–15, write 3 other mathematical sentences which express the same addends-sum relationship. For Ex. 1, the child would write:

$$300 + n = 807$$
$$807 - 300 = n$$
$$807 - n = 300$$

Page 230 (top); 242 (bottom); (244 top); 249 (bottom); 252 (top). The examples on these pages are organized very nicely for timed drill. The drill may be either oral or written. If it is written, have pupils write answers only on folded paper. The emphasis on time must follow understanding and partial mastery of the M. facts.

Page 233 (bottom). For Ex. 1–3, change each sentence which indicates A. to indicate S.; each sentence which indicates S. to indicate A. For Ex. 1a, change to: If $n = 285$, then $n - 166 =$ _ ? _. Then find the answers.

Page 236 (bottom). Change each sentence in any way you wish so that inserting = will make a *true* sentence. In this kind of activity the child should see that several possibilities are open to him. Perhaps you will want to give variety to this activity by asking children to change each sentence in two or even three different ways. Let *more capable children* check the work of others.

Page 242 (bottom). Ask pupils to add 3 to each of the products in column a, 5 to the products in column b, 8 to the products in column c, 17 to the products in column d, and so on. This could be either an oral or a written activity. If oral, the child would say, "seven times four plus three equals thirty-one." Or, he might say, "Seven fours equals twenty-eight, plus three equals thirty-one."

Page 247 (middle). Pupils may merely write the answers to the sentences in Ex. 1–10. For Ex. 1, write $(4 \times 4) + 4 = 20$. This would be an appropriate written assignment for *slower learners* after they have greater mastery of the M. facts. Later, they may do the matching part.

Page 251 (bottom). There are several possibilities in changing the nature of this written work to give pupils additional practice. Any time you adapt material, try to provide opportunity for the child to be creative in his thinking.

In many previous activities of this type it has been suggested that you have pupils rewrite each sentence in some way so that inserting = will make a *true* sentence. This is still a good alternative. Other alternatives are to ask pupils to change just one numeral in each sentence so that using the symbol > will make each a *true* sentence. The same could be done with the symbol <.

Supplementary Activities

• Replace the frames with numerals to show two factors and a product:

a. $2 \times 7 = (2 \times 5) + (2 \times 2)$
$= \square + \triangle = \bigcirc$

b. $4 \times 9 = (4 \times 3) + (4 \times 6)$
$= \square + \triangle = \bigcirc$

c. $5 \times 8 = (5 \times 4) + (5 \times 4)$
$= \square + \square = \bigcirc$

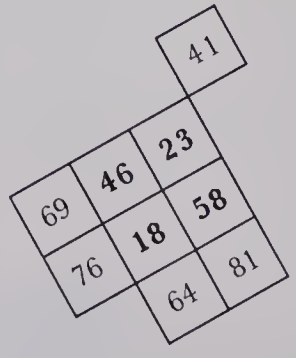
d. $3 \times 32 = (3 \times 30) + (3 \times 2)$
$= \square + \triangle = \bigcirc$

e. $4 \times 82 = (4 \times 80) + (4 \times 2)$
$= \square + \triangle = \bigcirc$

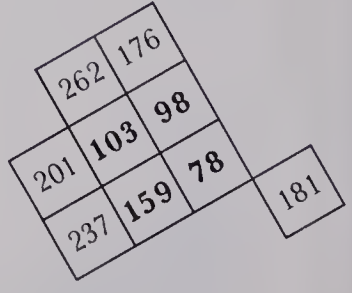
• *More capable children* should be able to work more difficult M. examples on the basis of the M. facts they already know.

• Have pupils draw a grid like the one below. The operation is addition. After pupils have completed some grids like *A* they may try completing some like *B*.

A



B



5. In his four games Jimmy made scores of 60, 64, 58, and 59 points. Find the total points for these games. 241 points

6. Jimmy's score was 7 for his first turn in a game. If he made this score in 4 turns, how many points would he make in 4 turns? 28 points

7. Jimmy's father scored 176 and 185 in two games. In the two games together how many points did he make? 361 points

8. His friend made 368 points in two games. If he made 189 in one game, what was his score in the other? 179 points

Do You Make Mistakes?

Diagnostic Test 6

Write answers on folded paper. Check carefully.

	a	b	c	d	e	Study Pages	Practice Sets
1.	9 × 3 = 27	6 × 4 = 24	3 × 8 = 24	4 × 7 = 28	8 × 4 = 32	221–227, 239–242	67, 68
2.	4 × 9 = 36	7 × 4 = 28	3 × 6 = 18	9 × 4 = 36	1 × 5 = 5		
3.	58 + 7 + 4 + 3 + 9 = 81	241 + 65 + 162 + 89 = 557	243 + 96 + 408 + 85 = 832	6 + 58 + 7 + 9 + 6 = 86	68 + 384 + 108 + 44 = 604	243	69
4.	21 × 3 = 63	31 × 7 = 217	10 × 6 = 60	73 × 3 = 219	52 × 4 = 208	246–249	70, 71

Do You Understand?

Test of Information and Meaning 6

Write answers for Ex. 1–8.

1. If you give \$1.00 for something that costs 58¢, you would get in change, with the fewest coins, _?_ 2 cents, _?_ 1 nickels, _?_ 1 dimes, and _?_ 1 quarters.

2. Which is the freezing point? 32° 32° 10° 15°

3. $3 \times 23 = 69$. Which numbers are the factors? 3, 23

4. Which digit in $\frac{3}{4}$ tells how many parts of the same size there are in a whole? 4

5. Which is the smallest? cup quart cup pint

6. In $2 \times 54 = 108$, is the product 54 or 108? 108

7. In $576 - 386$ do you rename to show more ones? No

8. If you are given $\frac{5}{6}$ of an orange, are you given 5 parts or 6 parts of the whole? 5

Write an *n*-example for each of these on page 252:

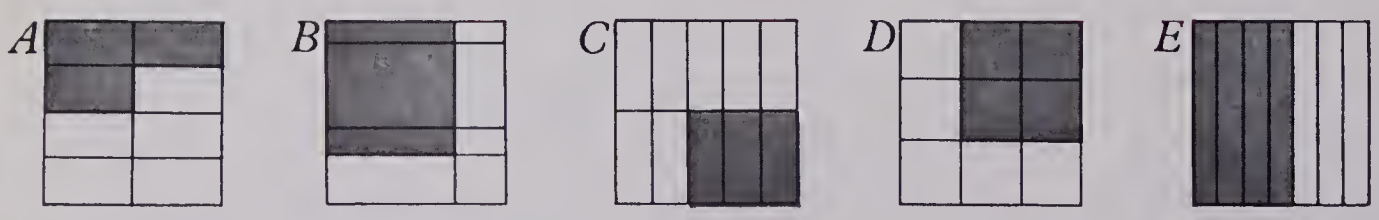
9. Problem 1 $3 \times 20 = n$ 10. Problem 2 $53 - 48 = n$ 11. Problem 3 $300 - 53 = n$

These fraction numbers are for parts of the same whole. Show each set of numbers from least to greatest.

12. $\frac{1}{2}, \frac{1}{7}, \frac{1}{10}, \frac{1}{6}$ $\frac{1}{10}, \frac{1}{7}, \frac{1}{6}, \frac{1}{2}$ 13. $\frac{1}{5}, \frac{1}{8}, \frac{1}{3}, \frac{1}{9}$ $\frac{1}{9}, \frac{1}{8}, \frac{1}{5}, \frac{1}{3}$

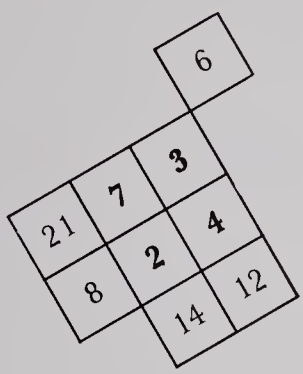
Write the letter of the picture below in which the colored part of the inside shows

14. $\frac{4}{9}$. D 15. $\frac{4}{7}$. E 16. $\frac{3}{8}$. A 17. $\frac{3}{10}$. C

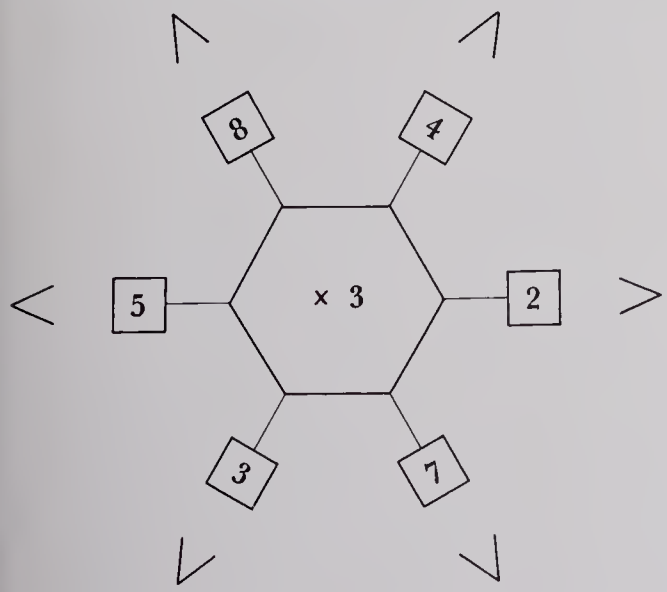


In grid *A*, each of the boxes to be filled is to show a sum. In grid *B*, each of the boxes to be filled is to show a sum but the numbers shown are greater. The numerals in gray should not be shown on the grid. They are given to help you guide the pupils.

• A variation of the above activity is shown below. Here pupils are to find products. They are not to try to add or multiply the products obtained.



• Duplicate a diagram like the one below on construction paper (about 6″ × 6″) or tagboard. Give each child several copies of the diagram. In the model of a hexagon (the shape in the center of the diagram) the child is to show an operation (for example ×) and a number. In each of the boxes show a number. If the operation is multiplication, find the results of multiplying the numbers shown in the boxes by the number shown in the hexagon. The operation may be addition, subtraction, or multiplication.



Pupils may write the correct answer for each operation near the ✓ mark.

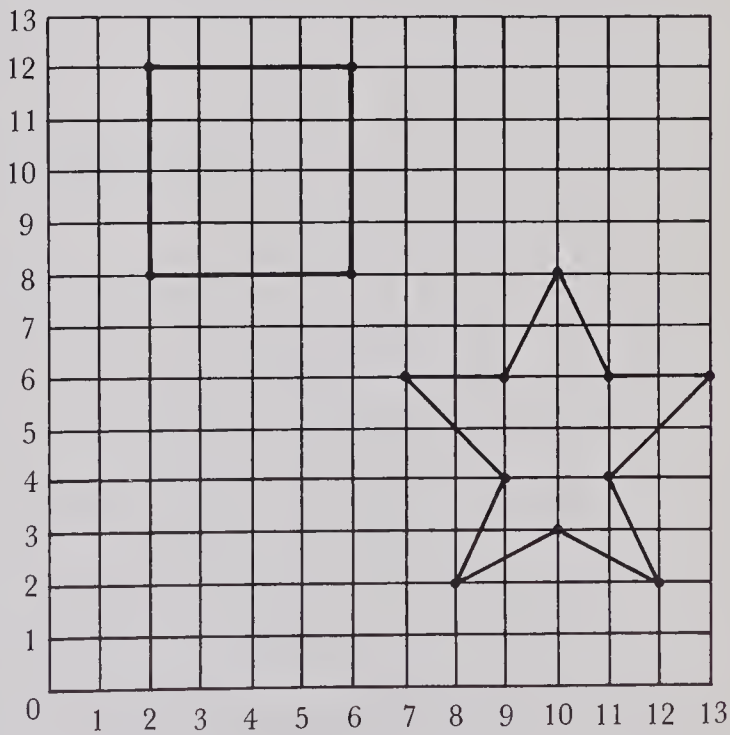
Coordinate Geometry

In previous activities the emphasis has been on the location of points by coordinate systems.

An activity that should tie in very nicely with the study of the school neighborhood or the local community would be to use a coordinate system in the construction of a simple map of the neighborhood (a city part, and so on). On the map, each pupil could identify by coordinates his home, the school, fire station, and so on. Ask questions like, "Who lives over 6 and up 4?"

A second activity in locating points which children will enjoy is making pictures on graph paper.

If $\frac{1}{2}$-inch graph paper is available, number the horizontal and vertical axes (number lines), as shown below. On the coordinate system below, two shapes were drawn from points that were plotted. To do this, plot the first point that is given. To plot the point (2,12), go over 2 and up 12. Plot the next point and then join the two points. Plot the next point and join it with the previous one. Continue this procedure until all points have been plotted.



The set of points for the picture of the square is: (2,12) (2,8) (6,8) (6,12)

The set of points for the star is: (10,8) (9,6) (7,6) (9,4) (8,2) (10,3) (12,2) (11,4) (13,6) (11,6)

Coordinates may be given for pictures of shapes, letters, animals, cars, and so on.

Teacher's Page 254

See Teacher's Page 252 for suggestions about the end-of-chapter tests.

NOTES

Write "Yes" or "No" for each of Ex. 18–26.

18. Does every M. fact have another M. fact to make a pair? No

19. If $n = 40$, does $80 - n = 40$? Yes

20. Is $(4 \times 3) + 6$ a name for the number 18? Yes

21. Is $\frac{1}{4}$ of a whole always the same size as $\frac{1}{4}$ of any other whole? No

22. Do you add the factors of a number to find the number? No

23. Are 4 quarts of liquid the same as 8 pints? Yes

24. $4 \times 42 = 168$ Does $42 \times 4 = 168$? Yes

25. In counting by sixths, does $\frac{4}{6}$ come just before $\frac{3}{6}$? Yes

26. Can we write, "606 = 59 tens and 16 ones"? Yes

How Well Can You Compute?

Computation Test 6

Copy in columns, and work. Check carefully.

1. 3×63 189

2. $700 - 161$ 539

3. $730 + 89$ 819

4. $139 - 67$ 72

5. $488 - 368$ 120

6. $124 + 694$ 818

7. $609 - 535$ 74

8. $571 - 248$ 323

9. 8×21 168

10. $900 - 299$ 601

11. $4 \times 32¢$ $1.28

12. $\$1.57 - \$0.88$ $0.69

13. $\$3.09 + \$3.67$ $6.76

14. $\$8.08 - \$6.54$ $1.54

15. $3 \times 92¢$ $2.76

16. 8, 6, 68, 8, 5 95

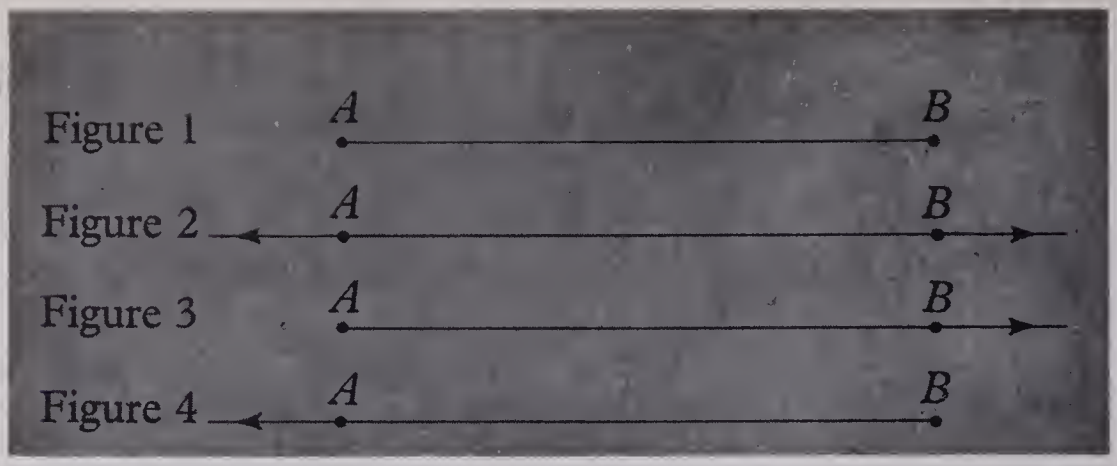
17. 75, 389, 34, 275 773

18. 500, 76, 235, 98 909

19. \$0.33, \$0.97, \$4.44, \$3.72 $9.46

20. 7¢, 6¢, 5¢, 39¢, 9¢ 66¢

21. \$1.09, \$0.72, \$0.40, \$6.99 $9.20



* Line Segments, Lines, and Rays

Geometry [O]

1. In the set of figures above, Figure 1 is a picture of line segment AB. Another name for line segment AB is line segment _?_ (BA). It has end points A and _?_ (B).

Think of going on and on beyond the end points of line segment AB, as in Figure 2 above. Figure 2 is a picture of line AB, or line BA.

2. Does line AB have end points? (No) If not, explain. (Line AB goes on and on.)

3. Why is line segment AB in Figure 2 a part of line AB? It has points A and B as its end points.

Now think of going on and on beyond only one of the end points of a line segment, as in Figure 3 and in Figure 4 above. Each of these pictures shows a ray.

4. How many end points does a ray have?

5. The way we name a ray tells which point is the end point. Ray AB is pictured in Figure 3. Was the end point given first or last in its name? First

6. Tell how to name the ray pictured in Figure 4. BA

7. In Figure 3, is line segment AB a part of ray AB? Yes
Is line segment AB a part of ray BA in Figure 4? Yes

* Now, more building of ideas on a developmental basis so that these ideas will be available for later formal work in geometry! Again, emphasize that pupils are not seeing lines, line segments, rays, and angles—only pictures of them.

Overview—Chapter 7

• In Chapter 7, work in multiplication is advanced to multiplying tens and ones or hundreds, tens, and ones, renaming the ones of the product and remembering one ten or more. Pupils also learn the multiplication facts with 1 as a factor (making use of the identity element) and with 0 as a factor. Again, the Commutative Property of Multiplication is put to use when pupils learn to check multiplication by multiplying when showing the factors in changed order.

Your pupils take one more large step in arithmetic for grade 3—learning the meaning of division, recognizing the two situations requiring division, and learning division tables for 2's and for 2 and for 3's and for 3.

Pupils recognize that dividing by a number (a factor) is the inverse of multiplying by that same number. They begin to make use of the factors-product relationship and, from use of arrays of dots, to think of sets of multiplication and division facts having the same product and factors.

• **Problem-Solving.** Problem-solving is advanced now to include division in the operations to be differentiated when deciding which operation is needed.

• **Maintenance.** The work with n in this chapter deals with the factors-product relationship and, because they know two factors in either multiplication or division, pupils are to find the number for n which stands for the product in both multiplication and division.

Work with geometry, purposely kept simple and intuitive in maintenance, is extended now to deal with lines, line segments, rays, and angles.

• **Enrichment.** The enrichment provides work with three unlike addends for which multiplication examples are to be written in order to find the number for the total quickly. Another exercise makes use of two multiplication examples in parentheses, such that in each case one factor shown in one pair of parentheses is like one factor shown in the other. Pupils, by considering the unlike factors, compare the two numbers represented in parentheses.

• **Testing.** The usual maintenance, mid-chapter review, and end-of-chapter tests are provided for purposes of evaluation.

Teaching Page 256

Pupil's Objective

To learn more about some of the beginning ideas in geometry: lines, line segments, and rays.

New Word: *beyond*

Background

See Background section on Teaching Page 257.

Teacher's Preparation

Have unlined paper and a ruler (sometimes called a straightedge) available for drawing pictures of lines and line segments.

Pre-Book Lesson

• Review the idea of a point. Have different pupils locate (and represent) a point with a dot.

• Review the idea of paths which join points. On the board have pupils show a path between two points. Bring out the idea that there are many different paths between two points. Then, discuss and illustrate the special path.

• Elicit from children and then illustrate the difference between a line and a line segment. (See Teaching Pages 120 and 121.)

• Distribute unlined paper and rulers. Direct pupils to draw a picture of a line on the paper and label two points J and K. Make sure pupils put arrowheads near the ends of the model of the line to indicate that the line is not limited by end points, but instead goes on indefinitely.

• Have pupils draw pictures of line segments and label the end points.

Using the Text Page

As you discuss Ex. 1–7 with your pupils, have them use the unlined paper and rulers to make pictures of lines, line segments, and rays.

Pupil's Objectives

(a) To understand the concept of an angle in general, and a right angle in particular; and (b) to learn to name angles in terms of the rays which form them.

New Words

formed, meet

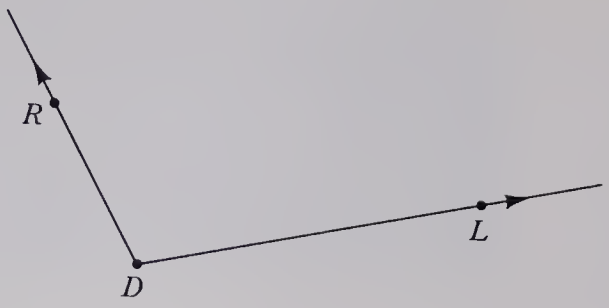
Background for Pages 256 and 257

In previous lessons pupils have learned about points, lines, simple closed curves, and polygons.

On page 256 the pupil is to learn more about line segments. A line segment is part of a line with two points of the line as its end points. A line segment is named by naming these points. The segment pictured in Figure 1 may be named line segment AB or line segment BA since its pair of end points are A and B. In the illustration below, different line segments may be named by different pairs of points. All labeled points are in line AE. Some of the line segments are line segment AB, AC, AD, AE, BC, BD, and so on.

Each of Figures 3 and 4 is a picture of a ray. The ray pictured in Figure 3 begins at point A and continues indefinitely to the right, while in Figure 4, the ray begins at point B and continues indefinitely to the left. Notice that a ray has only one end point. A ray is named by listing the end point first. In Figure 4, the ray is named ray BA to indicate that the end point is B and that the ray contains point B and all the points in line BA on the same side of point B as point A. In Figure 3, the ray is named ray AB to denote A as the end point and B as a point in the ray.

An understanding and knowledge of a ray is necessary before thinking about angles. An angle can be thought of as a geometric figure formed by two rays which are not part of the same line but which have a common end point.



In the illustration above, point D is shown to be the common end point of ray DR and ray DL. The angle includes all the points of both rays. Angles may be named by points in each ray and the common end point (vertex). The angle shown above may be named angle LDR or angle RDL. Notice that the vertex, point D, is listed in the middle. Sometimes, only the letter that names the vertex is used to name the angle.

The term *right angle* is introduced in this lesson to mean two rays that make a square corner at the point where they start.

Pre-Book Lesson

- Briefly discuss the difference between a line segment and a ray.
- Direct pupils to picture a ray and label the end point A and a point in the ray B. Then, have them picture another ray using point A as the end point, and label a point in the ray C. In your discussion bring out the idea of two rays having a common end point.
- Introduce the term *angle*. Be sure to develop the idea that the angle is the set of points in the two rays and not the portion between the rays.

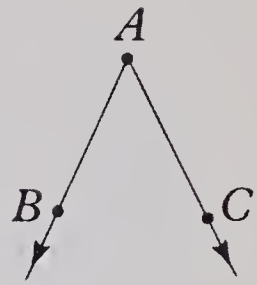
Using the Text Page

- Have pupils refer to the angles pictured in the book as well as picture their own as you guide the discussion of Ex. 1–7. In Ex. 5 a variety of answers may be given, for an angle has more than one name.
- When you have pupils show a right angle, it would be a good idea to have them use a card, book, or something similar which has a square corner. Otherwise, the proper impression of a *square corner* may not be created.

* Bring out that the rays extend on and on so the "opening," not the lengths of the rays, determines the angle.

Angles

Geometry: naming angles; right angles and others [O]

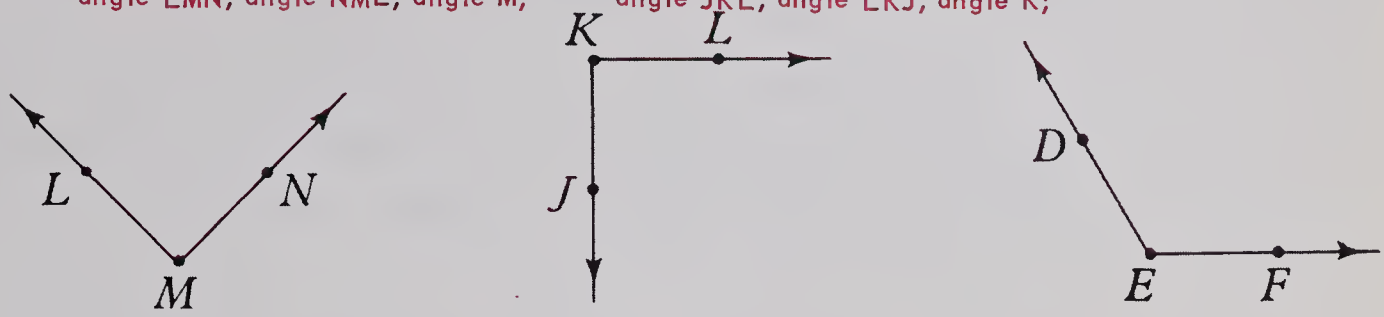


* At the right is a picture of an angle which we may name angle *BAC* or angle *CAB* or angle *A*.

** 1. Angle *A* is formed by two rays, ray AB and ray AC, with the same end point, point A.

2. Name in three ways each angle pictured below.

angle LMN, angle NML, angle M; angle JKL, angle LKJ, angle K;



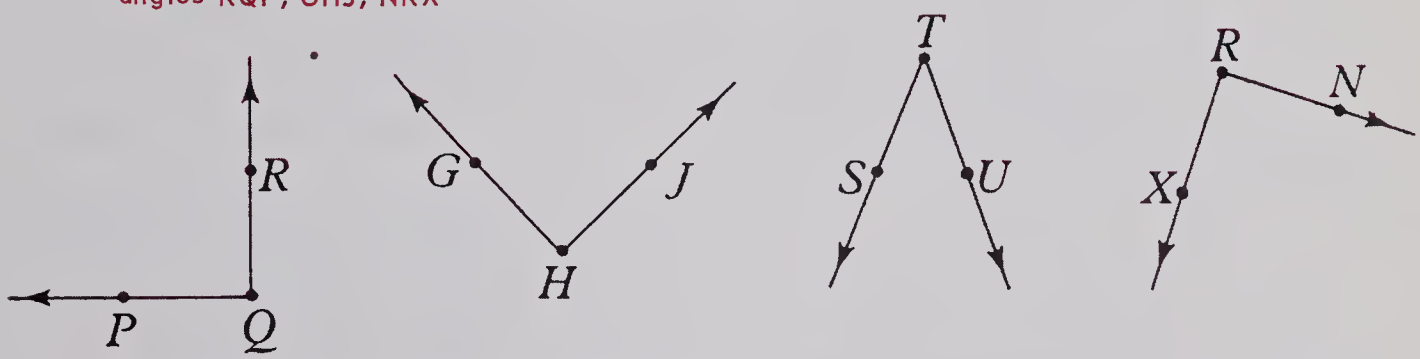
angle DEF, angle FED, angle E

3. Name the two rays that form each angle and the end point where the two rays meet. rays ML and MN, end point M; rays KJ and KL, end point K; rays ED and EF, end point E

4. In the picture of angle *LMN*, do the rays make a square corner where they meet at point *M*? Yes Test the corner. We call angle *LMN* a right angle.

5. Name the right angles pictured below.

angles RQP, GHJ, NRX



6. On the board draw a picture of an angle that is not a right angle. Name it angle *CDE*.

7. Now show a right angle and name it angle *FGH*.

** Make sure that pupils realize that the term angle does not include the space lying within the rays nor any space outside.

* The idea of a "remembered 1 ten" helps avoid the common error of adding 1 ten to the number of tens to be multiplied.

Multiplication with Renaming

Using the Commutative and Distributive Properties [O]

A
24
4
816
Wrong

1. Jack saw a box marked 24 cups. He wanted to know how many cups there were in 4 boxes. His father gave him the number sentence, $4 \times 24 = n$. Jack did the work as in Ex. A.

More than 1 ten must be shown

His father said that his answer was wrong. Why? Would 4×2 tens be 8 tens or 8 hundreds? 8 tens

2. Jack began again. $24 = 20 + 4$ or $4 + 20$. So he made two examples (Ex. B and Ex. C) and added the products (Ex. D). Explain his work in Ex. B–D.

B	C	D
4	20	16
×4	×4	+80
16	80	96

E	
Tens	Ones
2	4
	4
9	6

3. There is a much shorter way to multiply 24 by 4. Study the work in Ex. E. Multiply ones, then tens.

Ones: $4 \times 4 = 16$ We rename 16 to be 1 ten and 6 ones, so we write "6" in _?_ (one's) place and remember 1 ten.

* Tens: $4 \times 2 = 8$ Add 8 tens and the remembered 1 ten. $8 + 1 = 9$ Write "9" in _?_ (ten's) place for the product.

F	G	H
2		
47	47	47
47	×3	×3
+47	181	141
141	Wrong	

4. Ex. F. When do you add the 2 tens shown by the carried 2? First. 4+2=6; 6+4=10; 10+4=14

5. Ex. G. Why is this wrong? The 2 tens were added before multiplying.

6. Ex. H. Why is 141 the correct product? In multiplication, when do we add the remembered tens? After multiplying tens

3×7=21; 1 is shown in one's place; 2 is remembered; 3×4=12, 12+2=14

3×4=12, 12+2=14

Teaching Pages 258 and 259

Pupil's Objectives

(a) To extend understanding of the distributive property; and (b) to learn to multiply tens and ones when it is necessary to rename the product of the ones as tens and ones and remember the tens.

Background

Types of examples included on these pages are illustrated below. Note that, in some examples, after the product of the ones is renamed, 1 ten must be remembered and in others, 2 tens. In each multiplication, only the facts learned up to this point are involved.

53	32	67	43
×4	×7	×3	×8

The concept of renaming numbers is an important principle widely utilized in higher forms of mathematics. The renaming process together with the distributive property are two mathematical principles underlying the multiplication algorithm. For example, when we multiply 4 × 26 we rename 26 as 20 + 6, distribute the multiplication over the 20 and the 6, and add the partial products.

4 × 26	
4 × (20 + 6)	(renaming process)
(4 × 20) + (4 × 6)	(distributive property)
80 + 24	(adding partial products,
104	(product)

Ordinarily we commute the tens and ones, multiply the ones first, then the tens, and finally add the two partial products. Once children understand this basic relationship, they will be able to apply their knowledge to multiplying with 3-place numerals.

Other ways for finding the product for such an example follow:

26 = 2 tens 6 ones	26 = 20 + 6
×4	×4
×4	×4
8 tens 24 ones	80 + 24
10 tens 4 ones	104
104	

26
×4
24 (4 × 6 ones)
80 (4 × 2 tens)
104 (sum of partial products)

The algorithm involving remembering tens is merely an efficient form for showing what is illustrated in each example illustrated. When multiplication of the ones gives a product greater than 9, that product is renamed to be tens and ones. The ones are recorded and the tens remembered. After multiplying tens, the remembered tens are added. It is this process of renaming, recording, and remembering that is introduced and stressed in this lesson.

Pre-Book Lesson

• Review briefly the basic idea of multiplication as the joining of several equivalent sets and finding the number for the total set. Pose appropriate problems such as, "If Fred had a set of two pictures, Kay had a set of two pictures, and Ron had a set of two pictures, how many pictures did they have in all?"

Pose a problem involving *dozens*. Have pupils determine how many would be in 3 or 4 dozens. Do not present a problem involving carrying of ones.

• Review the idea of renaming numbers and the use of expanded notation to show the renaming. Show expanded notation for various 2-place numerals. Use the expanded notation illustrated below:

48 = 4 tens 8 ones
48 = (40 + 8)

• Pose and demonstrate a problem like the following:

There are three stacks of books on the shelf. If there are 15 books in each stack, how many books are there in all?

Let pupils suggest as many different ways as they can think of for finding the number for the total set.

• As different pupils' suggestions for finding a solution are made, use their responses to eventually arrive at a solution like the one below.

$$\begin{array}{rl} 15 = 1 \text{ ten } 5 \text{ ones} & \\ \times 3 \quad\quad\quad 3 & \\ \hline 3 \text{ tens } 15 \text{ ones} & \\ 4 \text{ tens } 5 \text{ ones} & \\ 45 & \end{array}$$

• Guide pupils to work the shorter way using remembering. (The pocket chart,* line abacus,* and so on will help them to *see* the process):

a. Multiply ones: $3 \times 5 = 15$. $15 = 1$ ten and 5 ones. Write "5" in one's place. Remember *1 ten.*

b. Multiply *tens*: $3 \times 1 = 3$. Add the remembered 1 ten to the 3 tens. Write 4 in ten's place. The answer is 45.

Using the Text Pages

• Use the chalkboard, pocket chart, or any materials deemed useful in discussing Ex. 1–6. Refer to any of the material developed in the Pre-Book Lesson.

Stress the distributive property. It may be helpful to review an example like "three 7's = ?" Seven may be renamed in different ways to help find the product. We may think of three 5's plus three 2's or, three 4's plus three 3's.

Adapt this type of thinking to working the example "three 15's = ?" Have children tell of the ways 15 may be renamed to find the product. We think most often of three 10's plus three 5's.

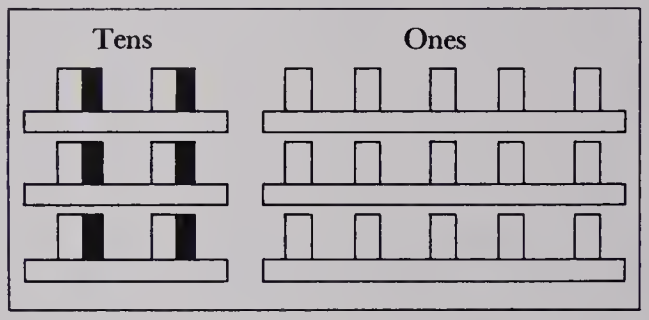
* See 21 and 10, page xix.

• Guide pupils to generalize in their own words about remembering one or more tens in multiplication. When adding, the carried digit may be used first in adding in ten's column. In multiplying, the child must multiply in the ten's column before adding the remembered number. This is a frequent source of error, for many pupils make the mistake of adding the remembered tens before multiplying.

• With all the preparation you will have given your pupils, they now should have little trouble with Ex. 7–11 and rows 12–14. Make sure that they understand what to do when a product is shown as in Ex. 11.

Individualizing Instruction

• *Slower learners* may need additional work with objects to form the correct understandings. Do not let them form incorrect habits of procedure. Use the pocket chart or a similar type teaching aid as shown below:



• *More capable children* may write the missing numerals in each example.

28	?5	23	31	5?
×3	×2	×8	×?	×3
?4	13?	1?4	??7	1?7

NOTES

For Ex. 7 below, you work this way:

Ones: *Think*, "3 × 9 = 27." 27 = 2 tens and 7 ones. So write "7" in one's place and remember 2 for 2 tens.

Tens: *Think*, "3 × 4 = 12, plus the remembered 2 equals 14." Write "14" for 14 tens, or 1 hundred and 4 tens.

Tell what to think in the multiplications for Ex. 8–11.

7.	8.	9.	10.	11.
49	24	58	$0.23	79¢
× 3	× 3	× 2	× 8	× 3
147	72	116	$1.84	237¢, or $2.37

If the product for the ones is greater than 9, write the one's digit and remember the tens.

Find the product for the tens and add the remembered tens to that product.

In row 12, when do you rename and remember 1 ten? Ex. a, b, e, f, g

	a	b	c	d	e	f	g
12.	36	32	83	23	32	24¢	$0.12
	× 3	× 6	× 2	× 3	× 5	× 4	× 9
	108	192	166	69	160	96¢	$1.08

What are the missing digits?

13.	45	15	32	78	23	19¢	$0.23
	× 2	× 4	× 7	× 4	× 5	× 2	× 6
	9 ?0	6 ?0	2 ?24	3?2 (1)	1?5 (1)	3 ?8¢	$?.?8 (1 3)

Which products are shown wrong? Why? See below.

14.	69	53	74	24	89	17¢	$0.34
	2	4	3	7	3	4	8
	128	212	212	168	247	58¢	$2.52

Ex. a, c, e, f, g: didn't add remembered tens.

Copy on the board the examples in row 12. Multiply.

Try These

[W]

Show products on folded paper.

	a	b	c	d	e	f	g	h
1.	42	34	24	43	86	27	25¢	$0.32
	8	7	5	6	4	4	3	8
	336	238	120	258	344	108	75¢	$2.56
2.	18	15	32	23	96	35	43¢	$0.76
	3	2	7	5	3	4	7	2
	54	30	224	115	288	140	$3.01	$1.52
3.	87	23	38	24	87	69	24¢	$0.54
	3	9	4	6	2	2	9	4
	261	207	152	144	174	138	$2.16	$2.16

Be careful! Sometimes there are no remembered tens.

4.	97	23	64	41	42	32	48¢	$0.95
	4	8	3	4	2	6	3	2
	388	184	192	164	84	192	$1.44	$1.90
5.	43	97	82	42	34	14	82¢	$0.43
	8	3	4	7	9	8	2	5
	344	291	328	294	306	112	$1.64	$2.15

Extra Examples. Work Set 73.

●**Extra Activity.** Work Set 103.

Working with *n*

A., S., M. [W]

Copy, and work.

1. $379 + 85 = n$ 464
2. $400 - n = 79$ 321
3. $3 \times 79 = n$ 237
4. $317 - 288 = n$ 29
5. $9 \times 42 = n$ 378
6. $4 \times 56 = n$ 224
7. $652 - 49 = n$ 603
8. $7 \times 32 = n$ 224
9. $450 + 448 = n$ 898
10. $4 \times 74 = n$ 296
11. $4 \times \$0.98 = \$n$ $3.92
12. $\$1.17 - \$0.18 = \$n$ $0.99
13. $\$2.58 + \$n = \$5.33$ $2.75
14. $3 \times \$0.86 = \$n$ $2.58
15. $\$7.00 - \$n = \$0.06$ $6.94
16. 536, 45, 8, 207 796
17. $2.07, $1.68, $0.97, $3.45 $8.17

Teaching Page 260

Pupil's Objectives

(a) To have practice in multiplying tens and ones with and without remembering; and (b) to have practice in finding the solution to addition, subtraction, and multiplication sentences.

Background

There are two types of mistakes children commonly make when renaming and remembering in multiplication is required. Frequently, pupils add the remembered tens to the number shown in ten's place before multiplying tens. Or, they multiply the remembered tens instead of the number of tens shown in ten's place and then they add the number of tens shown in ten's place. Be alert to the procedures used by each child in order to forestall or correct these errors.

Teacher's Preparation

Have paper ready to use for the folded-paper exercise at the top of page 260.

Pre-Book Lesson

Quickly review each of the following types of multiplication with 2-place numerals:

a. no remembering
b. remembering 1 ten only
c. remembering 2 or 3 tens

Be sure to illustrate the use of the distributive property in connection with each type of multiplication. Stress expanded notation as you show the renaming for the number to be multiplied and for the product of the ones.

Using the Text Page

- Show only the products for rows 1–5.
- Most pupils should be able to proceed with Ex. 1–17 independently. *Slower learners* may need some brief review of the inverse relationship between the addition and subtraction of a number and the interpretation of sentences involving A. and S.

Individualizing Instruction

- If *slower learners* cannot show products on folded paper, permit them to use a pocket chart to help *see* the process. Encourage them to use one of the forms of expanded notation to show multiplication of ones and tens.

a. $\begin{array}{r} 46 \\ \times 3 \\ \hline \end{array}$ = 4 tens 6 ones
$\times 3$
12 tens 18 ones
13 tens 8 ones
138

b. $\begin{array}{r} 46 \\ \times 3 \\ \hline \end{array}$ = $(40 + 6)$
$\times 3$
$(120 + 18)$, or 138

- Use Extra Activities Set 103 for *more capable children.*
- Use Extra Examples Set 73 as appropriate.

Pupil's Objectives

(a) To learn by principle the M. facts in which 0 is a factor; (b) to extend understanding of the Commutative Property of Multiplication; and (c) to have practice in finding products for M. facts mentally, while working in the motivating setting of inequalities and equalities.

Background

Zero often indicates the absence of something. It also indicates a point on a number line.

A number may be added to 0 and so increase the original number. This may be illustrated on a number line by starting at *0* and moving to the right the distance desired. However, for any multiplication having 0 as either factor, the product is 0. This can be best understood through trying to join any number of sets with 0 members or trying to join 0 equivalent sets. In each case, the resulting set would be the *empty set* (having 0 members). Understanding of these ideas best proceeds from concrete experiences.

Teacher's Preparation

Provide each pupil with some objects which he will use in the Pre-Book Lesson. It would be a good idea to have your permanent number line* ready for use in connection with the Pre-Book Lesson as well as when using the text pages. Otherwise, draw a picture of a number line on the chalkboard with the points for 0 through 10 labeled.

Pre-Book Lesson

• Engage pupils in a discussion about keeping score in games they play. Be sure to focus, in particular, on the use of "0" as a record of no score.

• Ask pupils to show 4 objects on their desks. Remove one at a time and each time one is removed have different pupils tell how many objects are left. After all objects have been removed, stress zero as the number for the set.

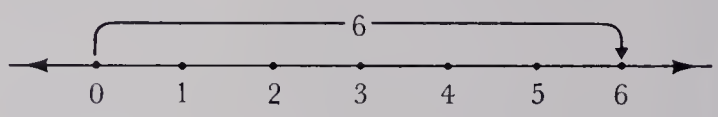
* See 6, page xix.

• Say, "If we have zero objects to begin with and we place six objects on the desk, how many objects will there be in all?"

Have children use their objects to demonstrate the situation. Then, ask a child to write a sentence on the board that will describe the demonstration.

Guide pupils to see and state in their own words that it is possible to add to zero and that the sum will be greater than zero.

• Use a number-line picture to illustrate the idea of beginning at zero and moving out to 6 with an arrow.



Write a sentence to go with the illustration.

Using the Text Page

• The Pre-Book Lesson should be adequate background for discussing Ex. 1–3. Ask children to show on the chalkboard a M. fact for the example "three 0's = ?" Do the same for Ex. 2 and 3.

• As children are asked to say the products in Ex. 4–6, be sure to discuss and emphasize the Commutative Property of Multiplication.

• Ask children to state a generalization about M. facts in which 0 is a factor. (This is a unique property of zero when multiplying.)

• The work at the bottom of the page provides an especially good setting for finding products. If pupils need to write this work, they should be permitted and encouraged to do so.

Individualizing Instruction

• For *slower learners* it may be helpful to use empty containers to demonstrate the ideas at the top of the page. Place three different empty boxes on the desk. Have pupils examine each box and tell about its contents. Then they may write and tell as many different sentences as possible about joining the contents of the three boxes.

• *More capable children* may write sentences of their own like the ones at the bottom of the page.

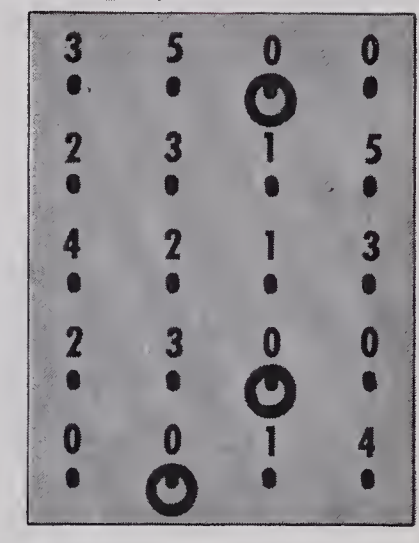
* Another group of M. facts which can be learned by principle—those facts having 0 as a factor!

When One of Two Factors Is 0

Using the Commutative Property of M. [O]

1. Each of the 3 rings on the board is on a peg that gives 0 points. What is the score?

$0 + 0 + 0 = 0$ Three 0's = ? 0



2. If each of 4 rings gives 0 points, what is the score?

$0 + 0 + 0 + 0 = 0$ Four 0's = ? 0

3. What is the score when each of
   a. 5 rings gives 0? Five 0's = ? 0
   b. 6 rings gives 0? Six 0's = ? 0

4. Say the products. a. 5×0 0 b. 8×0 0 c. 0×0 0

5. Because $3 \times 0 = 0$, then $0 \times 3 = 0$. Why? You can change the order of the factors without changing the product.

6. Say the products. a. 0×4 0 b. 0×8 0 c. 0×6 0

* **When 0 is a factor, the product is 0.**

Comparing Products

Mental M.; equalities and inequalities [O]

For each of Ex. 1–10, is the product for the first M. greater than or less than or equal to that for the second M.?

For Ex. 1 say, "36 is greater than 32, so 9×4 is greater than 4×8."

1. (9×4) is greater than (4×8)
2. (8×2) is less than (7×3)
3. (3×9) is greater than (4×5)
4. (6×4) is equal to (8×3)
5. (9×3) is less than (4×8)
6. (7×4) is equal to (4×7)
7. (3×8) is less than (4×7)
8. (9×2) is equal to (3×6)
9. (4×3) is less than (7×2)
10. (6×4) is less than (9×3)

Working Problems

Differentiating A., S., M. [W]

Write your work on your paper.

1. The baker at the Cake Shop works 42 hours a week. In 4 weeks how many hours does he work? 168 hours $4 \times 42 = n$ Multiply to find number for total; sets are equivalent.

2. One day he used all but 28 pounds of flour from a bag of 100 pounds. How many pounds of flour did the baker use? 72 lb. $100 - 28 = n$ Subtract to find number for other part.

3. The baker made 9 dozen round rolls. How many rolls was that? 108 rolls $9 \times 12 = n$ Multiply to find number for total; sets are equivalent.

4. He made 84 white, 124 filled, and 150 chocolate cookies. How many cookies did he make that day? 358 cookies $84 + 124 + 150 = 358$ Add to find number for total.

5. How many more chocolate cookies than white cookies did he make? 66 cookies Use numbers from problem 4. $150 - 84 = n$ Subtract to find number for other part.

6. At the end of the day only 18 of the 150 chocolate cookies were left. How many had been sold? 132 cookies $150 - 18 = n$ Subtract to find number for other part.

7. The store clerk put 36 loaves of bread on each of 3 shelves. How many loaves were on these shelves? 108 loaves $3 \times 36 = n$ Multiply to find number for total; sets are equivalent.

8. Mrs. Beck ordered 25 sugar doughnuts and 25 jelly doughnuts. How many doughnuts did she order? 50 doughnuts $25 + 25 = n$ Add to find number for total.

9. At 31¢ each, how much would 6 small cakes cost? $1.86 $6 \times 31¢ = n¢$ Multiply to find number for total; sets are equivalent.

10. One kind of cake the baker makes sells for 98¢. Mrs. Smith bought 4 of them. How much did they cost? $3.92 $4 \times 98¢ = n¢$ Multiply to find number for total; sets are equivalent.

Write an *n*-example for each problem.

[O]

* For each problem tell why you used A. or S. or M.

* Pupils move a step closer to independence by doing the written work first. However, you make a concluding oral check to pick up any deficiencies.

Teaching Page 262

Pupil's Objective

To learn to use the appropriate operation of addition, subtraction, or multiplication in solving a problem.

New Words

doughnuts, *shelves*

Background

Word problems provide opportunity to employ basic arithmetic skills in a variety of situations. In one respect problems may be considered as opportunities for motivated drill. More important, however, problems provide a setting in which to analyze and organize given data, to select pertinent from irrelevant data, and to determine a means for finding the solution.

Pre-Book Lesson

You may wish to review briefly the three mathematical operations studied to date. Use typical classroom situations for problems requiring each operation. Help children to state generalizations similar to the following:

a. Add to find the number for the total set when sets are not the same size.

b. Subtract to find the number for the part when you know the numbers for the total and one part.

c. Multiply to find the number for the total set when the sets are equivalent.

Using the Text Page

As part of the oral work have children try to justify the operation they have selected in each problem whether it is correct or not. If they have selected an incorrect process, guide their responses and thinking until they *see* the correct approach.

Individualizing Instruction

• It may be necessary to do each problem orally with the *slower learners*. For each problem have a child determine what information is given, what question is asked, and what operation is appropriate.

Emphasis should be on an analysis of each individual problem rather than on reliance upon a set of rules for solving all problems.

Write the following sentences on the chalkboard.

$$240 - n = 75$$
$$24 \times 7 = n$$
$$628 - 174 = n$$

Ask pupils to write a problem for each n-sentence.

• *All pupils* may be asked to explain the difference between problem situations for which addition is the appropriate operation and those for which multiplication is the appropriate operation.

• As pupils write an n-sentence for each problem, stress the importance of this process of translating the written problem into a number sentence, rather than merely stressing that they find the answer.

Pupil's Objectives

(a) To use the M. facts in which 3 or 4 is a factor in solving problems; and (b) to have practice in finding the solution to addition and subtraction number sentences.

Pre-Book Lesson

Arrange sets of objects in the room to illustrate situations for which the M. fact would involve 3 or 4 as a factor. In the discussion ascertain:

(a) Are all sets equivalent?

(b) What is the number of members in each set?

(c) How many sets are there?

Sometimes the factor 3 will tell the number for each set and sometimes it will tell how many sets there are.

Using the Text Page

• Have a child tell each number sentence and the answer for the problem, as suggested for Ex. 1.

• Before assigning the written work in addition and subtraction for Ex. 1–9, you may wish to use examples like the following for review. The review may be with *all pupils* or with only the *slower learners*.

$835 - 368 = n$ $\qquad$ $27 + n = 940$

$602 - n = 469$ $\qquad$ $14 + 6 + 37 + 15 = n$

Individualizing Instruction

• *All pupils* may write *n*-sentences for each problem in Ex. 1–6. An *n*-sentence for Ex. 1 could be written $3 \times 7 = n$.

• *All pupils* may write number sentences for M. facts in which either 2 or 3 or 4 is a factor. The sentences should be written on cards about 3″ by 9″ in size. Each of the following forms should be used in writing the sentences.

$$2 \times 7 = n$$
$$n \times 6 = 18$$
$$8 \times n = 32$$

Collect the sentences written by the pupils and use them for group practice in the *Show the Answer* game. Each child should have a 2″ by 3″ card for each of the numerals 1 through 9.

When a number-sentence card is held up, the pupils select the card that shows the number *n* represents in the sentence. Upon your command all cards are held up for you to see. You can see at a glance the incorrect responses.

• *Slower learners* may make number-sentence cards and answer cards to use for matching. On the back of each number-sentence card they should draw an array to illustrate the M. fact shown on the card.

• Duplicate a sheet with four headings as shown below. Write information in two columns. The children are to fill in the other columns.

Product	One Factor	Other Factor	M. Fact.
18	2	- ? -	- ? -
- ? -	8	3	- ? -
10	2	- ? -	- ? -
18	3	- ? -	- ? -
- ? -	4	9	- ? -

Reminder

Continue to maintain the following:

a. the ideas learned in connection with polygons

b. the ability to use a thermometer and to use the concepts learned related to "freezing" and "boiling"

c. the ability to recognize commonly used coins and how to make change

Oral Problems

Find the number for each total. For Ex. 1 *think* the number sentence, "Three 7's = 21." Say, "21 balloons."

1. Each of 3 clowns had 7 ballons. $3 \times 7 = n$ $\overset{21}{_?_}$ balloons
2. Five clowns each wore 3 hats. $5 \times 3 = n$ $\overset{15}{_?_}$ hats
3. On each of 6 horses were 3 girls. $6 \times 3 = n$ $\overset{18}{_?_}$ girls
4. Nine men each carried 4 flags. $9 \times 4 = n$ $\overset{36}{_?_}$ flags
5. Each of 7 horses carried 3 riders. $7 \times 3 = n$ $\overset{21}{_?_}$ riders
6. In each of 5 rows were 4 elephants. $5 \times 4 = n$ $\overset{20}{_?_}$ elephants

Say an *n*-example for each of problems 1–6.

Finding the Unknown Sum or Addend

* *Addends-sum relationship* [W]

Copy in columns and work. Check.

1. $752 - 469 = n$ 283	4. $356 - n = 297$ 59	7. $753 - 469 = n$ 284
2. $879 - n = 384$ 495	5. $930 - 68 = n$ 862	8. $800 - n = 267$ 533
3. $527 + 89 = n$ 616	6. $98 + n = 687$ 589	9. $633 + 84 = n$ 717

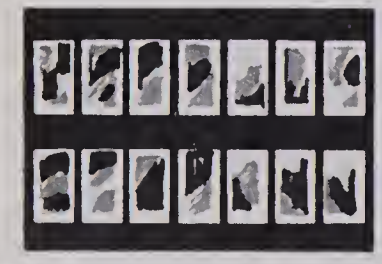
* Regardless of whether the numerals are 1-, 2-, 3- (or more) place numerals, the basic idea of subtracting when a sum and one addend are known provides a solution.

Finding How Many Equivalent Sets

Readiness for division [O]

1. Ann was putting pictures in a scrapbook. On one page she put 12 pictures in rows of 4. How many rows did the 12 pictures make? 3 How many equivalent sets of 4 pictures are there in a set of 12 pictures? 3 4's in 12 = ? 3

A



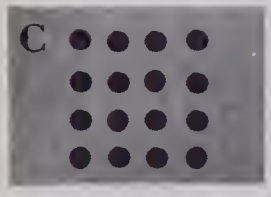
2. On another page in her scrapbook, Ann pasted 14 small pictures in rows of 7 (picture A). How many rows did she have? 2 How many 7's in 14? 2

3. Picture B shows 12 dots in rows of 3. How many rows are there? 4 How many 3's in 12? 4

B

4. Picture C shows 16 dots in rows of 4. How many 4's in 16? 4 4's in 16 = ? 4

C



5. Picture D shows 18 dots in rows of 6. How many 6's in 18? 3 6's in 18 = ? 3

D

For each of Ex. 6–8 make a dot picture on the board.

6. Use 12 dots in rows of 6. 6's in 12 = ? 2

7. Use 15 dots in rows of 3. 3's in 15 = ? 5

8. Use 10 dots in rows of 2. 2's in 10 = ? 5

The work on pages 264-265 prepares for work in division. On these pages the idea is associated with finding the number of equivalent sets in a total set. These equivalent sets are technically subsets of the total set.

Teaching Pages 264 and 265

Pupil's Objective

To develop understanding of the use of division in a measurement situation.

New Word

(page 264) *scrapbook*

Background

The division operation is used for different problem situations. One of these problem situations requires you to determine how many sets of a given size are contained in a known larger set. The question becomes, "How many sets of 4 numbers are contained in a set of 20 members?" You might then ask, "How many members are left over?" This is the *remainder* or the number for the *remainder set*. In the grade-3 work, the remainder is always zero. Division is used for this measurement situation. The material on these two pages provides readiness for beginning work on division which is introduced later in the chapter. In previous work, we learned that the answer to a multiplication example could be found through the use of addition. Similarly, we shall now learn that subtraction may be used to help find the answer to a division example. These pages are not for the purpose of learning any particular division facts, but to acquaint children with the general idea of division.

Teacher's Preparation

Have some objects available for each child to use at his desk. Twenty objects per child should be adequate. As suggested before, one-inch squares of construction paper will serve the purpose. You should have something to use for demonstration. A flannel board* with felt shapes* or a large abacus* on which sets of objects may be shown will serve very well.

Pre-Book Lesson

• If the opportunity arises, refer to a classroom situation in which the number for the total is known. Then determine how many subsets of a given size are contained in the larger set. You may point out that the number of members remaining is zero or that the remainder is zero. It is imperative at this time that the remainder set be the *empty set*, that is, the remainder be zero.

• Direct pupils to place 12 objects in a pile on their desks. Then, have them find how many sets (groups, piles, bunches) of 3 they find in the pile of 12. Continue this type of manipulation. Using objects, have pupils determine how many 4's in 16? 2's in 10? 5's in 20? and so on. It is very important to keep stressing the information that is known. First, they know the size of the set to begin with. Second, they know the size of the smaller sets to be removed. They are to find the number of the smaller sets contained in the larger one.

Using the Text Pages

• Encourage pupils to continue to use the objects in connection with Ex. 1–8. While some pupils are making dot pictures on the chalkboard, the rest of the class should use the objects at their desks.

• Discuss the subtraction in Boxes E, F, and G. As an understanding of division is being developed intuitively, its relationship to subtraction should be established.

When pupils were using objects to determine the number of subsets contained in the larger set, they physically removed each subset. We can associate subtraction with an operation on a set which involves removal of subsets. For example, the work in box E is comparable to removing sets of 5 objects from a pile of 15 objects. Be sure to draw the comparison between removing and subtracting.

Individualizing Instruction

• *All pupils* may use objects to illustrate each example in Ex. 18–31. Encourage pupils to use a variety of ways to illustrate the examples.

• *More capable children* may readily make use of M. facts they have learned in working these examples.

* See 2, 17, and 10, page xix.

In Ex. 26–31 you may permit *more capable children* to subtract the numbers for all of the subsets at once instead of one at a time.

One at a time	All at once
24	24
−8	−24
16	0
−8	
8	
−8	
0	

Do not be anxious to force *all pupils* into subtracting the number for all the subsets at once. It is extremely important to establish first the understanding of removing each subset.

• Duplicate a table like the one at the right for *all pupils* to complete. Discuss with pupils the M. facts in which 0 is a factor. Point out that they are shown in this table when it is completed.

After pupils have completed the table, guide them in forming generalizations by asking questions such as "Do you see something special when you write the numerals for the products?"

The generalizations listed at the right should be anticipated:

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

a. All the answers in the chart that are across from 0 and all the answers that are down from 0 are 0.

b. The answers across from 1 are the same as the factors shown across the top. The answers down from 1 are the same as the factors shown down the left side.

c. Each answer across from 2 and each answer down from 2 increases by 2.

Other generalizations can be made about 3 and 4.

NOTES

You can subtract to find the answers for Ex. 9–17.

9. How many 5's in 15? Use Ex. E. Starting with 15, how many 5's can you subtract? 3 5's in 15 = ? 3

10. Why does Ex. F show that there are four 2's in 8? 2 is subtracted 4 times

11. Ex. G. How many 3's in 9? 3 3 is subtracted 3 times. Explain. ∧ 3's in 9 = ? 3

E	F	G
15	8	9
−5 (1)	−2	−3
10	6	6
−5 (2)	−2	−3
5	4	3
−5 (3)	−2	−3
0	2	0
	−2	
	0	

On the board subtract, as shown in Ex. E–G, to find how many

12. 4's in 24. 6

13. 7's in 21. 3

14. 9's in 36. 4

15. 5's in 25. 5

16. 6's in 18. 3

17. 2's in 10. 5

[W]

For each of Ex. 18–21, draw the picture. For Ex. 18 write, "2's in 6 = 3." Work this way for Ex. 19–21.

18. 6 boats in rows of 2.

19. 10 tents in rows of 5. 5's in 10 = 2

20. 8 hats in rows of 4. 4's in 8 = 2

21. 12 balls in rows of 2. 2's in 12 = 6

For each of Ex. 22–25 make a dot picture. Count rows and then write the example with its answer.

22. 18 dots in rows of 9. 9's in 18 = 2

23. 14 dots in rows of 2. 2's in 14 = 7

24. 18 dots in rows of 3. 3's in 18 = 6

25. 16 dots in rows of 8. 8's in 16 = 2

For each of Ex. 26–31 subtract as in Ex. E–G above to find the answer. Then write the example with its answer.

26. 4's in 8 = ? 2

27. 8's in 24 = ? 3

28. 5's in 20 = ? 4

29. 6's in 24 = ? 4

30. 9's in 27 = ? 3

31. 7's in 28 = ? 4

Multiplying with 3-Place Numerals

[o]

1. A truck driver is unloading 4 bags of potatoes at Mr. Cook's little store. Each bag holds 100 pounds. In all, how many pounds will he unload? $4 \times 100 = n$

a. Count by 100's. Do you get 400? Yes

b. Add 100's (Ex. A). Is the sum 400? Yes In writing the numeral for the sum, where would you write "4"? in hundred's place *

* **c.** You can multiply, too. Why? The sets are equivalent. For 4×100, *think*, $4 \times 1 = 4$, so 4×1 hundred = 4 hundreds, or _?_. 400

d. The factors are _?4_ and _?100 The product is _?_. 400

A	
100	addend
100	addend
100	addend
100	addend
?	sum

B	
100	factor
×4	factor
400	product

To multiply, you write the numerals as in Ex. B. Why is the 4 for the product written in hundred's place? to show 4 hundreds Why are 0's written in ten's and one's places? to show 0 tens and 0 ones

Say the product for each of Ex. 2–6.

2.	3.	4.	5.	6.
100	100	200	400	300
× 5	× 9	× 3	× 2	× 3
500	900	600	800	900

You multiply hundreds in the same way that you multiply ones.

* If pupils understood earlier that tens are multiplied in the same way as ones, they should have no trouble in multiplying hundreds just as they multiplied ones.

Teaching Pages 266 and 267

Pupil's Objectives

(a) To learn to multiply with 3-place numerals; and (b) to make use of the distributive property and the commutative property so as to have the further extension of multiplication more meaningful.

New Word

(page 266) *unloading*

Background

On page 266 is the first introduction to multiplying with a 3-place numeral. This part of the lesson is limited to 3-place numerals showing hundreds only. Furthermore, no remembering is involved when multiplying hundreds.

In this introductory stage, the total is obtained first by counting by hundreds, then by adding hundreds, and finally by multiplying hundreds.

The relationship between addition and multiplication has been emphasized in many previous lessons. The distributive property was introduced on page 248 in connection with multiplication with 2-place numerals. It is being extended in the lesson on page 267 to multiplication with 3-place numerals. Consider the example 2×132. 132 is renamed $100 + 30 + 2$ making it possible to distribute the multiplication by 2 over each of several addends. The child has already learned to multiply 2×2, 2×30, and 2×100. Multiplying with a 3-place numeral provides an application for all of these learnings.

Pre-Book Lesson

• On the chalkboard, write a M. sentence like $3 \times 40 = n$. Ask a pupil to read it. Stress the reading of the sentence rather than giving the product. In fact, encourage several different ways of reading the sentence. Some pupils will say "3 times 40 equals what?" others "What does three 40's equal?" and others may say "Three 40's names what number?"

Ask pupils to tell ways in which they can find the answer. Pupils' responses probably will include the following:

(a) Add three 40's.

(b) Two 40's equal 80 and one more 40 equal 120.

(c) Three 4's equal 12, and 12 tens equal 120.

• On the chalkboard write the example

$$\begin{array}{r} 40 \\ \times 3 \\ \hline \end{array}$$

Now, when we multiply 3 times 4 tens how many tens will there be in the answer?

Review the meaning of 120: What does the 0 tell us? (There are no ones.) What does the 12 tell us? (There are 12 tens.) Another way of reading 12 tens is *one hundred twenty*.

Using the Text Pages

• Refer to the work of the Pre-Book Lesson as deemed appropriate in developing the material on page 266. You may wish to ask pupils to write Ex. 2–6 on the chalkboard as they explain the products.

• You are encouraged to use the chalkboard extensively in order to foster as complete an understanding as possible in connection with the exercises on page 267. For example, as the work in boxes C, D, E, F, and G is being discussed and analyzed you may wish to write on the board the same example in expanded form as shown below.

$$\begin{array}{rcl} 132 & = & 100 + 30 + 2 \\ \times 3 & & \times 3 \\ \hline & & 300 + 90 + 6 \\ & & 396 \end{array}$$

• You may wish to have selected pupils illustrate some of the examples in rows 10 and 11 as a check on their understanding. Permit the child to use any means of illustrating the example he wishes.

• Circulate among pupils as they proceed with the written work. Observe their work for lack of understanding or for incorrect procedures. Ask questions of individuals if you are uncertain of their level of understanding. It may be necessary to work with small groups of *slower learners* to clarify

meanings and to minimize the chance of poor procedural habits becoming fixed.

Individualizing Instruction

• To help *slower learners* move toward higher levels of mathematical maturity, use one or more of the following activities to foster understanding:

a. Write hundreds, tens, and ones as indicated.

$$\begin{array}{l} 243 = 2 \text{ hundreds} + 4 \text{ tens} + 3 \text{ ones} \\ \underline{\times 2} \qquad\qquad\qquad\qquad\qquad \times 2 \\ 4 \text{ hundreds} + 8 \text{ tens} + 6 \text{ ones}, \\ \qquad\qquad\qquad\qquad\qquad \text{or } 486 \end{array}$$

b. Write the 3-place numeral in expanded notation as shown below.

$$\begin{array}{l} 243 = (200 + 40 + 3) \\ \underline{\times 2} \qquad\qquad\quad \times 2 \\ 400 + 80 + 6, \text{ or } 486 \end{array}$$

c. Write the example and show each partial product as indicated.

$$\begin{array}{rl} 243 & \\ \underline{\times 2} & \\ 6 & (2 \times 3 \text{ ones}) \\ 80 & (2 \times 4 \text{ tens}) \\ \underline{400} & (2 \times 2 \text{ hundreds}) \\ 486 & \end{array}$$

• *More capable children* may be introduced to the associative property as a means of helping in the solution of some multiplication examples. The Associative Property of Multiplication provides that, of three factors, the third factor may be multiplied by the product of the first two; or the product of the second and third factors may be multiplied by the first without changing the final product. In multiplying 2×400, 400 may be thought of as 4×100. In turn, $2 \times (4 \times 100)$ may be considered as $(2 \times 4) \times 100$. Have these pupils write each of examples a–f as is shown for Ex. a.

$$2 \times 400 = 2 \times (4 \times 100) = (2 \times 4) \times 100 = 8 \times 100 = 800$$

a. 2×400	b. 2×300	c. 3×300
d. 4×200	e. 3×500	f. 4×300

All pupils may make a multiplication table like the one suggested at the end of the previous lesson. You may wish to duplicate a form so that pupils will show products only and not spend a great deal of time making the grid.

You may wish to use some of the following examples for additional practice for *all pupils:*

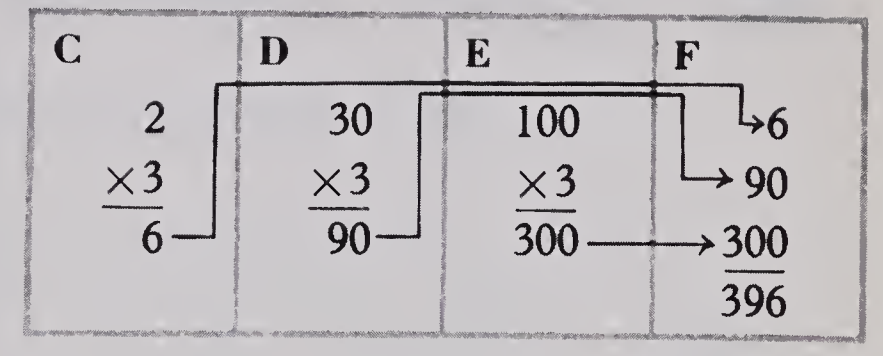
a. 3×30	2×40	4×20	3×20	
b. 300	100	200	700	300
$\times 2$	$\times 6$	$\times 4$	$\times 1$	$\times 3$
c. 131	444	403	423	333
$\times 3$	$\times 2$	$\times 2$	$\times 2$	$\times 3$

* Again note the steps so that we can use the idea of distributing multiplication over addition.

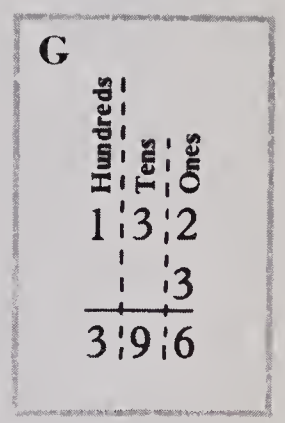
7. Three days a week, another truck brings Mr. Cook 132 packages of baked goods. What is the total number of packages Mr. Cook gets in the 3 days? $3 \times 132 = n$

* $132 = 100+30+2$ or $2+30+100$ $3\times(2+30+100)=n$

Ex. C–F. You multiply first ones, then tens, then hundreds. Then you add the products. Explain the work.



8. Explain the short way to multiply 132 by 3 (Ex. G).



9. Explain why 6, 9, and 3 were placed as shown. 6 is in one's place to show 6 ones, 9 is in ten's place to show 9 tens, 3 is in hundred's place to show 3 hundreds.

Explain the multiplications for Ex. 10a–10c.

	a	b	c	d	e	f	g
10.	122	223	423	243	213	212	\$3.22
	4	3	2	2	3	4	3
	488	669	846	4 ?86	6 ?39	8?4?8	\$?.?6
							9 6

See answer for Ex. 9 for sample.

Say the missing digits in Ex. 10d–10g above.

Which products are shown wrong in row 11? Why?

11.	212	324	224	112	121	111	\$1.14
	4	2	2	3	4	7	2
	216	648	446	136	125	118	\$2.28

a. $4\times2=8$, $4\times1=4$ c. $2\times4=8$ d. $3\times1=3$ e. $4\times1=4$, $4\times2=8$, $4\times1=4$
f. $7\times1=7$, $7\times1=7$, $7\times1=7$

[W]

Copy, and multiply.

12.	3×112 336	**16.**	2×242 484	**20.**	4×212 848	**24.**	$4 \times \$1.22$ \$4.88
13.	2×411 822	**17.**	4×121 484	**21.**	3×131 393	**25.**	$8 \times \$1.11$ \$9.36
14.	3×232 696	**18.**	3×212 636	**22.**	2×334 668	**26.**	$3 \times \$3.12$ \$8.42
15.	4×221 884	**19.**	7×111 777	**23.**	3×333 999	**27.**	$2 \times \$4.21$

More Multiplying with 3-Place Numerals

Remembering tens [O]

Hundreds	Tens	Ones
1	2	5
		3
3	7	5

1. Mr. Rice's fence is 125 feet long. It is 3 boards high. To build it, how many feet of the boards did he use? $3 \times 125 = n$

* For this multiplication, do you rename the product of the ones? Yes Why? The number of ones is greater than 9. Do you remember 1 ten? Yes

You multiply ones, then tens, then hundreds. Explain the work. Tell what to think.

Remember 1 ten for Ex. b, d, and g.

For which examples do you remember 1 ten? 2 tens?

Remember 2 tens for Ex. a and f.

	a	b	c	d	e	f	g
2.	113	139	111	325	213	214	$2.45
	×7	×2	×8	×3	×3	×5	×2

For which examples are products shown wrong? Why?

3.	× 423	× 126	221	114	× 215	324	$1.18
	2	3	4	6	4	3	4
2 × 2 = 4, 2 × 4 = 8	456	368	884	684	840	972	$4.72

3 × 2 = 6, 6 + 1 = 7 4 × 1 = 4, 4 + 2 = 6

Tell what to think for these multiplications:

4.	112	229	414	112	128	323	$4.17
	6	3	2	8	2	3	2
	672	687	828	896	256	969	$8.34

Copy each of Ex. 5–8 on the board and multiply.

5. 4×114 (456) 6. 5×113 (565) 7. 4×216 (864) 8. $3 \times \$2.24$ ($6.72)

* Pupils will find that remembering tens after renaming the product of the ones is the same whether the number multiplied is shown by a 2-place numeral or a 3-place numeral.

Teaching Pages 268 and 269

Pupil's Objectives

(a) To learn to remember one or more tens when multiplying with 3-place numerals; and (b) to learn to multiply with 0 in one's or ten's place.

Background

The idea of renaming numbers has been used extensively throughout this program. The naming of numbers in many different ways provides more flexibility in thinking and learning about numbers.

This is the first lesson that involves remembering tens after renaming the product of ones. Pupils have renamed a number so as to be able to distribute the multiplication over several addends and have, in turn, renamed the product arrived at. For example, for 2 × 241, the 241 was renamed (200 + 40 + 1). By multiplying each of these numbers by two, the product was named (400 + 80 + 2). Then, the product was renamed 482. Consider the following example of renaming in which remembered tens must be added.

$$\begin{array}{rl} 216 = 200 + 10 + 6 & \\ \times 4 \qquad\qquad \times 4 & \\ \hline 800 + 40 + 24 & \text{First naming of the product.} \\ 864 & \text{Product renamed. Notice that 2 tens had to be remembered and added to 4 tens.} \end{array}$$

The addition probably can be more readily seen in the following form.

$$\begin{array}{rl} 216 & \\ \times 4 & \\ \hline 24 & (4 \times 6 \text{ ones}) \\ 40 & (4 \times 1 \text{ ten}) \\ 800 & (4 \times 2 \text{ hundreds}) \\ \hline 864 & \end{array}$$

In this lesson and those to follow it is necessary to build a thorough understanding of the addition involved in this particular type of renaming. At the highest level of maturity, the pupil will generalize that it is necessary to multiply first and then add the number remembered. In the example cited above, the thought pattern would be: "Four times 1 ten equals 4 tens, plus the 2 tens remembered equals 6 tens."

At the bottom of page 269 multiplication of 0 by a number should not be a serious problem.

Pre-Book Lesson

• Try to pose a realistic problem for your pupils to solve. Record suggested solutions.

A package of paper contains 125 sheets. How many sheets of paper would be in 3 packages?

Pupils' suggestions should finally lead to recording something like the following:

125 may be renamed (1 hundred + 2 tens + 5 ones)

Multiply ones.

$$\begin{array}{r} 5 \text{ ones} \\ \times 3 \\ \hline 15 \text{ ones, or } 1 \text{ ten} + 5 \text{ ones} \end{array}$$

Multiply tens.	Multiply hundreds.
2 tens	1 hundreds
× 3	× 3
6 tens	3 hundreds

Add the partial products.

$$\begin{array}{r} 1 \text{ ten} \\ 3 \text{ hundreds } 6 \text{ tens } 5 \text{ ones} \\ \hline 3 \text{ hundreds } 7 \text{ tens } 5 \text{ ones} \end{array}$$

• In a chart like the one below, pupils show each partial product in the appropriate position. Call attention to the two numbers shown for tens. At this point stress that one is the result of multiplying and the other is the result of the product of the ones having been renamed.

	Hundreds	Tens	Ones
125		1	
× 3	3	6	5
	3	7	5

Pupils may use a similar chart to find solutions to the following examples:

317	247	114
× 3	× 2	× 6

Review briefly the role of 0 when it is an addend and when it is a factor. Have pupils tell the result when a number is added to 0. Then have them tell the result when 0 is multiplied by a number. Write some examples of each of the kinds on the chalkboard to confirm children's generalizations.

Write examples like the following on the chalkboard. Ask pupils to read each example and then tell the results. $(4 \times 0) + 3 = n$; $(2 \times 0) + 4 = n$; $(7 \times 0) + 2 = n$.

Using the Text Pages

• Pupils reveal their understandings as they discuss the examples in Ex. 2–4. From time to time have selected pupils illustrate the example in any way he wishes to confirm his statements.

• Circulate among pupils to spot-check their written work for procedural errors or evidence of misunderstanding. Be especially alert to the error of multiplying the remembered tens (tens to be added) and adding the tens to be multiplied. If you are in doubt as to a child's understandings, ask him probing questions about the example. For example, point to a specific digit in the numeral for the product and have him tell you how it was obtained.

• Review the M. facts with 0 as a factor as part of the oral work at the bottom of page 269. On the board, clarify the work in boxes B and C as follows:

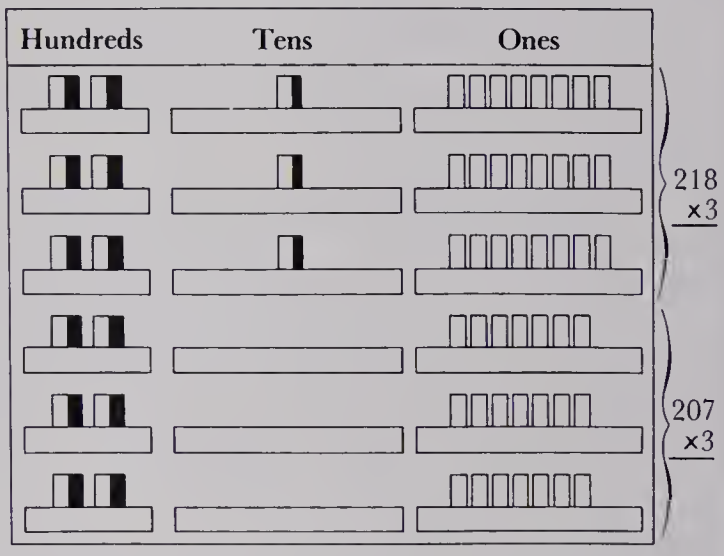
302	
×3	
6	(3 × 2 ones)
0	(3 × 0 tens)
900	(3 × 3 hundreds)
906	

208	
×4	
32	(4 × 8 ones)
0	(4 × 0 tens)
800	(4 × 2 hundreds)
832	

• Ask pupils to illustrate examples in this part of the lesson as is deemed appropriate.

Individualizing Instruction

• In the lesson dealing with 3-place numerals there has been less emphasis on the use of objects and representative materials. However, many pupils may gain greater insights if the concept is placed in a more objective setting. For this reason you may wish to review the principles involved in the multiplication presented on these pages. Two examples are shown illustrated in the pictures of the pocket charts. Each time ask pupils if more than 9 ones are shown. Follow this by having them explain the renaming and remembering.



• *All pupils* may find the following activities interesting and challenging.

a. Tell (or write) the numeral that belongs where ? appears in each example.

104	230	420	102	107
×3	×2	×2	×8	×3
3 ? 2	46 ?	84 ?	8 ? 6	3 ? 1

b. Tell which examples have wrong answers and why (or, copy the examples with wrong answers and do them correctly).

203	230	105	103	420
×4	×3	×2	×9	×2
802	693	210	907	840

• Use Extra Examples Set 74 and Set 75.

Reminder

Continue to maintain the ideas and terminology developed under the heading of Geometry.

[W]

Rows 9–12. Show products on folded paper.

	a	b	c	d	e	f	g
9.	113	223	112	328	317	115	$1.13
	× 6	× 4	× 5	× 2	× 3	× 6	× 4
	678	892	560	656	951	690	$4.52
10.	116	429	117	114	229	219	$2.15
	× 2	× 2	× 3	× 7	× 3	× 4	× 4
	232	858	351	798	687	876	$8.60
11.	218	421	328	117	133	447	$1.12
	× 4	× 2	× 3	× 4	× 3	× 2	× 7
	872	842	984	468	399	894	$7.84
12.	116	114	112	313	446	216	$2.14
	× 4	× 5	× 4	× 3	× 2	× 4	× 3
	464	570	448	939	892	864	$6.42

◆ **Extra Examples.** Work Set 74.

Multiplying When a Digit Is 0

[O]

Explain 0 in the numeral for the product a. in Ex. A. (2 × 0 = 0) b. in Ex. B. (3 × 0 = 0)

In Ex. C, why is there no 0 in the numeral for the product? (4 × 0 = 0, 0 + 3 = 3)

A	B	C
230	302	208
× 2	× 3	× 4
460	906	832

Tell what to think in Ex. 1–6.

1.	2.	3.	4.	5.	6.
408	120	107	110	202	$3.07
× 2	× 3	× 4	× 6	× 4	× 2
816	360	428	660	808	$6.14

[W]

Copy and multiply.

7. 2 × 408 816	10. 3 × 120 360	13. 4 × 107 428	16. 6 × $1.10 $6.60
8. 4 × 200 800	11. 2 × 207 414	14. 4 × 104 416	17. 4 × $2.06 $8.24
9. 2 × 206 412	12. 3 × 208 624	15. 7 × 104 728	18. 2 × $1.09 $2.18

◆ **Extra Examples.** Work Set 75.

* Note that, since the Commutative Property of Multiplication applies to the number as such, we have temporarily disassociated the numbers from the social situation.

Changing the Order of Factors

Using the Commutative Property of M. [O]

1. $2 \times 4 = n$ 8 $4 \times 2 = n$ 8 Does changing the order of the factors 2 and 4 change the product? No

A	B
2	34
34	2
n	68

2. Each of the 34 children in our room asked 2 friends to a picnic. How many friends were asked? $34 \times 2 = n$

You have not learned how to work the multiplication example shown in box A. But if you change the order of the factors, you can multiply, as in Ex. B.

[W]

Copy and multiply. Where you need to, change the order of the factors. Check by multiplying a second time.

3. $3 \times 231 = n$ 693
4. $2 \times 316 = n$ 632
5. $98 \times 3 = n$ 294
6. $2 \times 235 = n$ 470
7. $79 \times 4 = n$ 316
8. $7 \times 104 = n$ 728
9. $5 \times 43 = n$ 215
10. $103 \times 6 = n$ 618
11. $4 \times \$0.56 = \$n$ $2.24
12. $68 \times 4 = n$ 272
13. $104 \times 5 = n$ 520
14. $7 \times \$1.13 = \$n$ $7.91
15. $114 \times 6 = n$ 684
16. $4 \times \$2.06 = \$n$ $8.24
17. $340 \times 2 = n$ 680

To Keep in Practice

A., S., M. [W]

Copy each set of numerals in a column, and work.

1. $3.15 + $4.09 $7.24
2. $9.10 − $0.63 $8.47
3. $4 \times \$0.97$ $3.88
4. $8.00 − $5.49 $2.51
5. 4×208 832
6. 438×2 876
7. $320 + 86$ 406
8. 2×340 680
9. $7.00 − $4.00 $3.00
10. $8.69 − $7.99 $0.70
11. $1.87 − $0.92 $0.95
12. $6.00 − $0.28 $5.72
13. $8 + 9 + 4 + 6 + 35$ 62
14. $33 + 15 + 8 + 7 + 7$ 70

◆ **Extra Examples.** Work Set 76.

Teaching Page 270

Pupil's Objectives

(a) To learn to multiply the greater factor; and (b) to have practice in the various computational skills involved in addition, subtraction, and multiplication.

Background

When multiplying with a 3-place numeral and a 1-place numeral, we ordinarily multiply by the lesser number. There is nothing wrong with multiplying by the greater number; it is just more convenient to change the order of the factors and multiply by the lesser number. In this way multiplication can be distributed over the addends of the number to be multiplied. In fact, this is one occasion when the Commutative Property of Multiplication is very useful.

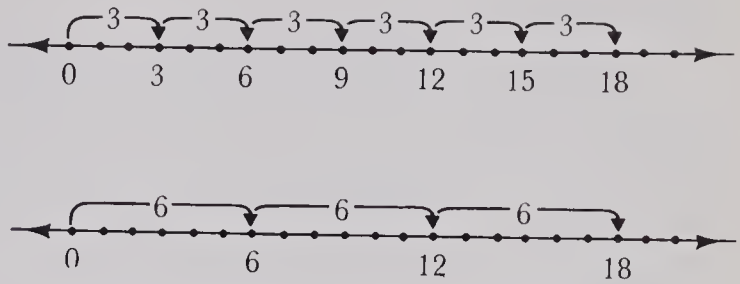
Pre-Book Lesson

• Use any one or more of the following activities to review and re-enforce the Commutative Property of Multiplication.

(a) Perhaps you have been making a large multiplication chart for the class as the various facts have been introduced and learned. (It would be a good idea.) Select a factor shown in the column at the left and one shown in the row at the top. Find the product. Then, select the same factors, but from the opposite places as just described and find the product. Have different pupils work this way with factors of their choice. They may use the multiplication charts they have made. At the same time, have each pupil write the number sentence appropriate for the factors selected and their product.

Guide pupils to the generalization (in their own words) that the order of the factors does not change the product.

(b) Write the number sentence $6 \times 3 = n$ on the chalkboard. Have a child illustrate finding the product on a number-line picture. Then write the sentence $3 \times 6 = n$. Ask another child to illustrate finding the product for this sentence on the same number-line picture. Through your questioning, guide pupils to the generalization.



• Write these two sentences on the board.

$$427 \times 8 = n$$
$$8 \times 427 = n$$

Ask, "Will the numbers for n in these two sentences be the same or will they be different?" Have pupils tell why.

Do not attempt to find the product. That is unimportant at this time. The important point is the understanding and application of the commutative property.

Using the Text Page

Some pupils may still need additional persuasion to apply the Commutative Property of Multiplication. It may help to work the example as written in box A to show that the answer will be the same as that in box B.

Individualizing Instruction

• It may not be necessary for *all pupils* to work all of the examples at the bottom of page 270. If a child has good grasp of the skills presented, other types of activities may be more appropriate and interesting to him.

A review and extension of the geometry activities on pages 256 and 257 may prove worthwhile.

• *More capable children* may make a multiplication chart and try to fill in numerals for the products. They have had enough of the basic principles of multiplication to be able to do this kind of thing independently.

• *All pupils* should find the following exercises useful in helping with the skill of multiplying tens and adding remembered tens.

$(3 \times 7) + 2$	$(4 \times 2) + 1$	$(6 \times 4) + 3$
$(7 \times 0) + 3$	$(9 \times 3) + 1$	$(5 \times 2) + 3$
$(5 \times 0) + 2$	$(8 \times 2) + 3$	$(2 \times 7) + 1$

Pupil's Objective

To practice solving problems using one of the three operations: addition, subtraction, or multiplication.

New Word

airport

Background

Developing the ability to solve problems encountered in our daily activities is one of the major purposes of a mathematics program. Pupils should be able to analyze the situation and then determine the particular skill needed to answer the given question. Because of the mixed problems it is necessary for the pupil to read each problem carefully so that he may decide how he should try to solve it. If you find that problems unique to your particular environment are more appropriate, by all means use them. Above all, however, do not neglect the problem-solving facet of the mathematics curriculum.

In later work in *Mathematics We Need*, pupils will work problems requiring the answering of a hidden question as a prerequisite to being able to answer the given question. Such problems may be called "2-step problems." Ex. 3 suggests this type of problem in that the answer to another question must be used to solve the given question.

Pre-Book Lesson

For purposes of review, have different children state a problem in which it would be necessary to use the operation: addition, subtraction, or multiplication. Have other pupils tell why the particular operation is appropriate.

Using the Text Page

- Pupils should proceed independently with the work. Circulate among children with reading difficulties to assist them with the vocabulary (if necessary).
- Wait until all pupils have completed the written work before doing the oral work suggested at the bottom of the page. Discuss the operation chosen. For problems that seem to be difficult, use dramatization, representative objects, or sketches to help build meaning.

Individualizing Instruction

Transportation units afford many opportunities for problem-solving. Collecting timetables, schedules, and maps may extend pupils' knowledge of travel by train, bus, auto, boat, and plane. Perhaps a classroom Travel Bureau may be established to stimulate the acquisition and use of such information.

NOTES

* Now you get an opportunity to check orally on difficulties with number sentences after pupils have independently written their operation selections and the work.

A. or S. or M. — Which?

Problem-solving [W]

For each of problems 1–7, write "A." or "S." or "M." Then write your work to find the answer.

1. Mr. and Mrs. Little rode with Dora to the airport when she left to visit her aunt. Each bus ticket cost $1.15. How much did the 3 tickets cost? M. $3.45
$3 \times \$1.15 = \$n$

2. Dora was to ride 365 miles on one plane and then 475 miles more on another. How long was the whole trip? A. 840 miles $365+475=n$

3. The trip to her aunt's would have been 85 miles longer by train. How many miles would the trip have been by train? Use your answer for problem 2. A. 925 miles $840+85=n$

4. After going 190 miles, the first plane landed. How many of its 365 miles did that plane still have to go? S. 175 miles $365-190=n$

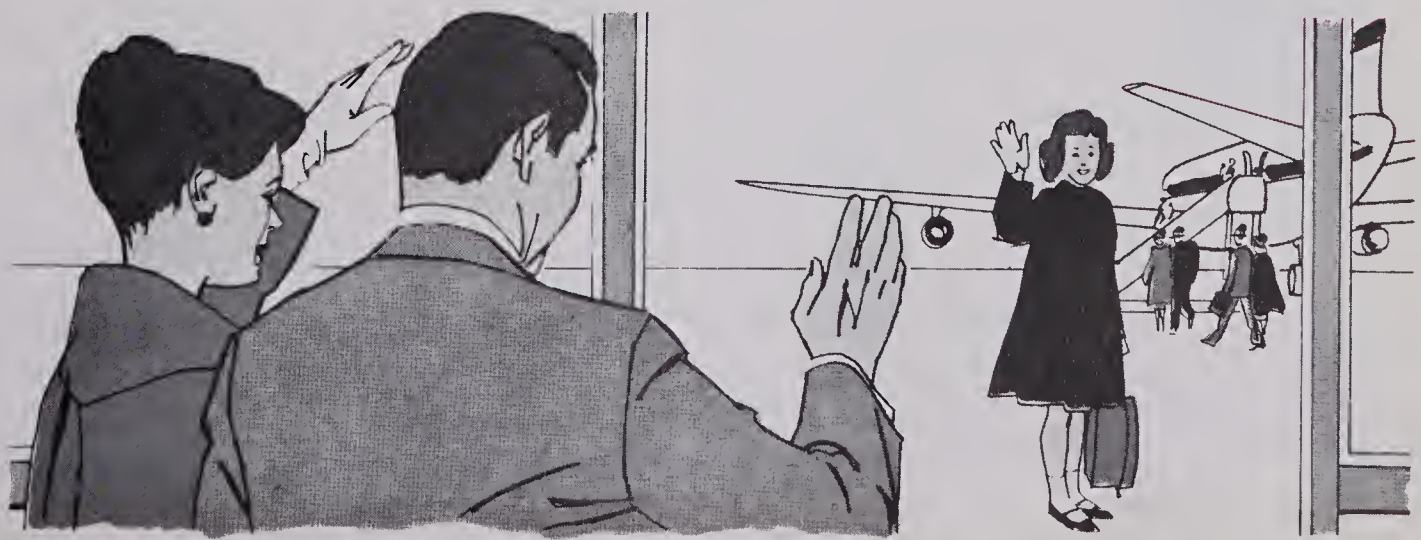
5. That plane flew about 205 miles an hour. Without stopping, how far would it go in 2 hours? M. 410 miles
$2 \times 205 = n$

6. By air the whole trip took 310 minutes. By train it would have taken 3 times as long, or how many minutes? M. 930 minutes
$3 \times 310 = n$

7. The bus ticket from the airport to her aunt's home cost $1.35. How much change did Dora get from $5.00? S. $3.65
$\$5.00-\$1.35=\$n$

[O]

* Now say an *n*-example for each problem.



* In these multiplications, just one factor is rounded, whereas in addition, both addends were rounded.

Estimating Products

Using rounded numbers [O]

1. Round each of these to the nearest ten:

64 60 73 70 91 90 46 50 29 30 12 10 37 40 48 50 85 90

* In estimating products, we use rounded numbers. For 2×39, we round 39 to 40 and *think* "$2 \times 40 = 80$."

Estimate the products for each of Ex. 2–9.

2. 2×39 80
3. 51×3 150
4. 4×41 160
5. 4×49 200
6. 81×3 240
7. 4×19 80
8. 2×78 160
9. 39×3 120

[W]

Ex. 10–21. Show your estimates of the answers. Estimated answers are shown above each example.

10. $48 \overset{90}{+} 42$ 90
11. $11 \overset{40}{\times} 4$ 44
12. $92 \overset{40}{-} 53$ 39
13. $17 \overset{70}{+} 49$ 66
14. $4 \overset{80}{\times} 19$ 76
15. $4 \overset{120}{\times} 29$ 116
16. $3 \overset{90}{\times} 29$ 87
17. $78 \overset{60}{-} 19$ 59
18. $12 \overset{60}{+} 45$ 57
19. $71 \overset{10}{-} 59$ 12
20. $19 \overset{60}{\times} 3$ 57
21. $24 \overset{80}{+} 58$ 82

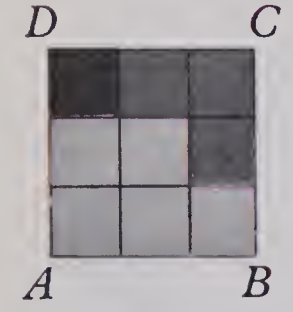
Now copy and work each of Ex. 10–21.

** Many Things

Maintenance [W]

Find the unknown addend. Write your work.

1. $300 - \overset{232}{n} = 68$
2. $89 + \overset{480}{n} = 569$
3. $\overset{85}{n} + 264 = 349$



Write the fraction for the part of the inside of square $ABCD$ that is

4. red. $\frac{1}{9}$
5. blue. $\frac{5}{9}$
6. yellow. $\frac{3}{9}$

Copy and finish Ex. 7–10.

7. 1 pound and 7 ounces is the same as $\overset{23}{_?_}$ ounces.
8. 1 week and 2 days is the same as $\overset{9}{_?_}$ days.
9. 1 yard and 6 inches is the same as $\overset{42}{_?_}$ inches.
10. 1 hour and 15 minutes is the same as $\overset{75}{_?_}$ minutes.

** Again, a mid-chapter review!

Teaching Pages 272 and 273

Pupil's Objectives

(a) To review and extend the understandings involved in rounding numbers to the nearest ten; (b) to learn to estimate products by using rounded numbers; and (c) to review important ideas and relationships.

Background

Rounding to the nearest ten was introduced and applied to addition and subtraction on pages 158, 159, and 160. You may wish to refer to these Teacher's Pages.

Skill in estimating gives the learner a quick means of determining the reasonableness of an answer. No basis for estimating the answer in multiplication has been introduced thus far, though some children may be using their knowledge of rounding numbers without formal instruction.

It must be emphasized that estimating does not give the exact answer, but merely an answer that is reasonable. Use of the term *about* should be encouraged.

Teacher's Preparation

You may wish to use your permanent number line to review the ideas of rounding to the nearest ten. It would be simple to draw a picture of a number line near the top of the board with only the points 0, 5, 10, 15, and so on labeled.

Pre-Book Lesson

• Write some 2-place numerals on the chalkboard and ask pupils to tell the ten which is nearest to each number shown. Then, have them use a number line to confirm their choice. This will help them to explain why each number is nearest to the ten chosen.

• The following terminology and idea should be reviewed in this activity:

a. Use the words *about* and *almost*. Recall the idea of *rounding* a number to the nearest ten.

b. Have pupils tell for each exercise if they are *rounding* to the greater number of tens or to the lesser number of tens.

c. In this program, a number shown ending in 5 is rounded to the greater number of tens.

• Review the use of rounding numbers to estimate the answer in addition and subtraction. The estimate will be *about* or *almost* the exact answer. In fact, it may be the exact answer.

On the chalkboard write the example $17 + 41$. Have pupils round each number and estimate the answer.

Write $83 - 56$ on the chalkboard. Develop the same type of thinking in connection with subtraction as was developed for addition.

• On the chalkboard write the example 3×67. Ask a child to tell how we could round the number to be multiplied to the nearest ten to estimate the answer to the multiplication example.

• In your discussion of rounding in multiplication see if pupils can determine whether the estimate will be greater than or less than the exact answer. (If the number is rounded to the lesser number of tens, then the estimate will be less than the exact answer. If the number is rounded to the greater number of tens, the estimate will be greater than the exact answer.)

Using the Text Pages

• As the oral lesson at the top of page 272 is developed you may wish to refer to a number line. It would be a good idea to illustrate finding the answer for one of the multiplication examples.

• Estimation is the important objective in the written work at the top of page 272. Be sure pupils first write estimated answers. Then, they should go back and write exact answers.

• The maintenance section could just as well be presented as an oral lesson. In this way you will have an opportunity to assess the knowledge and understandings attained by your pupils. After it has been discussed, then assign it as a written activity.

Individualizing Instruction

• *Slower learners* may use one or more of the following suggestions to assist them in building skill and understanding of rounding numbers:

48 → 50	73 → 70	52 → 50
+24 → 20	−46 → 50	×4 ×4
70	20	200

• Additional illustrations using a number line may help pupils having difficulty *seeing* the rounded number.

More capable children (perhaps *all pupils*) may decide in advance whether the estimated answer will be greater than, less than, or equal to the actual answer.

There are many understandings and ideas covered in the maintenance section. *All pupils* may select any ideas they wish that are least understood or are most appealing and find pages in the book covering that particular material. Perhaps they will find some exercises on those pages for review and written work. (Be sure the child selects pages which have already been presented.)

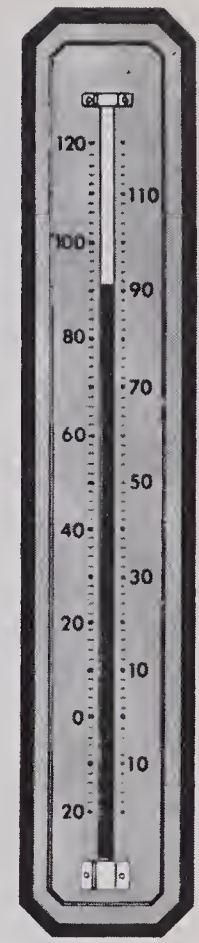
• *All pupils* should be able to do the enrichment exercises. Make an oral lesson of the materials as an introduction. Pupils should use "is greater than," "is less than," or "is equal to" to make each a *true* sentence. In Ex. 1, the pupil is to determine whether four 6's is greater than or is less than four 7's. Then he would read the sentence using the appropriate expression to make a *true* sentence.

In this activity, the knowledge of multiplication facts is essential. An understanding of the Commutative Property of Multiplication is also very important in thinking and determining relationships between two facts. In Ex. 2, for instance, the child may be encouraged to use the commutative property and read the sentence, "Three 9's is greater than three 8's."

The *more capable children* will be able to use a judgment without even thinking the products involved. Both multiplications have one factor the same. The judgment required is to decide that one product will be greater than, less than, or equal to the other product because the second factor is correspondingly greater or less or equal. These children will be stimulated by an introduction to this type of thinking and the judgment required.

NOTES

For Ex. 11–19 write "Yes" or "No" for your answers.



11. If the product of 23×3 is 69, then is 69 the product of 3×23? Yes

12. Is a square always a rectangle? Yes

13. In the answer for 3×108 will there be a 0? No

14. Is ray AB a part of line AB? Yes

15. Can you subtract to find how many sets of 4 dots there are in a set of 12 dots? Yes

16. Does water freeze at 90°? No

17. Is (4×9) greater than (3×8)? Yes

18. Is 120 a good estimate of the product for 4×39? No

19. Does the 4 in $\frac{4}{9}$ show the number of parts of the same size there are in a whole? No

Do You Like to Try New Things?

Enrichment [W]

Without multiplying, try to think which product is greater. In some examples the products are equal.

In Ex. 1, 7 is greater than 6 and 4 equals 4, so write, "(4×7) is greater than (4×6)." Work Ex. 2–10.

1. (4×6) or (4×7)
(4×7) is greater than (4×6)

2. (9×3) or (3×8)
(9×3) is greater than (3×8)

3. (9×2) or (2×7)
(9×2) is greater than (2×7)

4. (6×0) or (0×8)
(6×0) is equal to (0×8)

5. (1×9) or (9×1)
(1×9) is equal to (9×1)

6. (2×8) or (8×3)
(8×3) is greater than (2×8)

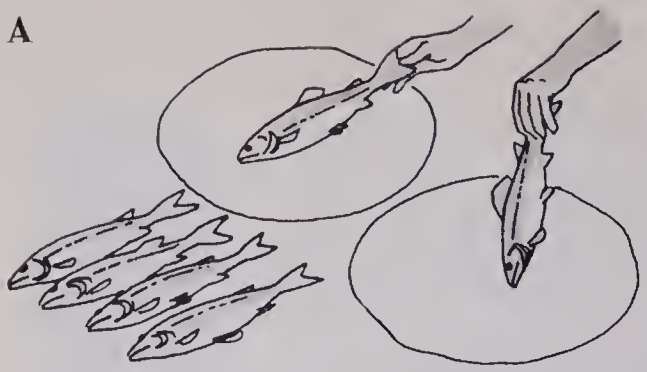
7. (5×4) or (4×6)
(4×6) is greater than (5×4)

8. (9×4) or (4×9)
(9×4) is equal to (4×9)

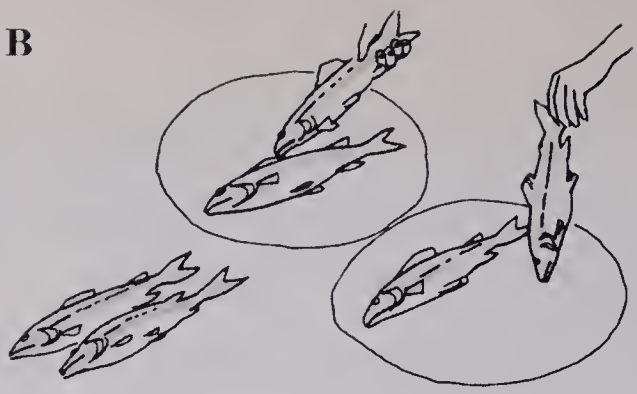
9. (4×5) or (7×4)
(7×4) is greater than (4×5)

10. (3×7) or (8×3)
(8×3) is greater than (3×7)

A



B



The Number for Each Equivalent Set

* *D. a binary operation; partitive situations* [O]

1. Ed and Tom decided to share their 6 fish equally. Ed separated the fish. How many fish did each boy get?

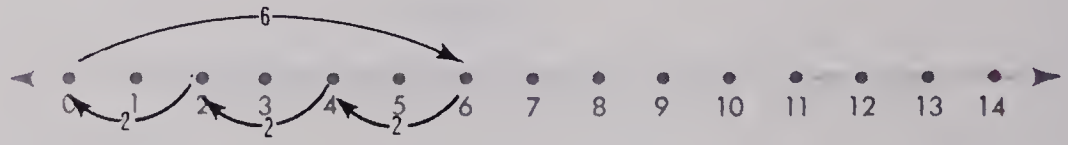
a. Ed picked up _?_ fish at a time (picture A). 2

b. How many times did he pick up 2 fish (picture B)? 3

c. In the end, how many fish did each boy get? 3

d. Find how many times 2 fish were taken away, by subtracting 2's: $6 - 2 = 4$; $4 - 2 = 2$; $2 - 2 = 0$.

e. We can subtract using the number-line picture. If we start with 6, how many times can we subtract 2? 3



f. If Ed had known that the number of fish for each set is 3 when you separate 6 fish into 2 equivalent sets, he could have said, "6 divided by 2 equals _?_." 3

2. When you separate 9 jacks into 3 equivalent sets, how many jacks are in each of the sets?

a. On the board, start as in picture C with 3 rings. Put 9 X's into 3 equivalent sets.

b. Then start with 9 and subtract 3's.

c. If you know that the number for each set is 3 when you separate 9 things into 3 equivalent sets, you can say "_?_ 9 divided by _?_ 3 equals _?_ 3."

* The operation of division is here associated with partitive situations—those in which the number of equivalent subsets is known and the number of members in each equivalent subset is to be found.

Teaching Pages 274 and 275

Pupil's Objectives

(a) To develop an understanding of the use of division in a partitive situation; and (b) to find ways of writing division examples.

New Words

(page 274) *divided*, *separated*
(page 275) *division*

Background

On pages 264 and 265 it was pointed out that different problem situations depend upon the division operation for solution. The measurement situation was described in that earlier lesson. The second situation, partitive, is presented in this lesson.

Measurement Situation. Basic question: "How many equivalent sets are contained in a larger known set?" To illustrate with numbers, "How many 4's in 12?"

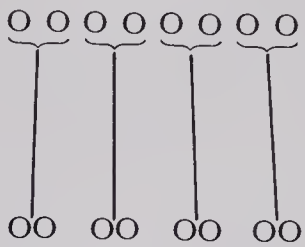
Partitive Situation. Basic question: "How many members are there in each of a given number of equivalent sets which are made from a known larger set?" To illustrate with numbers, "If a set of 12 members is separated into 4 equivalent sets, how many members will be in each set?"

In both situations described, the answer is 3. In the measurement situation, 3 tells us how many sets; while in the partitive situation, 3 tells us the number of members in each set.

Pupils are *not* to be taught the terms "measurement" and "partitive." This information is given to help you, the teacher, distinguish between situations which often are taken as synonymous merely because they may require the same operation. It is only important that pupils know there are two types of problem settings that use division and that they be able to identify them. The answer for one tells how many subsets of a given size are contained in a larger set; the answer for the other tells the number for each subset.

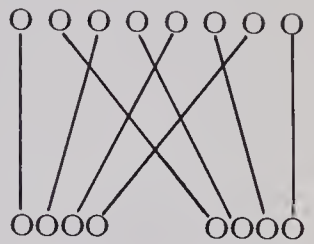
Subtraction helps us find the solution to both problem situations. Perhaps the following illustrations will clarify the two situations and may explain how subtraction may be used to find the answers.

MEASUREMENT



How many 2's in 8?

PARTITIVE



How many in each of the two equivalent subsets?

Notice that in the measurement situation we subtract by 2's since the number for each subset is known. In the partitive situation, we remove members one at a time until each is placed in either of the two equivalent subsets.

Teacher's Preparation

Have a flannel board and objects available for demonstration purposes. Each child should be provided with objects to be used at his desk.

Pre-Book Lesson

- Using objects in the room (chairs, pupils, books, and so on) and representative objects at the desks, demonstrate and review the measurement situation. Ask children to find how many 3's in 12? 2's in 10? 4's in 16?
- Have pupils show and describe "How many 2's in 8?" You may explain that the *remainder set* is the *empty set* and so the remainder for the division is 0. Then, tell them there is another kind of question that can be asked about a set of 8 members. If we separate 8 things into 2 equivalent sets, how many will be in each set?" Notice that we separate members of a set to illustrate division of the number of members in a set. This distinction is made through the series. For a partitive situation, you may wish to discuss a classroom where 8 pencils are being distributed. If we give the same number of pencils to 2 children, how many pencils will each child get?" Give the pencils to a pupil and have him work with them to show how he would find the answer. In all probability he will place one pencil at a time in each of two piles until all eight

pencils are gone. Have another child make marks on the chalkboard to show the same thing. All pupils may participate by using the representative objects at their desks.

• Repeat these activities with other classroom problems like 9 books in three piles, 12 children on 4 teams, 10 chairs at two tables. Notice that, for each of these examples, the remainder is zero. Have children illustrate each problem with marks on the chalkboard and with the objects at their desks.

Using the Text Pages

• As Ex. 1 and 2 are being discussed have pupils use the objects at their desks to illustrate the problems so as to clarify any points of confusion.

• As the oral lesson develops ask children to state a generalization in their own words about using the division operation to tell the number of members in each of a number of equivalent sets. All children understand sharing and may appreciate this work of separating a known set into a number of smaller sets each with the same number of members.

• Continue to use objects and illustrate on the chalkboard the ideas in Ex. 3–8 until you are certain the children are able to proceed independently.

• The two forms of writing a division example should both be used extensively. Both examples may be read in the same way: "8 divided by 4 equals what number?"

Individualizing Instruction

Slower learners will probably need extensive use of materials to help them in finding the solution to problems and examples. Encourage them to use the materials properly and effectively, and as they gain in maturity encourage the use of more abstract means of finding solutions.

NOTES

[W]

For Ex. 3, do these things:

a. Subtract to find the number for each equivalent set.

b. Write the number sentence you would say in dividing, like this: "6 divided by 3 equals 2."

For Ex. 4–8, work in the two ways.

3. 6 jacks, 3 sets — 6 − 3 = 3; 3 − 3 = 0
4. 9 bulbs, 3 sets — 9 − 3 = 6; 6 − 3 = 3; 3 − 3 = 0 — 9 divided by 3 equals 3
5. 8 bushes, 2 sets — 8 − 2 = 6; 6 − 2 = 4; 4 − 2 = 2; 2 − 2 = 0 — 8 divided by 2 equals 4
6. 10 bells, 2 sets — 10 − 2 = 8; 8 − 2 = 6; 6 − 2 = 4; 4 − 2 = 2; 2 − 2 = 0 — 10 divided by 2 equals 5
7. 12 flags, 4 sets — 12 − 4 = 8; 8 − 4 = 4; 4 − 4 = 0 — 12 divided by 4 equals 3
8. 10 stars, 5 sets — 10 − 5 = 5; 5 − 5 = 0 — 10 divided by 5 equals 2

When you know the number for a total set and how many equivalent sets there are, you can divide to find the number for each of the equivalent sets.

Two Ways to Write a Division Example

[O]

We can write division (D.) examples in the two ways shown in Ex. A and B. The signs ÷ and) mean to divide.

You read both Ex. A and Ex. B the same way: "8 divided by 4 equals 2."

A

$8 \div 4 = 2$

B

$4\overline{)8}$ (2)

Tell how to read each of Ex. 1–4.

1. $25 \div 5 = 5$ — 25 divided by 5 equals 5.
2. $27 \div 9 = 3$ — 27 divided by 9 equals 3.
3. $2\overline{)18}$ (9) — 18 divided by 2 equals 9.
4. $6\overline{)24}$ (4) — 24 divided by 6 equals 4.

[W]

Subtract to find each answer. Then write the division example with its answer two ways, as in Ex. A and B.

5. $10 \div 2$ 5
6. $3\overline{)6}$ 2
7. $5\overline{)15}$ (3)
8. $4\overline{)12}$ (3)
9. $18 \div 6$ 3
10. $7\overline{)21}$ (3)
11. $9 \div 3$ 3
12. $2\overline{)8}$ (4)
13. $32 \div 8$ 4
14. $16 \div 4$ 4

You could have written the answers above without subtracting, if you had known the division (D.) facts.

What Number for Each Set?

Partitive situations [W]

1. There are 12 birds distributed equally among 4 cages. How many birds are in each cage? 3

a. Find the answer for this problem by subtracting. 12 − 4 = 8; 8 − 4 = 4; 4 − 4 = 0

b. Write the D. example and its answer two ways, using ÷ and $\overline{)}$. 12 ÷ 4 = 3 $4\overline{)12}$ 3

For Ex. 2–9, work as you did in Ex. 1. First subtract and then write the D. example with its answer two ways.

2. If 21 boards were in 7 equivalent piles, how many boards would be in each pile? 3 boards

3. Eighteen sheep were put in 6 pens, the same number in each pen. How many sheep were in each pen? 3 sheep

4. Twenty-four people rode in 6 cars. If there were the same number in each car, how many were in a car? 4 people

5. If you put 24 cupcakes in 4 boxes, the same number in each box, how many cupcakes will there be in a box? 6 cupcakes

6. Patty distributed 12 pancakes equally among 3 plates. How many pancakes were on each plate? 4 pancakes

7. Thirty-two boards were put in 8 equivalent piles. How many boards were in each of the piles? 4 boards

8. Mrs. Moore had 28 candles. She put the same number on each of 4 cakes. How many were on a cake? 7 candles

9. If 20¢ is shared equally by 5 girls, how much money will each girl have? 4¢

Pupil's Objectives

(a) To solve problems having partitive situations; and (b) to practice using two different ways of writing division examples.

New Words

among, *distributed*

Background

In the last lesson the word *sharing* was used in connection with partitive situations. Either division example, $2\overline{)12}^{\,n}$ or $12 \div 2 = n$, may be used as a mathematical model. There is nothing about the model to suggest which division situation is involved. A division problem, however, can involve either a measurement or a partitive situation —one or the other, but never both. While the computation for both situations is the same, the answers mean different things. These differences were discussed in the previous lesson.

Pre-Book Lesson

- Ask three children to stand in front of the class. Give another child 12 pencils. Ask the child to count the pencils and then to make a division problem about the pencils and the children.
- Have a child go to the chalkboard and write a division example to go with the problem. Ask a different child to write the example in another way for the same problem. (Be sure to have both forms of the division example written on the board.)

On the board, have a child show how the problem may be solved by subtracting. See if his answer agrees with the one found through the demonstration at the front of the class.

Using the Text Page

- Refer to the Pre-Book Lesson in which subtraction was used to solve the problem. Also, refer to the two ways of writing division examples. Tell pupils that these forms will be used as models for each of the problems on page 276.
- You may need to read each problem for pupils having reading difficulties.

Individualizing Instruction

- Permit *slower learners* to use representative objects to assist in the solution of problems. Some pupils may find drawings satisfactory to aid them in seeing and understanding problems.
- Use as many classroom situations as possible which involve the partitive idea in division within limits of what has been developed thus far.
- Ask pupils to cite personal experiences in which a partitive situation was involved.
- *All pupils* (perhaps not the *slower learners*) may use the following information to write problems showing partitive situations.

1. 20 apples, 4 baskets
2. 12 birds, 3 cages
3. 18 cows, 2 pens
4. 8 wheels, 2 carts
5. 24 people, 4 cars
6. 32 boxes, 4 colors
7. 6 children, 2 wagons
8. 15 sticks, 3 piles
9. 21 candles, 3 cakes
10. 8 gumdrops, 4 children

Looking Ahead

After work on page 276, stress the observation that you can find the answer for such problems by subtracting, but it is much quicker to know the division facts so you can write the answers at once. In the very near future your pupils will begin to learn the division facts involving many different numbers.

Pupil's Objectives

(a) To have practice in solving problems using the operations of addition, subtraction, and multiplication; and (b) to identify unnecessary information in a problem.

New Words

cheaper, *fertilizer*, *hoe*

Background

The problem-solving facet of the mathematics program must not be neglected. The daily activities and experiences of your pupils provide opportunities for applications of many kinds. Always be on the alert to any situation which will extend or enhance learning in the regular program. If local problems are more interesting or appropriate, by all means use them freely.

Pre-Book Lesson

• Review with pupils the circumstances in which each of the operations addition, subtraction, and multiplication are used. Have pupils state problems if possible.

• Discuss with pupils that frequently information is given that is not necessary for the solution of the problem. State a problem which includes extraneous information. Ask pupils to identify the information that is important and that which is unnecessary.

Using the Text Pages

• Ask different pupils to read each problem. Discuss the information given for the problem. Have pupils tell which information is relevant and which is irrelevant. Have pupils tell why the information is or is not needed. Finally, have pupils tell what operation should be used and why.

• Following the discussion, all pupils should write the work for each problem.

Individualizing Instruction

• Below are given extra problems, which may be used with your *slower learners* or with the entire class if you wish.

a. To make pens for his 10 rabbits, Don got all the wood he needed from 4 boards each 12 ft. long. The total length of the boards was _ ? _ ft.

b. Don worked 2 hours on Monday, 2 hours on Tuesday, and 4 hours on Saturday to make the 3 rabbit pens. In all, how many hours did he spend on this job?

c. Four different kinds of vegetables cost Don 78¢ instead of the $1.25 he expected to pay for rabbit food. How much less than he expected did the food cost?

d. Don sold 17 rabbits for 75¢ each. The boy next door bought 3 of the rabbits. How much did he pay?

• *More capable children* may write an *n*-sentence for each problem.

Reminder

A wealth of data surrounds us in our daily activities. The gathering and organization of data into some useful form can be very interesting and rewarding. Continue to urge pupils to prepare charts, tables, and so on. These are quantitative experiences which are being explored in other facets of the school program as well.

* As in everyday situations, it is important for pupils to be able to choose only necessary data for solving problems.

Problems for Careful Readers

*Discovering extra data; A., S., M. [O]

1. One day Don worked 35 minutes. The next day he worked 45 minutes. How many minutes did he work these 2 days? 2 80 minutes (Ans.)

Which number for problem 1 is not needed?

For each problem tell what number, if there is one, you do not need in order to find the answer.

2. Don's father, Mr. Call, gave him $4.50 to buy a hoe. Don saw two hoes, one for $2.57 and one for $3.29. How much less did one hoe cost than the other? $4.50 $0.72 (Ans.)

3. Don bought the cheaper hoe for $2.57. How much of the $4.50 did he have left? $1.93 (Ans.)

4. Mr. Call bought 3 bags of fertilizer at $1.19 a bag and seeds for 90¢. How much did the fertilizer cost? 90¢ $3.57 (Ans.)

5. Mr. Call had with him a $5.00 bill and four $1.00 bills. If he used the $5.00 bill to pay for the things costing $4.47, what was his change from the bill? $1.00 $0.53 (Ans.)

6. One kind of rosebush sold for $1.25 and another kind for $1.69. How much were 3 of the cheaper bushes? $1.69 $3.75 (Ans.)

7. Don is paid 45¢ an hour and his little brother, 30¢ an hour. How much did Don earn for working 4 hours? 30¢ $1.80 (Ans.)

[W]

Now write your work for problems 1–7.

◆ **Extra Problems.** Work Set 13.

A

How Many Equivalent Sets?

* *Division; measurement situations* [O]

1. Billy is bunching carrots for his father to sell. He puts 4 carrots in each bunch. From 12 carrots how many bunches can Billy make (picture A)?

a. Billy took 4 carrots at a time. How many equivalent sets of 4 are there in 12?

b. To find the answer, you can subtract. $12 - 4 = 8$; $8 - 4 = 4$; $4 - 4 = 0$. How many 4's in 12?

c. If you know, without subtracting, that there are three 4's in 12, you can say, "12 divided by 4 equals _?3."

B

2. How many bunches of 7 onions each will 14 onions make (picture B)?

a. Start with 14. How many 7's can you subtract?

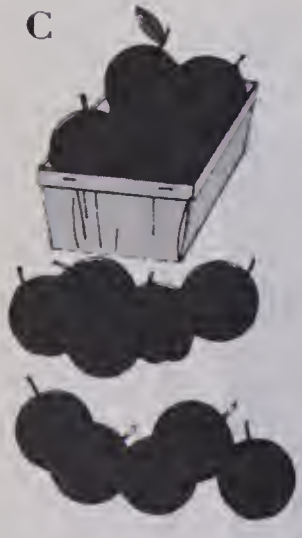
b. It is quicker to know and *think*, "14 divided by 7 equals _?2," without subtracting.

3. Billy put 5 large apples in each small basket. For 15 apples how many baskets did he need (picture C)?

a. Start with 15. How many 5's can you subtract?

b. So, 15 divided by 5 equals _?3.

C



In Ex. 1–3 you found how many equivalent sets. You subtracted, but it is quicker to divide, if you know the D. facts you need. In Ex. 1–3, you could have used $12 \div 4 = 3$, $14 \div 7 = 2$, and $15 \div 5 = 3$.

When you know the number for a total set and the number for each of the equivalent sets, you can divide to find how many equivalent sets there are.

* The operation of division is here associated with measurement situations—those situations in which the number of members in each equivalent subset is known and the number of equivalent subsets is to be found.

Teaching Pages 278 and 279

Pupil's Objectives

(a) To further use division in measurement situations; and (b) to extend understanding about division in general.

New Word: (page 278) *bunching*

Background

The difference between situations involving measurement and partitive ideas was discussed in previous material. These two pages present many different problem situations in which the measurement idea occurs. The basic question in all measurement situations may be stated as "How many 5's in 15?" We are trying to determine the number of equivalent subsets contained in a larger set. In this lesson you will note the use of "15 divided by 5 equals what number?" as the way of stating a division example.

The basic division facts are limited to the division of just certain whole numbers by other whole numbers. Earlier we learned that the product of any two whole numbers would always be a whole number. However, the quotient of two whole numbers is not always a whole number. Therefore, we say that the set of whole numbers is not closed with respect to division. Consequently, teachers must be especially alert to the division situations in the early stages of learning division facts to make sure that the answer for a division is not a fraction number. The children are not prepared to work within the set of fraction numbers and so the problems constructed should have a whole-number answer.

Using a number line has been suggested as a means of illustrating a division example. Actually, division cannot be illustrated as such on a number line, but the idea of a series of equal subtractions can. In effect, the subtractions shown on a number line indicate the number of equivalent subsets in the larger set. For example if, from 32, groups of 4 children are to go to the teacher's desk the number of 4's in 32 can be shown on a number line. When the division facts are mastered, pupils have arrived at a level of maturity whereby they can perform the division operation on certain whole-number pairs to yield a unique third whole number. Division, like the other operations learned thus far, is a binary operation.

Pre-Book Lesson

- Select a local problem such as:

There are 12 children in this group. If we make relay teams with 4 on a team, how many teams will there be?

- Dramatize the problem. Have 12 children come to the front of the class. Ask all children to count the number of children standing. Make a point that this is known information—the size of the entire group to begin with.

Then, have different pupils tell how they would determine the number of teams with 4 people on a team. Again, stress that the number of pupils on each team is known information.

In this type of problem we are to find the number of teams as we ask, "How many 4's in 12?"

In all probability, children will ask 4 pupils in the group of 12 to move to another location. Then, they will ask 4 more to move. Thus, sets of 4 pupils will be easily seen and readily counted.

Emphasize the meaning of 3 in this problem. What does the 3 tell us?

- Dramatize another classroom problem. The mathematical maturity and understanding of your pupils will determine the necessity for additional dramatizations.
- Using objects, direct all pupils to find:

a. "How many 2's in 10?"
b. "How many 5's in 15?"
c. "How many 3's in 12?"

Each time focus on the given information and the meaning of the answer.

- Say, "When we find 5's in 15, how many times can 5 members be taken from a set of 15?" Ask the same question about "3's in 12" and "2's in 10."
- Discuss with pupils the way to read a division example. $4\overline{)12}$ and $12 \div 4$ both are to be read "12 divided by 4." Do not allow terminology such as "4 goes into 12."

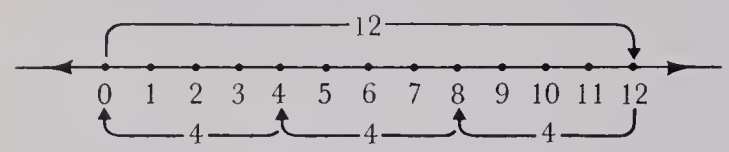
Using the Text Pages

• Read and discuss each problem on page 278. Use objects to represent the carrots, onions, and apples. Have pupils demonstrate the problem with the objects. Write on the chalkboard the work for each problem using subtraction.

Problem 1 may be illustrated by making dots as shown:

Each set of dots is encircled to show that there are three 4's in 12.

Problem 1 may be illustrated on a number line as shown.



• Your discussion coupled with the variety of ways of fostering understanding of the problem should pave the way for pupils to state in their own words the generalization at the bottom of page 278.

• In Ex. 4–11 on page 279, pupils are to make arrays to picture the division facts involved. Then they are to work the division using the two division forms introduced so far, $8 \div 4 = 2$ and $4\overline{)8}$. Pupils may use the thought pattern, "How many 4's in 8?" However, let them begin at once to use "8 divided by 4."

Teaching the alternate thought pattern can be done by asking pupils to read the example $4\overline{)8}$. If a pupil responds "How many 4's in 8?", then, ask if someone knows another way to read the same example.

Individualizing Instruction

• *Slower learners* will need to use one or more of the various ways suggested for finding the number of sets of a particular size contained in a larger set. Encourage the use of each of the following aids at one time or another.

a. Manipulate representative objects. Each subset is actually removed from the larger set.

b. Draw models of circles to represent the larger set. Then encircle each subset.

c. Draw an array in which each row of dots is one of the subsets.

d. Draw arrows on a number-line picture to show each subset removed.

• *More capable children* may use their new understandings and skills to work the following examples:

a. 8's in 32?
b. 4's in 28?
c. 9's in 36?
d. 3's in 24?
e. 4's in 36?
f. 3's in 27?

Many children will begin to see the relationship between the newly explored division facts and the multiplication facts they already know. This is not to be stressed at this time, but the child should be praised for *seeing* or *discovering* this relationship.

[W]

4. Picture D shows 8 toy tigers in rows of 4. How many sets of 4 are there?

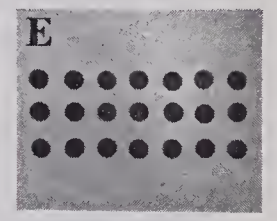
D

You can show the D. fact two ways.

Copy and finish: **a.** $8 \div 4 = ?$ 2 **b.** $4\overline{)8}$? 2

5. Picture E shows 21 dots in rows of 7. How many sets of 7 are there? 3 Show the D. fact two ways. $7\overline{)21}$; $21 \div 7 = 3$

E



Ex. 6–11. Make dot pictures. Under each picture show the D. fact in two ways.

6. 15 dots, rows of 5 $5\overline{)15}$; $15 \div 5 = 3$
7. 16 dots, rows of 4 $4\overline{)16}$; $16 \div 4 = 4$
8. 18 dots, rows of 6 $6\overline{)18}$; $18 \div 6 = 3$
9. 20 dots, rows of 4 $4\overline{)20}$; $20 \div 4 = 5$
10. 16 dots, rows of 8 $8\overline{)16}$; $16 \div 8 = 2$
11. 15 dots, rows of 3 $3\overline{)15}$; $15 \div 3 = 5$

For each of Ex. 12–17 show the D. fact in two ways. If you do not know the answers, use subtraction.

12. It takes Jim 4 minutes to read a page in his book. In 12 minutes how many pages can he read? 3 pages

13. If dolls are packaged 8 in a box, how many boxes will be needed for 16 dolls? 2 boxes

14. Jane pastes stars on a card, 3 stars in a row. How many rows can she make with 9 stars? 3 rows

15. Tom puts 2 nickels in his bank each week. How many weeks will it take to have 8 nickels in his bank? 4 weeks

16. To how many children can you give 6 sheets of paper if you have 12 sheets of paper in all? 2 children

17. Mrs. Bell uses 5 apples to make a pie. How many pies can she make from 15 apples? 3 pies

(2)

(4)

(6)

(8)

(10)

(12)

(14)

(16)

(18)

Division Facts, Dividing by 2

[W]

1. Ann and Jean are sewing 2 bows on each doll's dress. Six bows will be enough for how many dresses?

In the box find (6). Cover all the bows below (6). Now use the 6 bows to find how many 2's in 6.

Copy and finish: $6 \div 2 = ?_3$ $2\overline{)6}^{?_3}$

For each of Ex. 2–9, cover the bows you do not need. To find how many 2's in the number for each total, count by 2's the bows that show. Then show the D. fact two ways, with ÷ and with $\overline{)}$.

2. 8 bows 4 **4.** 2 bows 1 **6.** 12 bows 6 **8.** 10 bows 5

3. 16 bows 8 **5.** 14 bows 7 **7.** 18 bows 9 **9.** 4 bows 2

Division Facts Dividing by 2

$2 \div 2 = 1$
$4 \div 2 = 2$
$6 \div 2 = ?$
and so on to
$18 \div 2 = ?$

For Ex. 10, show both ways each D. fact for dividing by 2 that is needed to answer the question.

10. How many will each of 2 dolls have if Ann distributes equally between them

a. 6 bows? 3 **b.** 2 hats? 1 **c.** 4 shoes? 2 **d.** 8 dresses? 4

Copy and finish the table D. Facts, Dividing by 2.

$8 \div 2 = 4$, $10 \div 2 = 5$, $12 \div 2 = 6$, $14 \div 2 = 7$, $16 \div 2 = 8$

Teaching Page 280

Pupil's Objective

To discover and begin to learn the division facts when dividing by 2.

Background

The next six pages in the pupil's book deal with division in which 2 is a factor.

The inverse relationship between multiplication by a number and division by the same number is introduced for the express purpose of pulling together ideas learned previously and ideas now being taught.

Multiplication is defined as an operation on two numbers called factors which yields a unique third number called the product. When the two factors are known, then the product is determined by multiplication. However, if the product and one of its factors are known, then the other factor is determined by division. The lesson on page 280 deals with division facts in which 2 is the known factor. When this is the case, we are dividing by 2. The next lesson, page 281, presents division facts in which 2 is the unknown factor. In this case 2 is the answer.

Do not rush the development of the ideas and concepts on these pages. The basic principles of division are being introduced at this time and future learnings depend greatly on the establishment of depth in understanding at this time.

Pre-Book Lesson

• Under your direction each child should manipulate objects at his desk to explore various division facts for dividing by 2.

You may say, "Count out 12 discs. Now, find how many 2's there are in 12."

Do the same for 2's in 10, 14, 16, 18.

• Now say, "Count out just 12 discs. Separate the 12 discs into 2 piles each having the same number of discs. Tell how many discs there are in each pile." Do the same with other numbers like 10, 16, 14, 18.

• Discuss with pupils the possible meanings of the answer when dividing by 2. Sometimes the answer tells how many sets of 2 in a larger set. Sometimes it tells the number for each of 2 subsets of a larger set.

Using the Text Page

• For *slower learners* it may be necessary to use representative objects to find the answers to each of Ex. 2–10. Or, pupils may draw dot pictures and encircle the subsets.

• After all pupils have completed their table of division facts, have the table written on the chalkboard. Through analysis of the table and directed discussion, pupils should note the following relationships: (a) The numbers to be divided increase each time by 2; (b) The successive answers increase by 1; (c) All numbers divided are even numbers; and (d) Some answers are odd numbers and some are even numbers.

Individualizing Instruction

• *More capable children* may have some speed tests in writing the table of division facts for 2. Do not place the emphasis on speed, however, until you are confident the child understands the idea.

• *Slower learners* might receive practice in exercises similar to those shown below. It may help them gain greater understanding of the relationships mentioned in Using the Text Page.

a. Copy the following and then divide. 2's in 2; 2's in 4; 2's in 6; 2's in _ ? _; 2's in _ ? _; 2's in _ ? _; 2's in _ ? _; 2's in _ ? _; 2's in _ ? _.

b. Tell the answers. 2's in 12 = _ ? _; 2's in 8 = _ ? _; 2's in 14 = _ ? _; 2's in 16 = _ ? _.

Pupil's Objective

To discover and begin to learn the division facts in which 2 is the answer.

Background

In the D. facts shown on this page of the pupil's book, 2 is always the quotient, in contrast to the D. facts shown on page 280, where 2 is the divisor. The difference between the two sets of facts should be clear to pupils when the division is demonstrated.

Pre-Book Lesson

• Each child should have some representative objects. Direct children to count out 14 objects. Then ask them to find how many 7's in 14. Use the same approach to find how many 5's in 10; 6's in 12; 8's in 16; 4's in 8; 3's in 6; 9's in 18.

From time to time state a problem like:

How many groups of 6 chairs can be made from 12 chairs?

Use the representative materials.

• Direct children in the use of objects to find the solution to a problem situation involving the other division idea. Ask them to count out 12 objects by 1's. Then they are to determine how many will be in each of 6 groups of the same size. Do the same for separating a set of 16 objects into 8 groups of the same size; a set of 10 objects into 5 groups of the same size; and so on.

• Your pupils should soon discover that the answer to all the problems is 2. Discuss each meaning of 2 as the answer to each problem is given. Sometimes the 2 tells how many groups and sometimes it tells the size of each group.

Be sure to emphasize these two meanings of the answer. It is not necessary to use the terms *measurement* and *partitive*, but it is important to emphasize the ideas.

Using the Text Page

• Read and discuss problems 1 and 2 before assigning written work. Demonstrate these problems with representative materials and write the appropriate subtraction on the chalkboard.

• In Ex. 3–24, permit children to use any aid they wish (and need) to help in the solutions. They may (a) use representative objects, (b) draw models of circles, (c) make arrays, (d) illustrate on a number line, and (e) do subtractions.

• Be sure to have all pupils construct a table of division facts in which 2 is the answer. After all tables are completed, conduct a discussion about some relationships such as (a) Numbers being divided increase by 2; (b) The divisor increases by 1; (c) All answers are 2.

Individualizing Instruction

• *Slower learners* will need to continue extensive use of manipulative objects and a variety of aids to help build insight and understanding. They should, of course, always accompany any objective or pictorial solution to an example with the appropriate subtraction and the completed division example.

• *All pupils* would profit from a comparison and analysis of the two tables of division facts:

Division Facts Dividing by 2	Division Facts Answer 2
$2 \div 2 = 1$	$2 \div 1 = 2$
$4 \div 2 = 2$	$4 \div 2 = 2$
$6 \div 2 = 3$	$6 \div 3 = 2$
$8 \div 2 = 4$	$8 \div 4 = 2$
$10 \div 2 = 5$	$10 \div 5 = 2$
$12 \div 2 = 6$	$12 \div 6 = 2$
$14 \div 2 = 7$	$14 \div 7 = 2$
$16 \div 2 = 8$	$16 \div 8 = 2$
$18 \div 2 = 9$	$18 \div 9 = 2$

Ask children to look for similarities, relationships, and so on. In addition to the generalizations stated previously about each of the tables, you may expect answers like the following:

(a) The numbers shown in the first column of both tables are the same.

(b) The last two columns of both tables are the same but the columns are in different order.

(c) The product of the number divided by and the answer equals the number shown at the beginning of the sentence. (This idea will be developed more extensively in the next few pages.)

Division Facts, Answer 2

[W]

1. If Jean sews 3 buttons on each dress, 6 buttons will be enough for how many dresses?

Copy and finish:

$6 - 3 = 3$; $3 - 3 = ?$ 0 $6 \div 3 = ?$ 2

2. If Ann sews 4 buttons on each dress, 8 buttons will be enough for how many dresses?

Copy and finish:

$8 - 4 = {}^{4}?$; $-?^{4} - 4 = 0$ $8 \div 4 = ?$ 2

Subtract to find the answer for each of Ex. 3–11. Then show the D. fact.

3. $4 \div 2 = ?$ 2 **6.** $16 \div 8 = ?$ 2 **9.** $18 \div 9 = ?$ 2

4. $12 \div 6 = ?$ 2 **7.** $2 \div 1 = ?$ 2 **10.** $8 \div 4 = ?$ 2

5. $10 \div 5 = ?$ 2 **8.** $14 \div 7 = ?$ 2 **11.** $6 \div 3 = ?$ 2

Copy Ex. 12–21. Write all the answers you can.

12. $2\overset{2}{)}\overline{4}$ **13.** $5\overset{2}{)}\overline{10}$ **14.** $6\overset{2}{)}\overline{12}$ **15.** $1\overset{2}{)}\overline{2}$ **16.** $8\overset{2}{)}\overline{16}$

17. $3\overset{2}{)}\overline{6}$ **18.** $9\overset{2}{)}\overline{18}$ **19.** $2\overset{6}{)}\overline{12}$ **20.** $4\overset{2}{)}\overline{8}$ **21.** $7\overset{2}{)}\overline{14}$

For each of Ex. 22–24 show the D. fact two ways.

How many dimes will be in each set if you separate

22. 8 dimes into 4 equivalent sets? 2

23. 4 dimes into 2 equivalent sets? 2

24. 6 dimes into 2 equivalent sets? 3

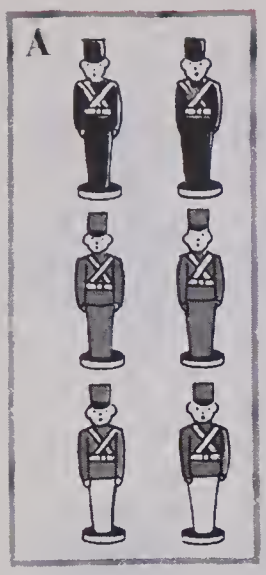
Write the table D. Facts, Answer 2.

8 ÷ 4 = 2
10 ÷ 5 = 2
12 ÷ 6 = 2
14 ÷ 7 = 2
16 ÷ 8 = 2

Division Facts Answer 2
$2 \div 1 = 2$
$4 \div 2 = 2$
$6 \div 3 = ?$
and so on to
$18 \div 9 = ?$

● **Extra Activity.** Work Set 104.

* A good summary of the two problem situations studied which require division!



Pairs of Division Facts

Oral work with D. facts having a factor 2 [O]

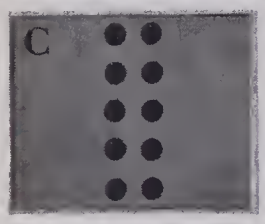
1. Box A shows Jim's 6 soldiers in equivalent sets.

a. How many are in each of 2 columns? $6 \div 2 = ?$ 3

b. How many are in each of 3 rows? $6 \div 3 = ?$ 2

$6 \div 2 = 3$ and $6 \div 3 = 2$ are a pair of D. facts. If you know one fact, you know the other.

2. Using rows and columns of dots, what pair of D. facts can you give 8÷4=2, 8÷2=4 a. for box B? 10÷2=5, 10÷5=2 b. for box C?



3. Does $4 \div 2 = 2$ have a D. fact to go with it? No Why? The number of equivalent sets and the number for each set are equal.

Most division facts go in pairs.

Tell each D. fact and the one that goes with it.

12÷6=2
4. $12 \div 2 = ?$ 6
10÷2=5
5. $10 \div 5 = ?$ 2
14÷7=2
6. $14 \div 2 = ?$ 7

16÷2=8
7. $16 \div 8 = ?$ 2
18÷9=2
8. $18 \div 2 = ?$ 9
8÷2=4
9. $8 \div 4 = ?$ 2

10. $7\overline{)14}$ 2
11. $3\overline{)6}$ 2
12. $2\overline{)10}$ 5

13. $9\overline{)18}$ 2
14. $2\overline{)16}$ 8
15. $6\overline{)12}$ 2

* Using Division for Problems

Partitive and measurement situations [O]

a. If 10 girls are in pairs, how many sets of 2 girls are there? 5

b. If 10 girls are separated equally into 2 sets, how many girls are there in each set? 5

1. Is problem a about equivalent sets? Yes ^ Is problem b? Yes v

2. In which problem do you find how many equivalent sets there are? a. Is the number sentence $10 \div 2 = 5$? Yes

3. In which problem do you find the number for each of the equivalent sets? b. What is the number sentence? 10÷2=5

Teaching Pages 282 and 283

Pupil's Objectives

(a) To learn about pairs of division facts; (b) to work problems in which both division situations are used; (c) to begin to learn the inverse relationship between division by a number and multiplication by that number.

Background

In studying the table on page 280 and that on page 281, the more adept pupil will intuitively sense that D. facts seem to go in pairs. Furthermore, all children have learned that there are pairs of addition, subtraction, and multiplication facts. Now pairs of division facts will be introduced.

It has been established intuitively that the operations of addition and multiplication are commutative but that subtraction is not commutative. Division, like subtraction, is not commutative. We cannot change the order of the numbers in a division and leave the result unchanged; that is, $8 \div 2 \neq 2 \div 8$. Whenever the opportunity arises, time could be well spent discussing the division operation and its properties. As children progress to new sets of numbers it will be important to consider which operations on each set of numbers are commutative.

In the first pages of this book, addition and subtraction of a number were introduced as inverses. Pupils learned that the addition of a number could be *undone* by the subtraction of the same number. At that time the terms "addend" and "sum" were introduced and were used to refer to numbers shown in both addition and subtraction sentences.

On page 283 multiplication and division by a number are introduced as inverses. The idea of inverses in a division sentence can be better understood as one notes that the same terms are used to identify the numbers shown in a division sentence as those shown in a multiplication sentence. Slowly, and carefully, children should begin to form the generalizations (a) When two factors are known we multiply to find the product; and (b) When a product and one of its factors are known we divide to find the unknown factor. As children become more mature in the development of their mathematical understandings they should arrive at the inverse idea—that we can divide by the factor multiplied, arrive at the other factor, and thus *undo* the multiplication by the first factor.

Pre-Book Lesson

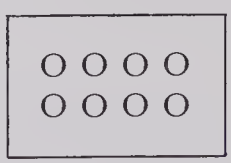
• The material at the top of pupil's page 282 follows logically from the material on pages 280 and 281. Therefore, a careful review of the important ideas presented on those two pages should be carried out. An excellent review would be to show both sets of division facts on the chalkboard and then analyze the two tables for similarities and differences. If pupils still have the sets of tables they prepared in connection with the two lessons concerned, it would be a good opportunity to use them for discussion and review.

• The analysis of the two tables of division facts in which 2 is a factor should result in the discovery that each number being divided is the product of the number divided by and the answer. Examine many of the division sentences in the two tables to see this. This idea of inverses is clearly brought out in the material at the bottom of page 283.

Using the Text Pages

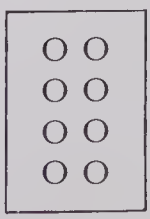
• Make extensive use of the arrays to show the pairs of division facts. A very effective way of involving pupils in the construction of arrays to help with this concept is shown below:

a. On a piece of construction paper use discs (or other manipulative objects) to make an array which shows *4's in 8.*



4's in 8 = 2
or
8 ÷ 4 = 2

b. Very carefully turn the construction paper so that the array shows a different D. fact. Have pupils tell about the array now.



2's in 8 = 4
or
8 ÷ 2 = 4

• Repeat the activity as long as you consider it necessary to establish the idea. Use it in connection with Ex. 1–15 at the top of page 282.

• The problems at the bottom of page 282 and the top of page 283 point out the different situations requiring division. Make it clear that the division fact is used to find the answer in both situations.

a. How many equivalent sets in a total set?

b. How many members in each of several equivalent subsets of a total set?

Pupils will have difficulty determining the kind of question asked in the problem. Draw pictures on the chalkboard and have pupils use objects at their desks to help them recognize the type of question. Then, the appropriate division sentence must be written.

• You will probably need to make extensive use of written work on the board in connection with the material at the bottom of page 283. Write on the board: $4 \times 3 = 12$. Ask pupils to tell the terms for each of the numbers shown in a multiplication sentence. Stress the idea that numbers multiplied are called *factors* and the answer is called the *product*.

Now write $12 \div 3 = 4$ on the board. Ask pupils if they think the same terms could be used for the numbers shown in this sentence.

Have pupils write the sentences in Ex. 5–12 on the board and then tell which numeral in the sentence shows a product and which numerals show factors.

Individualizing Instruction

• *Slower learners* may use a piece of construction paper on which to make arrays with discs to show the two division facts in Ex. 4–15 at the top of page 282. You may wish to have pupils identify each division fact shown on the construction paper.

• *All pupils* may write division sentences to go with problems 4–7 on page 283.

• *More capable children* may identify the related fact for each of Ex. 5–12 at the bottom of page 283.

• *More capable children* may make diagrams like the one at the bottom of page 283 for each of Ex. 5–12.

• Pupils may work with a partner. One child uses manipulative objects and finds the solution to questions asked by the other child. Children should use the tables of division facts they have made as a basis for asking questions like, "How many 2's in 14?" or "How many 6's in 12?" Each question should be one which uses for its solution a division fact having 2 as a factor.

NOTES

* Use considerable time in analyzing the factors-product relationship and the use of the terms factor and product for both multiplication and division.

Oral work in problem-solving

Tell the number question asked and the D. fact to use. For Ex. 4 say, "How many sets of 2? $12 \div 2 = 6$." For Ex. 5 say, "How many in each of 2 sets? $8 \div 2 = 4$."

4. Joan carried 12 plates from the table, 2 at a time. How many trips did she make?

5. She carried away 8 cups in 2 trips, the same number each time. How many cups did she take each trip?

6. Jack has 16 spelling words in 2 lists, with the same number in each. How many words are in each list? How many in each of two sets? $16 \div 2 = 8$

7. Mr. Hook set out 18 lettuce plants, 9 in each row. How many rows of lettuce plants did he have? How many sets of 9? $18 \div 9 = 2$

Can a Division Undo a Multiplication?

Inverses; factors-product relationship [O]

1. Adding a number can be undone by subtracting that number. Can multiplying by a number be undone by dividing by that number? Yes Study the work in the box.

2. When we multiply two factors, we get a _?_. product

3. When we divide, why can we say that the number we divide is the product? It is the product of the two factors.

4. When we divide, why can we say that the answer and the number we divide by are factors? They were multiplied to get the product.

For each example tell the product and its factors.

5. $4 \times 6 = 24$ factors: 4 and 6 product: 24

6. $18 \div 2 = 9$ factors: 2 and 9 product: 18

7. $5 \times 3 = 15$ factors: 5 and 3 product: 15

8. $10 \div 2 = 5$ factors: 2 and 5 product: 10

9. $9 \times 4 = 36$ factors: 9 and 4 product: 36

10. $5 \times 2 = 10$ factors: 5 and 2 product: 10

11. $6 \div 3 = 2$ factors: 3 and 2 product: 6

12. $8 \times 3 = 24$ factors: 8 and 3 product: 24

* factor factor product — product factor factor

$2 \times 7 = 14 \longleftrightarrow 14 \div 2 = 7$

* Now pupils may be guided to realize that a set of M. and D. facts may be associated with three numbers just as was the case for sets A. and S. facts.

A

Discovering Sets of M. and D. Facts

2 as a factor; use of arrays [O]

You know there are sets of A. and S. facts. Most of them have four facts. Are there sets of M. and D. facts?

1. Picture A shows 8 apple trees Dick's father set out on the farm. There are

a. _?_ (4) rows, with _?_ (2) trees in a row.

b. _?_ (2) columns, with _?_ (4) trees in each column.

Use the rows and columns of trees for c.

c. Finish the pairs of M. facts and D. facts.

M. facts: $4 \times 2 = ?$ (8) $\quad 2 \times 4 = ?$ (8)

D. facts: $8 \div 2 = ?$ (4) $\quad 8 \div 4 = ?$ (2)

*2. Tell the facts that make the set of M. and D. facts about the product 8 and its factors 4 and 2. $4 \times 2 = 8$ $8 \div 4 = 2$ $2 \times 4 = 8$ $8 \div 2 = 4$

B

3. Picture B. Dick's father set out 10 more apple trees in _?_ (2) rows, with _?_ (5) trees in each row.

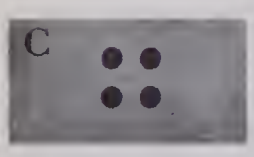
a. What two factors of 10 can you find? (2 and 5)

b. Finish the set of M. and D. facts about the product 10 and these factors.

M. facts: $2 \times$ _?_ (5) $= 10 \quad 5 \times$ _?_ (2) $= 10$

D. facts: $10 \div 2 = ?$ (5) $\quad 10 \div 5 = ?$ (2)

C



4. Picture C. What two factors of 4 can you find?

M. fact: $2 \times$ _?_ (2) $= 4 \quad$ D. fact: $4 \div 2 = ?$ (2)

When the two factors of a product are equal, the set of M. and D. facts has only one M. fact and one D. fact.

When the two factors are not equal, there are two M. facts and two D. facts in the set.

Teaching Pages 284 and 285

Pupil's Objective

To discover sets of multiplication and division facts through work with sets of M. and D. facts in which 2 is a factor.

Background

This lesson completes the building of the basic generalizations regarding the inverse relationship between multiplication and division by a number. You should reread the background information for the preceding three lessons if you are uncertain as to these relationships. The generalizations to be developed in this lesson are stated at the bottom of page 284, and are illustrated below.

9; factors 3 and 3	$3 \times 3 = 9; 9 \div 3 = 3$
8; factors 2 and 4	$2 \times 4 = 8; 4 \times 2 = 8$
	$8 \div 2 = 4; 8 \div 4 = 2$

Pre-Book Lesson

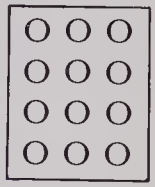
• On the chalkboard, write the numerals 3, 4, 7. Ask pupils to tell an addition sentence that may be written with these numerals. Write it on the chalkboard. Then, ask if there is any other addition sentence that may be written with these numerals. Write it on the board. Use the same questioning to have pupils give two subtraction sentences. Finally you have four facts shown on the board, two addition facts and two subtraction facts.

Bring out in discussion that there is a set of four A. and S. facts for the particular set of numbers shown. Ask if there is any set of numbers for which there is only 1 A. fact and 1 S. fact. Pupils should choose numbers such as 4, 4, and 8.

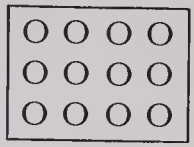
On the chalkboard, write the numerals 2, 3, 6. Ask pupils to tell and then show on the board two M. facts and two D. facts using the numbers shown. Bring out in discussion that there are four facts that make up this set of M. and D. facts. Then, write the numerals 2, 2, 4. Have pupils tell the related M. and D. facts and determine how many facts are in this set.

• Write and discuss as many different sets of M. and D. facts as you consider to be necessary for the idea of the number of facts in a set to be firmly established in the pupils' minds.

• You may find it appropriate to form arrays on a large classroom flannel board.* For example, show the array for four 3's. This would be the same as the array for 3's in 12. Have children tell the M. fact and D. fact that the array demonstrates.



Turn the flannel board to show a different M. and D. fact. This reveals that there are three 4's in 12. Ask pupils to tell the two facts demonstrated by this array.



As children compare the two arrays and see their likeness, it will help them realize that the M. and D. facts shown are part of the same set.

Using the Text Pages

• The oral work for Ex. 1–10 should be accompanied by extensive use of manipulative objects as well as the pictures and arrays in the child's book. Many times throughout the discussion, have pupils state in their own words the generalizations at the bottom of page 284.

• For *slower learners* who need to do so, encourage them to reconstruct arrays for the written exercises 11–19. The use of arrays on a piece of construction paper would be most valuable.

• If you have available from previous lessons the tables of divisions facts in which 2 is a factor, it would be good to compare those tables with the tables at the bottom of page 285.

Have pupils find the example in the table on page 285 that goes with the one selected from their own tables.

Individualizing Instruction

• *Slower learners* may work with partners to study the D. facts shown in the tables at the bottom of page 285. One pupil can hold the book and ask questions such as "How many 2's in 12?" "How many 8's in 16?" and so on. Since the answer is

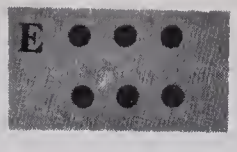
* See 2, page xix.

recorded in the book this pupil will be able to check his partner's work. The pupil being questioned may use objects to find the answer he does not know. Or, he may use any aid he wishes. Pupils should alternate asking each other questions.

• *More capable children* should find exercises like those below more challenging.

$12 \div n = 2$	$n \div 4 = 2$
$16 \div n = 8$	$n \div 2 = 7$
$10 \div n = 2$	$n \div 6 = 2$

NOTES



5. Say the set of M. and D. facts for each of D–F.

Say the set of M. and D. facts for product

6. 10; factors 2 and 5. 7. 16; factors 8 and 2.

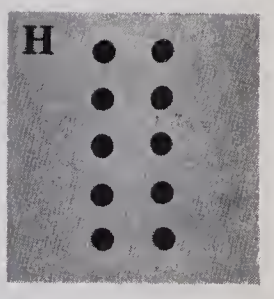
Finish each set of M. and D. facts for Ex. 8–10.

8. $14 \div 7 = 2$ 9. $9 \times 2 = 18$ 10. $2\overline{)4}$

Show the set of M. and D. facts you can make

11. from picture G. 12. from picture H.

13. Make a dot picture for the product 8 and its factors 2 and 4. Show the set of M. and D. facts.



Show the set of M. and D. facts for product

14. 6; factors 2 and 3. 15. 14; factors 7 and 2.

Copy and finish Ex. 16–19. Under each, show the other facts in the same set of M. and D. facts.

16. $2 \times 6 = ?_{12}$ 17. $9 \times 2 = ?_{18}$ 18. $4\overline{)8}$ 19. $5\overline{)10}$

Be sure you know each D. fact. Cover the answers and say the facts. Do not take them in order.

Division Facts, Dividing by 2								
$2\overline{)2}$	$2\overline{)4}$	$2\overline{)6}$	$2\overline{)8}$	$2\overline{)10}$	$2\overline{)12}$	$2\overline{)14}$	$2\overline{)16}$	$2\overline{)18}$
Division Facts, Answer 2								
$1\overline{)2}$	$2\overline{)4}$	$3\overline{)6}$	$4\overline{)8}$	$5\overline{)10}$	$6\overline{)12}$	$7\overline{)14}$	$8\overline{)16}$	$9\overline{)18}$

* Pupils have been differentiating among two, then three, and now four operations for solving problems. Again, the introduction is oral.

Try These

[W]

Write the answers on folded paper.

	a	b	c	d	e	f
1.	$2\overline{)16}^{8}$	$6\overline{)12}^{2}$	$2\overline{)10}^{5}$	$2\overline{)4}^{2}$	$4\overline{)8}^{2}$	$3\overline{)6}^{2}$
2.	$5\overline{)10}^{2}$	$2\overline{)2}^{1}$	$2\overline{)12}^{6}$	$8\overline{)16}^{2}$	$9\overline{)18}^{2}$	$1\overline{)2}^{2}$
3.	$7\overline{)14}^{2}$	$2\overline{)8}^{4}$	$2\overline{)18}^{9}$	$2\overline{)6}^{3}$	$2\overline{)14}^{7}$	$8\overline{)16}^{2}$

Make Study Cards for facts you do not know. Use them for practice. Then write your answers again for rows 1–3.

*A., S., M., or D. — Which?

Oral problem-solving [O]

1. The 10 buffaloes in the picture are shown in pairs. How many pairs are there? 5

If you had no picture, could you find the answer by subtracting? Yes by dividing? Yes Which way is quicker? Dividing

2. If 6 of the 10 buffaloes run off, how many will be left? 4

Can you find the answer by dividing? No Why, or why not? Subtract to find number for other part.

For each of problems 3–9 tell if you should add or subtract or multiply or divide, and tell why.

3. Indian hunters saw one group of 6 buffaloes and another group 2 times as large. How many buffaloes were in the larger group? Multiply to find number for total because sets are equivalent. 12 buffaloes

Teaching Pages 286 and 287

Pupil's Objectives

(a) To have written practice on division facts learned; (b) to have oral practice in analyzing and solving problems in which one of the four operations A., S., M., or D. may be used; and (c) to have practice in mixed computational examples.

New Word

(page 287) *rugs*

Background

For the first time in this book, a set of problems includes work with all four operations—addition, subtraction, multiplication, and division. Pupils have encountered a set of problems in which just one operation was used to solve all problems. In other lessons pupils found it necessary to differentiate and select the appropriate one of two or three operations. But now they must read carefully and select from among four different possible operations.

The problems should be analyzed orally with the pupils. Adequate time should be given to discussing and analyzing the problems to permit each pupil to clarify in his mind the difference among all types of problems.

Pre-Book Lesson

• Pupils have had some previous experiences in differentiating between A., S., and M. In this activity pupils must also be able to distinguish between the need for S. or D. in a problem situation.

On the chalkboard write problems similar to *a* and *b* below.

a. Joan cut out 8 pictures and pasted 2 in her scrapbook. How many pictures did she still have to put in the scrapbook?

b. If Joan pastes 2 pictures on a page, how many pages in the scrapbook will the 8 pictures fill?

Give a child 8 objects to represent the pictures (slips of paper, and so on) and have him dramatize the situation in problem *a* and find the answer. Write the arithmetic sentence under the problem and write the letter S. after it.

Next, have problem *b* represented with objects. Write two completed D. examples on the board with the letter D. after them.

Compare the two problems and the two solutions. Guide your pupils to see that in problem *a* only one set of 2 is taken from the set of 8; therefore, subtraction is the appropriate operation. In *b*, on the other hand, as many sets of 2 as possible are taken from the set of 8, and division is the appropriate operation.

• You may wish to do a dramatization and analysis of two other problems before turning to the text exercises.

Using the Text Pages

• The written material at the top of page 286 may be used as a test of pupils' mastery of the D. facts having 2 as a factor. Show pupils how to fold paper so that answers may be written to division examples. In these examples they merely place the paper above the example rather than below it. Emphasize writing answers only. For *more capable children*, you may wish to make it a speed test. Do not stress speed to the detriment of accuracy.

• The oral work with the problems should stress at every opportunity the similarities in problems as well as the differences. Use pictures, representative objects, drawings on the chalkboard, and so on to dramatize each problem. Be certain that your pupils understand each problem and why a particular operation is appropriate.

• The written practice at the bottom of page 287 includes examples of A., S., and M. Note the use of *n* in some examples.

Individualizing Instruction

• *All pupils* may pick out the problems which use addition or subtraction. Ask them to write an *n*-sentence to go with the problem.

• *More capable children* may try to make problems very much like the ones studied in this lesson. If practical, suggest situations which are pertinent to current study in other facets of the curriculum. Or, suggest items of special interest to pupils.

• *Slower learners* may need further practice in

differentiating between division and subtraction problems. The following pairs may be used:

a. How many 2¢ fishhooks can Jim buy for 12¢?

b. After spending 2¢ of his 12¢ for a fishhook, how much did Jim have left?

c. Joe has to move 14 chairs. He can carry 2 chairs at a time. How many chairs will be left after Joe has made one trip?

d. Joe can carry 2 chairs at a time. How many trips must he make to carry away 14 chairs?

e. Sally used 2 small oranges to make a glass of orangeade. How many oranges were left in the bag if there were 18 oranges at first?

f. How many glasses of orangeade can be made from 18 oranges if 2 oranges are used to make each glass of orangeade?

Reminders

For the most part the material developed in the next several lessons deals with division. It is appropriate to maintain the skills already established through appropriate practice.

a. Have pupils make an addition-and-subtraction-facts chart from time to time.

b. A multiplication facts chart like the one below should be prepared by pupils for reviewing and maintaining the M. facts learned to date.

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

c. For practice in multiplying involving remembering a number of tens, the following type of practice would be appropriate:

$(4 \times 7) + 3 = n$ $\quad$ $(3 \times 5) + 2 = n$

$(2 \times 6) + 1 = n$ $\quad$ $(8 \times 3) + 3 = n$

NOTES

4. The hunters killed 5 of the 20 buffaloes in a herd. The others ran away. How many buffaloes ran away? Subtract to find number for other part. 15 buffaloes

5. After killing the 5 buffaloes, the hunters killed 7 more. How many buffaloes did they kill in all? Add to find number for total. 12 buffaloes

6. It took 2 ponies to carry all the meat from one buffalo. How many ponies were needed for the meat from 12 buffaloes? Multiply to find number for total because sets are equivalent. 24 ponies

7. Indian women made rugs from buffalo skins, 3 skins for each rug. How many skins did they use for 4 rugs? Multiply to find number for total because sets are equivalent. 12 skins

8. Ten women made some of the rugs. Six more women made the others. How many women made the rugs? Add to find number for total. 16 women

9. If they had used 2 skins for each rug, how many rugs could they have made from 8 buffalo skins? Divide to find number of equivalent sets. 4 rugs

Read problems 1–9 again, and say the answers.

To Keep in Practice

A., S., M. [W]

Copy and work each of Ex. 1–16. Check when you can.

1. 104×8 832	4. $314 - 270$ 44	7. $321 + 79$ 400	10. $9 \times \$1.10$ \$9.90
2. $600 - 78$ 522	5. $435 + 205$ 640	8. 320×3 960	11. $\$4.21 - \$3.19$ \$1.02
3. 2×447 894	6. $589 - 490$ 99	9. 6×114 684	12. $\$8.67 - \$2.62$ \$6.05

13. $22 + 35 + 8 + 16 + 20$ 101

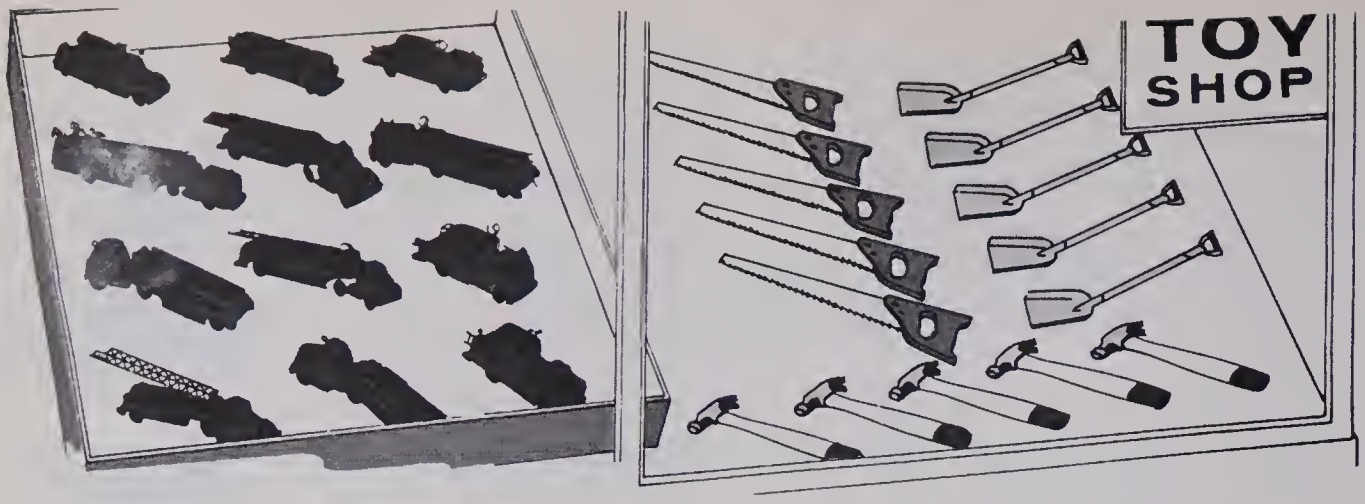
14. $8¢ + 9¢ + 5¢ + 7¢ + 8¢$ 37¢

15. $206 + 366 + 58$ 630

16. $\$0.79 + \$2.46 + \$1.19$ \$4.44

Copy each of Ex. 17–22, writing a numeral in place of n.

17. $n - 75 = 200$ 275	19. $605 + n = 665$ 60	21. $n - 463 = 291$ 754
18. $n + 188 = 311$ 123	20. $643 - n = 115$ 528	22. $4 \times 124 = n$ 496



Sets of Facts with 3 as a Factor

Use of arrays; factors-product relationship [O]

1. This big box of fire trucks is in a window at the store. How many fire trucks are in the box?

a. _?_ rows; _?_ fire trucks in each row. $4 \times 3 = n$ (4, 3)

b. The product for the M. fact is _?_. (12)

c. The factors of the product are _?_ and _?_. (4, 3)

d. If you know a product and its factors, you know a set of M. and D. facts. Finish the facts in this set:

$4 \times 3 = ?$ 12 $\quad 3 \times 4 = ?$ 12 $\quad 12 \div 3 = ?$ 4 $\quad 12 \div 4 = ?$ 3

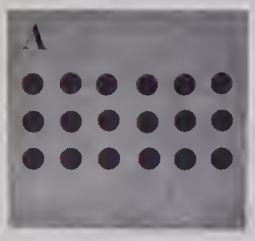
e. Say the pair of new D. facts you have found. 12 ÷ 3 = 4 12 ÷ 4 = 3

2. In the window are also these 3 sets of play tools: 5 shovels, 5 hammers, and 5 saws. How many play tools are there all together? $3 \times 5 = n$ 15

a. The product is _?_. (15) The factors are _?_ and _?_. (3, 5)

b. Say the set of M. and D. facts for this product and factors. 3 × 5 = 15 5 × 3 = 15 15 ÷ 3 = 5 15 ÷ 5 = 3

c. Say the pair of new D. facts you have found. 15 ÷ 3 = 5 15 ÷ 5 = 3



3. In box A there are 3 rows of 6 dots each.

a. How many dots are there in all? $3 \times 6 = n$ 18

b. Say the product; 18 the factors. 3 and 6

c. Say the set of M. and D. facts. 3 × 6 = 18 18 ÷ 3 = 6 6 × 3 = 18 18 ÷ 6 = 3

d. Say the pair of new D. facts. 18 ÷ 3 = 6 18 ÷ 6 = 3

Teaching Pages 288 and 289

Pupil's Objective

To gain further insight into factors-product relationships through discovery of sets of M. and D. facts in which 3 is a factor.

New Word

(page 288) *tools*

Background

The purpose of these two pages is to extend the understandings about the relationships between multiplication and division. The focus is not on the learning of division facts for 3. (This is stressed in the next lesson.)

A great deal of time should be given to the oral part of this lesson. Be absolutely certain that your pupils are building an understanding of the following relationships between multiplication and division:

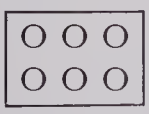
a. When two factors are known, we multiply to find the product.

b. When the product and one of its two factors are known, we divide to find the other factor.

c. There are sets of M. and D. facts. When the two factors are equal, the set of M. and D. facts has 1 M. fact and 1 D. fact. If the two factors are not equal, then there are 2 M. facts and 2 D. facts in the set. It is very important to continue to use and understand the distributive property. This property, you will recall, relates two operations. Thus, it is possible to use addition to help find the solution to a multiplication example and also to use addition to help find the solution to a division example.

Pre-Book Lesson

• Use your flannel board* (or some means of demonstrating with objects) to show an array of 2 rows and 3 columns.



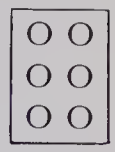
Ask "Two 3's equal what?"

"Who can write a sentence for this question?"

Be sure all pupils see the relationships between the array, the sentence, and the question.

* See 2, page xix.

• Turn your flannel board so the array is changed.



Ask: "Three 2's equal what?"

"Who can write a sentence for this question?"

• Continue to turn the flannel board and ask a question for each array referred to above.

"How many 2's in 6?"

"How many 3's in 6?"

• Finally, you should have written on the chalkboard four different sentences in which 2, 3, and 6 appear. Discuss the sentences telling ways in which they are alike and ways in which they are different. Have children tell the product and factors shown in each sentence.

• Do the same kind of demonstration for 3, 3, 9. Guide pupils to a generalization about the number of facts in the set of M. and D. facts shown.

Using the Text Pages

• A great deal of discussion should be given to the oral lesson on page 288. Use your flannel board as needed to foster greater insight into each of the problems and examples. At each opportunity relate the two new division facts to the two known multiplication facts.

• You may wish to discuss Ex. 4–5 with pupils before assigning it as written work.

• In Ex. 6, make certain your pupils know what *rows* are in forming an array. You may wish to ask pupils to draw arrays for Ex. 7 and 8.

Individualizing Instruction

• Some pupils find greater security in using objects to illustrate a particular example. You may provide pupils with discs, and so on instead of asking them to make dot arrays.

• Some *slower learners* will probably find it necessary to make an array for each example in Ex. 13–22. Perhaps you will want to conduct this as a directed oral activity instead of a written assignment. You may have *more capable children* work with *slower learners* on these examples.

• *More capable children* may check other pupils' work in Ex. 7–12.

• Continued practice with making a multiplication chart would be valuable for *all pupils*.

• The following matching game should be good drill for *all pupils*.

Cut strips of paper (construction paper, tagboard) about 2″ × 5″. Pupils may show a M. or D. fact on each strip, as at the right.

$6 \times 3 = 18$	$18 \div 3 = 6$	$24 = 3 \times 8$
	$24 \div 3 = 8$	

a. Pupils may match M. and D. facts.

b. Pupils may group their strips to show sets of M. and D. facts.

NOTES

[W]

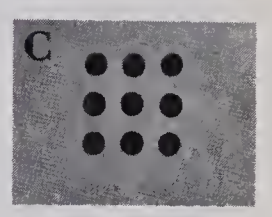
4. Box B. The M. example is $9 \times 3 = n$.

a. Show the product. 27

b. Show the factors. 9 and 3

c. Show the set of M. and D. facts for this product and factors. $9 \times 3 = 27$ $3 \times 9 = 27$ $27 \div 9 = 3$ $27 \div 3 = 9$

d. Ring the two new D. facts shown.



5. Box C. The M. example is $3 \times 3 = \overset{9}{n}$.

a. Show the product. 9

b. Show the factors. 3 and 3

c. Show the set of M. and D. facts. $3 \times 3 = 9$ $9 \div 3 = 3$

d. How many facts are there in this set of facts? 2

6. Draw 21 dots in 3 equivalent sets.

a. Show the product and the factors. 21, 3 and 7

b. Show just the pair of new D. facts. $21 \div 3 = 7$ $21 \div 7 = 3$

c. Show the set of M. and D. facts. $3 \times 7 = 21$ $7 \times 3 = 21$ $21 \div 7 = 3$ $21 \div 3 = 7$

Show the sets of M. and D. facts for product

7. 12; factors 3 and 4.
$3 \times 4 = 12$ $12 \div 3 = 4$
$4 \times 3 = 12$ $12 \div 4 = 3$

8. 24; factors 3 and 8.
$3 \times 8 = 24$ $24 \div 3 = 8$
$8 \times 3 = 24$ $24 \div 8 = 3$

Copy and finish Ex. 9–12. Then under each, show the pair of D. facts in the same set of M. and D. facts.

9. $3 \times 2 = 6$
$6 \div 3 = 2$
$6 \div 2 = 3$

10. $3 \times 6 = 18$
$18 \div 3 = 6$
$18 \div 6 = 3$

11. $9 \times 3 = 27$
$27 \div 9 = 3$
$27 \div 3 = 9$

12. $7 \times 3 = 21$
$21 \div 7 = 3$
$21 \div 3 = 7$

Some of the D. examples below have wrong answers. To check each answer, draw a dot picture. Then copy each D. example and write its correct answer.

13. $5\overline{)15}$ $\quad \overset{3}{2}$

14. $8\overline{)24}$ $\quad 3$

15. $3\overline{)12}$ $\quad 4$

16. $3\overline{)21}$ $\quad \overset{7}{6}$

17. $6\overline{)18}$ $\quad \overset{3}{2}$

18. $9\overline{)27}$ $\quad \overset{3}{4}$

19. $7\overline{)21}$ $\quad 3$

20. $3\overline{)18}$ $\quad 6$

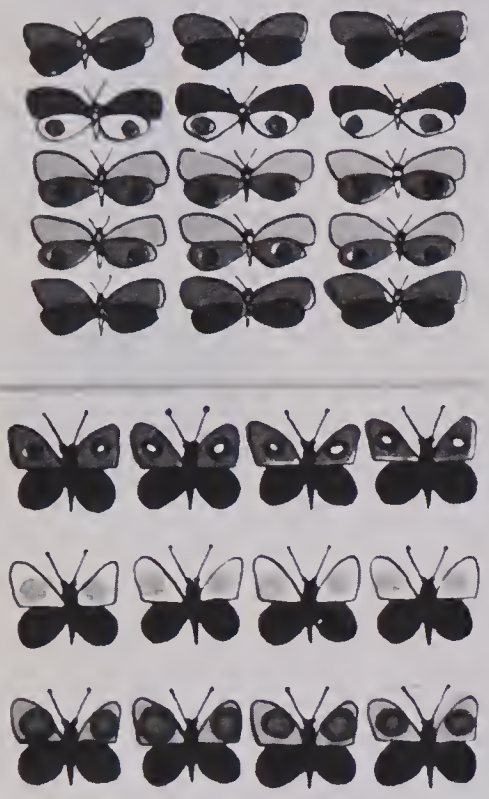
21. $3\overline{)27}$ $\quad 9$

22. $3\overline{)24}$ $\quad \overset{8}{7}$

Division Facts, Dividing by 3

[O]

Our class is painting paper butterflies in school and putting them on boards to hang on the walls.



1. Tom put his 15 butterflies in sets of 3. How many equivalent sets did they make?

a. Is the D. example, $15 \div 3 = ?$ Yes

b. If you had no picture, could you find the answer by subtracting, as at the right? Yes Explain. You may repeatedly subtract to find how many equivalent sets of 3 members.

c. Why is 5 the answer? 3 is subtracted 5 times.

15
−3
12
−3
9
−3
6
−3
3
−3
0

2. Ruth made 3 equivalent sets from 12 butterflies she painted. How many were in each set? 4

On the board use subtraction to find the answer. Then show the D. fact. $12 \div 3 = 4$

[W]

Find the answer for each of Ex. 3–8 by subtracting. Then copy and finish the D. example.

3. $24 \div 3 = ?$ (8) 4. $15 \div 3 = ?$ (5) 5. $27 \div 3 = ?$ (9)

6. $18 \div 3 = ?$ (6) 7. $9 \div 3 = ?$ (3) 8. $21 \div 3 = ?$ (7)

Division Facts Dividing by 3

$3 \div 3 = 1$

$6 \div 3 = 2$

$9 \div 3 = ?$ (3)

and so on to

$27 \div 3 = ?$ (9)

Copy and finish Ex. 9–16. To find an answer you do not know, draw a picture or subtract or use a M. fact.

9. $3\overline{)15}$ (5) 11. $3\overline{)21}$ (7) 13. $3\overline{)12}$ (4) 15. $3\overline{)27}$ (9)

10. $3\overline{)9}$ (3) 12. $3\overline{)24}$ (8) 14. $3\overline{)18}$ (6) 16. $3\overline{)6}$ (2)

17. Write the table of D. Facts, Dividing by 3.

$12 \div 3 = 4$ $15 \div 3 = 5$ $18 \div 3 = 6$ $21 \div 3 = 7$ $24 \div 3 = 8$

18. Show the new D. facts in a list.

$9 \div 3 = 3$ $12 \div 3 = 4$ $15 \div 3 = 5$ $18 \div 3 = 6$ $21 \div 3 = 7$ $24 \div 3 = 8$ $27 \div 3 = 9$

Teaching Page 290

Pupil's Objectives

(a) To extend understanding of the use of subtraction in finding the solution to division examples; and (b) to find and begin to learn the D. facts when dividing by 3.

New Word

butterflies

Background

The next four pages in the pupil's book consider all aspects of learning the division facts in which 3 is a factor. This page considers those division facts when 3 is the divisor. Each number sentence shows a product and a known factor of 3. The task is to find the unknown factor.

Pupils will find the table of D. facts far more reasonable if you, the teacher, help them to make use of the distributive idea. For $24 \div 3$, guide pupils in thinking of 24 as $15 + 9$. They will then be able to distribute their division as follows: $(15 \div 3) + (9 \div 3)$. Thinking of division as subtraction, we can say that five 3's and three 3's can be subtracted. Developing a sound understanding of the distributive property as it is applied to division provides the learner with a powerful tool with which to work any division example.

The oral exercises must be developed slowly and carefully. This will give you the opportunity to check understanding as well as correct and incorrect answers. Correct answers are important, of course, but at this time the development of ideas and understandings is equally important.

Pre-Book Lesson

- Using objects in the classroom (pupils, chairs, books, and so on) show the work for the question, "How many 3's in 12?"
- Have each pupil use objects of his own at his desk and find how many 3's in 15; 24; 9; 18; 27; 21.
- With classroom objects once more, begin with 12 objects and place them in three piles of the same size. Ask pupils to do the same thing with their own discs for 21, 18, 27, 15.
- Discuss with pupils the meaning of the answer each time they divide by 3. Sometimes the answer tells how many groups of 3. Sometimes it tells how many in each of three groups.

Using the Text Page

- In the oral work make an array on the flannel board, draw an array of dots on the chalkboard, and/or have pupils make an array with objects and dots at their desks. Be sure *all pupils* understand the ideas involved in each exercise. Ask pupils to use subtraction on the chalkboard to find the solution to the problem.
- *Slower learners* may find it more helpful to use objects to make arrays for each of the written exercises.

NOTES

Teaching Page 291

Pupil's Objective

To find and begin to learn the D. facts when 3 is the answer.

Using the Text Page

After all pupils have made the two tables of division facts in which 3 is a factor, conduct a discussion to analyze each table for important generalizations. The following generalizations should be stated by pupils in their own words.

a. In both tables the numbers to be divided increase by 3.

b. In one table the number is divided by 3 while in the other table the answer is 3.

c. In one table the answer is the same as the number divided by in the other table and increases by 1 throughout each table.

Relate the facts in the table of facts in which 3 is the divisor to the table of facts having answer 3, by working with sets of facts. Ex. 13–22 provide practice in this activity.

Individualizing Instruction

- *All pupils* may show M. and D. facts on strips of paper (see Teaching Page 289). Use them for the matching game and for finding sets of M. and D. facts.
- *More capable children* (perhaps *all pupils*) may engage in a written activity called *Name the Number*. Have each pupil select one number and write the standard numeral for that number at the top of a piece of paper. Then, below that numeral write ways of naming the number. For example, 17 may be named $(4 \times 4) + 1$, $18 - 1$, and so on.
- *More capable children* may respond to the "puzzle" element in exercises like the following:

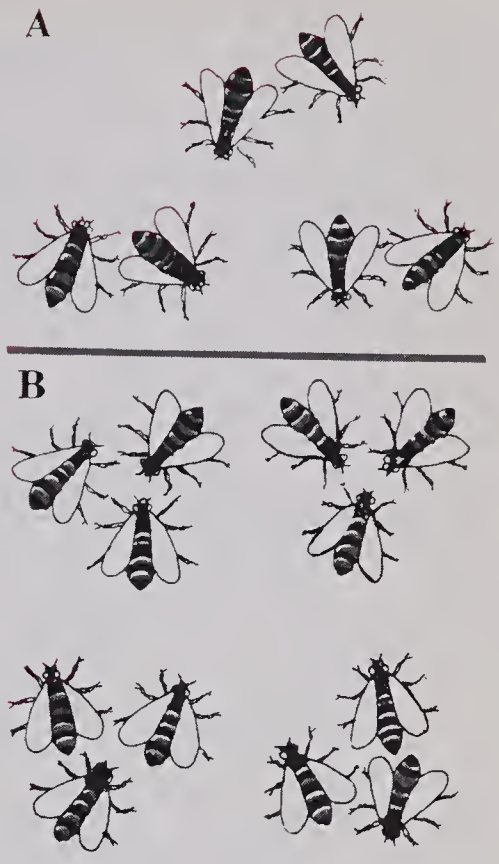
40?	44?	4?2	207
+76	×2	−127	×?
483	898	345	621
212	1?3	103	?83
×4	×5	×9	+99
8?8	565	9?7	682
624	263	4?2	7?6
−??	+??5	×2	−8?
580	438	864	694

NOTES

*As various ways of solving the problem are brought out, constantly emphasize that by knowing the D. fact, the efficient method of division can be used.

Division Facts, Answer 3

[O]



1. Picture A. 6 bees are in sets of 2. How many equivalent sets are there?
   - a. Is the D. example, $6 \div 2 = ?$ Yes
   - b. Can you subtract to find the answer? Yes
   - c. What is the answer for $6 \div 2$? 3

2. Picture B. 12 bees are in 4 equivalent sets. How many are in each set? On the board,
   - * a. write the D. example. $12 \div 4 = n$
   - * b. find the answer by subtracting. $12-4=8,\ 8-4=4,\ 4-4=0$
   - * c. draw a dot picture or use a M. fact.
   - * d. show the D. fact. $12 \div 4 = 3$

[W]

Copy and finish Ex. 3–10. To find answers you do not know, subtract or draw a dot picture or use a M. fact.

3. $3\overline{)9}$ 3	4. $9\overline{)27}$ 3	5. $7\overline{)21}$ 3	6. $3\overline{)9}$ 3
7. $6\overline{)18}$ 3	8. $5\overline{)15}$ 3	9. $8\overline{)24}$ 3	10. $4\overline{)12}$ 3

11. Write the table D. Facts, Answer 3.

12. Show the new D. facts in a list.

$12 \div 4 = 3$
$15 \div 5 = 3$
$18 \div 6 = 3$
$21 \div 7 = 3$
$24 \div 8 = 3$

$9 \div 3 = 3$ $12 \div 4 = 3$ $15 \div 5 = 3$ $18 \div 6 = 3$ $21 \div 7 = 3$ $24 \div 8 = 3$ $27 \div 9 = 3$

Division Facts Answer 3

$3 \div 1 = 3$
$6 \div 2 = 3$
$9 \div 3 = ?$
and so on to
$27 \div ? = 3$

Copy and finish each of Ex. 13–22. Then under it show the D. fact, if any, that goes with it.

$(12 \div 3 = 4)$ 3	$(21 \div 3 = 7)$ 3	$(27 \div 9 = 3)$ 9	$(15 \div 5 = 3)$ 5	$(24 \div 3 = 8)$ 3
13. $4\overline{)12}$ 3	14. $7\overline{)21}$ $(18 \div 6 = 3)$ 6	15. $3\overline{)27}$ $(6 \div 3 = 2)$ 3	16. $3\overline{)15}$ $(3 \div 3 = 1)$ 3	17. $8\overline{)24}$ $(27 \div 3 = 9)$ 3
18. $3\overline{)9}$	19. $3\overline{)18}$	20. $2\overline{)6}$	21. $1\overline{)3}$	22. $9\overline{)27}$

Now for each of Ex. 13–22, show the M. facts or fact in the same set of M. and D. facts.

13. $4 \times 3 = 12$, $3 \times 4 = 12$ 14. $7 \times 3 = 21$, $3 \times 7 = 21$ 15. $3 \times 9 = 27$, $9 \times 3 = 27$ 16. $3 \times 5 = 15$, $5 \times 3 = 15$ 17. $8 \times 3 = 24$, $3 \times 8 = 24$

18. $3 \times 3 = 9$ 19. $3 \times 6 = 18$, $6 \times 3 = 18$ 20. $2 \times 3 = 6$, $3 \times 2 = 6$ 21. $1 \times 3 = 3$, $3 \times 1 = 3$ 22. $9 \times 3 = 27$, $3 \times 9 = 27$

Subtracting to Check Division Answers

[o]

A

$$\begin{array}{r} 21 \\ -7 \\ \hline 14 \\ -7 \\ \hline 7 \\ -7 \\ \hline 0 \end{array}$$

1. $21 \div 7 = ?$ Why does subtracting 7's in Ex. A show that 3 is the answer for $21 \div 7$? 7 is subtracted 3 times.
Did you subtract 7's one at a time until 0 was left? Yes

*2. $21 \div 7 = ?$ Study the work in Ex. B.
First you divide 21 by 7, and write "3."
Next you *think*, "Three times 7 equals 21."
Then you subtract. $21 - 21 = 0$.
Did you subtract 7's one at a time or did you subtract three 7's all at once? all at once What was left after you subtracted? 0

B

$$\begin{array}{r} 3 \\ 7\overline{)21} \\ 21 \text{ (three 7's)} \\ \hline 0 \end{array}$$

C

$$\begin{array}{r} 5 \\ 3\overline{)18} \\ 15 \\ \hline 3 \end{array}$$

D

$$\begin{array}{r} 8 \\ 2\overline{)12} \\ 16 \\ \hline ? \end{array}$$

Wrong

3. $18 \div 3 = ?$ Why does the work in Ex. C show that 5 is not the answer we want? 3 is left after subtracting.
What is the answer we want? 6 Why? Six 3's can be subtracted.

4. $12 \div 2 = ?$ Why does the work in Ex. D show that 8 is the wrong answer? 2 cannot be subtracted 8 times.
What is the answer? 6 Why? Six 2's can be subtracted.

5. On the board, show that 7 is not the answer we want for $24 \div 3$. First subtract seven 3's one at a time. Then subtract seven 3's all at once.

[w]

For Ex. 6–15, check each answer in two ways as for Ex. 5. Then write each example with the correct answer.

3	3		3	6
2	2	5	4	7
6. $9\overline{)27}$	7. $8\overline{)24}$	8. $3\overline{)15}$	9. $7\overline{)21}$	10. $2\overline{)12}$
8	9	3	9	8
11. $3\overline{)24}$	12. $2\overline{)18}$	13. $6\overline{)18}$	14. $3\overline{)27}$	15. $2\overline{)16}$

* Remember that the division operation, as such, is associating 3 with the numbers 7 and 21. We can find this 3 by using the algorithm in box B which shows subtracting three 7's all at one time.

Teaching Page 292

Pupil's Objective

To build further understanding of the relationship of division to subtraction.

Background

Progression toward maturity in using division is brought about through an understanding of what the operation involves. It is used to find the number of times we can remove a set of a given size or to find the size of a given number of equivalent sets. We can think of the answer as the number of subtractions possible. As the text material shows, the answer for the division is incorrect if the number it shows is either too great or not great enough.

Pre-Book Lesson

• Write a division sentence on the chalkboard such as $21 \div 3 = n$. Ask a pupil to read the sentence.

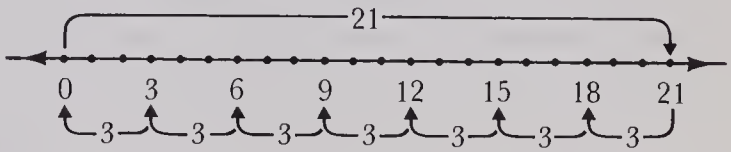
Frequently, pupils want to read the answer when reading an *open* sentence—a sentence which does not show an answer. Do not permit children to read the answer with the sentence. At this time, the focus is on reading the sentence so a question is asked in many different ways. In this instance the sentence may be read in many ways: "How many 3's in 21?" "3's in 21 equals what?" "21 divided by 3 equals what number?"

• Ask children to tell ways in which the answer to the question may be found. Record and illustrate each way suggested by a child. If your pupils have developed the understandings and insights which have been taught in the lessons dealing with division, you should have responses including the following:

a. If we count 21 objects (blocks, children, and so on) and then make equivalent sets of 3, there will be 7 sets.

b. If we subtract 3 at a time until we can't subtract any more, we will subtract seven 3's.

• You may wish to illustrate response *b* on a number-line picture on the chalkboard.



• Write two or three division examples on the board ($2\overline{)16}$, $3\overline{)18}$, $8\overline{)24}$). Ask a pupil to work the example any way he wishes. Then, ask another pupil to work the example another way. Emphasize the use of the second way as a means to check the first.

Using the Text Page

Discuss the material very carefully. Relate the examples and exercises to the Pre-Book Lesson, as deemed appropriate. The written work at the bottom of page 292 is an opportunity to put into practice all the discussion in this lesson. Call the attention of your pupils to the methods in Ex. 2 of subtracting the 7's one at a time and subtracting three 7's all at once.

NOTES

Teaching Page 293

Pupil's Objective

To extend understanding of the factors-product relationship.

Using the Text Page

• You are given an opportunity to observe the pupils' understanding of the relationships between factors, products, pairs of facts, sets of facts, and so on introduced in connection with division facts in which either 2 or 3 is a factor.

• Ex. 11 on page 293 is designed to guide pupils to thinking about other types of division. The set of whole numbers is not closed for the division operation. There are many times when the result of dividing two whole numbers is not another whole number. At this stage of mathematical development it is adequate for children to begin to realize this possibility.

Individualizing Instruction

• It would be good practice for *all pupils* to match the slips of paper they have prepared (see Teaching Page 289) to show pairs of D. facts; sets of M. and D. facts.

• You may wish to extend the matching practice by having pupils make answer cards for the M. and D. facts that have been studied. Then, they may play a game of matching each sentence with an answer.

NOTES

Table of D. Facts, Dividing by 3

1	2	3	4	5	6	7	8	9
$3\overline{)3}$	$3\overline{)6}$	$3\overline{)9}$	$3\overline{)12}$	$3\overline{)15}$	$3\overline{)18}$	$3\overline{)21}$	$3\overline{)24}$	$3\overline{)27}$

Table of D. Facts, Answer 3

3	3	3	3	3	3	3	3	3
$1\overline{)3}$	$2\overline{)6}$	$3\overline{)9}$	$4\overline{)12}$	$5\overline{)15}$	$6\overline{)18}$	$7\overline{)21}$	$8\overline{)24}$	$9\overline{)27}$

Factors-product relationship [O]

1. Cover the answers in the tables and try to say the facts. Make Study Cards for the facts you do not know.

2. In $3\overline{)15}$, which factor of 15 is known? 3 What is the unknown factor? 5

3. In the table for dividing by 3, why are the answers greater by 1 as you move to the right in the table? Each product increases by 3.

4. In the other table, why is 3 always the answer? Each product increases by 3 and the known factor increases by 1.

5. Are answers odd or even numbers when you divide

a. odd numbers by 3? odd **b.** even numbers by 3? even

6. What would the product be in a D. example with 10 as one factor and 3 as the other factor? 30

7. Which fact for the two tables is the same? $9 \div 3 = 3$ Why? The factors are equal.

8. Is 3 a factor in all the D. facts for both tables? Yes

9. Was 2 a factor in all the D. facts about 2? Yes

10. Which pair of D. facts about 3 did you learn when you learned the D. facts about 2? $6 \div 2 = 3$ $6 \div 3 = 2$

11. Why are examples like $3\overline{)7}$ and $4\overline{)13}$ not in the table? The number left after subtracting is greater than 0.

● **Extra Activity.** Work Set 105.

These questions about relationships lead children to think about why numbers behave in certain patterns.

Working with M. and D. Facts

Oral problem-solving [O]

Read problems 1–5, and say a M. or D. fact for each.

1. Trucks like that in the picture carry 5 big logs at a time. In 4 trips, how many logs can the truck carry?

2. How many loads of 5 logs each will 15 logs make? $4 \times 5 = 20$

3. It takes a truck about 3 hours to go to the mill and back. How many hours will 8 round trips take? $15 \div 5 = 3$ $8 \times 3 = 24$

4. Dick Smith worked in the woods 21 hours one week. If he worked 7 hours a day, how many days did he work?

5. A 24-foot log was cut into 3 pieces of the same length. How many feet long was each of the pieces? $21 \div 7 = 3$ $24 \div 3 = 8$

Inequalities [W]

For each of Ex. 6–11, do the work in both ( )'s. Then copy the number name for the greater number.

a	b		a	b
6. $(24 \div 3)$	(2×6) 2×6	9. (8×2)	(3×4) 8×2	
7. (2×8)	(7×3) 7×3	10. (3×6)	(2×7) 3×6	
8. $(27 \div 3)$	$(14 \div 2)$ $27 \div 3$	11. $(21 \div 3)$	$(18 \div 2)$ $18 \div 2$	

Pupil's Objectives

(a) To differentiate between needs for multiplication or division in problem-solving; and (b) to work with M. and D. facts within the context of problem-solving and inequalities.

Background

The application of M. and D. facts in problem-solving is an important facet of a balanced mathematics program. Even though your pupils may be arriving at a level of mastery of the M. and D. facts studied, it is no assurance that they can apply this knowledge to problems. Therefore, it is very important to state and analyze many different problem situations in which multiplication and division may be used.

Once again, you are cautioned not to stress the answer as much as the operation or operations. Analyzing the situation, identifying pertinent information, and stating and writing an appropriate mathematical sentence for the problem must precede finding the answer.

It should also be mentioned that several different ways of solving a problem are usually open to us. What one child may use is not necessarily what another child will use or prefer. Permit as much flexibility in thinking as the situation will allow.

The material at the bottom of page 294 is one more opportunity to provide practice in a setting dealing with inequalities. The appropriate answer to each example involves the *is greater than* idea. It has been pointed out many times previously that a number may be named in many ways. The child's goal in these exercises is to determine which number named is greater, while your goal is to give the child practice in M. and D. facts which he is to master.

Pre-Book Lesson

• Engage in an oral lesson accompanied by written work on the chalkboard which requires the pupil to name a number in different ways.

• Write (2×3) and $(12 \div 3)$ on the board. Ask pupils to tell which number named is greater. Write two or three other number names for which pupils must select the greater number shown.

For some pupils it may be necessary to construct arrays with objects or dots to help them *see* each number.

Using the Text Page

• Discuss each of problems 1–5 carefully and thoroughly. Use objects to demonstrate or illustrate each situation. *Slower learners*, in particular, will need some means of helping them *see* the situation.

Ask pupils to tell the appropriate M. or D. fact for each problem. Then have an appropriate mathematical sentence written on the board to show each fact.

• The Pre-Book Lesson should be adequate preparation for most children to proceed independently with Ex. 6–11. You may wish to discuss each exercise with the *slower learners* before assigning it as written work.

Individualizing Instruction

• *All pupils* may write an *n*-sentence for each of the problems.

• Many lessons and enrichment activities throughout this book have dealt with the notion of inequality. On pupil's page 90 the symbols for *is greater than* and *is less than* ($>$, $<$) were introduced and they have been maintained in many exercises since then. At this time, it would be worthwhile to complete sentences which use the symbols for *is greater than* and *is less than*.

Prepare a duplicated sheet similar to the following:

------ > ------
------ < ------
------ > ------
and so on

Then, have the pupil name numbers, in any way he wishes, to make each a *true* sentence showing inequality.

You may require that the number names show two factors, such as 6×3; or a product and a factor, such as $18 \div 3$.

Teaching Page 295

Pupil's Objectives

(a) To check on skills learned to date; and (b) to have further practice where needed.

Using the Text Page

The *Try These* exercises may be used as a test of division facts which have been introduced to date.

The practice section includes practically every skill that has been covered in the program to date. Observe pupils' work for common sources of difficulty. It may be necessary to review or reteach certain concepts or skills with small groups of children.

Many of the *n*-sentences at the bottom of the page are written with addition symbols but require subtraction since *n* represents an unknown addend. Other sentences written with subtraction symbols require addition since *n* stands for the sum.

Individualizing Instruction

- Permit *slower learners* to use objects or construct arrays to help find the solution to the division practice exercises.
- In working Ex. 11–19, *more capable children* may write sentences using $>$ and $<$ to make *true* sentences. Emphasize the renaming of numbers in many different ways.

NOTES

Try These

[W]

Write answers on folded paper.

	a	b	c	d	e	f	g
1.	$3\overline{)18}^{6}$	$4\overline{)12}^{3}$	$9\overline{)18}^{2}$	$7\overline{)14}^{2}$	$3\overline{)6}^{2}$	$6\overline{)12}^{2}$	$3\overline{)21}^{7}$
2.	$3\overline{)12}^{4}$	$3\overline{)27}^{9}$	$2\overline{)12}^{6}$	$5\overline{)10}^{2}$	$6\overline{)18}^{3}$	$3\overline{)9}^{3}$	$8\overline{)16}^{2}$
3.	$3\overline{)24}^{8}$	$2\overline{)18}^{9}$	$7\overline{)21}^{3}$	$2\overline{)8}^{4}$	$9\overline{)27}^{3}$	$2\overline{)16}^{8}$	$3\overline{)15}^{5}$
4.	$5\overline{)15}^{3}$	$3\overline{)3}^{1}$	$8\overline{)24}^{3}$	$2\overline{)6}^{3}$	$2\overline{)14}^{7}$	$4\overline{)8}^{2}$	$1\overline{)3}^{3}$

◆**Extra Examples.** Work Set 77.

To Keep in Practice

A., S., M. [W]

Row 1. Write answers on folded paper.

	a	b	c	d	e	f	g	h
1.	530	102	709	32	\$9.21	\$2.19	600	\$1.03
	-88	$\times 9$	$+89$	$\times 6$	-5.78	$\times 4$	-247	$\times 7$
	442	918	798	192	\$3.43	\$8.76	353	\$7.21

Ex. 2–10. Copy and work. Check.

2. 207×4 828
3. $807 - 789$ 18
4. $515 - 206$ 309
5. $6 + 7 + 0 + 7$ 20
6. $18 + 16 + 39 + 37$ 110
7. \$6.11 − \$0.89 \$5.22
8. $456 + 86 + 305$ 847
9. $7 + 83 + 215 + 9$ 314
10. \$3.24 − \$2.15 \$1.09

Ex. 11–19. Find the unknown number. Then copy each example using a numeral for that number.

11. $841 - n = 64$ 777
12. $n + 108 = 216$ 108
13. $2 \times 125 = n$ 250
14. $n - 600 = 175$ 775
15. $89 + 117 = n$ 206
16. $3 \times 75 = n$ 225
17. $87 - \square = 59$ 28
18. $93 + n = 166$ 73
19. $4 \times 238 = n$ 952

◆**Extra Problems.** Work Set 14.

* Now pupils will find that n may stand for a product in either a multiplication example or a division example.

Finding the Product *n* Stands For

Factors-product relationship [O]

Tell what product *n* stands for in each of Ex. 1–3.

1. $2 \times 6 = n$ 12 2. $8 \times 3 = n$ 24 3. $4 \times 2 = n$ 8

4. In the D. example, $n \div 3 = 2$,

a. one factor is _?_. 3 The other factor is _?_. 2

b. we know two factors, so *n* stands for the _?_. product

c. $2 \times 3 = 6$ so, in the example $n \div 3 = 2$, $n =$ _?_. 6

Tell what product *n* stands for. Explain.

5. $n \div 4 = 2$ 8; $4 \times 2 = 8$

6. $7 \times 2 = n$ 14

7. $3 \times 6 = n$ 18

8. $n \div 6 = 2$ 12; $6 \times 2 = 12$

9. $n \div 5 = 3$ 15; $5 \times 3 = 15$

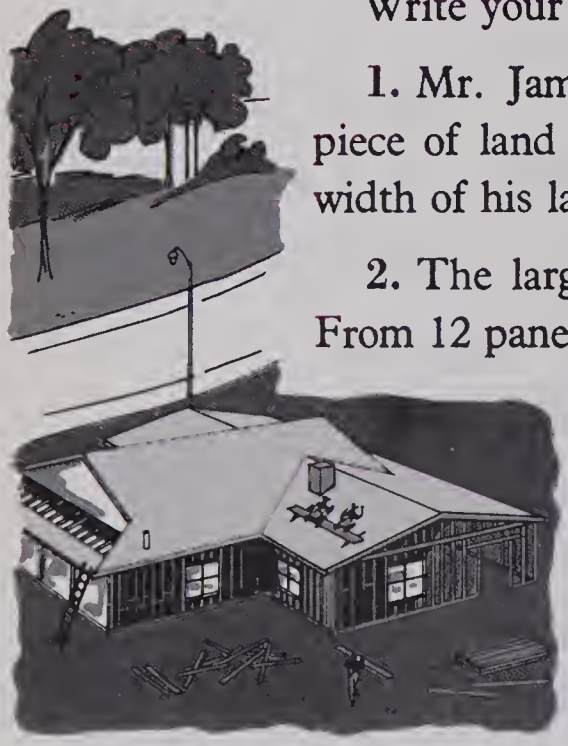
10. $n \div 2 = 5$ 10; $2 \times 5 = 10$

When you know two factors, you multiply to find the product.

Can You Solve Problems?

Problem Test 7

Write your work for each of problems 1–8.



1. Mr. James is building a house 52 feet wide on a piece of land 125 feet in width. How many feet in the width of his land will be left at the sides of the house? 73 feet

2. The large windows will each use 4 panes of glass. From 12 panes of glass how many windows can be made? 3 windows

3. The front walk will be 35 feet long, the back walk, 28 feet, and the walk around the house, 196 feet. How many feet of walk will there be? 259 feet

4. Five men each worked 32 hours one week. How many hours was that in all? 160 hours

Teaching Pages 296 and 297

Pupil's Objectives

(a) To extend understanding and knowledge of the factors-product relationship; (b) to work the end-of-chapter test on problem-solving; and (c) to take the Diagnostic Test for Chapter 7.

New Words

(page 296) *panes, width*
(page 297) *concrete, paid*

Background

Several previous lessons have dealt with multiplication and division by the same number as inverses. In all sentences introduced to date, the multiplication symbol ($\times$) has been used when the two factors were shown and the division symbol ($\div$) has been used when a product and one factor were shown.

Consider the question, "What number divided by 4 equals 3?" The appropriate sentence could be $n \div 4 = 3$. Although a division symbol is given, two factors are shown and the task is to find the number for n, the product. Or, consider the question, "Two times what number equals 16?" A number sentence for this question is $2 \times n = 16$. Here, a multiplication symbol is given but a product and one factor are shown and the task is to find the number for n, a factor. The material at the top of page 296 extends all that has been learned about a product and its factors to include these two types of mathematical sentences.

Pupils should begin to state in their own words something to the effect that regardless of the operation symbol indicated in the mathematical sentence, when both factors are given, multiplication is used; and when one of two factors and the product are given, division is used to find the other factor.

Pre-Book Lesson

• Before discussing the work at the top of the page you may wish to say something similar to what follows.

"Who can tell me what number times 4 equals 8?" Most pupils will probably know the answer. Then, ask a pupil to write a number sentence on the chalkboard for the question you just asked. Tell the child you want the number sentence to be written in exactly the same way you stated the question. Have different children try until the sentence is written $n \times 4 = 8$. Then ask pupils if n stands for a factor or a product.

Continue with this kind of questioning and writing of sentences until you are satisfied that your pupils can identify factors and a product. Sentences such as: $3 \times n = 15$ and $n \div 5 = 3$ provide worthwhile experience in such identification.

• For Problem Test 7, it would probably be a good idea to review orally the four operations—addition, subtraction, multiplication, and division, as well as to demonstrate their use in a problem setting. It may be desirable to demonstrate with objects in the classroom or with representative materials at each child's desk.

Using the Text Pages

• The oral work at the top of page 296 is very short but the ideas there are tremendously important. Use the chalkboard extensively in discussing Ex. 4–10. Questions a, b, and c for Ex. 4 should be adapted for each of Ex. 5–10. Have pupils tell and write on the chalkboard other M. and D. sentences to show a pair of M. and D. facts for each of these sentences.

• *More capable children* should be able to proceed with both tests independently.

• You may wish to read each problem with pupils having reading difficulties before assigning it as written work.

After *all pupils* have completed the problem test, discuss each problem. Have pupils tell what operation they used for the problem and why.

Remind pupils how to use folded paper in writing answers for the diagnostic test.

Individualizing Instruction

You may wish to duplicate a page of practice exercises like the following for *all pupils*.

$3 \times n = 9;\ n = ?$ $\quad n \div 6 = 2;\ n = ?$
$n \times 7 = 14;\ n = ?$ $\quad 21 \div n = 3;\ n = ?$

See Page 296 for suggestions about Diagnostic Test 7.

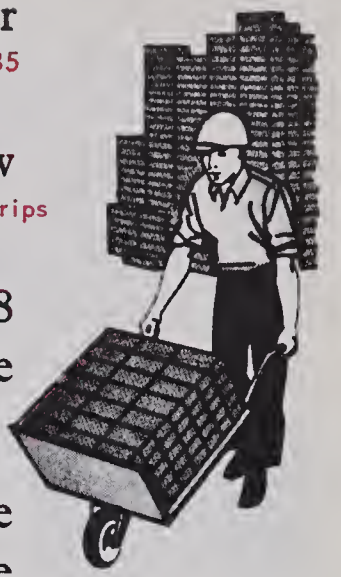
NOTES

5. One man was paid $9.00 for some work. His helper was paid $2.65 less. How much was the helper paid? $6.35

6. A worker can carry 3 concrete blocks at a time. How many trips must he make to move 27 concrete blocks? 9 trips

7. In three loads, a worker moved 127 bricks, 98 bricks, and 105 bricks. How many bricks in all did he move in the three loads? 330 bricks

8. Men have been working on the house 46 days. The builder says that it will be 3 times that many days before the house is finished. How many days is that? 138 days



Do You Make Mistakes?

Diagnostic Test 7

Write answers on folded paper. Work carefully.

	a	b	c	d	Study Pages	Practice Sets
1.	$3\overline{)24}$ 8	$2\overline{)18}$ 9	$7\overline{)14}$ 2	$9\overline{)27}$ 3	280–281, 290–291	77
2.	$12 \div 6$ 2	$21 \div 7$ 3	$16 \div 2$ 8	$18 \div 6$ 3		
3.	$27 \times 4 = 108$	$32 \times 6 = 192$	$58 \times 2 = 116$	$\$0.87 \times 3 = \$2.61$	258–259	73
4.	$300 \times 3 = 900$	$243 \times 2 = 486$	$\$3.08 \times 3 = \$9.24$	$\$1.13 \times 7 = \$7.91$	266–269	74, 75

Do You Understand?

Test of Information and Meaning 7

Write *n*-examples for these on page 296:

125 − 52 = n
1. Problem 1

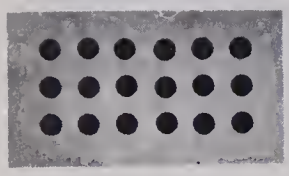
35 + 28 + 196 = n
2. Problem 3

5 × 32 = n
3. Problem 4

4. Copy the examples in which you remember tens.

a. 4×202 **b.** 5×104 **c.** 3×228 **d.** 7×110

Ex. b. and c.



5. For the dot picture show

a. the two M. facts. 3 × 6 = 18, 6 × 3 = 18

b. the factors. 3 and 6

c. the product. 18

d. the two D. facts. 18 ÷ 3 = 6, 18 ÷ 6 = 3

Check by subtraction. Then show the correct fact.

16 ÷ 8 = 2
6. $21 \div 7 = 3$

7. $16 \div 8 = 3$

15 ÷ 5 = 3
8. $15 \div 5 = 4$

Copy, putting a correct numeral in place of *n*.

9. $8 \times 4 = n$ 32

10. $n \div 3 = 7$ 21

11. $n \div 2 = 8$ 16

12. $n - 27 = 31$ 58

13. $n + 8 = 21$ 13

14. Which is greater, (4×7) or (9×2)? 4 × 7

15. Show the D. fact for this subtracting:

$20 - 5 = 15$; $15 - 5 = 10$; $10 - 5 = 5$; $5 - 5 = 0$.

20 ÷ 5 = 4

Show the set of M. and D. facts for product

3 × 3 = 9, 9 ÷ 3 = 3
16. 9; factors 3 and 3.

8 × 3 = 24, 3 × 8 = 24, 24 ÷ 8 = 3, 24 ÷ 3 = 8
17. 24; factors 8 and 3.

Write "Yes" or "No" to answer questions 18–27.

18. $3 \times 26 = 78$ Does $26 \times 3 = 78$? Yes

19. Does a line have end points? No

20. Can you find, by adding, how many equivalent sets there are in a total set? No

Teaching Pages 298 and 299

Pupil's Objective

To evaluate the extent to which the skills and understandings presented in Chapter 7 have been developed.

Using the Text Pages (pages 298–299)

A child's performance on the Test of Information and Meaning may or may not indicate his level of understanding. Therefore it is imperative that you discuss each item with your pupils. Ask *Why?* frequently. Ask pupils to verify their answers. Ask other pupils if they can show or tell the same idea in another way.

Using the Text Page (page 299)

The results on this Computation Test should give you a reasonably good idea of the ability of your pupils to deal with all skills that have been introduced in addition, subtraction, multiplication, and division.

Use the tables below to find the per-cent scores for the individual record cards (see Teaching pages 50 and 51.)

Table of Per Cents for Chapter 7 Scores

Problem Test		Computation Test					
Score	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*
1	13	1	3	11	37	21	70
2	25	2	7	12	40	22	73
3	38	3	10	13	43	23	77
4	50	4	13	14	47	24	80
5	63	5	17	15	50	25	83
6	75	6	20	16	53	26	87
7	88	7	23	17	57	27	90
8	100	8	27	18	60	28	93
		9	30	19	63	29	97
		10	33	20	67	30	100

Suggestions for Material to Accompany End-of-Chapter 7 Tests

After administering the tests on pages 296–299, in all probability you will find that some pupils need additional help or review before proceeding to the work of the next chapter. Meanwhile, you may find one or more of the following suggestions valuable for use with those children who are ready to go to the work in Chapter 8.

Alternate uses of Pages in Chapter 7

Page 261 (bottom). Have *all pupils* rewrite each of these exercises as follows:

1. $36 > 32$
2. $16 < 21$

Page 269. Copy Ex. 7–18 as they are written with the answer. For Ex. 7: $2 \times 408 = 816$.

Page 270. On folded paper write answers for Ex. 3–17.

Page 273 (bottom). In each sentence pretend the word "or" is + and then find the solution.

1. $(4 \times 6) + (4 \times 7) = 24 + 28 = 52$

Page 286. Use the exercises at the top of the page for a timed test.

Page 287 (bottom). On folded paper write answers only for Ex. 1–12.

Page 294. In Ex. 6–11, change "or" to +. Then find the answer for each example.

6. $(24 \div 3) + (2 \times 6) = 8 + 12 = 20$

Page 295. Use the division examples at the top of the page for a timed test.

Supplementary Activities

• In the following exercises the child is to indicate the appropriate operation A., S., M., or D. with its symbol. You may write other examples.

$3 \bigcirc 4 = 7$
$3 \bigcirc 4 = 12$
$18 \bigcirc 3 = 15$
$18 \bigcirc 3 = 6$
$18 \bigcirc 3 = 21$
$18 \bigcirc 3 = 54$

• *More capable children* should be able to find the number for n in each of the following sentences. You will probably want to write more sentences for pupils to work.

$(2 \times n) + 1 = 17$ $\quad$ $(21 \div 3) + n = 15$
$(n \div 4) - 5 = 3$ $\quad$ $(8 \times 4) - n = 23$
$47 - (6 \times 4) = n$ $\quad$ $n - (18 \div 2) = 15$
$(9 \times 3) - (15 \div n) = 22$

• From what you have learned about the factors-product relationship, copy and complete:

a. $2 \times 242 = 484$, so $484 \div 2 = ?$
b. $2 \times 495 = 990$, so $990 \div 2 = ?$
c. $24 \times 25 = 600$, so $25 \times 24 = ?$
$600 \div 24 = _ ? _$, so $600 \div 25 = ?$
d. $17 \times 43 = 731$, so $43 \times 17 = ?$
$731 \div 43 = _ ? _$, so $731 \div 17 = ?$

• *All pupils* have written tables of D. facts for 2 through $18 \div 2 = 9$ (and $18 \div 9 = 2$) and for 3 through $27 \div 3 = 9$ (and $27 \div 9 = 3$). Now complete the tables started below.

Dividing by 2

$20 \div 2 = 10$	$20 \div 10 = ?$
$22 \div 2 = ?$	$22 \div _ ? _ = 2$
$24 \div 2 = ?$	$24 \div _ ? _ = 2$
$26 \div 2 = ?$	$26 \div _ ? _ = 2$
through	
$36 \div 2 = ?$	$36 \div _ ? _ = 2$

Dividing by 3

$30 \div 3 = 10$	$30 \div 10 = 3$
$33 \div 3 = ?$	$33 \div _ ? _ = 3$
$36 \div 3 = ?$	$36 \div _ ? _ = 3$
$39 \div 3 = ?$	$39 \div _ ? _ = 3$
through	
$54 \div 3 = ?$	$54 \div _ ? _ = 3$

• The following activity puts the distributive property to use in working with some multiplications and divisions.

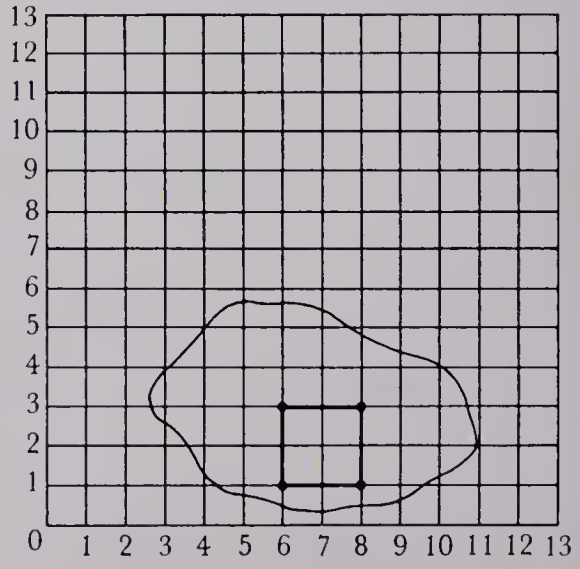
a. $(2 \times 5) + (2 \times 3) = (2 \times n)$; $n = ?$
b. $(4 \times 8) + (4 \times 2) = (4 \times n)$; $n = ?$

c. $7 \times 3 = 21$	d. $8 \times 4 = 32$
$7 \times 2 = 14$	$8 \times 3 = 24$
$7 \times 5 = ?$	$8 \times 7 = ?$
e. $9 \times 3 = 27$	f. $12 \div 3 = 4$
$9 \times 3 = 27$	$15 \div 3 = 5$
$9 \times 6 = ?$	$27 \div 3 = ?$

Coordinate Geometry

The material on coordinate geometry at the end of Chapter 6 suggested the construction of pictures on a coordinate grid. With this background pupils should be able to engage in a game like *Find the Treasure*. In this game, two children (or two teams) hide a treasure on an island. The island is concealed on the grid as is the treasure on the island. The location of the treasure must be identified by naming three or four points which may be in a line (vertical, horizontal, or diagonal) or a square.

The children take turns naming points with coordinates and making x's at the points named. The object is to locate the treasure but, of course, finding the island will narrow the region of the grid in which the treasure is located. Each time a point is called that falls on part of the island, the opponent must say, "You are on my island." If any portion of the treasure is "hit" by a point called, the game is over.



21. Is $15 \div 5$ a name for the number 3? Yes

22. In D. facts is the answer called the product? No

23. When 0 is one factor, is the product equal to the other factor? No

24. Can you find how many 8's in 24 by subtracting? Yes

25. Do the rays of every angle make a square corner? No

26. If 2 boys share 12 pieces of candy equally, do you multiply to find how many pieces each boy gets? No

27. In the example $27 \div 3 = 9$, are the factors 9 and 27? No

How Well Can You Compute?

Computation Test 7

Write answers on folded paper. Work carefully.

	a	b	c	d	e	f
1.	104	609	48	87¢	\$7.00	\$3.58
	×8	+99	×4	×3	−1.65	+2.89
	832	708	192	\$2.61	\$5.35	\$6.47
2.	437	86	200	34¢	\$4.01	\$0.65
	×2	+79	×4	×6	−3.98	×4
	874	165	800	\$2.04	\$0.03	\$2.60
3.	875	106	352	82¢	\$1.18	\$7.28
	−305	×3	+408	−39¢	×2	+0.72
	570	318	760	43¢	\$2.36	\$8.00
4.	$3\overline{)18}_{6}$	$6\overline{)12}_{2}$	$3\overline{)24}_{8}$	$2\overline{)16}_{8}$	$3\overline{)21}_{7}$	$9\overline{)27}_{3}$

Copy in columns. Add, and check.

5. 22, 61, 235, 76 394

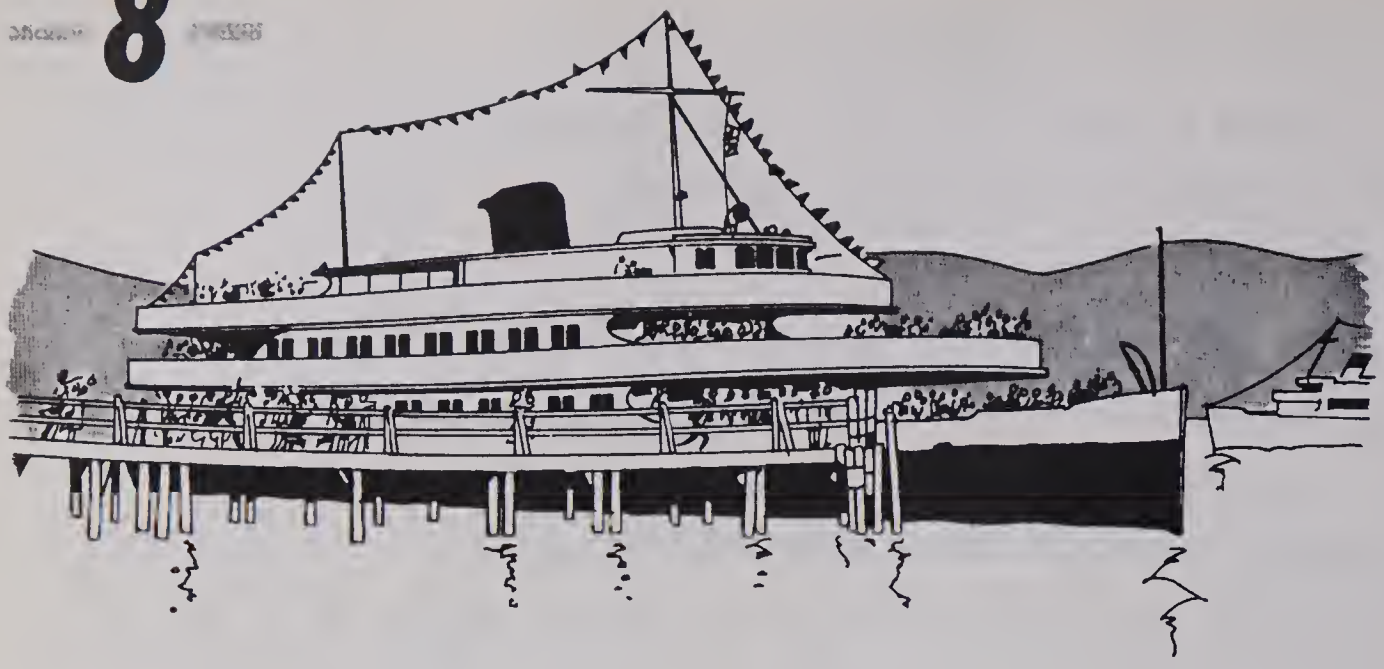
6. 4, 67, 6, 8, 7 92

7. 9, 4, 7, 8, 53 81

8. 206, 67, 58, 395 726

9. 25, 296, 34, 109 464

10. 5, 8, 40, 6, 6, 65



Problems for Careful Readers

*Discovering data missing; A., S., M., D. [O]

1. The picture shows boats that carry people on a large lake. At one stop many of the 70 people on one boat got off. How many people stayed on the boat?

Can you find the answer? No If not, why not? You are not told the number of people that got off.

Which of problems 2–10 can you not work? Tell why? Ex. 2,4,5,6,7,8, and 9.

2. At this same stop, 35 people got on the boat. With those who stayed on, there were how many people on the boat then? You don't know the number of people that stayed on the boat.

3. One boat makes a trip to a boat museum 18 miles across the lake. How far does the boat go in making the trip and coming back to the starting place?

4. Mr. White bought tickets for himself and 3 other men. How much did he pay for the tickets? You are not told the cost of each ticket.

5. Two groups of people went across the lake to visit the boat museum. How many people in all was that? You are not told the number of people in the groups.

*Some problems have too many data. These problems have some data missing. These types and other variations stress the importance of careful reading.

Overview — Chapter 8

Chapter 8 is the last chapter of this text. From it your pupils will complete their arithmetic work in grade 3. The work in multiplication and division is extended through sets of multiplication and division facts with 4 as a factor and with 5 as a factor. Through the use of arrays, the factors-product relationship is clearly emphasized and your pupils should come to a full realization that division by a number is the inverse of multiplication by that number and vice versa and, for that reason, a good check for one operation is its inverse.

The work in multiplication using 2-place and 3-place numerals is extended now to renaming the tens in a product and remembering one hundred or more and also to renaming the products of both ones and tens and remembering the necessary tens and hundreds. As an added item in division, facts with 1 as a factor are studied.

Two new facets of the study of number are introduced in this chapter as preparation for further work along these lines in grade 4. The reading and writing of 4-place numerals is handled in terms of using many names for a number. Work with graphs is begun with a simple presentation of the pictograph. This initial treatment of function should be well received by those pupils who are looking for something new!

• **Problem-Solving.** In this chapter problem-solving is extended to tie in with the other development so pupils are required to differentiate among needs for any of the four operations for solving problems, to write questions in order to complete problems requiring multiplication or division for their solution, to use data from other problems, to deal with situations having too many data and not enough, and to judge whether an answer for a multiplication or a division problem is wrong by making use of one of these generalizations:

If the factor multiplied by is greater than 1, the product is greater than the other factor.

If the product is divided by a factor greater than 1, the answer is less than the product.

• **Maintenance.** The work with fraction numbers is maintained and extended now to the finding of the number for a fractional part of a set, and to use division for the operation. More work is done also in the use of fractions with measurements of time, weight, length, and liquids.

Inequalities and work with them are maintained in the use of the symbols $>$, $<$, and $\neq$.

• **Enrichment.** For the more capable children, still more difficult work with parentheses and with n is provided. Another exercise suggests adding mentally by using addends of the number to be added. In a third assignment, from addition examples using 2-place and 3-place numerals, these children are to make multiplication examples having the same answer.

• **Testing.** The maintenance, mid-chapter review, and end-of-chapter tests provide the usual over-all picture of pupils' achievements during this period of work.

Teaching Pages 300–301

Pupil's Objectives

(a) To have practice in reading problems to determine whether or not all necessary information is given; (b) to have practice in finding the solution to written problems; and (c) to have an oral lesson in multiplication and addition as preparation for multiplication with remembering hundreds.

New Word

(page 300) *museum*

Background

Problem-solving is a multi-faceted skill for in it mathematics finds its applications in many different sources and ways. Finding the solution to a prob-

lem frequently requires the ability to read. Of course, merely reading the problem is not enough. The reader must determine what is being asked, what operation or operations may be used, whether or not all necessary information is given, and whether or not the information which is given is necessary. The primary purpose of the material on these two pages is to give pupils practice in analyzing the problem situation, determining what operation is appropriate and, finally, deciding whether or not it is possible to solve the problem with the information given.

Pre-Book Lesson

• The idea of missing data may not be new to your pupils but on the chalkboard, write problems similar to the following. Note data missing in each one.

a. Jan had 7 puppies. She gave some of the puppies to her friends. How many puppies did Jan keep?

b. Fred shot 4 arrows at each target. How many arrows did he shoot in all?

Ask pupils to read each problem and find the answer. Have pupils tell what is wrong or what is missing. Occasionally a child has difficulty expressing the missing information. You may help by asking what appear to be ridiculous questions: "Would it help if we knew the color of Jan's puppies?" "Could we find the answer if we knew the color of Fred's shirt?" "Then, what would help us find the answer?"

• Make certain your pupils get the idea that data may be missing and that appropriate information must be supplied before the solution can be found.

Using the Text Pages

• Problems 1–10 should be worked in the same way as the problems just discussed in the Pre-Book Lesson. Have pupils determine what information is missing and what operation should be used to find the solution.

• The written work requires that additional data be supplied. The following quantitative information is suggested. Make sure that pupils understand "pair" in Ex. 10.

Ex. 1, 22 for the number of people who got off.

Ex. 2, 48 for the number of people who stayed on the boat.

Ex. 4, 18¢ for the cost of a ticket.

Ex. 5, 21 and 18 for the number's of people in the two groups.

Ex. 6, 18 for the number of people in the total group.

Ex. 7, 21 people for the number of people in the other total group.

Ex. 8, 35¢ for the cost of the post cards.

Ex. 9, 18 for the number of water glasses she bought.

• The oral work at the bottom of page 301 is valuable preparation for multiplication with remembering hundreds. Pupils may work with partners and alternate in saying answers.

If you make up additional examples of this type for board work be sure the remembered number is realistic. For example, when multiplying by 4, the greatest possible remembered number would be 3, because the greatest product is 36 (4×9).

Individualizing Instruction

• *All pupils* may make up problems of their own in which information is missing. They must be able to give the missing data and be able to tell the correct operation.

• Use the *Show the Answer* cards with *all pupils* for the exercises at the bottom of page 301. You may wish to change the order of examples.

• On folded paper pupils may write the answers only to the exercises at the bottom of page 301, either by rows or by columns.

6. One group of people was separated equally into 3 small groups to see the museum. How many were in each small group? You are not told the number of people in the total group.

7. The other people walked in small groups of 7. How many small groups were there? You are not told the number of people in the total group.

8. One man bought a model of a boat for $4.75 and some post cards. How much did these things cost? You are not told the cost of the post cards.

9. A woman bought water glasses to take home. The glasses were put in boxes, 6 to a box. How many boxes were needed? You are not told the number of glasses she bought.

10. Each of a pair of boat models weighed 3 pounds. How much did the pair of models weigh?

[W]

After your teacher gives you the numbers you need, write your work for each of problems 1–10.

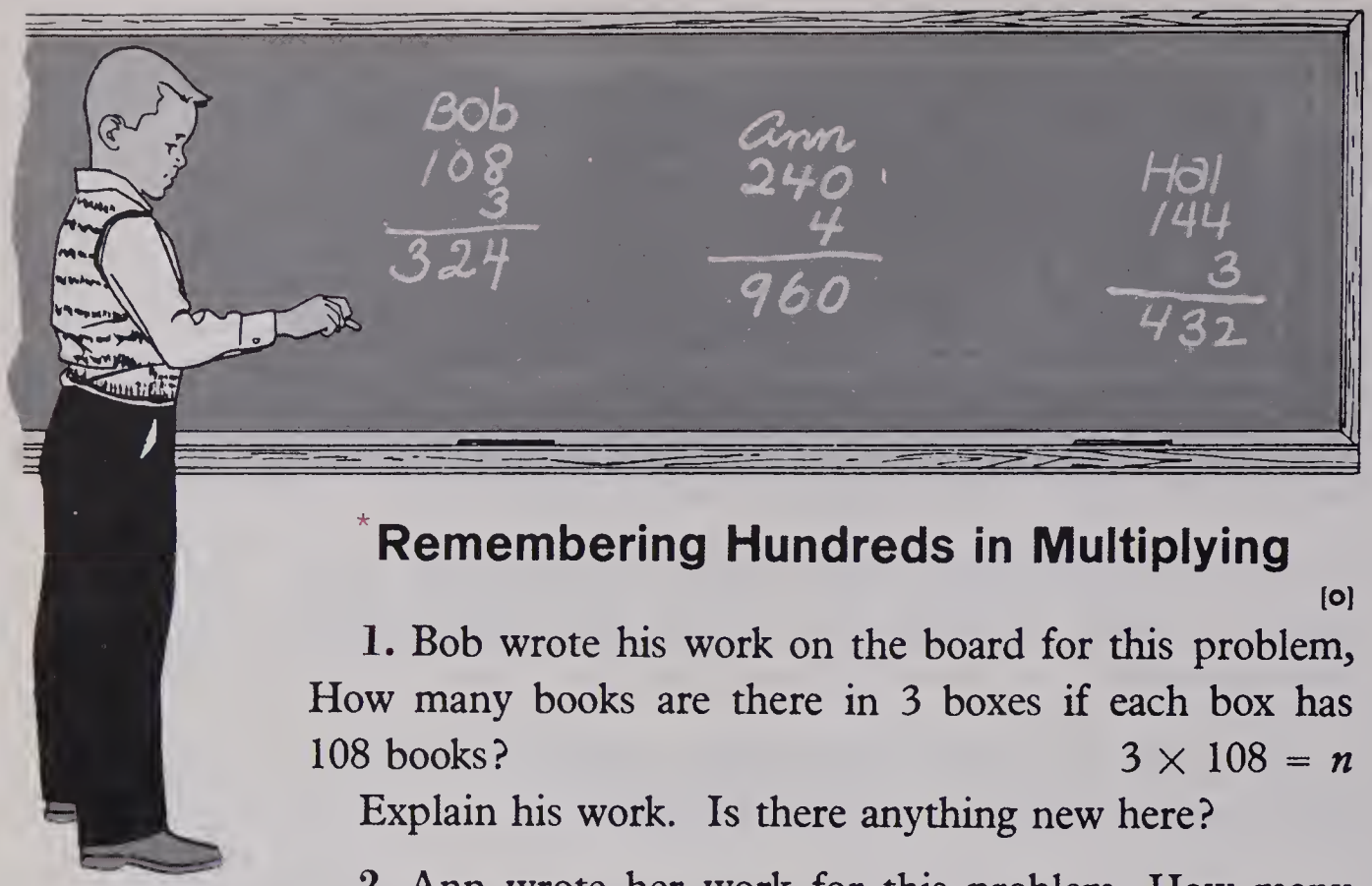
* Practice in Adding Remembered Numbers

Multiplication; mental procedures [O]

For each example say only the answer. For Ex. 1a, *think*, "12, 14." Say, "14."

	a	b	c	d	e
1.	$(4 \times 3) + 2_{14}$	$(6 \times 2) + 4_{16}$	$(4 \times 9) + 3_{39}$	$(8 \times 4) + 4_{36}$	$(4 \times 4) + 2_{18}$
2.	$(4 \times 8) + 3_{35}$	$(9 \times 3) + 2_{29}$	$(7 \times 3) + 1_{22}$	$(3 \times 9) + 2_{29}$	$(6 \times 3) + 3_{21}$
3.	$(3 \times 5) + 2_{17}$	$(4 \times 6) + 3_{27}$	$(2 \times 9) + 1_{19}$	$(7 \times 4) + 5_{33}$	$(4 \times 5) + 2_{22}$
4.	$(9 \times 2) + 4_{22}$	$(2 \times 8) + 1_{17}$	$(4 \times 7) + 3_{31}$	$(8 \times 3) + 2_{26}$	$(2 \times 6) + 1_{13}$
5.	$(6 \times 4) + 4_{28}$	$(5 \times 3) + 3_{18}$	$(2 \times 7) + 1_{15}$	$(5 \times 4) + 4_{24}$	$(5 \times 2) + 4_{14}$
6.	$(3 \times 8) + 2_{26}$	$(8 \times 2) + 1_{17}$	$(9 \times 4) + 4_{40}$	$(3 \times 7) + 2_{23}$	$(3 \times 6) + 2_{20}$

* Since multiplication often involves multiplying a number of tens, or whatever, and then adding the remembered number, it is worthwhile to have special practice for these two stages.



* Remembering Hundreds in Multiplying

[o]

1. Bob wrote his work on the board for this problem, How many books are there in 3 boxes if each box has 108 books? $3 \times 108 = n$

Explain his work. Is there anything new here?

2. Ann wrote her work for this problem, How many booklets are there in 4 boxes of 240 booklets each?

$4 \times 240 = n$

a. The product of the tens is 16. Do you rename? Yes

b. 16 tens = 1 hundred and 6 tens.

c. Did Ann have to remember 1 hundred? Yes Why?

d. Did Ann add the remembered 1 hundred before or after she multiplied the hundreds? after How can you tell? 4 × 2 = 8; 8 + 1 = 9

You remember and add hundreds in multiplying in the same way that you remember and add tens in multiplying.

3. Study Hal's work for this problem, If each box holds 144 pencils, how many pencils are there in 3 boxes?

$3 \times 144 = n$

a. Did Hal have to remember 1 ten? Yes 1 hundred? Yes Why?

b. Explain Hal's work. 12 ones = 1 ten and 2 ones; 12 tens = 1 hundred and 2 tens

* Renaming the product of the tens and remembering a number of hundreds is similar to renaming the product of the ones and remembering a number of tens. Pupils should have little trouble with this skill.

Teaching Pages 302 and 303

Pupil's Objectives

(a) To learn to remember one or more hundreds when multiplying with 3-place numerals; and (b) to learn to remember both tens and hundreds when multiplying with 3-place numerals.

Background

Your pupils have learned to remember tens in multiplying with 2-place and with 3-place numerals. They should be ready to remember hundreds since the principles of remembering and adding hundreds are the same as those of remembering and adding tens. Some of your pupils may be able to recognize the need for remembering hundreds and may be able to explain the procedure in the exercises on page 302 without a long and careful development of the new skill.

Pre-Book Lesson

• On the chalkboard, write examples like each of the following. Ask pupils to tell how to work each example. Be sure all children understand the procedures involved in each example.

a. $\begin{array}{r} 32 \\ \times 3 \\ \hline \end{array}$ no remembering

b. $\begin{array}{r} 27 \\ \times 3 \\ \hline \end{array}$ remembering tens

c. $\begin{array}{r} 206 \\ \times 4 \\ \hline \end{array}$ remembering tens

d. $\begin{array}{r} 428 \\ \times 2 \\ \hline \end{array}$ remembering a ten

• After working and discussing each of the types listed above, write an example like that below:

Ask pupils if they can tell how this example is like any of the examples already discussed.

$\begin{array}{r} 172 \\ \times 4 \\ \hline \end{array}$

Your pupils should see a similarity in that this example and examples b, c, and d above involve remembering and adding. Be sure to develop with the children the idea that in this example it is necessary to rename the product of the tens, remember and add hundreds while in the previous examples tens were remembered and added.

• Write an example like the one below in which tens and hundreds are both remembered and added:

Discuss and work this example with your pupils. Be certain that each step in completing the operation is understood by all pupils. A frequent error made by pupils is to multiply the remembered ten and add the number of tens to be multiplied. Your questions should give you the opportunity to determine whether or not this type of error is committed.

$\begin{array}{r} 132 \\ \times 6 \\ \hline \end{array}$

Using the Text Pages

• The oral work on these two pages should give you plenty of opportunity to identify the types of difficulties encountered by your pupils. *Slower learners* may need extensive work in multiplication with numbers shown in expanded form. See the Individualizing Instruction section for suggestions.

• Before assigning the written work you may wish to have pupils turn to the bottom of page 301 and do some examples orally again. Be sure to discuss the similarity between doing these examples and doing multiplication examples involving remembering and adding.

In Ex. 13–24 the 3-place numeral is sometimes written first and sometimes written last. Ask why we can do this. Suggest that the latter approach is easier. This will indicate the desirability of copying each example in a certain way.

Individualizing Instruction

• It may be necessary to demonstrate the principle involved in multiplying with remembering by using a pocket chart.

• *Slower learners* may gain additional insight through multiplication using these expanded forms:

a. $\begin{array}{r} 483 \\ \times 2 \\ \hline \end{array}$ = 4 hundreds 8 tens 3 ones

$\times 2$

8 hundreds 16 tens 6 ones = 966

In this example 16 tens must be renamed. As a result, 1 hundred must be remembered and added to the 8 hundreds.

b. $483 = 400 + 80 + 3$

$\underline{\times 2} \qquad \underline{\times 2}$

$800 + 160 + 6 = 966$

• Perhaps *slower learners* will profit from a brief review of renaming numbers by doing exercises like those below either orally or as written work.

a. 4 tens 15 ones may be renamed _ ? _ tens 5 ones.

b. 6 tens 28 ones may be renamed 8 tens _ ? _ ones.

c. $20 + 18 =$ _ ? _ $+ 8 =$?

d. $400 + 150 + 7$ _ ? _ $+ 50 + 7 =$?

• *More capable children* may try to determine whether answers are reasonable. They may then show other pupils their findings. For example, Ex. 11c on page 303 may be tested in the two ways shown below.

390 is greater than 300

$\underline{\times 2} \qquad \underline{\times 2}$

780 is greater than 600

390 is less than 400

$\underline{\times 2} \qquad \underline{\times 2}$

780 is less than 800

• Use Extra Examples Set 78 as needed.

NOTES

For which of Ex. 4–7 do you remember and add hundreds after multiplying? Explain.

Ex. 5 and 6

18 tens = 1 hundred and 8 tens

4. 5×112 **5.** 4×247 **6.** 3×182 **7.** 4×208

24 tens = 2 hundreds and 4 tens

Tell the missing digits. Explain your work.

	a	b	c	d	e	f	g	h
8.	143	131	236	131	496	196	120	453
	×2	×5	×4	×5	×2	×3	×8	×2
	2?86	6?55	9?4 (4)	6?55	??2 (99)	5?? (88)	9?60	??6 (90)

Which multiplications are wrong? Explain why.

9.	141	138	132	229	124	269	108	432
	7	4	6	3	8	2	5	2
	987	522	692	687	962	438	940	964

b. $4 \times 3 = 12$; $12 + 3 = 15$ c. $6 \times 1 = 6$; $6 + 1 = 7$ e. $8 \times 2 = 16$; $16 + 3 = 19$

f. $2 \times 2 = 4$; $4 + 1 = 5$ g. $5 \times 1 = 5$ h. $2 \times 4 = 8$ [W]

Show products on folded paper.

	a	b	c	d	e	f	g
10.	374	140	192	190	141	152	$1.80
	2	5	4	2	6	2	4
	748	700	768	380	846	304	$7.20
11.	282	141	390	121	171	130	$4.62
	3	7	2	8	4	6	2
	846	987	780	968	684	780	$9.24
12.	132	297	246	132	489	156	$1.55
	7	3	4	5	2	3	4
	924	891	984	660	978	468	$6.20

Copy and multiply. Sometimes you do not rename.

13. 405×2 810 **16.** 6×102 612 **19.** 7×120 840 **22.** $5 \times \$1.32$ ($6.60)

14. 2×403 806 **17.** 332×3 996 **20.** 4×121 484 **23.** $8 \times \$1.14$ ($9.12)

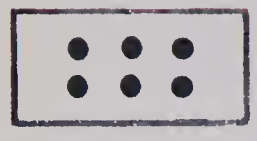
15. 3×184 552 **18.** 267×2 534 **21.** 104×6 624 **24.** $7 \times \$1.30$ ($9.10)

◆ **Extra Examples.** Work Set 78.

* This is the other use of n for a number in the factors-product relationship. Now n stands for a factor in both M. and D. examples.

*Finding an Unknown Factor

n; factors-product relationship [O]



1. From the dot picture, say the M. and D. facts.

$2 \times 3 = \overset{6}{n}$ $\quad 3 \times 2 = \overset{6}{n}$ $\quad \overset{6}{n} \div 2 = 3$ $\quad \overset{6}{n} \div 3 = 2$

When you know the two factors, 2 and 3, you can _multiply_ to find the product.

2. From the dot picture, say the two D. facts.

$6 \div 2 = \overset{3}{n}$ $\quad 6 \div 3 = \overset{2}{n}$

Because you know the product, 6, and one factor, 2 or 3, you can _divide_ to find the unknown factor.

3. Tell what you must find, product or factor.

a. $2 \times 4 = \overset{\text{product}}{n}$ $\quad$ b. $12 \div 3 = \overset{\text{factor}}{n}$ $\quad$ c. $6 \times \overset{\text{factor}}{n} = 18$

d. $21 \div \overset{\text{factor}}{n} = 3$ $\quad$ e. $\overset{\text{factor}}{n} \times 2 = 16$ $\quad$ f. $\overset{\text{product}}{n} \div 3 = 7$

4. Tell how to find the unknown factor in these:

a. $2 \times n = 10$ $\quad^{10 \div 2 = n}$ $\quad$ b. $7 \times n = 14$ $\quad^{14 \div 7 = n}$ $\quad$ c. $n \times 3 = 15$ $\quad^{15 \div 3 = n}$

5. Tell how to find the unknown factor in these:

a. $8 \div 2 = n$ $\quad^{8 \div 2 = n}$ $\quad$ b. $12 \div 4 = n$ $\quad^{12 \div 4 = n}$ $\quad$ c. $24 \div 3 = n$ $\quad^{24 \div 3 = n}$

6. Tell how to find the unknown factor in these:

a. $18 \div n = 3$ $\quad^{18 \div 3 = n}$ $\quad$ b. $16 \div n = 2$ $\quad^{16 \div 2 = n}$ $\quad$ c. $12 \div n = 4$ $\quad^{12 \div 4 = n}$

** **When you know the product and one factor, not zero, you divide to find the unknown factor.**

[W]

Write your work to find the number n stands for.

7. $16 \div 2 = n$ $\quad^{16 \div 2 = 8}$

8. $2 \times n = 18$ $\quad^{18 \div 2 = 9}$

9. $n \times 3 = 24$ $\quad^{24 \div 3 = 8}$

10. $14 \div n = 2$ $\quad^{14 \div 2 = 7}$

11. $27 \div 3 = n$ $\quad^{27 \div 3 = 9}$

12. $n \div 8 = 3$ $\quad^{8 \times 3 = 24}$

13. $5 \times n = 15$ $\quad^{15 \div 5 = 3}$

14. $18 \div 9 = n$ $\quad^{18 \div 9 = 2}$

15. $18 \div n = 6$ $\quad^{18 \div 6 = 3}$

16. $21 \div 7 = n$ $\quad^{21 \div 7 = 3}$

17. $8 \times n = 16$ $\quad^{16 \div 8 = 2}$

18. $3 \times 7 = n$ $\quad^{3 \times 7 = 21}$

● **Extra Activity.** Work Set 106.

** Zero must not be included as a known factor. In the factors-product relationship $4 = \frac{12}{3}$ and $3 \times 4 = 12$. However, for $\frac{4}{0} = n$, $4 = 0 \times n$, or $4 = 0$, which is a false sentence. It is agreed that we do not divide by zero.

Teaching Page 304

Pupil's Objective

To learn more about the factors-product relationship.

Background

The relationship between two factors and their product has been developed rather extensively in previous pages of this book. On page 296 there was a brief lesson in which pupils were to determine whether n in a sentence denoted a product.

The material on page 304 extends the introduction on page 296. Now, pupils are expected to generalize about the appropriate operation to use to find the product or factor represented by n.

In the two sentences $4 \times 3 = n$ and $n \div 4 = 3$, two factors are known so the appropriate operation is multiplication.

In the two sentences $15 \div n = 5$ and $n \times 5 = 15$, a product and one of two factors are known so the appropriate operation is division.

Children should begin to express in their own words the two generalizations about the use of multiplication and division.

Pre-Book Lesson

• Write incomplete sentences like the following on the board and have pupils make true sentences:

$8 \bigcirc 2 = 6$	$8 \bigcirc 2 = 16$
$8 \bigcirc 2 = 10$	$8 \bigcirc 2 = 4$

Through your questioning and discussion guide your pupils to understand that the operation used on a pair of numbers determines the result.

• Focus attention on only multiplication and division sentences by writing examples like the following: $4 \times 5 = n$; $8 \times n = 24$; $21 \div 3 = n$; $n \div 6 = 4$; and so on. For each sentence ask pupils to tell whether n stands for a factor or a product. Then, have children tell which of the other numbers shown in each sentence are products and which are factors. Ask each individual to tell why he thinks the number is a factor or a product.

It may be desirable to construct arrays on the chalkboard or to use objects on the desks to help children *see* the mathematical sentence.

Using the Text Page

The questions and ensuing discussion for Ex. 1–6 should provide adequate basis for pupils to state in their own words the generalization about which operation to use when a factor and a product are known and the generalization about which operation to use when two factors are known.

Individualizing Instruction

• Write examples in the forms shown below to use for the same type of discussion developed throughout this lesson:

n	6	7	n
$\times 3$	$\times n$	$n\overline{)28}$	$5\overline{)15}$
24	24		

• *More capable children* should be able to work exercises like the following independently:

$36 \bigcirc 9 = (2 + 2)$	$(3 + 2) \times 3 = 12 \bigcirc 3$
$36 \bigcirc 9 = (3 \times 9)$	$12 \bigcirc 12 = 6 \times 4$

• Use Extra Activity Set 106 as you deem appropriate. Guide pupils in realizing that neither 0 nor any number greater than 0 can be divided by 0. You may approach division of 0 by asking, "How many 0's are there in 0?" Because of the factors-product relationship, you may rephrase the question and ask "0 times what number equals 0?" $(0 \times n = 0)$ Help pupils realize that n in this sentence can stand for any number because 0 times any number equals 0. And so there is no unique number for n—i.e., the solution is indeterminate.

You may then approach division of a number greater than 0 by 0. "How many 0's are there in 2?" or "0 times what number equals 2?" $(0 \times n = 2)$. From what has been said about the product when one factor is 0, pupils should come to realize that 0 times any number equals 0 so therefore, there is no unique number that n can represent so that $0 \times n = 2$ will be a true sentence.

Encourage pupils to state the following ideas:

a. It is meaningless to divide 0 by 0 because there is an infinite number of results possible.

b. It is meaningless to divide a counting number by 0 because there isn't any result possible.

Teaching Page 305

Pupil's Objective

To extend what is known about sets of M. and D. facts to facts in which 4 is a factor.

Background

Two major ideas have already been developed. Every M. and D. fact is part of a set of M. and D. facts. When both factors are equal there are just two facts in the set (one M. fact and one D. fact). When the factors are not equal there are four facts in the set (two M. facts and two D. facts).

An array may be constructed to illustrate a set of M. and D. facts. The number of rows and the number of columns of the array show the factors and the total number of elements in the array shows the product.

Pre-Book Lesson

• With a large chart like the one shown below, review the M. facts that have been learned so far.

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20					
6	6	12	18	24					
7	7	14	21	28					
8	8	16	24	32					
9	9	18	27	36					

Give a pupil a pointer. Have the child point out each number called for. "One factor is 4, the other factor is 2; point to the product shown."

"The product I am thinking of is 21, one of its factors is 7, point to the other factor shown."

• You may wish to construct an array as each M. fact in the chart is being discussed. This may be done by each pupil at his desk either with dots on paper or with discs.

Some pupils may note that an array is formed on your classroom chart for each M. fact discussed. For example, an array of square shapes can be seen in working with the M. fact $4 \times 2 = 8$.

×	1	2
1	1	2
2	2	4
3	3	6
4	4	8

• Ask a child to show a fact on the chalkboard for numbers pointed out on the chart.

Another child can find the same factors shown in different places on the chart. Discuss with pupils that when the factors of a pair are not equal, they can be found in two different places.

• Conduct the same kind of discussion for three other numbers. Though pupils have not studied the D. facts involving 4, they should be able to show these D. facts from what they know.

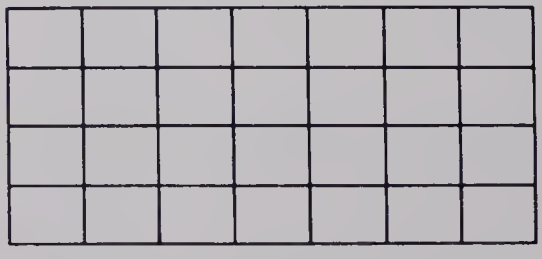
Using the Text Page

• The oral part of this lesson should develop very nicely. Make use of arrays.

• If your pupils can construct arrays and understand sets of M. and D. facts, they should have no difficulty with the written work.

Individualizing Instruction

• *Slower learners* may use graph paper with squares $\frac{1}{2}$ inch or larger to outline or cut out arrays for M. and D. facts. On the back of the array the child should write sentences showing the set of M. and D. facts represented.

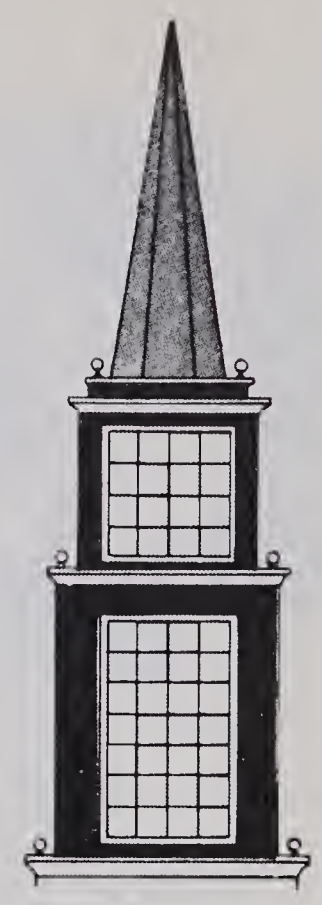


$4 \times 7 = 28$
$7 \times 4 = 28$
$28 \div 7 = 4$
$28 \div 4 = 7$

• *All pupils* may make their own M. charts for M. facts in which one factor is not greater than 4.

M. and D. Facts with 4 as a Factor

Use of arrays; factors-product relationship [O]



1. The lower set of windows in the picture has 7 rows with 4 windows in each row.

a. 7 and 4 are factors of what product? 28

b. Say the two M. facts. $7 \times 4 = 28$ $4 \times 7 = 28$

c. Dividing by 7 undoes multiplying by 7, and dividing by 4 undoes multiplying by 4. So, what two D. facts go with the two M. facts?

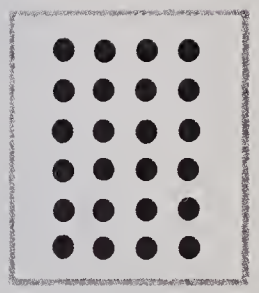
d. On the board show the set of M. and D. facts for 28 and its factors 7 and 4. $28 \div 7 = 4$ $28 \div 4 = 7$

$7 \times 4 = 28$ $28 \div 7 = 4$
$4 \times 7 = 28$ $28 \div 4 = 7$

2. The top set of windows has $_\,\overset{4}{?}\,_$ rows with $_\,\overset{4}{?}\,_$ windows in each row.

a. What product and factors do you find? 16; 4 and 4

b. Say the D. fact in this set of facts. $16 \div 4 = 4$



3. The dot picture has $_\,\overset{6}{?}\,_$ rows of $_\,\overset{4}{?}\,_$ dots.

a. The product is $_\,\overset{24}{?}\,_$. Its factors are $_\,\overset{6}{?}\,_$ and $_\,\overset{4}{?}\,_$.

b. Say the M. and D. facts in this set of facts.

$6 \times 4 = 24$ $4 \times 6 = 24$ $24 \div 6 = 4$ $24 \div 4 = 6$ [W]

4. Draw a dot picture with 5 rows of 4 dots each. Show the set of M. and D. facts. $5 \times 4 = 20$ $20 \div 5 = 4$
$4 \times 5 = 20$ $20 \div 4 = 5$

Show the M. and D. facts in the set with product

5. 32; factors 4 and 8.
$4 \times 8 = 32$ $8 \times 4 = 32$ $32 \div 4 = 8$ $32 \div 8 = 4$

6. 24; factors 4 and 6.
$4 \times 6 = 24$ $6 \times 4 = 24$ $24 \div 4 = 6$ $24 \div 6 = 4$

7. 28; factors 4 and 7.
$4 \times 7 = 28$ $7 \times 4 = 28$ $28 \div 4 = 7$ $28 \div 7 = 4$

8. 36; factors 4 and 9.
$4 \times 9 = 36$ $9 \times 4 = 36$ $36 \div 4 = 9$ $36 \div 9 = 4$

Copy and finish Ex. 9–16. Then show the other facts in each set of M. and D. facts you started.

$5 \times 4 = 20$ $4 \times 5 = 20$ $20 \div 5 = 4$ $20 \div 4 = 5$ $2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$

9. 5×4 11. 3×4 13. 2×4 15. 6×4

$3 \times 4 = 12$ $4 \times 3 = 12$ $12 \div 3 = 4$ $12 \div 4 = 3$ $6 \times 4 = 24$ $4 \times 6 = 24$ $24 \div 6 = 4$ $24 \div 4 = 6$

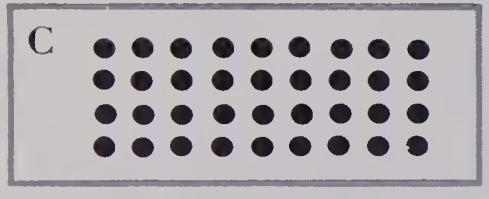
10. 4×4 12. 7×4 14. 8×4 16. 4×9

$4 \times 4 = 16$ $16 \div 4 = 4$
$7 \times 4 = 28$ $28 \div 7 = 4$ $4 \times 7 = 28$ $28 \div 4 = 7$
$8 \times 4 = 32$ $32 \div 4 = 8$ $4 \times 8 = 32$ $32 \div 8 = 4$
$4 \times 9 = 36$ $36 \div 4 = 9$ $9 \times 4 = 36$ $36 \div 9 = 4$

A

B

C



D. Facts with a Factor 4

Pairs of D. facts; use of arrays [O]

1. Mrs. Fry put 5 large roses in each bouquet. How many bouquets did she make from 20 roses? $20 \div 5 = n$

a. Picture *A* shows ? 4 bouquets.

b. If you had no picture, could you find the unknown factor by subtracting 5's? Yes by using a M. fact? Yes Explain. The number of 5's subtracted is the unknown factor. Use corresponding M. fact.

c. On the board subtract or use a M. fact to find the unknown factor for $20 \div 5 = n$.

d. Say the D. fact with its answer. $5 \times 4 = 20$

e. Has the D. fact another D. fact $20 \div 5 = 4$ to make a pair? Yes If so, say it. $20 \div 4 = 5$

2. Picture *B*. Mrs. Fry asked Alice to make bouquets with 28 zinnias, putting the same number in each of 4 vases. How many zinnias will be in each vase? $28 \div 4 = n$ 7

a. In this D. example, what is the product? 28 What is the known factor? 4

b. On the board draw a dot picture or use a M. fact to find the unknown factor. $4 \times 7 = 28$

c. Say the D. fact. $28 \div 4 = 7$

d. Does this D. fact have another D. fact to make a pair? Yes If so, say it. $28 \div 7 = 4$

3. Say two M. facts from picture *C*. $4 \times 9 = 36$ $9 \times 4 = 36$

Then say the pair of D. facts in the set of M. and D. facts. $36 \div 4 = 9$ $36 \div 9 = 4$

4. On the board show two D. facts for the product 32 and its factors 4 and 8. $32 \div 4 = 8$ $32 \div 8 = 4$

Teaching Pages 306 and 307

Pupil's Objective

To begin to learn the division facts in which 4 is a factor.

New Words

(page 306) *bouquet*, *vases*, *zinnias*

Background

The next three pages in the child's book afford the opportunity to study, learn, and organize the division facts in which 4 is a factor. Sometimes 4 is the known factor and sometimes 4 is the unknown factor. The two tables of D. facts on page 307 require the child to organize what he knows and what he is learning into a form in which many generalizations can emerge. Analyzing the tables and discussing the various relationships is an important part of learning these particular facts. Ideas and generalizations which have been introduced in previous learnings should be reinforced at this time.

The idea that we divide in measurement and partitive situations should also be reinforced. Sometimes the answer to a division example tells us the number of subsets contained in a larger known set. This is the measurement situation. At other times the answer tells us the number of members in each of a given number of subsets. This is the partitive situation. As was noted earlier, these terms should not be given to the learner, but rather the concepts involved.

Teacher's Preparation

Each child should have some objects for constructing arrays. Discs, squares of paper, bottle caps, and so on will do.

Pre-Book Lesson

• Ask your pupils to place 24 objects in a pile. Each child must be certain that he has just 24 objects in the pile. Then, ask the children to find "how many 4's there are in the pile of 24."

Each child should realize that he has six sets of four objects on his desk. Discuss the question asked and the answer to the question, "What does the 6 mean?" (Pupils have more difficulty with the meaning of the answer in division than with the other operations.)

• Continue finding the number of subsets of 4 in a given set of objects (16, 20, 28, 32). Since division is associated with separating or identifying sets of a given size, pupils should have considerable experience with the idea through the use of objects.

• Direct pupils to construct an array as they work with objects to find how many 4's are contained in 24. First, the child will have a pile of 24 objects. Then, he removes a set of 4 from the pile. These should be arranged in a row. The next set of 4 should be arranged in a row under the first one. Continue removing 4's and arranging them in rows until the array is completed.

The activity just completed should help establish to a greater degree the relationship between division and multiplication. Through guided discussion, help pupils *see* and state the four related M. and D. facts that go with this array.

Using the Text Pages

• The oral work on page 306 is very important preparation for the written work on page 307. You may find it necessary to ask the questions in different ways to assure the development of understanding. Construct an array for each problem. Give variety in the construction of arrays by having some pupils use objects on their desks, others use dots on paper or the chalkboard, and others use felt cut-outs on the flannel board.*

• The written work includes a variety of activities and learnings. First, in Ex. 5–9, the child directly relates facts he has shown to an array he has just constructed.

Ex. 10–17 provide some beginning drill work with these particular facts. Call attention to the fact that some answers are wrong. Permit pupils to detect the errors by any of the methods they have been learning in connection with the M. and D. facts.

In Ex. 18, the children are to complete the two tables of D. facts in which 4 is a factor. Completing the tables is important but it is equally

* See 17 and 2, page xix.

important to analyze the tables and discuss them for the same generalizations which were identified for similar tables for factors 2 and 3.

Finally, Ex. 19–21 are problems which use the division operation. Pupils may use either subtraction or division to check their work.

Individualizing Instruction

- Adapt the written work on page 307 as may be necessary for *slower learners*. It may be necessary to discuss the work orally before assigning it as written work.
- *Slower learners* (and possibly *all pupils* who are having difficulty with 5 or more D. facts) may construct arrays with dots, find answers by subtraction, show sets of M. and D. facts, and make matching cards for facts and answers. Provide a model for one set of facts and then supervise the pupils' work for the next one or two sets of facts. Then, pupils may work independently or with pupil helpers.
- You may have *all pupils* begin a notebook for the division facts. Each table of facts should be written on a separate page of the notebook. This is an activity pupils may work on during any spare time or at home.

Extra Problems:

a. If each child sells 7 tickets, how many children will be needed to sell 28 tickets?

b. If 5 children can sit on one bench, how many benches will be needed for 20 children?

c. If each child eats 4 sandwiches at Ann's birthday party, 32 sandwiches will be enough for how many children?

d. If a quart of ice cream is enough for 6 children, how many quarts must be bought to serve 24 children?

e. A pan of candy can be cut into 9 large pieces. If you want 36 large pieces, how many pans of candy will you need?

f. If you can cut out 8 gold stars from one sheet of paper, how many sheets of paper are needed to cut out 32 gold stars?

NOTES

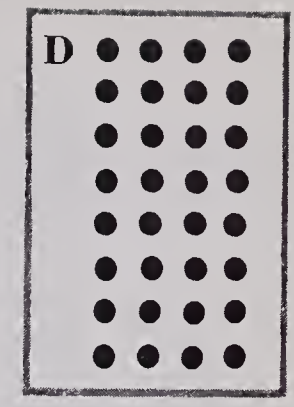
[W]

Show the set of M. and D. facts you can get

5. from picture D. $8\times4=32$ $4\times8=32$ $32\div8=4$ $32\div4=8$

6. from picture E. $4\times4=16$ $16\div4=4$

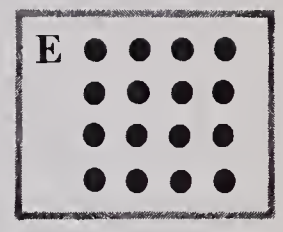
7. Make a dot picture F for 24 and its factors 4 and 6. Then show the set of M. and D. facts.



Show the set of M. and D. facts

8. for 36 and its factors 4 and 9. $4\times9=36$ $9\times4=36$ $36\div4=9$ $36\div9=4$

9. for 20 and its factors 4 and 5. $4\times5=20$ $5\times4=20$ $20\div4=5$ $20\div5=4$



Copy each of Ex. 10–17 and find which of pictures A–F you can use to check the answer.

a. Put $\surd$ after your answer if it is correct.

b. If your answer is not correct, show the correct D. fact from the picture.

10. $28 \div 4 = 7$ $\surd$

11. $20 \div 5 = 3$ $20\div5=4$

12. $32 \div 8 = 4$ $\surd$

13. $24 \div 6 = 3$ $24\div6=4$

14. $36 \div 9 = 4$ $\surd$

15. $16 \div 4 = 5$ $16\div4=4$

16. $28 \div 7 = 7$ $28\div7=4$

17. $36 \div 4 = 6$ $36\div4=9$

18. Copy and finish the tables of D. facts with 4 as a factor. $16\div4=4$ $20\div4=5$ $24\div4=6$ $28\div4=7$ $32\div4=8$ $16\div4=4$ $20\div5=4$ $24\div6=4$ $28\div7=4$ $32\div8=4$

Division Facts Dividing by 4

$4 \div 4 = 1$
$8 \div 4 = 2$
$12 \div 4 = ?^{3}$
and so on to
$36 \div 4 = ?^{9}$

Division Facts Answer 4

$4 \div 1 = 4$
$8 \div 2 = 4$
$12 \div 3 = ?^{4}$
and so on to
$36 \div ?^{9} = ?^{4}$

For Ex. 19–21 write answers only. For help, subtract or use a picture or a M. fact.

19. How many bags will you need to put 28 pieces of candy in bags, 7 pieces to a bag? 4 bags

20. If 9 pencils cost 36¢, how much does one cost? 4¢

21. If 32 boys rode to a ball game in trucks, 8 boys to a truck, how many trucks did they use? 4 trucks

Table of D. Facts, Dividing by 4

1	2	3	4	5	6	7	8	9
$4\overline{)4}$	$4\overline{)8}$	$4\overline{)12}$	$4\overline{)16}$	$4\overline{)20}$	$4\overline{)24}$	$4\overline{)28}$	$4\overline{)32}$	$4\overline{)36}$

Table of D. Facts, Answer 4

4	4	4	4	4	4	4	4	4
$1\overline{)4}$	$2\overline{)8}$	$3\overline{)12}$	$4\overline{)16}$	$5\overline{)20}$	$6\overline{)24}$	$7\overline{)28}$	$8\overline{)32}$	$9\overline{)36}$

Factors-product relationship [O]

1. In the tables above, cover the answers and try to say them. Make Study Cards for facts you do not know.

2. What D. fact is the same for both tables? The factors are equal. Why? $16 \div 4 = 4$

3. When one factor is 4, is the product odd or even? even

4. When you divide by 4, why are the answers greater by 1 as you go from left to right in the table? Each product increases by 4.

5. How many new D. facts are shown in the two tables? 11

Greater Than? Less Than? Equal To?

Equalities and inequalities [W]

*Remember: For is greater than, we can write > and for is less than, <. Do the work in ( )'s for each of Ex. 1–10. Then copy the example using > or < or =.

1. $(28 \div 4) \;_?_\; (27 \div 9)$ — $>$
2. $(32 \div 4) \;_?_\; (300 - 295)$ — $<$
3. $(3 \times 179) \;_?_\; (900 - 147)$ — $>$
4. $(986 - 400) \;_?_\; (501 - 97)$ — $<$
5. $(24 \div 6) \;_?_\; (18 \div 2)$ — $>$
6. $(115 \times 4) \;_?_\; (710 - 89)$ — $<$
7. $(560 + 398) \;_?_\; (7 \times 134)$ — $=$
8. $(32 \div 4) \;_?_\; (24 \div 3)$ — $<$
9. $(4 \times 39) \;_?_\; (5 \times 44)$ — $=$
10. $(206 - 197) \;_?_\; (36 \div 4)$ — $>$

● **Extra Activity.** Work Set 107.

* By associating repeatedly the symbols > and < with their meanings, you avoid the confusion that often occurs with these symbols.

Teaching Pages 308 and 309

Pupil's Objectives

(a) To organize the D. facts with 4 as a factor; (b) to have practice in the understandings and skills that have been developed in working with multiplication and division facts; (c) to maintain and use the symbols ($>$ and $<$) for *is greater than* and *is less than*; and (d) to check understandings of important ideas studied to this point.

Background

As each new table of M. and D. facts is introduced and developed, pupils should begin to learn relationships which exist among the various tables. One such generalization has to do with the number of new facts that actually are included in each new table encountered.

After a child has learned the M. tables in which 2, 3, and 4 are factors, he knows most of the facts which will appear in tables not yet introduced to him. When he learns the M. facts in which 2 is one factor he is also learning facts which will appear in tables for the other factor. For example, 2×8 will appear in the table for 8's; 8×2 will appear in the table for 8. By the time he is actually introduced to the tables for 9 and 9's, there will be just one new fact to learn—namely, $9 \times 9 = 81$.

The same is true for work with D. tables which correspond to the M. tables because of the factors-product relationship. In the table for dividing by 9, the only new fact is $81 \div 9 = 9$.

It would be a very worthwhile experience to have pupils list tables of facts, and each time encircle all new facts being shown.

Many lessons and enrichment activities throughout this book have used the ideas of *is greater than* and *is less than* and the mathematical symbols. The use of the symbols is sometimes confusing to your pupils. Remind them that each symbol has an open end which is greater than the closed end. Or, conversely, the closed end is less than the open end. Generally speaking, the end pointing to the left points to the term we read first. The sentence $(2 + 5) > (9 - 4)$ would read, "Two plus five is greater than nine minus four" or it could be read, "Seven is greater than five."

Pre-Book Lesson

• Write a division sentence on the chalkboard such as $14 \div 2 = n$. Ask a child to read the sentence. Ask, "Does this sentence *tell* us something or does it *ask* us something?" Emphasize the difference between sentences that tell and sentences that ask. Then, ask a child to show another way that this example may be written.

$$2\overline{)14}\ n$$

Continue this activity of having pupils write division examples in these two different forms. Your pupils should be thoroughly familiar with and use both forms of writing a division example.

• Write on the chalkboard, "$4 \times 2 > 2 \times 3$." Ask pupils if the sentence on the board is true or false. Have them tell why.

Write on the board, $4 \times 2 < 3 \times 3$. Ask pupils if this sentence is true or false. Ask them to explain why.

Remind pupils that these symbols are more meaningful than most symbols in that the greater end of each symbol points to the greater number and the lesser end points to the lesser number.

Ask different pupils to write a sentence on the chalkboard using two pairs of parentheses and either $>$ or $<$. Then, have the child read the sentence he has written. Or, he may ask another child to read the sentence he has written.

Using the Text Pages

• Ask pupils to study the two tables at the top of page 308. Then, read and discuss each of the questions carefully and thoroughly. In Ex. 5, be sure to guide your pupils to see that not all the facts in the tables are new. Direct children to find and state the facts they already know from learning the 2's and 3's.

• The written work at the bottom of page 308 permits pupils to review the symbols for *is greater than* and *is less than* and at the same time have some practice in many skills which have been developed so far. Probably, *all pupils* will want to try these exercises. Perhaps you can select certain ones

which your *slower learners* can do successfully and ask them to work only those exercises at this time.

• The Try These exercises at the top of page 309 include D. facts in which 2, 3, or 4 is a factor. As an oral lesson, pupils may work in pairs.

• The maintenance work at the bottom of page 309 should provide you with some insights as to the progress being made by each of your pupils. If you find weak spots, now is the time to reteach, if necessary.

Individualizing Instruction

• You may wish to duplicate some exercises like the following for your *slower learners*:

Use $>$ or $<$ to make each a true sentence.

a. $(43 + 17)$ ___ (24×2)
b. $(18 \div 3)$ ___ $(12 - 7)$
c. (3×5) ___ $(25 - 7)$
d. (4×2) ___ $(18 \div 2)$
e. $(3 + 5 + 8)$ ___ (7×3)
f. $(20 \div 4)$ ___ $(12 \div 2)$

• The Try These exercises may be used as a practice test. *All pupils* who missed 7 or more facts should construct a dot array and show the set of M. and D. facts for each of the examples he missed on the test.

• *Slower learners* may use the matching cards they prepared previously (see Teacher's Page 307) to gain additional practice in working with the division facts in which 4 is a factor.

• Use Extra Activity Set 107 as you deem appropriate. In guiding pupils, you may wish to note some of the following ideas:

a. An even number is a number of 2's. For example, 6 is three 2's, 8 is four 2's, 14 is seven 2's, and so on.

b. When we multiply an even number by either an even number or an odd number, the result is another number of 2's—i.e., another even number. For example, 6 is three 2's, so 4 times three 2's is twelve 2's or 24. Also, 10 is five 2's, so 3 times five 2's is fifteen 2's or 30.

NOTES

*Another possible quick test on D. facts! This time one factor is no greater than 4.

*Try These

D. facts with a factor 2 or 3 or 4 [O]

Say the answers by rows, then by columns.

	a	b	c	d	e	f	g
1.	6)24 (4)	3)27 (9)	4)16 (4)	7)21 (3)	4)24 (6)	9)18 (2)	4)4 (1)
2.	7)28 (4)	2)16 (8)	3)15 (5)	4)32 (8)	7)14 (2)	4)20 (5)	3)21 (7)
3.	6)18 (3)	5)20 (4)	4)8 (2)	3)24 (8)	5)15 (3)	8)24 (3)	3)9 (3)
4.	9)27 (3)	8)16 (2)	3)12 (4)	2)14 (7)	8)32 (4)	2)18 (9)	4)12 (3)
5.	2)12 (6)	4)28 (7)	3)18 (6)	2)8 (4)	4)36 (9)	9)36 (4)	6)12 (2)

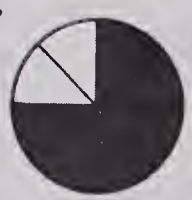
[W]

Now write answers on folded paper.

**Many Things

Maintenance [W]

For each of Ex. 1–4 write a missing numeral.



1. _?_ $\frac{6}{8}$ of the inside of this circle is colored.

2. Factors of 28 are 4 and _?_. 7

3. In the fraction $\frac{4}{7}$, the _?_ 7 shows how many parts of the same size there are in a whole.

4. There are _?_ 2 facts in the set of M. and D. facts for 16 and its factors 4 and 4.

Using rounded numbers, write estimated answers.

5. $27 + 29$ 60 6. $91 - 49$ 40 7. $32 + 39$ 70 8. 2×41 80

Write the work to find the number for n.

9. $28 \div n = 4$ (28÷4=7) 10. $n \times 4 = 36$ (36÷4=9) 11. $n \div 8 = 3$ (8×3=24)

12. What polygon has 4 sides and 4 square corners? rectangle

**Another mid-chapter review of information and meanings!

*A set may have just 1 member. Consequently, the work in Ex. 2, 3, and so on is actually just an extension of ideas pertaining to sets having more than 1 member.

To Keep in Practice

A., S., M. [W]

Copy and work Ex. 1–20.

1. 4×237 948
2. 143×6 858
3. $342 + 538$ 880
4. $800 - 269$ 531
5. 89×4 356
6. $88 + 465$ 553
7. $864 - 703$ 161
8. 7×140 980
9. $711 - 284$ 427
10. $5 \times \$1.44$ \$7.20
11. 44×9 396
12. $721 - 469$ 252
13. $\$9.40 - \$8.36$ \$1.04
14. $\$6.13 - \$3.94$ \$2.19
15. $701 - 469$ 232
16. $4 \times \$1.24$ \$4.96
17. $5 + 9 + 8 + 6 + 7$ 35
18. $46 + 8 + 9 + 7 + 6$ 76
19. $348 + 37 + 420$ 805
20. $\$2.41 + \$1.88 + \$3.48$ \$7.77

Part of a Set

[O]

A

*1. Picture *A*. Each part of the candy stick is $\frac{1}{2}$ of the whole stick. Why? The candy is separated into two parts of the same size.

a. Does $\frac{1}{2}$ name the number for each part? Yes

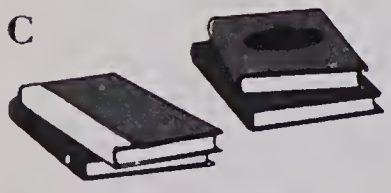
b. Does the fraction number, one half, mean 1 of the 2 parts? Yes

B

2. Picture *B*. This set of 8 books is separated into _?_ 2 parts having the same number of books.

a. Does the fraction number, one half, mean 1 of the 2 parts? Yes

C



b. $\frac{1}{2}$ of 8 books is _?_ 4 books.

3. Picture *C*. This set of 8 books is separated into _?_ 4 parts having the same number of books.

a. Does the fraction number, one fourth, mean 1 of the 4 parts? Yes

b. $\frac{1}{4}$ of 8 books is _?_ 2 books.

Teaching Pages 310 and 311

Pupil's Objectives

(a) To review computational skills which have been learned; (b) to discover the idea that a fraction number may represent a part of a set of things as well as a part of one thing; and (c) to use division to find the number in each part represented by a fraction number.

New Word

(page 311) *cones*

Background

Children have heard and used "half a dozen" which involves the fraction-number idea representing part of a set of things. Informally, they have probably been exposed to this idea in a variety of ways. However, the only formal development of fraction numbers thus far has been limited to a fraction number representing part of one thing.

In this lesson, children must think of equivalent parts of a set and how many objects are in each of the parts. This idea ties in very nicely with the partitive idea mentioned in an earlier lesson where a group was separated into several parts of the same size and the task was to determine how many objects were in each of the parts. Now, too, the child determines how many parts of the same size there are (determined by the fraction number) and then finds how many objects there are in each of these equivalent parts. This situation, like the partitive situation studied earlier, requires division.

It is true that we divide to find the answer for an example such as $\frac{1}{2} \times 12$. But, we do not divide by $\frac{1}{2}$. When the child comes to understand the partitive situation involved—namely that a set of 12 members is separated into 2 sets with the same number of members, he will realize that division by 2 is necessary in order to find how many members there are in each of the two equivalent sets.

Teacher's Preparation

• Refer to pupil's pages 140–143 and the corresponding teaching pages in order to determine the appropriate method of review of fractions showing the numerator 1. In the event you have extended this material, you will know what has been taught and how much review is necessary.

• Consider the real objects and representative materials you have available for developing the idea that sets of objects may be separated into parts of the same size. Be sure to provide materials such that each pupil may participate in *discovering* the idea.

Pre-Book Lesson

• Direct pupils' review of fraction numbers representing one part of one thing. ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, . . . $\frac{1}{10}$) Use string, ribbon, folded paper, circular and rectangular cut-outs, and so on.

• The following generalizations should be stated:

a. Three thirds (four fourths, six sixths, and so on) of the same object make one whole object.

b. One half (one third, one seventh, and so on) of a small whole object will not be the same size as one half (and so on) of a larger object. Folded strips of paper are effective to help develop this idea.

c. The more parts there are of one whole thing, the smaller the parts are (one eighth of it is smaller than one half).

• Place a set of four objects on the flannel board. Ask a child to identify the number of objects in the set. Then, have the child arrange the objects into two subsets (each with the same number of objects). You may ask the child to place one subset in one location on the flannel board and the other subset in another location. Discuss with pupils the way in which one half of a set of four things involves the same idea as one half of one thing.

• Most children at this age know and understand the idea of one half of a set of objects. They begin to have trouble identifying one fourth and one third of a set. Place six objects on the flannel board. Have pupils separate the set of six objects into two subsets (halves). Then, have them arrange the set of six objects into three subsets (thirds). Be sure pupils understand the idea that each of the three subsets necessarily have the same number of members.

Continue this activity with sets of objects containing more members. Separate them into halves, thirds, fourths. In this activity it is extremely important that pupils grasp the idea of arranging a set into subsets each having the same number of objects.

Using the Text Pages

• The written work at the top of page 310 may be used as an achievement test. Or, you may make a diagnostic analysis of the results on the test. If pupils miss two or more items of the same type, perhaps they are in need of some review or reteaching.

• All the material on pages 310 and 311 should be developed orally. You should make extensive use of manipulative materials to help pupils build the correct understandings. Each pupil should have materials of his own to use. From time to time, use sets of objects in the classroom to dramatize the particular part of a set under consideration.

The suggested board work at the bottom of page 311 is for the purpose of relating what has already been learned with the idea in this lesson. When the idea of the fraction number representing a part of a set and the idea of finding how many in each of the equivalent parts are clearly established, then guide children to *see* that division may be used to find how many are in each of the equivalent parts.

Direct pupils to use objects to confirm the results obtained in Ex. 9–12.

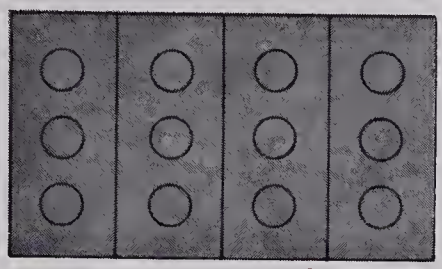
Individualizing Instruction

Slower learners will need much more work with sets of objects in finding how many are in each of the parts of a set. Pupils may be paired and work together in finding a part of a set of objects. Be sure to pair a *more capable child* with a *slower learner* to minimize the opportunity of incorrect learnings.

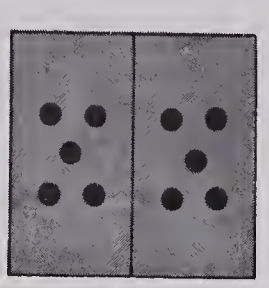
NOTES

For each of Ex. 4–6, what is the fraction number that tells about each part of the whole set pictured? Finish the sentence to tell how many in one part of the set.

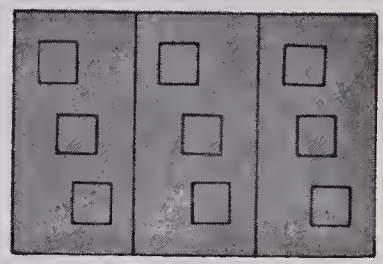
D



E



F



4. Picture D. $\frac{1}{4}$ of 12 circles is 3 circles.

5. Picture E. $\frac{1}{2}$ of 10 dots is 5 dots.

6. Picture F. $\frac{1}{3}$ of 9 squares is 3 squares.

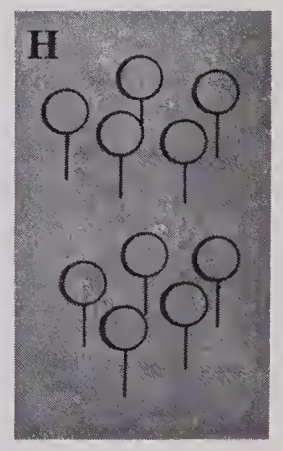
G

7. Picture G. $\frac{1}{3}$ of 6 cones is 2 cones.

The set of cones is separated into 3 parts, each part with the same number of cones. So each part is $\frac{1}{3}$ of the whole set.

To find how many in $\frac{1}{3}$ of 6, can you divide 6 by 3? Remember, you can write your work these two ways:

H



a. $6 \div 3 = 2$ **b.** $3\overline{)6}\ 2$

8. Picture H. How many lollipops are there in $\frac{1}{2}$ of 10 lollipops? 5

Use division to find the answer. On the board write your work two ways, as shown above.

Check your answer by using the picture.

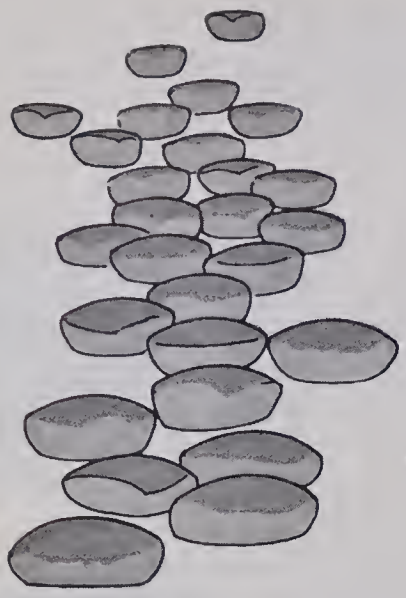
Copy on the board and finish Ex. 9–12. Use division to find the answers. Write each D. example two ways.

9. $\frac{1}{4}$ of 16 $16 \div 4 = 4$ $4\overline{)16}\ 4$

10. $\frac{1}{5}$ of 20 $20 \div 5 = 4$ $5\overline{)20}\ 4$

11. $\frac{1}{7}$ of 28 $28 \div 7 = 4$ $7\overline{)28}\ 4$

12. $\frac{1}{6}$ of 18 $18 \div 6 = 3$ $6\overline{)18}\ 3$



Finding How Many in One Part

[O]

1. Betty's mother made 27 rolls. If she baked the same number of rolls on each of 3 tins, how many rolls were on each tin?

a. What is the fraction number for each one of the 3 parts? $\frac{1}{3}$

b. On the board write in two ways the D. example to use in finding how many there are in one of the 3 parts. $27 \div 3$ $3\overline{)27}$

c. Say the D. fact. $27 \div 3 = 9$

2. Betty distributed 36 cupcakes equally among 4 plates.

a. What is the fraction number for each of the parts? $\frac{1}{4}$

b. On the board write and finish the D. example two ways for finding how many are in one part. $36 \div 4 = 9$ $4\overline{)36}$ [W]

What is the fraction number for each of the parts of a set separated equally into

3. 4 parts? $\frac{1}{4}$ 5. 5 parts? $\frac{1}{5}$ 7. 3 parts? $\frac{1}{3}$ 9. 10 parts? $\frac{1}{10}$

4. 9 parts? $\frac{1}{9}$ 6. 8 parts? $\frac{1}{8}$ 8. 7 parts? $\frac{1}{7}$ 10. 6 parts? $\frac{1}{6}$

Copy and finish Ex. 11–22. Use D. facts to find the answers. Write your work for each D. example two ways.

For Ex. 11 write, "$12 \div 2 = 6$ $2\overline{)12}$ $\frac{1}{2}$ of 12 is 6."

For Ex. 12 write, "$18 \div 6 = 3$ $6\overline{)18}$ $\frac{1}{6}$ of 18 is 3."

11. $\frac{1}{2}$ of 12 6
12. $\frac{1}{6}$ of 18 3
13. $\frac{1}{7}$ of 21 3
14. $\frac{1}{8}$ of 32 4
15. $\frac{1}{8}$ of 16 2
16. $\frac{1}{2}$ of 18 9
17. $\frac{1}{6}$ of 24 4
18. $\frac{1}{7}$ of 28 4
19. $\frac{1}{2}$ of 16 8
20. $\frac{1}{9}$ of 27¢ 3¢
21. $\frac{1}{8}$ of 24¢ 3¢
22. $\frac{1}{5}$ of 20¢ 4¢

Teaching Page 312

Pupil's Objectives

(a) To extend what has been learned about the fraction number for each of the equivalent parts of a set; and (b) to find by D. how many are in one part.

New Word: *tins*

Background

This lesson continues the work begun on pages 310 and 311. The oral work presents the ideas in problem settings. It is very important to utilize the many applications of the mathematical skills your pupils are building day by day. These opportunities arise in all facets of the school curriculum as well as in daily activities in general. It is important for you to explore informally various problems with which your pupils may be confronted outside the mathematics lessons.

The written work at the bottom of page 312 should give your pupils practice in writing a division example in two different mathematical forms. Bring out in your discussions with children that either of the D. forms may be used to find how many in each part. In the past, we have tended to use just one of these forms, $4\overline{)12}$.

Make certain your pupils have a good grasp of the ideas represented by the different forms before requiring them to use the forms. For many pupils, this means a great deal of work with objects.

Pre-Book Lesson

• State a classroom problem like the following.

"I have 24 pieces of construction paper. I want to place the same number of pieces of paper on each of the 4 tables. How many pieces of paper will be on each table?"

• Ask a child the fraction number for each part of the set and how he would solve the problem. Permit him to distribute the papers in any way he wishes. Then complete a division example on the chalkboard to go with the problem.

• Then, ask pupils to state the problem in another way. Be sure to elicit from your pupils both of the following ways of describing the situation and asking the question.

a. "I have 24 pieces of construction paper. I want to separate them equally among the 4 tables. How many pieces are on each table?"

b. "I have 24 pieces of construction paper. If I place $\frac{1}{4}$ of the pieces on each table, how many pieces will there be on each table?"

• After the problem has been stated in several different ways, have pupils complete another division example on the chalkboard. Thus, you should have both forms on the board:

$$24 \div 4 = 6 \qquad \begin{array}{r} 6 \\ 4\overline{)24} \end{array}$$

An analogy like the following may help pupils to understand that different mathematical forms may be used to express the same idea.

State: "My father took me to the zoo." Ask pupils if they can tell the same thing using different words or arranging the words in another way. Other possibilities are: "I went to the zoo with my father." "My father and I went to the zoo together." and so on.

Using the Text Page

• While discussing the problems at the top of the page, have pupils use representative objects to help find the solutions. Be sure the children understand that for Ex. 11–22, sets of the given sizes are intended.

• Leave the two mathematical forms on the board for pupil reference (if needed) in doing the written work. Permit the children to use objects if they wish in connection with the written work.

Individualizing Instruction

• Use every classroom situation possible in the distribution or organization of materials to apply what is being learned about a part of a set.

• Distribute different types of geometric designs for pupils to mount on charts to show separation of sets of objects into equivalent subsets.

$$\begin{array}{r} 5 \\ 3\overline{)15} \end{array}$$

$15 \div 3 = 5$

$\frac{1}{3}$ of a set of 15 is 5.

Teaching Page 313

Pupil's Objective

To learn more about using fractions in recording measurements.

Background

The question "How many?" is answered by considering a set of objects and telling how many elements are in the set.

On the other hand, questions such as "How far?" and "How long?" are answered by using units of measurement such as inches, feet, and hours.

The approximate length of an object is expressed by a measurement which is shown by a numeral for the measure and the name of a unit of measurement. The measurement is determined through the application of a unit of measurement to the object being measured. The number of times the unit is applied is considered the measure of the length.

We say that all measurements are approximate. When we give the measurement 7 inches, we have given the length to the nearest inch. If we wish to measure with greater precision, we may give the length in terms of the nearest half inch (then it would be necessary to use a fraction number for the measure—i.e., $1\frac{1}{2}$, $3\frac{0}{2}$, and so on). As we increase our unit of precision, we increase the precision of the measurement. Teaching Pages 64–65 on measurement further elaborate on this idea.

This discussion in connection with the various units of measurement presented on page 313 should help pupils see the need for using fractions as a means of showing greater precision in measurement. For example, $9\frac{1}{4}$ feet is a more precise measurement than *about* 9 feet.

Pre-Book Lesson

- Depending on how much work you have done in connection with the material presented on fractions and units of measurement, review the equivalents taught for linear, liquid, time, and weight measurements.

inches, feet, yards
pints, quarts, gallons
ounces, pounds
minutes, hours, days, weeks

- Ask pupils to tell ways in which they use fractions to describe distances. (halfway to the store, halfway home, and so on)

Ask them to tell ways in which they use fractions to describe weight and time.

Using the Text Page

All the measurements referred to in this lesson may be demonstrated in the classroom with the appropriate instruments.

a. Perhaps you have a large cardboard clockface. Use it with Ex. 1. Move the large hand to positions to show *quarter past*, *quarter before*, and so on. Be sure to bring out the idea that using a fraction permits us to give an answer that is more precise.

b. Use a scale, yardstick, gallon container, and so on to enhance the development of understanding in Ex. 2–12.

Individualizing Instruction

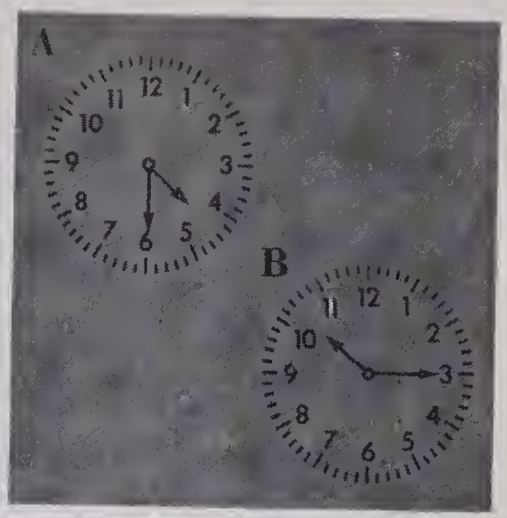
- *All pupils* may make a list of relationships which may be described with fractions ($\frac{1}{4}$ yd. = 9 in., and so on).
- *More capable children* may construct and show on the chalkboard some relationships as follows:

9 in.	9 in.	9 in.	9 in.

four 9's = 36 (or, 4 × 9 is 36)
so, $\frac{1}{4}$ of 36 in. is 9 in.

Fractions in Measurement

[O]



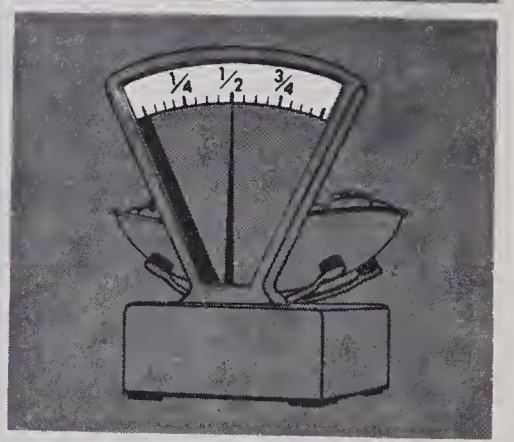
1. Why can we say that

a. clock A shows half past 4? The minute hand has moved one half around the clockface.

b. clock B shows quarter past 10? The minute hand has moved one quarter around the clockface.

2. One week is 7 days. Then 1 day is what part of a week? $\frac{1}{7}$

3. The scale shows that the candy weighs $\frac{1}{2}$ pound.



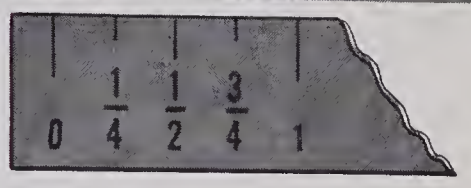
a. How many ounces does the candy weigh? 8 Use the little black marks to help you.

b. 1 pound is _?_ 16 ounces. $\frac{1}{2}$ of 16 is _?_ 8

4. How many ounces are there in

a. $\frac{1}{4}$ pound? 4 **b.** $\frac{1}{8}$ pound? 2

5. Explain the fractions in the picture of 1 inch on the ruler.



Use your ruler to measure these pictures of line segments. You may need to use fractions.

6. _____

7. _____

8. 1 yd. = 3 ft. What part of a yard is 1 foot? $\frac{1}{3}$

9. How many feet are there in $\frac{1}{3}$ of a yard? 1

10. 1 gal. = 4 qt. $\frac{1}{4}$ of a gallon is _?_ 1 quart.

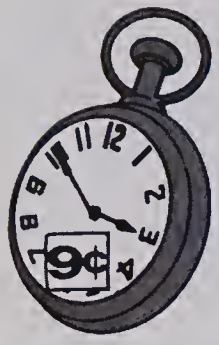
11. There are 8 pints in a gallon. How many pints are there in **a.** $\frac{1}{8}$ of a gallon? 1 **b.** $\frac{1}{4}$ of a gallon? 2

12. 12 things = 1 dozen. How many things are in

a. $\frac{1}{2}$ dozen? 6 **b.** $\frac{1}{4}$ dozen? 3 **c.** $\frac{1}{3}$ dozen? 4

Multiply or Divide or Subtract — Which?

Oral problem-solving [O]



Ex. 1 and 2. Tell whether to multiply or divide.

1. At 9¢ each, how much will 4 toy watches like that in the picture cost? Multiply $4 \times 9 = n¢$

2. Is $\frac{1}{2}$ of 18¢ enough to buy one of the watches? Divide $18¢ \div 2 = n¢$

Ex. 3–5. Tell whether to divide or subtract and why.

3. Ann put black paint on $\frac{1}{4}$ of 12 masks. How many masks did she paint black? Divide to find the number for each equivalent set. $12 \div 4 = n$

4. Ann's friends wore all but 3 of the 12 masks. How many were used? Subtract to find the number for the other part. $12 - 3 = n$

5. The 12 masks would make how many piles of 6 each? Divide to find the number of equivalent sets. $12 \div 6 = n$

Tell how to work each problem. Say each answer.

6. If you use $\frac{1}{4}$ of 24 sheets of paper for each notebook, how many sheets will each notebook have? $24 \div 4 = 6$ 6 sheets $24 \div 4 = n$

7. Peg made 9 notebooks in all. Jean made 16. How many fewer notebooks did Peg make than Jean? $16 - 9 = 7$ 7 notebooks $16 - 9 = n$

8. Jack found 2 eggs in each of 5 nests. How many eggs did he find in all the nests together? $5 \times 2 = 10$ 10 eggs $5 \times 2 = n$

9. If children sit 2 on a bus seat, how many seats will be needed for 14 children? $14 \div 2 = 7$ 7 seats $14 \div 2 = n$

10. Five boys shared 20 bus tickets equally. How many tickets did each boy have? $20 \div 5 = 4$ 4 tickets $20 \div 5 = n$

Now say an *n*-example for each of problems 1–10.

♦ **Extra Problems.** Work Set 15.

Pupil's Objective

To have practice in recognizing the differences between problems which use the operations of subtraction, multiplication, and division.

New Words

masks, *notebook*, *whether*

Background

The problems in Ex. 1–2 in the pupil's book are planned to indicate the difference between a problem which requires division and one which requires multiplication. Then, in Ex. 3–5, differences between problems requiring division and subtraction are noted. When sets each have the same number of members, we divide to find this number or to find the number of sets.

The necessary operations do not follow a definite sequence beginning with Ex. 6; thus, a choice of operation is required. The analysis of problem situations in this lesson affords a good opportunity to discuss the differences between the addends-sum relationship and the factors-product relationship. It also gives an opportunity to measure the degree of retention of previous mathematical experiences. Your pupils have learned addition and multiplication as *binary* operations, and have learned the terms associated with each. But, at no time so far has there been a formal discussion of a group of problems in which numbers may be either addends or factors.

Pre-Book Lesson

Write the following sentences on the chalkboard:

$$8 \bigcirc 4 = 12$$
$$8 \bigcirc 4 = 4$$
$$8 \bigcirc 4 = 32$$
$$8 \bigcirc 4 = 2$$

Ask pupils to tell the operation that is needed to make each a true sentence and then have them write the appropriate symbol in the sentence.

Ask other children to think about a problem for each sentence. Then have them discuss the problems and suggest ways in which they are alike and ways in which they are different. Bring out the idea that sometimes the numbers are addends and sometimes they are factors.

Using the Text Page

- Ask your pupils to compare the situations in Ex. 1 and 2 and explain why they must multiply or divide.
- The problems in Ex. 3, 4, and 5 seem to be much alike, but there is a basic reason why it is necessary to divide in Ex. 3 and 5 but not in Ex. 4. Help pupils to *see* that the parts referred to in Ex. 3 and 5 are equivalent, unlike those in Ex. 4. You may also use this discussion to discover how familiar your pupils are with the idea that division is used to answer the question, "How many members are there in each part?"
- Continuing with Ex. 6, probe beyond the answers given by different children. Find *why* one or another of the operations has been selected (whether the operations be correct or incorrect). Pupils may dramatize or draw sketches to help make meanings clear. Throughout the lesson, encourage pupils to state their own generalizations.
- After the discussion about the problems, pupils are asked to state the *n* sentence for each problem. These sentences may be written on the chalkboard. Be sure to emphasize understanding in this activity.

Individualizing Instruction

- Duplicate (or write on the chalkboard) exercises like the following. Pupils must write the correct operation symbol to make each a true sentence.

$9 \bigcirc 4 = 13$	$7 \bigcirc 3 = 21$
$3 \bigcirc 8 = 24$	$7 \bigcirc 3 = 4$
$21 \bigcirc 7 = 14$	$24 \bigcirc 4 = 6$
$15 \bigcirc 5 = 3$	$24 \bigcirc 8 = 16$

- *Slower learners* may need additional work in differentiating between the two situations requiring division—the partitive and measurement. As was noted earlier, these terms need not be introduced, but rather the concepts involved.

Teaching Page 315

Pupil's Objective

To have practice in computation with A., S., M., and D.

Background

Consider the mathematical sentence $7 = 16 - n$. The task in this particular sentence is to find an unknown addend. Now consider the same sentence rewritten $7 = (2 \times 8) - (3 \times n)$. Two numbers have been renamed as product expressions. However, the task is still to find an unknown addend. Now that one of its factors is shown, the other factor must be found.

The work at the bottom of the page is an excellent activity for many of your pupils. It involves practice in using addition and subtraction skills as well as multiplication and division. Working these sentences involves two operations. First, the child must use his knowledge of addition or subtraction to help determine what the unknown addend is. Then, since this addend is shown as the product of a known and an unknown factor, the next step is to use either multiplication or division to find the number for this unknown factor.

Pre-Book Lesson

You may wish to review briefly the two division forms. For Ex. 3, 5, 7, and 8 you may wish to review the idea that a part of a set represented by a fraction may be found by dividing. The work on pages 311 and 312 may have been adequate background for doing these exercises.

Using the Text Page

Pupils should be able to proceed independently with the written work at the top of the page. You may need to read the directions for each section of the work. It would be a good idea to circulate among the pupils while they are working to check their procedures. If a child has a wrong answer, ask him to tell you how he got the answer. You should question and guide him to find his own errors.

Individualizing Instruction

- A diagnostic analysis of the results on the work at the top of the page should provide you with additional information for grouping pupils for presentation of new experiences as well as identifying skills or concepts which need reviewing or reteaching.
- *More capable children* (or, all whom you consider ready) should find the enrichment activity at the bottom of the page not only challenging but exciting.

The directions for doing the work must be very carefully presented. You may wish to use a step-by-step presentation as follows:

a. Write the sentence: $27 = (3 \times 6) + (3 \times n)$.
b. Rewrite: $27 = 18 +$ what number.
c. Think: $27 - 18 =$ what number.
d. In the first sentence $(3 \times n)$ stands for 9.
e. So, think: $(3 \times$ what number$) = 9$.
f. Think: $9 \div 3 =$ what number.
g. Therefore, $n = 3$.
h. So, $27 = (3 \times 6) + (3 \times 3)$.

To Keep in Practice

A., S., M., and D. [W]

Write your answers on folded paper.

1. $9\overline{)18}$ 2
2. $28 \div 7$ 4
3. $\frac{1}{3}$ of 27 9
4. $7\overline{)21}$ 3
5. $\frac{1}{6}$ of 24 4
6. $8\overline{)32}$ 4
7. $\frac{1}{9}$ of 36 4
8. $\frac{1}{5}$ of 20 4

Copy and work each of Ex. 9–20.

9. 104×9 936
10. $715 - 68$ 647
11. 3×259 777
12. $440 - 389$ 51
13. $206 + 195$ 401
14. 9×32 288
15. $908 - 309$ 599
16. 43×8 344
17. \$8.00 − \$4.62 \$3.38
18. $46 + 175 + 16 + 77$ 314
19. $6 \times \$1.23$ \$7.38
20. $98 + 456 + 389$ 943

Copy each example using a numeral instead of n.

21. $153 - n = 78$ 75
22. $n + 215 = 411$ 196
23. $n - 572 = 68$ 640
24. $n \times 9 = 27$ 3
25. $36 \div 9 = n$ 4
26. $n \div 2 = 8$ 16

Do You Like to Try New Things?

Enrichment [W]

1. $27 = (3 \times 6) + (3 \times n)$.
$3 \times 6 = 18$. Then 18 is one addend of 27.
$27 - 18 = 9$. We know that $3 \times n = 9$, so $n = 3$.
Write, "$27 = (3 \times 6) + (3 \times 3)$."

Write your work in this way for Ex. 2–11.

2. $9 = (3 \times 8) - (3 \times n)$ 5
3. $26 = (2 \times 7) + (2 \times n)$ 6
4. $68 = (4 \times 8) + (4 \times n)$ 9
5. $40 = (8 \times 3) + (8 \times n)$ 2
6. $16 = (4 \times 9) - (4 \times n)$ 5
7. $32 = (4 \times 4) + (4 \times n)$ 4
8. $20 = (4 \times 5) + (4 \times n)$ 0
9. $13 = (7 \times 3) - (4 \times n)$ 2
10. $18 = (6 \times 4) - (2 \times n)$ 3
11. $50 = (9 \times 4) + (7 \times n)$ 2

Choosing the Numbers to Use

Unnecessary data; using data from other problems [W]

Write your work. Use only the numbers you need.

1. Mr. Brown has 106 feet of train track. Dick Brown has 39 fewer feet of track. How many feet does a train go if it goes 4 times around Mr. Brown's track? 424 feet

2. How many feet long is Dick's track? See Ex. 1. 67 feet

3. If the two train tracks are put together, how many feet long will the new track be? Use numbers from problems 1 and 2. 173 feet

4. Mr. Brown said that for Dick's birthday he will give him a tunnel costing 75¢, 3 cars costing 65¢ each, and track costing $1.50. How much will the cars cost? $1.95

5. How much will the tunnel and the new track together cost? Use numbers from problem 4. $2.25

6. Dick runs his trains about 35 minutes a day on school days and 2 times as many minutes on Saturday. About how many minutes did he spend one week when he ran the trains Monday, Thursday, and Friday? 105 minutes

7. About how many minutes does Dick run his trains on Saturday? Use numbers from problem 6. 70 minutes

Teaching Page 316

Pupil's Objectives

(a) To learn to use only pertinent information in solving problems; and (b) to have practice in using information from another problem in solving a problem.

New Word

tunnel

Background

The daily activities of all people encompass many encounters with quantitative situations. These encounters frequently entail the application of mathematical skills and concepts. Of course, extraneous data and information are almost always present. Consequently, it becomes necessary to select only those data pertinent to the solution of a problem and then use the appropriate operation. Very frequently, the solution of one problem is dependent upon the answer to another problem.

The problems on page 316 are to give pupils experience in distinguishing between relevant and irrelevant data as well as to use information given in one problem in finding the solution to another problem.

Estimation is a skill that should be encouraged in connection with solving written or verbal problems. Your pupils have been taught several techniques for determining whether or not an answer is reasonable. Encourage the use of this type of thinking as each problem is being considered.

Pre-Book Lesson

• Write on the board quantitative information about a situation in your classroom. Use the information for stating problems. For example:

2 P.M.
April 24
5 tables
42 chairs
17 boys
15 girls
29 science books

• Ask, "If only the boys are sitting on chairs, how many empty chairs will there be?" Have a child identify the given information needed to solve the problem.

Ask such questions as: "Do we need to know what time it is? Do we need to know the date?" and so on. Bring out in the discussion that often information is given that is not needed in solving a problem.

• Have one pupil state a problem using information written on the chalkboard. Then, another pupil may tell what information is needed to solve that problem.

Using the Text Page

More capable children should be able to proceed with the written work. For those pupils who have reading difficulties, it may be necessary to read through each problem orally. Make sure pupils know what each problem is about. You may assign someone to help each pupil having reading difficulties.

Individualizing Instruction

All pupils may write problems in which extraneous information is given. Select some for reading aloud to the class. At all times, stress the information that is needed and that information which is unnecessary.

Teaching Page 317

Pupil's Objectives

(a) To learn to organize mathematical information in the form of a picture graph; (b) to learn to read and interpret a picture graph; and (c) to learn to solve problems based on information recorded in a picture graph.

New Word

graph

Background

The idea of *pairing* information about two or more known variables was discussed on Teaching Page 81. (Refer to that page for a discussion of the relation concept.) At that time, pupils were introduced to a chart as a means of recording paired information. In this lesson, pupils are paired with the number of books they have read. The new idea in this lesson is the technique of presenting the information. The picture graph is often a more attractive and interesting means of portraying information than merely writing numerals.

Pre-Book Lesson

Engage in an activity which involves pairing information. For example, you could ask four children to stand at the front of the classroom. Give each child a different number of pencils (rulers, erasers, books, and so on). Have each child tell how many pencils he has.

Ask pupils to tell "good" ways of keeping a record of this information. (In all probability a chart will be suggested on which each child's name is written with a numeral for the number of pencils.)

Discuss with the class the possibility of drawing pictures of pencils to record the information.

If you have a flannel board and appropriate materials you may construct a picture graph.

Using the Text Page

The picture graph should be easy for pupils to interpret. Since the lesson is developed orally, there should be no great difficulty for any child with any of the material. You may wish to assign this page as written work for *more capable children.*

Individualizing Instruction

• Your pupils should be encouraged to observe phenomena which occur about them. In addition, it is important to keep an accurate record of the events or facts associated with the particular phenomenon. For example, children may plant seeds and then keep a record of the growth, both in time and in height. They may keep a record of the number of different kinds of automobiles that pass a given location during a given period of time.

Engage in some type of observing and record-keeping activity. Record the information in the form of a picture graph.

• *More capable children* may keep records and make picture graphs in which one picture represents a group of incidents. For example, in the picture on pupil's page 317, each book in the graph could represent five, ten, or any number of books read by each pupil.

• *More capable children* may write the answers to questions 4–6 at the bottom of page 317 as they imagine that each book in the graph represents

a. 5 books. b. 8 books.

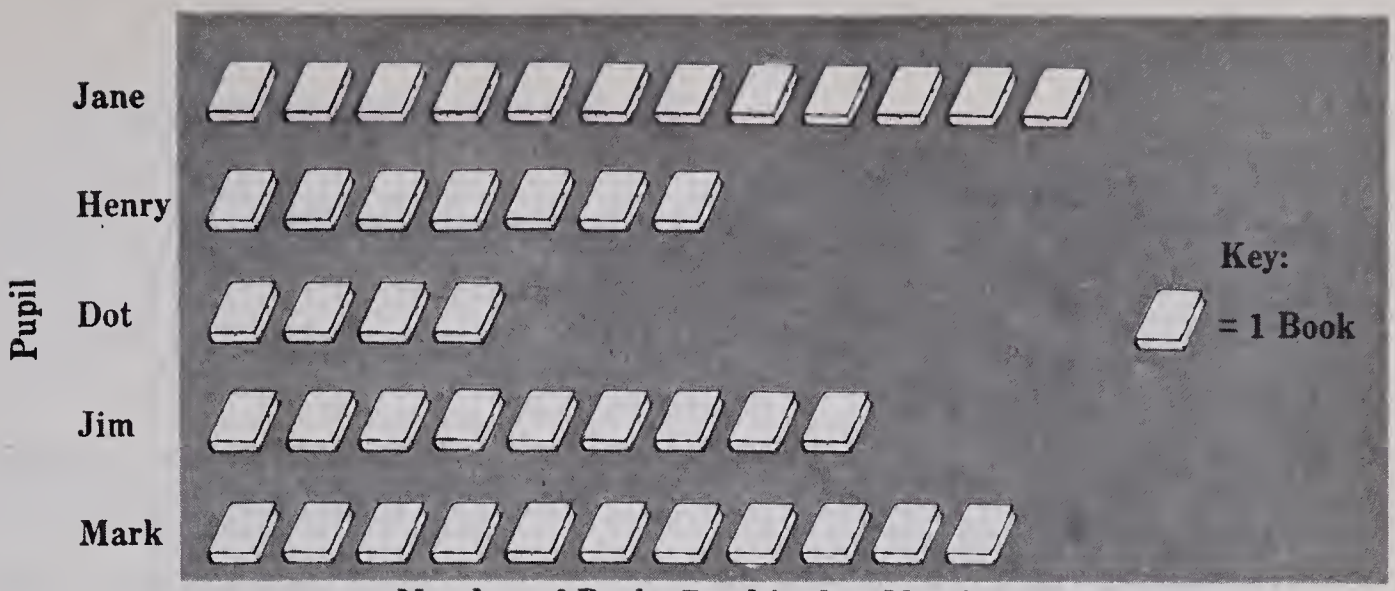
Looking Ahead

The multiplication facts in which 5 is a factor will be learned soon. A good readiness activity follows. Each pupil should show the whole numbers from 1 to 50 with 1–10 in the top row, 11–20 in the second row, and so on. Then, the child should place an X on 5 and every 5th numeral in the chart.

Have the children record the numerals that have been covered with an X. Discuss with them the generalizations they may *see.*

a. All numerals covered by X end in 5 or 0.

b. The numerals covered by X are in two columns only.



Number of Books Read in One Month

Reading a Picture Graph

[o]

The picture graph above shows how many books some of the children in Dot's room read one month.

1. What does each row of books on the graph show? How many books each child read

2. How many books were read

a. by Dot? 4 b. by Henry? 7 c. by Jim? 9 d. by Mark? 11

3. Tell the name of the pupil who read

a. the greatest number of books. Jane

b. the next greatest. Mark d. the next greatest. Henry

c. the next greatest. Jim e. the least. Dot

4. Write on the board in order the names you found for Ex. 3a–e. Why does Jane's name come first? She read the greatest number of books. Why does Dot's name come last? She read the least number of books.

5. How many more books were read

a. by Jane than by Jim? 3

b. by Mark than by Dot? 7

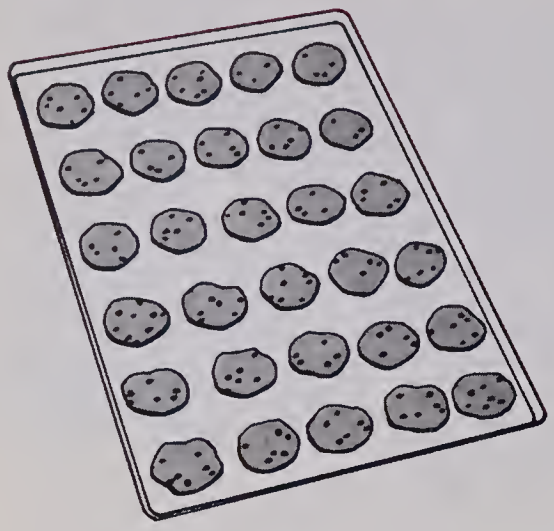
6. How many books were read

a. by the girls? 16 b. by the boys? 27

* Although M. and D. facts in grade 3 are confined to those in which 1 factor is no greater than 5, it should be noted that the other factor may be as great as 9.

*M. Facts with a Factor 5

Discovering pairs of M. facts [W]



1. How many cookies can Mother get from this pan of cookies? There are 6 rows of 5 cookies each. Count by 5's if you need to.

a. Copy and finish the *n*-example $6 \times 5 = n$, 30 to show the M. fact.

b. Show the M. fact that goes with it to make a pair. 5×6=30

2. Suppose the pan had 7 rows of 5 cookies. How many cookies would there be in all? Count by 5's.

a. Copy and finish $7 \times _ ? 5 = n$ 35

b. Show the M. fact that goes with it. 5×7=35

For each of Ex. 3 and 4, show a pair of M. facts. Suppose that the pan of cookies has

3. 8 rows of 5 cookies. 8×5=40 5×8=40 4. 9 rows of 5 cookies. 9×5=45 5×9=45

5. Another pan has 5 rows of 5 cookies. Show the M. fact. 5×5=25 Is there another M. fact to go with it? No

A	B	C	D	E
9	8	6	7	5
9	8	6	7	5
9	8	6	7	5
9	8	6	7	5
9	8	6	7	5
45	40	30	35	25

6. Copy and finish $5 \times 9 = n$. 45 Use Ex. A to find the product. Show the M. fact that goes with it. 9×5=45

What M. fact can you make from 7. B? 5×8=40 8. C? 5×6=30 9. D? 5×7=35 10. E? 5×5=25

11. Show the M. fact, if there is one, to go with each of Ex. 7–10 to make a pair.

12. 2 and 5 are factors of 10. 8×5=40 6×5=30 7×5=35 Show two M. facts. 2×5=10 5×2=10

Teaching Pages 318 and 319

Pupil's Objective

To discover and begin to learn the M. facts in which 5 is a factor.

New Word

(page 319) *soda*

Background

Your pupils have been learning some multiplication facts. At the same time they have been learning some properties which govern the multiplication operation in one way or another. Learning the properties provides a foundation upon which children may rely for learning other facts more independently than could otherwise be expected. It also releases youngsters to attack problems or examples intelligently rather than by rote memory exclusively.

By this time your pupils should have a good grasp of the commutative property as it applies to multiplication. It is not necessary that they know the term as such, but they should be able to understand it in their work with multiplication in that they realize that changing the order of the factors will not change the product.

A second important property which children should understand and use is the Distributive Property of Multiplication over Addition. This property makes it possible for a child to find the product for a M. fact involving 5 using only the M. facts he has learned to date. One example should suffice to illustrate this point. To find the product of 7×5 (seven 5's) a pupil may use what he knows about seven 2's and seven 3's.

Therefore,

$$(7 \times 5) = (7 \times 2) + (7 \times 3)$$
$$7 \times 5 = 14 + 21$$
$$7 \times 5 = 35$$

Application of the commutative property tells pupils that the product for 5×7 is also 35.

The inverse relationship of multiplication by a number and division by the same number offers additional insights which will help pupils arrive at mastery of the M. and D. facts.

Through the use of arrays, pupils should begin to find it easy to think of *factors* and *products*. The numbers of elements in a row and a column of the array illustrate the factors. The number of elements in the entire array illustrates the product.

The distinction between M. facts for 5's and M. facts for 5 has been discussed before. Briefly, in facts for 5's, *5* represents the number of members in each equivalent set, while in facts for 5, *5* denotes the number of equivalent sets.

Pre-Book Lesson

• Have pupils construct arrays to show what they know about M. facts in which 5 is a factor. They should be able to show and explain the relationship between five 1's and one 5; five 2's and two 5's; five 3's and three 5's; five 4's and four 5's. Rely heavily upon the arrays constructed by each child to review what has been already learned about the Commutative Property of Multiplication.

• Ask a pupil (or pupils) to show each of the M. facts learned so far in which 5 is a factor. The following should be shown:

$$1 \times 5 = 5$$
$$2 \times 5 = 10$$
$$3 \times 5 = 15$$
$$4 \times 5 = 20$$

As was noted earlier, pupils should begin to understand that, as these facts which were once learned in other tables are being listed in this table for 5's, so each new table of M. facts being learned contains M. facts of other tables yet to be listed. Eventually, when they explore the table of M. facts for 9, there will actually be only one new M. fact to be learned, $9 \times 9 = 81$.

• Under the facts just shown on the chalkboard, show the M. facts involving 5 which are yet to be learned. Have pupils tell the factors.

We Know	We Must Learn
$1 \times 5 = 5$	$5 \times 5 = ?$
$2 \times 5 = 10$	$6 \times 5 = ?$
$3 \times 5 = 15$	$7 \times 5 = ?$
$4 \times 5 = 20$	$8 \times 5 = ?$
	$9 \times 5 = ?$

• Many children (perhaps most) should be able to use the known facts to help find the products of the unknown facts. (This is putting the distributive property to work.) Ask if anyone can find the product for six 5's by using known facts.

There are several possibilities in solving this:

a. Six 3's + six 2's
b. Six 2's + six 2's + six 1's
c. Six 4's + six 1's

• Be sure to permit pupils to utilize what they have discovered as products for the unknown facts. For example, after the children have found the product for 6×5, encourage them to use this information as well as other known information to find the products for 8×5 and 9×5.

Using the Text Pages

• Your *more capable children* should have no difficulty with the written work. They may wish to construct arrays with objects at their desks for some of these exercises.

• You may find it desirable to read the material with the *slower learners* as they do the written work. It is very important that they understand what to do in each written exercise.

• In order to reinforce the idea of pairs of M. facts, it would be desirable to write the two tables of M. facts side by side. This enhances the possibility of seeing the relationship between any pair of M. facts in the two tables.

• The oral work could just as well be assigned as written work. The primary advantage in doing it orally is to give you the opportunity to assess the understanding and mastery of your pupils.

Individualizing Instruction

• Duplicate a sheet with headings and information as suggested below:

Product	One Factor	Other Factor	M. Fact.
35	5	7	_ ? _
30	5	6	_ ? _

• *All pupils* may write sentences in the following form for extra practice with the M. facts:

Let △ represent one of two factors and □ represent the other factor of the product represented by ○.

• Pupils may find products on a number-line picture for pairs of M. facts in which 5 is a factor.

• Have *all pupils* make a chart like the following in which all M. facts learned so far are shown.

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

Copy and finish each of Ex. 13–18 to show a M. fact. Then show the other M. fact to make a pair.

13. $20 = 5 \times _?_$ 4
14. $35 = _?_ \times 5$ 7
15. $45 = 5 \times _?_$ 9
16. $30 = 5 \times _?_$ 6
17. $15 = _?_ \times 5$ 3
18. $40 = _?_ \times 5$ 8

M. Facts for 5's	M. Facts for 5
$1 \times 5 = 5$	$5 \times 1 = 5$
$2 \times 5 = 10$	$5 \times 2 = 10$
$3 \times 5 = 15$	$5 \times 3 = 15$
and so on to	and so on to
$9 \times 5 = 45$	$5 \times 9 = 45$

19. Copy and finish the tables started above.
4×5=20 5×5=25 6×5=30 7×5=35 8×5=40
5×4=20 5×5=25 5×6=30 5×7=35 5×8=40

Oral problems [O]

Think and say the answers for Ex. 20–22.

20. If you put 5 small ice cubes in each glass, how many ice cubes will you need for

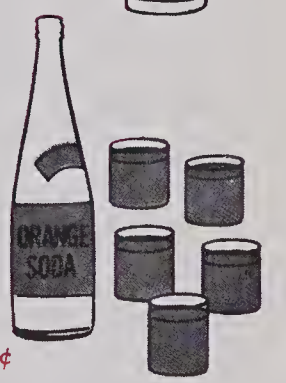
a. 6 glasses? 30 b. 9 glasses? 45 c. 4 glasses? 20

21. A large bottle of orange soda fills 5 glasses. How many glasses can you fill with

a. 7 bottles of soda? 35 b. 5 bottles of soda? 25

22. If a small glass of orange soda costs 5¢, how much must you pay for

a. 8 glasses? 40¢ b. 4 glasses? 20¢ c. 9 glasses? 45¢



Say the products by rows and then by columns.

	a	b	c	d	e	f	g	h
23.	5×5 25	4×9 36	5×2 10	5×0 0	5×8 40	9×3 27	5×7 35	9×5 45
24.	3×9 27	0×5 0	5×9 45	1×5 5	4×6 24	7×5 35	8×4 32	4×7 28
25.	6×4 24	2×5 10	5×3 15	3×7 21	9×4 36	3×8 24	6×5 30	5×6 30
26.	3×5 15	3×6 18	5×4 20	4×8 32	8×5 40	7×4 28	5×1 5	4×5 20

Table of M. Facts for 5's

5	5	5	5	5	5	5	5	5
1	2	3	4	5	6	7	8	9
5	10	15	20	25	30	35	40	45

Table of M. Facts for 5

1	2	3	4	5	6	7	8	9
5	5	5	5	5	5	5	5	5
5	10	15	20	25	30	35	40	45

Factors-product relationship [O]

1. For the tables of M. facts with 5 as a factor there are nine new M. facts. Which facts are they? $5 \times 5 = 25$ $5 \times 6 = 30$ $5 \times 7 = 35$ $5 \times 8 = 40$ $5 \times 9 = 45$ $6 \times 5 = 30$ $7 \times 5 = 35$ $8 \times 5 = 40$ $9 \times 5 = 45$

2. Starting with the answers for 5×1 and 1×5, one of which two digits is always used in one's place? 0 and 5

3. $6 \times 5 = 30$ Why must 35 be the product of 7×5? 1 more 5 is shown.

4. For these tables of M. facts, when are the products odd numbers? When both factors are odd numbers. When are they even numbers? When one factor is an even number

5. If we should show 10×5, 11×5, and 12×5 at the end of the table of M. facts for 5's, what products would be shown? 50, 55, and 60

Tell which multiplications are shown wrong, and why.

6.	**7.**	**8.**	**9.**	**10.**
57 ×	105	149 ×	152	$0.68 ×
5	8	5	6	5
275	840	706	912	$3.00

Ex. 6: $5 \times 5 = 25$, $25 + 3 = 28$ Ex. 8: $5 \times 9 = 45$, $5 \times 4 = 20$, $20 + 4 = 24$ Ex. 10: $5 \times 6 = 30$, $30 + 4 = 34$

11. Cover the products shown in the tables above. Say the products. Do not take the facts in order. Then make Study Cards for the facts you do not know.

Teaching Page 320

Pupil's Objectives

(a) To bring together M. facts for 5's and for 5; and (b) to review what has been learned about M. facts in earlier lessons.

Background

This page follows logically the material that has been covered in which 5 is a factor. These are the last tables of M. facts that will be studied formally in this book. Take this opportunity to focus attention on the following ideas:

a. *Commutative.* The order in which the factors are multiplied will not affect the product.

b. *Distributive.* We may use addition to help find a product. Adding products of two particular known facts gives the product of an unknown fact.

c. *Identity element.* When 1 is one factor, the product is equal to the other factor.

Pre-Book Lesson

• Before turning to the oral work in the text, you may use either of the following activities:

a. Have pupils refer to the M. charts they have prepared which include all the M. facts they have learned to date. Ask questions about known factors and unknown products. Have pupils ask questions of each other. Be sure each child can use the chart correctly.

b. Or, you may prefer to use the *Show the Answer* game. Write sentences on the chalkboard, each showing an unknown product. Have pupils hold up, on a signal, the correct numeral to show the product.

• Review briefly, but carefully, the ideas and skills involved in remembering tens in multiplication with 2- and 3-digit numerals.

Using the Text Page

• You may prefer to have each pupil work with a partner for a short time so as to help him identify those facts needing additional study. Emphasize that the role played by the partner is that of *helper* who assists the pupil to help himself, rather than that of a *critic* who tries to find his partner's mistakes. Suggest that pupils use the tables of facts at the top of page 320 and vary the order.

• In discussing the questions on page 320, be sure to emphasize the properties discussed in the Background section.

Individualizing Instruction

• Pupils who have three or more errors in saying the products at the top of page 320 may make cards for matching (the fact on one card and the answer on another). On the back of each fact card pupils may draw an array (dots, x's, and so on) to illustrate the M. fact.

• *Slower learners* may work these problems:

a. How many pennies are there in 4 piles of 7 pennies?

b. If 6 children can ride on a bobsled, how many can ride on 5 bobsleds of the same kind?

c. Nine flowers, each with 4 petals, have how many petals in all?

d. If it takes 5 nails to fasten down a board, how many nails will be needed for 8 of the boards?

e. Tom's team made 5 runs. Dick's team made 4 times that number of runs. How many runs did Dick's team make?

f. Ann is going to learn 7 new spelling words a day. How many words will she learn in 5 days?

g. If 9 large strawberries fill a dish, how many such strawberries will you need for 4 dishes?

Teaching Page 321

Pupil's Objective

To have practice in using the symbols $>$, $<$, and $=$ to make true sentences.

Background

The symbols $>$ and $<$ were introduced very early in the text and ideas they represent have been used a great deal throughout the entire program. The primary purpose of the written work on page 321 is to give pupils practice in using the many operations, facts, and concepts they have learned thus far. The pupil should think of each sentence as containing two names. Each name is written in parentheses. The pupil determines the numbers being named. Then he must decide whether the first number named is greater than, less than, or equal to the second number named, and use the appropriate symbol to denote this relationship. Emphasize the idea that to write true sentences he must use the correct symbols.

Pre-Book Lesson

Depending upon the extent you deem it necessary, review the four operations, and the various forms in which each may be shown. Also, discuss the general idea of *renaming* numbers. Be sure your pupils understand the skill of *remembering* as it applies to multiplication and *carrying* as it applies to addition.

Using the Text Page

Observe the work of *slower learners*. If they are having difficulty with the written work as it stands, modify the assignment so that pupils show the numbers named in each sentence. Perhaps you will assign only selected exercises. If an assignment seems unusually long, *slower learners* feel a sense of defeat and so never really get started.

Individualizing Instruction

• It may be necessary to prepare simpler exercises for *slower learners* to have practice in using the symbols $<$ and $>$. If so, try some like the following.

(6×4) ___ (5×5)
$(36 \div 4)$ ___ (4×2)
(5×7) ___ (8×4)

• The enrichment material is for all those who may be ready to find relationships among numbers. In working these exercises, they are actually using *compensation* in that subtracting from one addend compensates for adding to another. Permit *all pupils* to try these exercises. If it is obvious that some pupils are having difficulty, suggest some alternate work. (Perhaps some of the suggestions at the end of the chapter would be more appropriate.)

NOTES

* Although finding the mean average is deferred until grade 4, the essence of the idea is used here.

Using > or < or =

Equalities and inequalities [W]

Do the work in the ( )'s. Then copy each example, writing > for is greater than or < for is less than or = for is equal to. Then copy again, using only = or ≠.

1. $(\frac{1}{4}$ of 24) $\overset{=}{_?_}$ (24 ÷ 4)
2. (9 × 4) $\overset{>}{_?_}$ (5 × 5)
3. (412 − 380) $\overset{=}{_?_}$ (8 × 4)
4. (508 − 487) $\overset{<}{_?_}$ (215 − 88)
5. (32 ÷ 8) $\overset{<}{_?_}$ (18 ÷ 3)
6. (3 × 87) $\overset{>}{_?_}$ (4 × 65)
7. $(\frac{1}{3}$ of 27) $\overset{=}{_?_}$ (36 ÷ 4)
8. (206 × 4) $\overset{>}{_?_}$ (258 × 3)
9. (513 − 98) $\overset{>}{_?_}$ (2 × 188)
10. (7 × 4) $\overset{>}{_?_}$ $(\frac{1}{2}$ of 18)
11. (179 + 288) $\overset{<}{_?_}$ (3 × 287)
12. (6 × 143) $\overset{=}{_?_}$ (2 × 429)
13. (84 × 5) $\overset{>}{_?_}$ (6 × 22)
14. (56 × 3) $\overset{<}{_?_}$ (4 × 48)

Do You Like to Try New Things?

Enrichment [W]

1. 23 + 23 + 23 = 69. Write a M. example you can use to find the answer. 3 × 23

2. **a.** 10 + 11 + 12 = 33 **b.** 3 × 11 = 33

* Change 2a to a new addition example from which the multiplication example in 2b can be written. 11 + 11 + 11

3. Change each of Ex. a–f to a M. example.

a. 10 + 12 + 14 3×12 **b.** 8 + 16 + 12 3×12 **c.** 12 + 9 + 6 3×9

d. 15 + 13 + 11 3×13 **e.** 11 + 9 + 13 3×11 **f.** 20 + 24 + 22 3×22

4. Make 6 A. examples which can be changed to M. examples having the same answer. Use 2-place numerals.

5. Write 6 A. examples using 3-place numerals. Make 3 of them so they can be written as M. examples.

Finding Unknown Products

M. facts with 2, 3, 4, 5 as factor [W]

Turn to page 319. Use rows 23–26, and show the products on folded paper.

Practice in Multiplication

[W]

Copy and multiply.

1. 5×108 540	6. 55×6 330	11. 32×9 288	16. $5 \times \$1.90$ $9.50
2. 54×9 486	7. 8×115 920	12. 54×7 378	17. $4 \times \$1.76$ $7.04
3. 6×154 924	8. 57×5 285	13. 6×132 792	18. $9 \times \$1.05$ $9.45
4. 7×140 980	9. 4×159 636	14. 5×32 160	19. $7 \times \$1.35$ $9.45
5. 78×5 390	10. 238×3 714	15. 43×8 344	20. $3 \times \$2.46$ $7.38

♦ **Extra Problems.** Work Set 16.

D. Facts with 5 as a Factor

Discovering pairs of D. facts [O]

1. The 30 badges on the board make how many rows of 5 badges each? $30 \div 5 = n$

a. You can use the picture to find the D. fact. $30 \div 5 = ?$ 6

b. For $30 \div 5 = n$ 6 the product is _? 30. The known factor is _? 5. To find the unknown factor, you can use what M. fact?

c. The D. fact that goes with the D. fact you found is $30 \div$ _? 6 $= 5$. 5×6=30

d. Say the set of M. and D. facts for 30 and its factors 5 and 6.

$5 \times 6 = n$ 30 $\quad 30 \div 5 = n$ 6

$6 \times 5 = n$ 30 $\quad 30 \div 6 = n$ 5

Teaching Pages 322 and 323

Pupil's Objectives

(a) To have practice in all multiplication skills developed to this time; (b) to begin to learn the division facts in which 5 is a factor; and (c) to become acquainted with sets of M. and D. facts in which 5 is a factor.

New Word

(page 322) *badges*

Background

Your pupils have had experiences with solving division problems by using subtraction. They have learned that in the sentence $35 \div 5 = n$, n represents the number of times that five can be subtracted from 35. As an initial approach to a division problem, subtraction can be used to make work more meaningful.

The children have also learned to solve division problems by using known multiplication facts. If children know the M. facts, then any division sentence of the type $35 \div 5 = n$ may be solved by thinking of the appropriate M. fact.

Each of these ideas should be emphasized as children are continuing to learn new M. and D. facts in order to give them maximum understanding and mastery of those M. and D. facts.

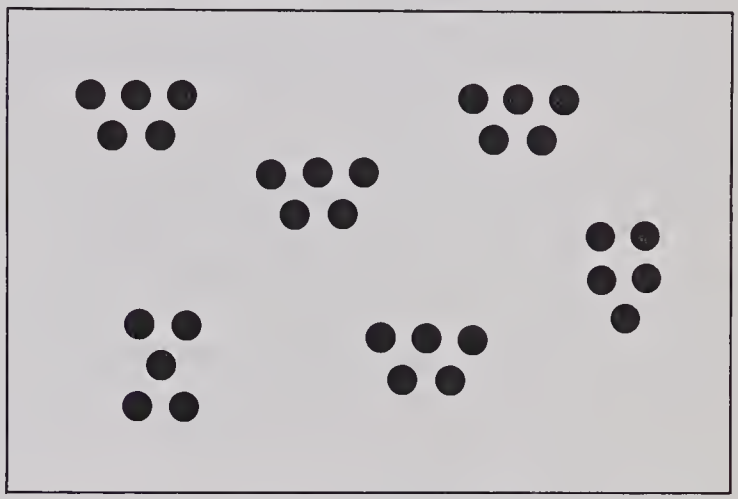
Pre-Book Lesson

• Write the sentence $30 \div 5 = n$ on the chalkboard. Ask a child to read the sentence. Emphasize that they read the sentence as it appears; that they should not try to give the answer. Ask pupils to write another division example on the board which asks the same question.

• After all children understand what they are to find, discuss the use of subtraction to find the unknown factor. Do this as you place cut-outs* on the flannel board. Make certain that all pupils know there are 30 objects on the flannel board. Again, have a child read the sentence. Say, "Let's see how many times we can remove 5 things (stars, rabbits, birds, and so on) from the set of 30."

* See 17, page xix.

Have a child come to the flannel board and remove 5 objects. Encourage children to place each subset removed so that it appears separate from each other subset. Say, "Ann removed one set of 5. How many objects are left?" Continue removing 5's until all objects of the original set have been organized into subsets of 5. Your flannel board should look something as follows:



Discuss with children how many sets of 5 have been removed from the set of 30. Help pupils realize that the number of 5's we can remove from 30 is the answer for the division sentence written on the board, $30 \div 5 = n$.

• This is the logical time to relate division to the known multiplication facts. Ask children to tell the multiplication facts that could be used to help find the solution to a particular division sentence. After the M. facts are shown, review the factors-product relationship involved and show the complete set of M. and D. facts as is done below for the D. sentence $30 \div 5 = n$.

$5 \times 6 = 30$	$30 \div 5 = 6$
$6 \times 5 = 30$	$30 \div 6 = 5$

Using the Text Pages

• The written work at the top of page 322 may be used to diagnose previous learnings in multiplication or may be used for additional practice. Doing these exercises should provide a basis for pupils to decide whether or not they know the M. facts that have been taught. This material may be used at any time. It does not necessarily have

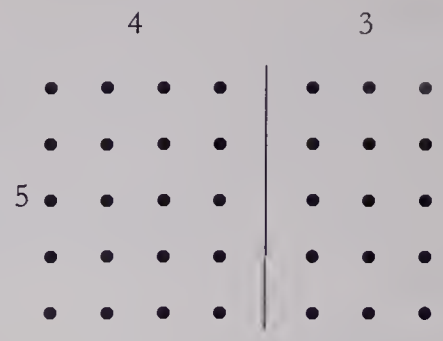
to precede the oral work on the remainder of these two pages.

• Have pupils construct arrays in connection with the oral work on pages 322 and 323. This should help pupils in discovering and stating the pairs of division facts with 5 as a factor.

• Use any teaching aid or method you consider appropriate in helping children visualize the divisions for Ex. 15–22 at the bottom of page 323. Children must develop understanding of each fact while they are acquiring mastery of that fact. Constructing arrays with objects or making dot arrays usually helps pupils to *see* the set of facts associated with any given pair of factors. For example, in an array for factors 4 and 5, the product can be found by counting the number of dots. For a given factor and product, the unknown factor can be found by making an array in which the total number of dots shows the product and the number of dots in each row shows the known factor. By counting the rows, the unknown factor can be found.

Individualizing Instruction

• Pupils may also make arrays to illustrate the distributive property. The following array for 5×7 shows that you can distribute multiplication of 7 over its addends 4 and 3, and not change the product. This is realized when we note that five rows of four dots on the left side plus five rows of three dots on the right side make the total number of dots in both arrays. Thus, we can say that $5 \times 7 = (5 \times 4) + (5 \times 3)$.



• *More capable children* may discover that the above array also shows that division of 35 can be distributed over its addends 20 and 15, and not change the answer. Ask questions such as the following to help pupils use the array in a meaningful way:

a. How many 5's in 20? (4)
b. How many 5's in 15? (3)
c. How many 5's in 35? (7)

The sum of the answers for the two divisions is the same as that for $35 \div 5$. Thus, we can say that $35 \div 5 = (20 \div 5) + (15 \div 5)$.

• Have children draw arrays and write sentences in as many different ways as they can to illustrate 5×7. Do the same for other M. facts in which 5 is a factor.

• *All pupils* should be able to write each of Ex. 15–22 using another division form.

• Use Extra Problems Set 16 as needed.

NOTES

2. If you pin 40 badges on 8 sheets of paper, the same number on each sheet, how many badges will you put on each sheet?

a. Is the number sentence, $40 \div 8 = n$? Explain.

b. What is the product? 40 the known factor? 8

c. Draw a dot picture or show a M. fact on the board to find the unknown factor. $8 \times 5 = 40$

d. Say the set of M. and D. facts for the product 40 and its factors 8 and 5. $8 \times 5 = 40$ $5 \times 8 = 40$ $40 \div 8 = 5$ $40 \div 5 = 8$

Say the two D. facts in the set of M. and D. facts for

3. 10; factors 2 and 5. $10 \div 2 = 5$ $10 \div 5 = 2$

4. 20; factors 4 and 5. $20 \div 4 = 5$ $20 \div 5 = 4$

5. $5 \times 5 = 25$, so $25 \div 5 = 5$. Is there another D. fact to go with this one to make a pair? No Why? The factors are equal.

[W]

Copy and finish each of Ex. 6–14. If you need help, draw dot pictures. Then show the D. facts in the same set.

6. $5 = 5 \times _?\ 1$ $5 \div 5 = 1$ $5 \div 1 = 5$

7. $30 = _?\ 5 \times 6$ $30 \div 5 = 6$ $30 \div 6 = 5$

8. $45 = 5 \times _?\ 9$ $45 \div 5 = 9$ $45 \div 9 = 5$

9. $35 = 7 \times _?\ 5$ $35 \div 7 = 5$ $35 \div 5 = 7$

10. $20 = _?\ 4 \times 5$ $20 \div 4 = 5$ $20 \div 5 = 4$

11. $10 = 5 \times _?\ 2$ $10 \div 5 = 2$ $10 \div 2 = 5$

12. $40 = 5 \times _?\ 8$ $40 \div 5 = 8$ $40 \div 8 = 5$

13. $15 = _?\ 5 \times 3$ $15 \div 5 = 3$ $15 \div 3 = 5$

14. $25 = 5 \times _?\ 5$ $25 \div 5 = 5$

Study the work in Ex. A and B. Ex. B shows how to check Ex. A.

Show work as in box B to check each of Ex. 15–22. Then write the example with the correct answer.

A	B
$3\overline{)15}$	$3\overline{)15}$
	$\underline{15}$
	0

15. $6\overline{)30}$ (5)

16. $5\overline{)45}$ (9)

17. $7\overline{)35}$ (5)

18. $5\overline{)40}$ (8)

19. $9\overline{)45}$ (5)

20. $5\overline{)35}$ (7)

21. $8\overline{)40}$ (5)

22. $5\overline{)25}$ (5)

* Of course, in finding a fractional part, sets language is understood. Pupils are reminded here that, as on pages 310-311, they can find the number for a part of a set by using division.

Working with D. Facts

With a factor 5 [W]

Division Facts Dividing by 5

$5 \div 5 = 1$
$10 \div 5 = 2$
$15 \div 5 = ?$
and so on to
$45 \div 5 = ?$

Copy and show the products. Under each, show the D. facts or fact in the set of M. and D. facts.

1. 5×8 40 — $40 \div 5 = 8$ $40 \div 8 = 5$
2. 9×5 45 — $45 \div 9 = 5$ $45 \div 5 = 9$
3. 7×5 35 — $35 \div 7 = 5$ $35 \div 5 = 7$
4. 5×5 25 — $25 \div 5 = 5$
5. 5×6 30 — $30 \div 5 = 6$ $30 \div 6 = 5$
6. 4×5 20 — $20 \div 4 = 5$ $20 \div 5 = 4$
7. 5×9 45 — $45 \div 5 = 9$ $45 \div 9 = 5$
8. 3×5 15 — $15 \div 3 = 5$ $15 \div 5 = 3$

* $\frac{1}{4}$ of 20 is _?_. Remember, for Ex. 9 you can divide to find the answer. $20 \div 4 = 5$, so $\frac{1}{4}$ of 20 is 5.

Copy and finish Ex. 10–26.

Division Facts Answer 5

$5 \div 1 = 5$
$10 \div 2 = 5$
$15 \div 3 = ?$ 5
and so on to
$45 \div 9 = ?$ 5

9. $\frac{1}{4}$ of 20 5
10. $\frac{1}{6}$ of 30 5
11. $\frac{1}{4}$ of 36 9
12. $\frac{1}{3}$ of 27 9
13. $\frac{1}{5}$ of 35 7
14. $\frac{1}{7}$ of 28 4
15. $\frac{1}{8}$ of 32 4
16. $\frac{1}{5}$ of 30 6
17. $\frac{1}{7}$ of 35 5
18. $\frac{1}{5}$ of 20 4
19. $\frac{1}{5}$ of 25 5
20. $\frac{1}{3}$ of 24 8
21. $\frac{1}{5}$ of 45 9
22. $\frac{1}{8}$ of 40 5
23. $\frac{1}{5}$ of 40 8
24. $\frac{1}{2}$ of 10 5
25. $\frac{1}{3}$ of 21 7
26. $\frac{1}{8}$ of 16 2

27. Copy and finish the tables of D. facts at the left.
$20 \div 4 = 5$ $25 \div 5 = 5$ $30 \div 6 = 5$ $35 \div 7 = 5$ $40 \div 8 = 5$

Mental work [O]

Say the answers by rows and by columns.

	a	b	c	d	e
28.	$9\overline{)36}$ 4	$35 \div 5$ 7	$\frac{1}{5}$ of 10 2	$24 \div 8$ 3	$9\overline{)45}$ 5
29.	$5\overline{)40}$ 8	$\frac{1}{5}$ of 25 5	$3\overline{)24}$ 8	$\frac{1}{4}$ of 24 6	$27 \div 9$ 3
30.	$\frac{1}{6}$ of 24 4	$3\overline{)27}$ 9	$\frac{1}{9}$ of 45 5	$4\overline{)28}$ 7	$8\overline{)40}$ 5
31.	$35 \div 7$ 5	$30 \div 6$ 5	$36 \div 4$ 9	$\frac{1}{9}$ of 27 3	$5\overline{)30}$ 6
32.	$\frac{1}{6}$ of 30 5	$\frac{1}{8}$ of 40 5	$20 \div 5$ 4	$8\overline{)32}$ 4	$\frac{1}{7}$ of 28 4
33.	$5\overline{)25}$ 5	$32 \div 8$ 4	$32 \div 4$ 8	$21 \div 7$ 3	$7\overline{)35}$ 5

Pupil's Objectives

(a) To review all that has been learned about multiplication and division facts in which 5 is a factor; (b) to write the tables of division facts in which 5 is a factor; and (c) to have oral practice with D. facts learned so far.

Background

The factor 5 is the last one to be developed formally at this grade level. In all probability many pupils will learn most if not all of the multiplication and division facts with factors greater than 5. However, those facts will not be considered as a formal part of the program at this time.

Your pupils have learned that for any given pair of unequal factors there is a set of four M. and D. facts. When the factors are equal, there are only two facts in the set. The inverse relationship of multiplication and division by a number makes it possible to think of any M. or D. fact and, in turn, determine the other fact or facts in its set of M. and D. facts.

Two different forms of division examples have been introduced—$24 \div 4 = n$ and $4\overline{)24}^{\,n}$. In connection with sets of items, a third form of sentence is used—"$\frac{1}{4}$ of 24 marbles is _ ? _ marbles" or, in short form, "$\frac{1}{4}$ of 24 is _ ? _." The solution requires division. Pupils should be helped to read sentences of this type before they are to be expected to determine the operation for solving. Only when they realize that they are to find 1 of the 4 parts of the same size in a set of 24, will they come to understand that division can be used to find the solution. In this particular instance, we divide 24 by the whole number 4 since 4 is the number of each of the equivalent sets.

You, as the teacher, will realize that we can find the solution to a sentence of this type by dividing by the reciprocal. Four is the reciprocal of $\frac{1}{4}$. Therefore we can divide by 4 to find the solution for the sentence $\frac{1}{4} \times 24 = n$. We do not say that $\frac{1}{4}$ of 24 is _ ? _ is a division sentence. But, we say that we can use division to find the solution to this type of sentence.

Pre-Book Lesson

Review and summarize in any way you deem appropriate for your pupils so that you are satisfied with their understanding of the following:

a. For two unequal factors there is a set of four M. and D. facts.

b. For two equal factors, there is a set of two M. and D. facts.

c. There are two different forms of division examples.

d. We can find the answer to a sentence for finding one part of a set by dividing.

Using the Text Page

• By this time your pupils should be able to do these written exercises with no great difficulty. You may wish to have the children complete the two tables of division facts before doing Ex. 9–26. Be certain they see how they can use the facts in the first table in working some of these exercises.

• Have pupils work in pairs for rows 28–33 and say rows or columns of answers to each other. *More capable children* may work with those who are not sure about a particular answer.

Pupil's Objectives

(a) To write questions for M. and D. problems based on given information; (b) to have practice in translating problem situations into number sentences; (c) to show the M. or D. fact for each number sentence; and (d) to have mixed computational practice in finding the number for n.

New Words

corks, *popguns*

Background

The role of translating a problem situation into a number sentence has been discussed previously. In this lesson, the act of translating must be preceded by the making of an M. or D. question for the problem situation as was done in connection with addition and subtraction. Pupils should be told to think about the information for a problem in an effort to arrive at a question which relates it in a meaningful way.

Pre-Book Lesson

• Use some appropriate classroom, school, or community information for problems using either multiplication or division. Write on the chalkboard information like the following:

a. 21 children; 7 cars.

b. 6 cars; 5 children in each car.

Ask pupils if they can *make up* a problem question using any of the information written on the board. Have pupils discuss the information, how it may be related to state a question, and the various types of questions that may be stated.

• Write two types of information on the chalkboard, one for a partitive situation, and one for a measurement situation and have pupils explain the difference.

a. 28 children; 7 cars
How many in each of the cars?

b. 28 children; 4 children in each car.
How many cars are needed?

• Have pupils write an n-sentence for each of the problems mentioned in this Pre-Book Lesson.

Using the Text Page

• Be sure the children understand the three distinct parts of the written assignment at the top of the page.

a. Write a division or multiplication question.

b. Write an n-sentence.

c. Show the correct M. or D. fact.

• For the exercise at the bottom of the page, caution pupils to do the work inside the parentheses first. You may wish to have pupils follow the procedure shown below for Ex. 1.

1. $(\frac{1}{4} \text{ of } 36) + (\frac{1}{3} \text{ of } 27) = n$

$9 + 9 = n$

$18 = n$

Individualizing Instruction

• Ask selected pupils to read questions they have written for the information given at the top of the page. Discuss these questions and have pupils tell why they are (or are not) appropriate.

• Most pupils will find it interesting and challenging to make sentences of their own like those at the bottom of the page. Permit pupils to be as creative as they wish. However, they must be prepared to explain their work.

Writing M. and D. Problem Questions

[W]

For Ex. 1, a D. question might be, How many monkeys are on each stick? The *n*-example is $16 \div 4 = n$. The D. fact is $16 \div 4 = 4$.

1. 16 monkeys, the same number on each of 4 sticks

Write a question to make a D. problem for each of Ex. 2–4. Then write the *n*-example and show the D. fact.

2. 40 chicks, 8 chicks in each box
   How many boxes are needed? $40 \div 8 = n$ $40 \div 8 = 5$
3. 45 chicks, distributed equally among 5 boxes
   How many chicks are in each box? $45 \div 5 = n$ $45 \div 5 = 9$
4. 35 cows, 5 in each of the groups
   How many groups of cows are there? $35 \div 5 = n$ $35 \div 5 = 7$

Write a question to make a M. problem for each of Ex. 5–8. Then write the *n*-example and show the M. fact to use. For Ex. 5, you might write, "How many popguns are in the 5 piles? $5 \times 7 = n$ $5 \times 7 = 35$"

5. 5 piles, 7 popguns in each pile
6. 8 corks in each of 5 bags How many corks are there in all? $5 \times 8 = n$ $5 \times 8 = 40$
7. 5 shelves, 9 popguns on each shelf How many popguns are on the 5 shelves? $5 \times 9 = n$ $5 \times 9 = 45$
8. 6 boxes, 5 corks in each box How many corks are in the 6 boxes? $6 \times 5 = n$ $6 \times 5 = 30$

Working with *n*

A., S., M., D. [W]

Copy each example showing the correct number for *n*.

1. $(\frac{1}{4} \text{ of } 36) + (\frac{1}{3} \text{ of } 27) = n$ 18
2. n 21 $= (3 \times 5) + (18 - 12)$
3. $(45 \div 5) + (28 \div 4) = n$ 16
4. $(27 + 46) + (8 \times 5) = n$ 113
5. $(5 \times 8) - (40 \div 5) = n$ 32
6. $(82 - 46) + (22 + 19) = n$ 77
7. $(107 + 215) - (480 - 296) = n$ 138
8. $(7 \times 5) + (6 \times 0) = n$ 35

*Here, too, in finding a fractional part, sets language is understood.

Table of D. Facts, Dividing by 5

1	2	3	4	5	6	7	8	9
$5\overline{)5}$	$5\overline{)10}$	$5\overline{)15}$	$5\overline{)20}$	$5\overline{)25}$	$5\overline{)30}$	$5\overline{)35}$	$5\overline{)40}$	$5\overline{)45}$

Table of D. Facts, Answer 5

5	5	5	5	5	5	5	5	5
$1\overline{)5}$	$2\overline{)10}$	$3\overline{)15}$	$4\overline{)20}$	$5\overline{)25}$	$6\overline{)30}$	$7\overline{)35}$	$8\overline{)40}$	$9\overline{)45}$

M. and D. relationships [O]

1. Study the tables above. Find ways to help you to learn the facts, as you did with the tables on pages 293 and 308. Make Study Cards if you need them.

For each of Ex. 2–19, tell how to use the given fact to find the unknown product or factor.

2. $5 \times 7 = 35$, so $7 \times 5 = n$ 35

3. $36 \div 4 = 9$, so $32 \div 4 = n$ 8

4. $40 \div 5 = 8$, so $45 \div 5 = n$ 9

5. $6 \times 3 = 18$, so $3 \times n = 18$ 6

***6.** $5 \times 9 = 45$, so $\frac{1}{9}$ of $45 = n$ 5

7. $3 \times 7 = 21$, so $21 \div 3 = n$ 7

8. $20 \div 5 = 4$, so $20 \div n = 5$ 4

9. $4 \times 7 = 28$, so $28 \div 7 = n$ 4

***10.** $8 \times 3 = 24$, so $\frac{1}{3}$ of $24 = n$ 8

11. $5 \times 6 = 30$, so $4 \times 6 = n$ 24

12. $25 \div 5 = 5$, so $5 \times \bigcirc = 25$ 5

***13.** $\frac{1}{4}$ of $36 = 9$, so $n \times 4 = 36$ 9

14. $5 \times 7 = 35$, so $5 \times 8 = \square$ 40

***15.** $\frac{1}{2}$ of $14 = 7$, so $n \times 2 = 14$ 7

16. $16 \div 2 = 8$, so $2 \times 8 = n$ 16

17. $27 \div 3 = 9$, so $27 \div \triangle = 3$ 9

18. $4 \times 9 = 36$, so $36 \div 4 = n$ 9

19. $5 \times 7 = 35$, so $35 \div 7 = n$ 5

◆ **Extra Examples.** Work Set 79.

Teaching Page 326

Pupil's Objectives

(a) To organize the D. facts in which one factor is 5; and (b) to use a helping fact to find an unknown factor or product.

Background

Oral work affords the opportunity for you to observe pupils' understandings. Make certain your questions and guidance encourage pupils to show their understanding rather than merely the ability to give a numerical answer.

In this lesson you should look for evidence of the following understandings. Illustrative examples are identified below.

a. Factor $\times$ factor = product

The order of the factors doesn't affect the product. (Ex. 2, 5)

b. Product $\div$ factor = factor

A decrease in the product is associated with a decrease in the unknown factor. (Ex. 3)

An increase in the product is associated with an increase in the unknown factor. (Ex. 4)

c. Pairs of M. facts (Ex. 2, 5)

d. Pairs of D. facts (Ex. 3, 8, 17)

e. Sets of M. and D. facts (Ex. 7, 9, 12, 16, 18, 19)

f. There is more than one mathematical sentence which requires division for solving. (Ex. 6, 10, 13, 15)

Pre-Book Lesson

• Before opening the pupil's text, have pupils make two tables of D. facts with 5 as a factor. You may wish to duplicate a page for pupils to work like that shown below.

$5\overline{)5}$	$5\overline{)10}$	$5\overline{)15}$	$5\overline{)20}$	$5\overline{)25}$	$5\overline{)30}$
$1\overline{)5}$	$2\overline{)10}$	$3\overline{)15}$	$4\overline{)20}$	$5\overline{)25}$	$6\overline{)30}$
$5\overline{)35}$	$5\overline{)40}$	$5\overline{)45}$	$7\overline{)35}$	$8\overline{)40}$	$9\overline{)45}$

• Discuss the two tables of D. facts. One table shows D. facts in which 5 is the answer. The other shows D. facts in which you are dividing by 5. Be sure pupils understand this difference.

Using the Text Page

Use the pupil-prepared tables for reference in discussing the exercises on this page. Use the chalkboard and construct arrays as needed to enhance understanding.

Individualizing Instruction

This would be an appropriate time to have *all pupils* make a multiplication chart in which all M. facts studied to date are shown. Such a chart was suggested in connection with Teaching Pages 318 and 319. It would save time if you duplicated an empty chart for this purpose.

Reminder

Continue to maintain the following:

a. Ability to recognize and use the geometric shapes and terms that have been introduced

b. Ability to recognize and use the common units of measurement that have been taught

c. Ability to gather, organize, and present information in graphs, tables, and charts

Teaching Page 327

Pupil's Objective

To learn to add and subtract mentally.

Background

As with all suggested enrichment activities in this program, this page of exercises is designed only for those pupils in your class who are ready and able. This exercise requires that pupils be flexible enough to adopt quickly newer and shorter ways of computation.

The method of addition suggested on this page is an application of the Commutative and Associative Properties of Addition. Consider the example $43 + 35$. After renaming 35 as $30 + 5$, the associative property allows us to write $(43 + 30) + 5$; the commutative property allows us to write $43 + (5 + 30)$. Again, applying the associative idea, we write $(43 + 5) + 30$.

In adding or subtracting with 2-place numerals, one of the numbers is renamed to show tens and ones. As a result, two subtractions or two additions follow—one for the ones and one for the tens.

Pre-Book Lesson

For those pupils you consider ready to do work of this type, write the example $43 + 35$ on the chalkboard. Discuss with them the renaming of one of the numbers and then the addition as suggested in the Background section.

Using the Text Page

- Go over Ex. 1 and 2 orally with all pupils and then assign the remainder of the page as written work.
- Note that the pupil is to work each example *in his head*, and then to copy it with the answer. Then he is to copy each example again and work it the way he has been working it up to this time.

Individualizing Instruction

You may wish to give some of the *more capable children* a column of three or four 2-place numerals to work with in this way. For an example such as $52 + 27 + 84 + 16$, the thinking might be:

$$52 + 20 = 72$$
$$72 + 7 = 79$$
$$79 + 80 = 159$$
$$159 + 4 = 163$$
$$163 + 10 = 173$$
$$173 + 6 = 179$$

NOTES

* This is an excellent plan for computing mentally. Its use should be encouraged.

Do You Like to Try New Things?

Enrichment [W]

* Try adding in your head.

1. $43 + 35 = n$ $35 = 5 + 30$, so to 43 you can add first 5, then 30.

$43 + 5 = 48$ $48 + 30 = 78$ So, $43 + 35 =$ _?_ 78

2. $27 + 28 = n$ *Think* this way: "$28 = 8 +$ _?_ 20

$27 + 8 = 35$ $35 +$ _?_ $20 =$? 55 So, $27 + 28 =$ _?_ 55."

Work each of Ex. 3–8 in your head as you did Ex. 1 and 2. Then write each example with its answer.

3. $42 + 15 = n$ 57 **4.** $33 + 27 = n$ 60 **5.** $62 + 27 = n$ 89

6. $19 + 18 = n$ 37 **7.** $25 + 36 = n$ 61 **8.** $39 + 35 = n$ 74

Now copy Ex. 3–8 and add to see how many correct answers you got when you added in your head.

* Now try subtracting in your head.

9. $58 - 37 = n$ $37 = 7 + 30$, so you can subtract first 7, then 30.

$58 - 7 = 51$ $51 - 30 = 21$ So, $58 - 37 =$ _?_ 21

10. $62 - 28 = n$ *Think* this way: "$28 = 8 +$ _?_ 20

$62 - 8 = 54$ $54 -$ _?_ $20 =$? 34 So, $62 - 28 =$ _?_ 34"

Work each of Ex. 11–16 in your head, as you did Ex. 9 and 10. Then write each example with its answer.

11. $89 - 42 = n$ 47 **12.** $91 - 65 = n$ 26 **13.** $72 - 25 = n$ 47

14. $73 - 19 = n$ 54 **15.** $58 - 24 = n$ 34 **16.** $69 - 34 = n$ 35

Now copy Ex. 11–16 and subtract to see how many correct answers you got when you subtracted in your head.

*A good chance for a quick test! One factor is no greater than 5.

*Finding Unknown Factors

D. facts with a factor 2, 3, 4, or 5 [W]

Write answers on folded paper.

	a	b	c	d	e	f	g
1.	7 4)28	2 5)10	4 9)36	8 5)40	7 5)35	9 4)36	8 3)24
2.	5 5)25	3 9)27	8 2)16	4 5)20	3 7)21	7 2)14	5 9)45
3.	4 8)32	5 3)15	4 6)24	2 7)14	6 2)12	5 6)30	9 3)27
4.	9 2)18	7 3)21	1 5)5	3 6)18	5 4)20	2 9)18	4 4)16
5.	5 7)35	2 6)12	8 4)32	6 5)30	4 3)12	5 8)40	6 3)18
6.	4 7)28	5 2)10	2 8)16	3 5)15	3 8)24	9 5)45	6 4)24

**Many Things

Maintenance [W]

Show the set of M. and D. facts for each of Ex. 1–3.

1. $35 = 5 \times 7$
   $5 \times 7 = 35$ $35 \div 5 = 7$
   $7 \times 5 = 35$ $35 \div 7 = 5$

2. $36 = 4 \times 9$
   $4 \times 9 = 36$ $36 \div 4 = 9$
   $9 \times 4 = 36$ $36 \div 9 = 4$

3. $30 = 6 \times 5$
   $6 \times 5 = 30$ $30 \div 6 = 5$
   $5 \times 6 = 30$ $30 \div 5 = 6$

Copy each example showing the number in place of n.

4. $32 - (8 \times 3) = n$ 8

5. $12 + (24 \div 6) = n$ 16

6. $n = (18 \div 6) + (24 \div 4)$ 9

7. $n = (3 \times 7) + (5 \times 5)$ 46

Show the D. facts that go with

8. $8 \times 5 = 40$.
   $40 \div 8 = 5$
   $40 \div 5 = 8$

9. $5 \times 9 = 45$.
   $45 \div 5 = 9$
   $45 \div 9 = 5$

10. $8 \times 4 = 32$.
    $32 \div 8 = 4$
    $32 \div 4 = 8$

Write answers estimated to the nearest ten.

11. $48 + 32$ 80
12. $85 - 39$ 50
13. 4×49 200
14. 31×4 120
15. $93 - 62$ 30
16. 5×39 200
17. $54 + 27$ 80
18. 62×2 120

For Ex. 19–28 write "Yes" or "No" for each answer.

19. Are 18 ounces less than 1 pound? No

**Another mid-chapter review of information and meanings!

Teaching Pages 328 and 329

Pupil's Objectives

(a) To have written practice with D. facts; (b) to nave practice with the many understandings and computational skills that have been learned; and (c) to extend understanding about 1 as a factor within the context of a division example.

Background

Practice is an essential ingredient in a well-balanced mathematics program. Many skills, concepts, and understandings have been carefully introduced throughout this program. Practice has helped build a level of mastery dependent, of course, upon the degree of understanding of the concept involved. But, maintenance helps to insure mastery as well as extend it.

All the written work on pages 328 and 329 is to help establish a minimum level of mastery as well as maintain the skills that have been established.

At the bottom of page 329 is some oral work with division sentences showing 1 as a factor. Since division by a number is the inverse of multiplication by that number, your pupils have already been exposed to these notions within the context of the multiplication sentence. You should help pupils state in their own words generalizations like the following:

a. When 1 is the known factor, the unknown factor will be equal to the product.

b. When the product and the known factor are equal, the unknown factor will be 1.

Your pupils have been introduced to the idea that it is impossible to divide by zero. There is no need to stress the exception of zero at this time, though it is mentioned in the generalization on the pupil's page.

Pre-Book Lesson

• You may wish to engage in a *Show the Answer* game for review and practice. It will give you the opportunity to make a quick check on your pupils' mastery of D. facts before assigning the written work at the top of page 328.

• You may quickly review M. facts in which 1 is a factor. Then help pupils to realize that, because of the inverse relationship of multiplication by 1 and division by 1, their knowledge of M. facts will help them in their work with the associated D. facts.

Using the Text Pages

• Use the written work at the top of page 328 as a progress test. After writing the answers on folded paper, pupils should check their own work. (Have the answers written on answer sheets which you place in two or three locations in the room.)

Each child should then make a detailed record of the types of examples not worked correctly. This would give him a better idea of the areas in which he is having difficulty. On the chalkboard, write the following categories which pupils may use for their individual records. Each child should show the number of errors for each type of example.

Answer 5 _____
Dividing by 5 _____
Answer 4 _____
Dividing by 4 _____
Others _____

If a child misses more than 2 examples in a category, guide him in using appropriate study helps (making study cards of facts and answers, constructing arrays, or making tables of facts).

• The maintenance section may be used as a progress test. In working this exercise, also, each child should be aware of concepts or understandings that are not well established. You would find it most useful to make a class diagnosis of this material. If you find that several children are having difficulty with the same exercise, then this is the time to reteach or review. If the foundation is not strong, future learning will be jeopardized.

• There are no new principles involved in the oral work at the bottom of page 329. Your pupils should have very little difficulty with these facts.

Individualizing Instruction

• Be sure to go over the maintenance section orally after the written work has been completed.

A careful discussion of each item will provide an excellent review of important ideas that have been developed so far. It will also provide you with information as to which items may need further clarification or reteaching.

- *All pupils* may make a list of D. facts having the factor 1, D. facts when the answer is 1, and D. facts when dividing by 1. Some pupils may use the form $1\overline{)4}^{\,n}$ while others may use the form $4 \div 1 = n$. After all tables have been completed, discuss and compare the two forms.

NOTES

*Division facts having 1 as one factor can all be learned by the two principles stated here.

20. Can you divide to find $\frac{1}{4}$ of 32? Yes

21. Is $\frac{1}{3}$ of a piece of string longer than $\frac{1}{2}$ of the same piece of string? No

22. To solve a problem must you always use all the numbers in the problem? No

23. Can you subtract to check D. answers? Yes

24. Do you remember 2 hundreds in 4×205? No

25. Is the red part $\frac{4}{5}$ of the inside of this square? No

26. Can angle *RST* be named angle *TSR*? Yes

27. Does dividing by 3 undo multiplying by 3? Yes

28. Do you multiply hundreds in the same way you multiply ones? Yes

*D. Facts with 1 as a Factor

[o]

You know the answers for Ex. 1–3. Say them.

1. $3 \div 3 = n$ 1 **2.** $5 \div 5 = n$ 1 **3.** $2 \div 2 = n$ 1

When you divide a product, not zero, by a factor equal to the product, the answer is 1.

Say the answers for Ex. 4–8.

4. $8\overline{)8}^{1}$ **5.** $6\overline{)6}^{1}$ **6.** $9\overline{)9}^{1}$ **7.** $7\overline{)7}^{1}$ **8.** $1\overline{)1}^{1}$

You know the answers for Ex. 9–11. Say them.

9. $4 \div 1 = n$ 4 **10.** $2 \div 1 = n$ 2 **11.** $5 \div 1 = n$ 5

When you divide a product by the factor 1, the answer is equal to the product.

Say the answers for Ex. 12–16.

12. $1\overline{)6}^{6}$ **13.** $1\overline{)9}^{9}$ **14.** $1\overline{)7}^{7}$ **15.** $1\overline{)1}^{1}$ **16.** $1\overline{)8}^{8}$

* Only 16 spaces not containing numerals remain in the chart on page 331, so now most of the M. and D. facts have been taught!

* A Chart for M. and D. Facts

Use of Commutative Property; inverses [O]

The chart on page 331 is for some M. and D. facts. It shows products and their factors.

1. Find a black 10 twice in the chart.

a. From one 10, run your finger along the same row and find the numeral in the blue column. Then from that 10 again, run your finger up the same column and find the numeral in the red row.

b. Do this same thing for the other 10 in the chart.

c. You have found that 2 and 5 are factors of 10.

d. Say the set of M. and D. facts about the product 10 and its factors 2 and 5. $2 \times 5 = 10$ $5 \times 2 = 10$ $10 \div 2 = 5$ $10 \div 5 = 2$

2. In the same way, find a black 14 twice. Then find the factors shown for 14.

a. On the board show the factors 2 and 7 and the product. 14

b. Say the set of M. and D. facts. $2 \times 7 = 14$ $7 \times 2 = 14$ $14 \div 2 = 7$ $14 \div 7 = 2$

3. Find a black 12 twice in the chart. On the board show the factors, 4 and 3, the product, 12, and the set of M. and D. facts. $4 \times 3 = 12$ $3 \times 4 = 12$ $12 \div 4 = 3$ $12 \div 3 = 4$

4. Find the other two 12's in the chart. What are the factors? 2 and 6 Say the set of M. and D. facts. $2 \times 6 = 12$ $6 \times 2 = 12$ $12 \div 2 = 6$ $12 \div 6 = 2$

5. Find 16 and the factors shown.

a. Say the set of M. and D. facts. $4 \times 4 = 16$ $16 \div 4 = 4$

b. Why are there only two facts in this set? The factors are equal.

Tell how to use the chart to find

6. the factors shown for the 18 's; 3 and 6 for the 18 's. 2 and 9

** 7. the product for a. 5×4. 20 b. 4×7. 28 c. 9×5. 45

** Have pupils form the habit of reading the chart beginning with the factor shown in the left column.

Teaching Pages 330 and 331

Pupil's Objectives

(a) To learn to use a chart for M. and D. facts; (b) to organize and summarize all that has been learned about multiplication and division facts; and (c) to prepare for learning the remaining M. and D. facts.

New Word

(page 330) *twice*

Background

Every property governing the operations of multiplication and division may be illustrated through the use of a chart like the one shown below:

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30				
7	7	14	21	28	35				
8	8	16	24	32	40				
9	9	18	27	36	45				

5 × 7

7 × 5

Inverse Idea. Multiplication can be "undone" by dividing a product by the number by which you multiplied.

For example, $5 \times 7 = 35$ and $35 \div 5 = 7$.

Identity Element for Multiplication. If 1 is a factor, the product is equal to the other factor. The products shown in the chart across from the numeral 1 will be exactly the same as the factors shown across the top of the chart.

Commutative Property. Changing the order of the factors does not affect the product. For example, the product shown across from 7 and below 5 is the same as that product shown across from 5 and below 7.

Associative Property. Since multiplication is a *binary* operation, only two of three numbers may be multiplied at once. The associative property allows us to associate the first two or the second two of the three factors. The product remains unchanged. Consider the example $4 \times 2 \times 3$. Associating 4 and 2 and multiplying their product by 3 yields the same result as associating 2 and 3 and multiplying their product by 4.

Distributive Property. The distribution of multiplication over addition is illustrated for all M. facts. Consider the M. fact $5 \times 7 = 35$. We can direct children to use the chart to find five 7's in many different ways. For example, they can find five 2's and then find five 5's and add the results. By renaming 7 in a variety of ways, they can find the product 35 in a variety of ways.

Note that the lower right portion of the chart is blank. This portion of the chart is to show the products of the remaining M. and D. facts to be learned. A careful, systematic study of the chart should impress your pupils that not many facts are still to be learned even though the 6's, 7's, 8's, and 9's haven't been given formal consideration. In fact, some children may discover that although there are 16 blanks, only 10 facts must be known in order to complete the chart. Because of the Commutative Property of Multiplication, several pairs of facts have the same product. As an enrichment activity for later work in this chapter, have pupils discuss how the commutative property is illustrated in the chart.

Teacher's Preparation

Construct squares for a chart on a large piece of tag board like the one on page 331. Show the factors across the top and down the left side of the chart. (You may wish to show the factors across the top in red and those down the side in blue.) Pupils will show the products for facts learned so far. Do not show products in the squares in the lower right portion of the chart. Thus, you will have a multiplication chart with all the factors shown, but limited to the products for those facts which have been studied so far.

Pre-Book Lesson

• Guide the class discussion and participation as the children show the products on the chart (use a felt pen or crayons). Use statements and questions like the following:

a. The factors I am thinking of are shown by the red 4 and the blue 7. Who can show the product in the proper place?

b. Is there another place in the chart where we can show the product 28?

c. The product I am thinking of is 18. One of its factors is 6. Who can point to two different squares for showing this product?

d. I am thinking of a product whose pair of factors are equal. Where is one place this product could be shown? Are there any other places?

• After you have asked some questions of this type, be sure to guide pupils to *see* the Commutative Property of Multiplication.

• After a child has successfully shown a product in the chart, have him discuss factors of some other product and then ask another child to show the product in the chart.

• Complete the chart and then use it in connection with the oral discussion on the text page.

Using the Text Pages

• Your pupils should have no difficulty with the oral exercises on page 330. It would be a good idea to use the class chart just prepared as well as the chart on page 331 in the pupil's book.

• For Ex. 8–10 on page 331, have children work with the class chart while they are telling the information requested. It is more meaningful for children to illustrate the idea on the chart than merely to have them tell about it.

• Your *more capable children* will probably not have difficulty with the written exercises. However, with the rest of your class it is recommended that you do Ex. 11–15 orally before assigning it as written work. In these exercises, it would be most appropriate to emphasize the factors before determining the product. In fact, it would be a good idea to show each pair of factors on the chalkboard before attempting to find the product. Bring in the use of the distributive property to determine a product on the basis of facts that are known. For example, pupils may find the product of 6×6 through their knowledge of the facts $6 \times 4 = 24$ and $6 \times 2 = 12$.

Individualizing Instruction

Working with the chart to foster greater insight into the relationships between M. and D. facts and between the operations of multiplication and division should be a basic part of your pupils' learning experiences. You must decide the level of mastery of each child and the type of activity most appropriate for him.

Some children will be able to learn quickly all the remaining M. and D. facts as a result of thoughtful study of this chart; others will continue to establish greater mastery of just the facts presented in the chart. Whatever the level may be, employ the chart so as to enhance understanding.

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	**10**	12	**14**	16	**18**
3	3	6	9	**12**	15	**18**	21	24	27
4	4	8	**12**	**16**	20	24	28	32	36
5	5	**10**	15	20	25	30	35	40	45
6	6	12	18	24	30	A	?	?	?
7	7	**14**	21	28	35	?	B	?	?
8	8	16	24	32	40	?	?	C	?
9	9	**18**	27	36	45	?	?	?	D

Factors-product relationship

8. Tell how to work with the chart when you know a product and one factor and must find the unknown factor.

9. Tell how to find the unknown factor for

a. $3\overline{)15}^{\,5}$. **b.** $9\overline{)27}^{\,3}$. **c.** $8\overline{)16}^{\,2}$. **d.** $5\overline{)30}^{\,6}$. **e.** $7\overline{)28}^{\,4}$.

10. Say the D. fact that goes with each fact for Ex. 9.

$15 \div 5 = 3$ $27 \div 3 = 9$ $16 \div 2 = 8$ $30 \div 6 = 5$ $28 \div 4 = 7$

[W]

11. Copy and finish: In the 5-row and the 5-column the numerals for all products end in ?0 or ?5.

12. Find the space showing A in the chart.

a. A stands for what product? 36

b. What are the factors? 6 and 6

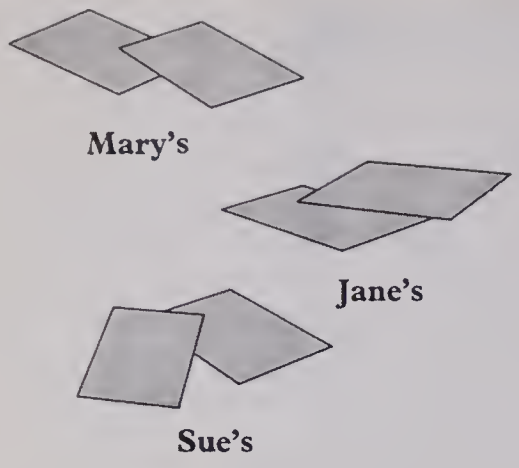
c. Show the set of M. and D. facts. $6 \times 6 = 36$ $36 \div 6 = 6$

13–15. As in Ex. 12, for each of letters B, C, and D,

a. show the product the letter stands for. 49 64 81

b. show the factors for each product. 7 and 7 8 and 8 9 and 9

c. show the set of M. and D. facts. $7 \times 7 = 49$ $8 \times 8 = 64$ $9 \times 9 = 81$
$49 \div 7 = 7$ $64 \div 8 = 8$ $81 \div 9 = 9$



* Dividing Tens

Factors-product relationship [O]

1. Mary, Jane and Sue shared 6 sheets of paper equally.

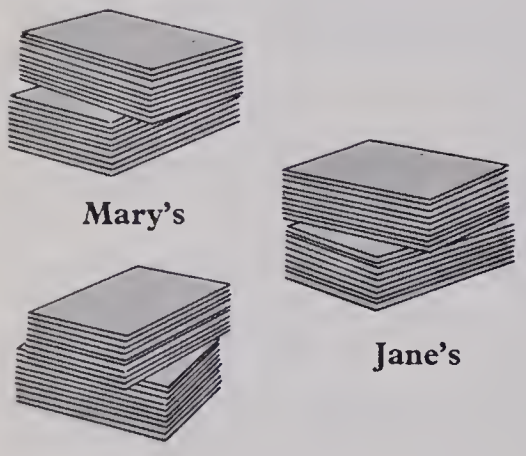
a. Use the picture to tell how they distributed 6 sheets.

b. In Ex. A, when you divide 6 ones by 3 the answer is 2 _?_. ones

A

Product ↓ Factors ↙ ↘

$6 \text{ ones} \div 3 = 2 \text{ ones}$ or $3\overline{)6 \text{ ones}}$ $\quad 2 \text{ ones}$



c. Factors of 6 are 3 and _?_. 2

2. The girls shared 6 piles of paper equally. Each pile had 10 sheets, so there were 6 tens, or 60, sheets.

a. Use the picture to tell how the girls distributed 60 sheets.

b. In Ex. B, when you divide 6 tens by 3 the answer is 2 _?_. tens

c. Is 20 another name for 2 tens? Yes

B

Product ↓ Factors ↙ ↘

$6 \text{ tens} \div 3 = 2 \text{ tens}$ or $3\overline{)6 \text{ tens}}$ $\quad 2 \text{ tens}$

d. Factors of 6 tens are 3 and 2 _?_ tens, so factors of 60 are _?_ 3 and 20.

You divide tens in the same way that you divide ones.

* Now for division of tens, as for multiplication of tens, pupils begin slowly and find that they can divide tens just as they divide ones.

Teaching Pages 332 and 333

Pupil's Objectives

(a) To begin to learn to divide tens; and (b) to be able to differentiate between dividing tens and dividing ones.

Background

This is the first in a series of lessons in which the children learn to divide tens.

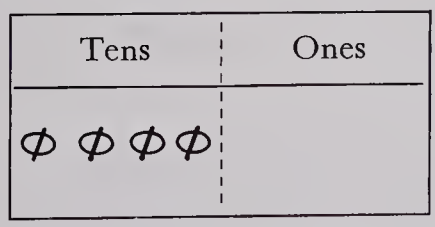
Your pupils have had experience with facts in finding the solution to a D. by determining a missing factor. In other words, they have made use of the factors-product relationship. If the child knew his multiplication facts, then any division example could be solved by using the appropriate M. fact.

The factors-product relationship should be developed in dividing tens just as it was developed in dividing ones. In working the example 60 ÷ 2, the child realizes that the product 60 has the factors 2 and 30, or 2 and 3 tens. In turn, the answer for the division, 30 or 3 tens, should become evident.

Pre-Book Lesson

• Review with pupils the reading and meaning of numerals like 40, 70, 50, and so on. Use any of the place-value teaching aids* to help illustrate the idea.

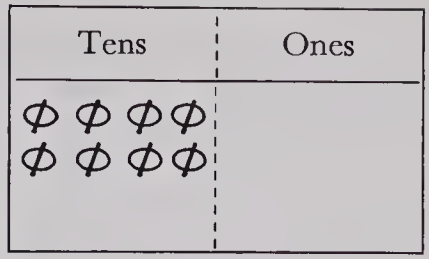
On a flannel board you may show 40 as follows:



Have pupils explain that 40 equals 4 tens and no ones.

• After discussing the meaning of various numerals showing tens, show 80 on the flannel board. Make sure pupils show 8 tens and no ones.

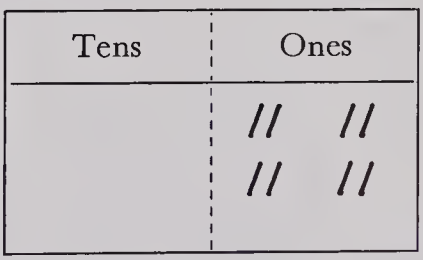
Then, ask a child to organize the 8 tens into two equivalent sets.



Ask pupils to tell how many tens in each of the two sets.

At this point you may wish to have one or two pupils find the solution to the question "80 ÷ 2 = _ ? _." Have them work with a set of 80 objects and separate them into 2 piles until all objects are gone. This could be done as an independent activity while you continue the lesson with the rest of the children. The pupils should find that there are 40 objects in each of two subsets made from a set of 80. This will confirm the result found in the flannel-board demonstration. Then write the example, $2\overline{)80}$ (40). Is 40 a reasonable answer? Help pupils to realize that the 0 in the answer means there are no ones.

• On the flannel board, show 8 ones. Then, place the 8 ones into two equivalent sets.



Discuss with pupils that there are 4 ones in each of the two sets of ones just as there are 4 tens in each of the sets of tens. This Pre-Book Lesson should lead pupils to the discovery that you divide tens in the same way that you divide ones.

When pupils have learned the likeness of dividing ones and dividing tens, a big step has been taken toward mastery of the division operation.

Using the Text Pages

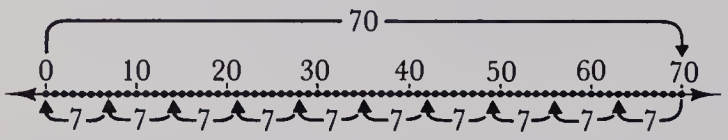
• As the oral part of this lesson is developed, make extensive use of the teaching aids used previously to illustrate the factors-product relationship.

* See 2, 3, 10, and 21, page xix.

No mention was made of it in the Pre-Book Lesson. Also have pupils write examples as shown in boxes A and B.

• If you have a permanent number line,* demonstrate to pupils how Ex. 8–22 may be illustrated on it. For Ex. 8, $7\overline{)70}$, you might think of two questions:

"How many 7's in 70?" This question involves the measurement idea mentioned in an earlier lesson. The number of times you can move 7 units on the number line from 70 to 0 is the number of 7's in 70.



"How many members in each equivalent set when 70 things are separated into 7 equivalent sets of things?" A pupil may demonstrate this partitive idea by separating a set of 70 objects into 7 subsets, placing one object in each subset, in turn, until the 70 are distributed. The number of objects in each subset is the answer for the division $70 \div 7$. The partitive idea may also be shown by drawing an array of dots having 7 rows and finding the number of dots that can be placed in each row if the array has 70 dots in all.

* See 6, page xix.

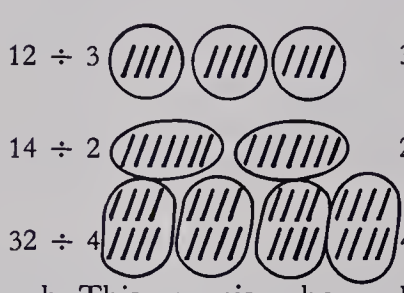
Individualizing Instruction

• Permit *slower learners* to use whatever device they wish to help in finding the solution to the examples in the written work.

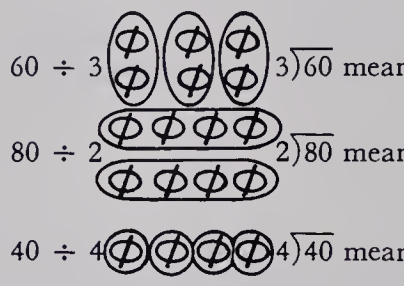
• *All pupils* may check the answers on a number line.

• *Slower learners* may profit from the following:

a. Division facts are shown below. Each fact involves division of ones.



b. This exercise shows division of tens. You divide tens in the same way that you divide ones.



NOTES

3. If 80 shells are shared equally by 4 girls, each girl will get how many? $80 \div 4 = n$ Finish the work.

a. 80 is another name for 8 _?_. tens

b. 8 tens $\div$ 4 = 2 _?_, or 20. Factors of 80 are 4 and 2 tens, or 4 and _?_. 20

In each example say the answer and finish the work.

4. $60 \div 2 = ?$ 30 Factors of 60 are 2 and _?_. 30

5. $90 \div 3 = ?$ 30 Factors of 90 are 3 and _?_. 30

6. $80 \div 2 = ?$ 40 Factors of 80 are _?_ 2 and _?_ 40.

7. $60 \div 3 = ?$ 20 Factors of 60 are _?_ 3 and _?_ 20.

Tell how the answer was found for each of Ex. 8–12. Dividing tens

8. $7\overline{)70}$ 10 9. $3\overline{)90}$ 30 10. $6\overline{)60}$ 10 11. $2\overline{)40}$ 20 12. $3\overline{)60}$ 20

In some of Ex. 13–22 a digit in the answer is missing and the other digit is in the wrong place or a 1-digit answer is in the wrong place. Explain what to do.

13. $2\overline{)60}$ 30 $4\overline{)80}$ 20 14. $4\overline{)36}$ 9 $5\overline{)40}$ 8 15. $5\overline{)50}$ 1 $5\overline{)50}$ 10 16. $3\overline{)30}$ 10 $9\overline{)90}$ 10 17. $5\overline{)20}$ 4 $5\overline{)20}$ 4

18. $4\overline{)80}$ 2 19. $5\overline{)40}$ 8 20. $9\overline{)45}$ 5 21. $9\overline{)90}$ 1 22. $2\overline{)20}$ 10

[W]

Copy and work each of Ex. 23–42.

23. $4\overline{)40}$ 10 24. $5\overline{)40}$ 8 25. $4\overline{)36}$ 9 26. $2\overline{)80}$ 40 27. $8\overline{)16}$ 2

28. $9\overline{)27}$ 3 29. $4\overline{)28}$ 7 30. $7\overline{)35}$ 5 31. $4\overline{)24}$ 6 32. $9\overline{)90}$ 10

33. $3\overline{)90}$ 30 34. $6\overline{)24}$ 4 35. $6\overline{)60}$ 10 36. $2\overline{)16}$ 8 37. $7\overline{)28}$ 4

38. $4\overline{)32}$ 8 39. $5\overline{)50}$ 10 40. $5\overline{)45}$ 9 41. $8\overline{)80}$ 10 42. $6\overline{)30}$ 5

A

B

Dividing Tens and Ones

Using the Distributive Property [O]

1. Sue, Sally, and Jane shared 36 flowers equally. How many flowers did each girl get? $36 \div 3 = n$

a. Picture A. From 30 flowers each girl gets 10 flowers. Picture B. From 36 flowers each girl gets 12.

b. For $36 \div 3 = n$, tell the product and known factor. Product: 36 Known factor: 3

c. Explain Ex. C. Is 12 the unknown factor? Yes

C

$36 = 30 + 6$

$10 + 2$, or 12

$3\overline{)30 + 6}$

*d. Ex. D shows another way to find the unknown factor.

Divide tens: $3 \div 3 = 1$. In what place is 1 written in the answer? ten's Why? You are dividing tens.

Multiply to find how many you subtract. Since each of the 3 girls gets 1 set of 10 flowers, you subtract 3×1 ten, or 3 tens.

Subtract. $36 - 30 = 6$

D

Tens	Ones
1	2
3)3	6
3	0 (3×1 ten)
	6
	6 (3×2)
	0

Divide ones: $6 \div 3 = 2$. Where is 2 written in the answer? one's place Why? You are dividing ones.

Multiply to find how many you subtract. Since each of the 3 girls gets 1 set of 2 flowers, you subtract 3×2, or 6.

Subtract. $6 - 6 = 0$

* Using place-value ideas helps pupils to realize that they use the same steps in showing division whether tens are being divided or ones.

Teaching Pages 334 and 335

Pupil's Objective

To learn to divide tens and ones.

New Word

(page 334) *since*

Background

In the preceding lesson, your pupils learned to divide with a 2-place numeral showing tens. In this lesson, the children will be introduced to dividing with a 2-place numeral showing both tens and ones.

Under certain circumstances we may use the distributive property when dividing. Consider the example $36 \div 3 = n$. For some time now your pupils have been thinking of numbers like 36 as $30 + 6$ (3 tens plus 6 ones). With 36 renamed as $30 + 6$, the child can use the distributive property to find the solution. First, he may divide 30 by 3 and then 6 by 3 so as to distribute the division over the two addends.

Learning to rename a number so that the distributive property may be used efficiently is very important. The renaming should always be such that both the tens and the ones, as renamed, are products of the known factor and some unknown factor.

The first example in the pupil's book involves division of tens and ones. First, the use of a series of pictures is suggested as a method of solution. Then, renaming and use of the distributive property allows for working division the child already knows. And finally, the division algorithm as in box D on the pupil's page uses digits in their places (place-value idea) to show the unknown factor for the division as well as the number that has been divided.

Pre-Book Lesson

• Present a problem similar to the one in the pupil's book for the children to solve. Use representative objects in the room to assist in finding the answer. For an example such as $46 \div 2$, 46 objects could be separated into two groups of the same size.

If a child does not suggest the possibility of grouping the total into tens and ones, do so yourself. Show bundles of tens and ones on the flannel board.

Tens	Ones
⌀ ⌀ ⌀ ⌀	// / // /

Then ask a child to separate the tens into two groups of the same size. Do the same with the ones.

• Record the solution on the chalkboard as shown in the two steps below.

a. 46 is renamed $40 + 6$.

b. $2\overline{)40 + 6}$

$20 + 3$, or 23

Using the Text Pages

• You may wish to explain the subtraction shown in box D on page 334.

Be sure that all pupils understand that the number of tens that have been divided is subtracted so as to find the number of ones left to be divided. The number of ones is then divided, as usual.

• All the work on these two pages should be done orally. When you have a pupil do an example on the chalkboard, have the other members of the class work the same example on paper. Use their written work as well as the board work as a basis for discussion. The understanding that is built in this lesson is essential to success in future work of this type.

Individualizing Instruction

• All children should come to understand that each multiplication results in the number of tens or ones which are subtracted at one time from the number to be divided.

• Practice exercises may be found at the top of page 336.

See Teacher's Page 334 for suggestions for teaching page 335.

NOTES

2. For the 48 spelling words she had to learn, Sue made 2 lists with the same number in each. How many words were in each list? $48 \div 2 = n$

a. The product is _?48_. The known factor is _?2_.

b. Explain Ex. E as one way to find the unknown factor. 48 = 40 + 8 40 ÷ 2 = 20 8 ÷ 2 = 4 20 + 4 = 24

c. Explain Ex. F as a better way to find the unknown factor.

E

$48 = 40 + 8$

$20 + 4$, or _?_

$2\overline{)40 + 8}$

Tens: *Think*, "4 ÷ 2 = 2." Write "2" in ten's _?_ place in the answer.

Multiply. 2 × 2 tens = 4 tens, or _?40_. Write "40."

Subtract. 48 − 40 = _?8_

Ones: *Think*, "8 ÷ 2 = 4." Write "4" in one's _?_ place in the answer.

Multiply. 2 × 4 = 8. Write "8."

Subtract. 8 − 8 = 0

F

Tens	Ones	
2	4	
4	0	(2 × 2 tens)
	8	
	8	(2 × 4)
	0	

d. When 2 is one factor of 48, the other factor is _?24_.

Explain how to finish the work in Ex. 3–5.

3. $3\overline{)96}$ (3?)

90 (3 × 3 tens)

6

6 (_?2_ × _?_)

0

4. $2\overline{)68}$ (34)

60 (_?2_ × _?3 tens_)

8

8 (_?_ × _?4_)

0

5. $4\overline{)88¢}$ (22¢)

80 (_?4_ × _?2 tens_)

8

8 (_?4_ × _?2_)

0

Work each of Ex. 6–10 on the board. Explain your work.

6. $2\overline{)84}$ (42) **7.** $3\overline{)69}$ (23) **8.** $4\overline{)48}$ (12) **9.** $7\overline{)77¢}$ (11¢) **10.** $3\overline{)39¢}$ (13¢)

*Now a quick review test in division of tens and ones!

*Try These

[W]

Divide. Check by dividing a second time.

1. $3\overline{)99}^{33}$	4. $2\overline{)66}^{33}$	7. $3\overline{)63}^{21}$	10. $3\overline{)36¢}^{12¢}$	13. $4\overline{)88¢}^{22¢}$
2. $4\overline{)84}^{21}$	5. $3\overline{)69}^{23}$	8. $2\overline{)46}^{23}$	11. $2\overline{)64¢}^{32¢}$	14. $3\overline{)93¢}^{31¢}$
3. $6\overline{)66}^{11}$	6. $2\overline{)84}^{42}$	9. $4\overline{)44}^{11}$	12. $8\overline{)88¢}^{11¢}$	15. $2\overline{)68¢}^{34¢}$

◆ **Extra Examples.** Work Set 80.

Multiplying to Check Division Answers

**Inverses; factors-product relationship [O]

A

Factors

$$3\overline{)96}\quad \text{Product}$$

32

90

6

6

0

B

Check

32 } Factors

3 }

96 Product

Is the answer correct for Ex. A? How can you tell?

The division in Ex. A shows that 3 and 32 are factors of 96. So, the product of 3×32 should be 96, the product you divided.

Explain Ex. B. Why is it a check for the work in Ex. A? It shows that 96 is the product of the two factors. Does Ex. B show that 3 and 32 are factors of 96? Yes Explain. It shows that their product is 96.

> **To check an answer in division, you can multiply the answer and the factor you divided by. That product should be equal to the product you divided.**

On the board check these answers by multiplying:

1. $84 \div 2 = 42$ ($2 \times 42 = 84$)
2. $69 \div 3 = 32$ ($3 \times 32 = 96$)
3. $48 \div 4 = 21$ ($4 \times 21 = 84$)

[W]

Copy and divide. Check by multiplying, as in Ex. B.

4. $5\overline{)55}^{11}$	7. $3\overline{)66}^{22}$	10. $4\overline{)84}^{21}$	13. $9\overline{)99¢}^{11¢}$	16. $7\overline{)77¢}^{11¢}$
5. $3\overline{)99}^{33}$	8. $2\overline{)48}^{24}$	11. $2\overline{)62}^{31}$	14. $2\overline{)22¢}^{11¢}$	17. $5\overline{)50¢}^{10¢}$
6. $2\overline{)44}^{22}$	9. $3\overline{)30}^{10}$	12. $4\overline{)48}^{12}$	15. $3\overline{)69¢}^{23¢}$	18. $4\overline{)88¢}^{22¢}$

** Previously, pupils have checked their division by subtraction. Now, a better check is to make use of the inverse idea—multiplication by a number undoes division by that number.

Teaching Page 336

Pupil's Objectives

(a) To have practice in dividing tens and ones; and (b) to learn to check division answers by multiplication.

Background

In general, your pupils have learned to check their division by subtracting. Often, they have used a number line to illustrate the subtracting.

The factors-product relationship has shown that multiplication and division by a number are inverses—one undoes the other. Your pupils have learned that if they know a product and one of its factors, they can find the unknown factor by dividing. Now, they can check the division answer by multiplying it by the known factor to undo the division. If the product found by this multiplication is the same as the number divided, then the pupil knows that the division answer is correct.

A word of caution may be appropriate. Checking for checking sake is not particularly motivating or exciting to most children. Studies have shown that unless otherwise motivated, many pupils *force* the check. Consider the following example.

a.
$$\begin{array}{r} 42 \\ 2\overline{)86} \\ \underline{80} \\ 6 \\ \underline{6} \\ 0 \end{array}$$

b.
$$\begin{array}{r} 42 \\ \times 2 \\ \hline 86 \end{array}$$

As shown in *a*, the pupil made a mistake in dividing. He is required to check the answer which he does in *b*. It is obvious from inspecting *b* that the child did not actually multiply, but merely showed the product he knew he was supposed to.

If pupils are multiplying to check their division answers, the activity may be worthwhile. However, if they are merely *forcing* answers in doing this form of check, then some other activity would be more appropriate.

Pre-Book Lesson

• After the children have completed the practice exercises at the top of the page, engage in an activity similar to the following:

a. Write examples on the chalkboard like these:

$$\begin{array}{ccc} n & 3 & 4 \\ 4\overline{)20} & 5\overline{)n} & n\overline{)28} \end{array}$$

b. Have pupils tell whether n in each example stands for a factor or a product. Be sure they can explain why.

c. Ask different pupils to tell the number for n in each of the above examples, and then tell how they determined the numbers. In the middle example in particular, be sure to emphasize that multiplication may be used to find the product represented by n.

• On the board, write an example like $3\overline{)93}$. Ask a pupil to find the unknown factor. Have another pupil tell the product and the two factors. Then have still another multiply the two factors to see if he gets the product.

Guide pupils to state in their own words that if the product is the same as the one we divided, then the factor found must be the correct one.

Try one or two other examples of this type to confirm the generalization.

Using the Text Page

• You should be able to cover the oral work very quickly as a follow-up of the Pre-Book Lesson.

• The written work provides two types of practice. First, the children must divide and then they must multiply as a check. Be certain the pupils actually multiply when they are checking their division answer, not simply *force* the answer.

Individualizing Instruction

• *All pupils* may check some of the answers for the practice exercise at the top of the page by multiplying the two factors.

• *All pupils* may be shown that division may be used to check the result of multiplication examples. This would be one more way of stressing the inverse relationship. Multiplication can *undo* division and division can *undo* multiplication.

• Use Extra Examples Set 80 as needed.

Pupil's Objectives

(a) To have mixed practice in all computational skills which have been taught; (b) to find the number for *n* in mathematical sentences; and (c) to have practice in using equality and inequality symbols to write true sentences.

Background

The practice exercises on this page cover almost every skill and idea that has been taught in this book. The material at the top of the page is mixed practice in all the operations. At the middle of the page are some exercises which offer practice in computation in the setting of mathematical sentences. As an added challenge, these sentences are solved by using operations not directly indicated by the given symbols. Choosing the correct operation requires knowledge and application of the addends-sum relationship and the factors-product relationship. At the bottom of the page is more practice with all four operations but placed in a setting in which the child must use his knowledge of inequalities. The emphasis in these exercises is to write *true* sentences after the computation has been completed.

Pre-Book Lesson

• Review quickly with pupils the addends-sum relationship. Have pupils state in their own words and write examples of the ideas:

a. If we know two addends we find the sum by adding.

b. If we know one addend and the sum, we find the other addend by subtracting.

• Review quickly the factors-product relationship. Have pupils state in their own words and write examples of the ideas:

a. If we know two factors, we find the product by multiplying.

b. If we know one factor and the product, we find the other factor by dividing.

• Ask a child to write on the chalkboard the number sentence for *Seven is greater than four*. He should have written $7 > 4$. If he did not, ask another child to write the same sentence. Through this type of activity review with pupils the meanings of the symbols $>$ and $<$.

Then, write (7×4) ___ $(35 - 8)$. Ask a child to write the symbol that will make a true sentence. Have the child explain what he is doing and why he used the symbol that he did. Be sure that pupils understand that they must do two computations before they can decide what symbol to use in completing the sentence.

Using the Text Page

Make certain that all pupils understand what they are to do in each set of written exercises, particularly the set at the foot of the page.

Individualizing Instruction

• *More capable children* may work mentally in Ex. 1–9 at the foot of the page. Have them write the sentences after finding the number for *n* mentally.

• *Slower learners* may not be able to complete all exercises. Assign only selected items for written work.

* Pupils are not given the meaning of > and < here. Check to see how well they can work without this help.

To Keep in Practice

A., S., M., D. [W]

To check, work each example a second time.

1. $36 \div 3$ 12
2. 6×103 618
3. $89 + 472$ 561
4. 4×243 972
5. $900 - 78$ 822
6. $506 + 394$ 900
7. $84 \div 4$ 21
8. 498×2 996
9. $311 - 285$ 26
10. $66 \div 6$ 11
11. 106×5 530
12. $48¢ \div 2$ 24¢
13. $760 - 358$ 402
14. $6 \times \$1.42$ $8.52
15. $345 + 454$ 799
16. $88 \div 8$ 11
17. 5×167 835
18. $96¢ \div 3$ 32¢
19. 98×4 392
20. $80 \div 4$ 20
21. $78 + 6 + 89 + 117$ 290
22. $255 + 138 + 390$ 783
23. $\$4.00 + \$0.68 + \$3.17$ $7.85
24. $\$0.69 + \$0.75 + \$5.62$ $7.06

Now turn each division number sentence around. For Ex. 1 write, "$36 \div 3 = 12$, so $12 = 36 \div 3$."

More Practice

n-examples; equalities and inequalities [W]

Work Ex. 1–9, to find the number for n. Then copy each example with the correct numeral.

1. $n + 205 = 311$ 106
2. $n - 293 = 146$ 439
3. $n \div 3 = 4$ 12
4. $173 = n - 58$ 231
5. $n = 800 - 67$ 733
6. $246 + n = 500$ 254
7. $153 - n = 47$ 106
8. $3 \times n = 66$ 22
9. $\frac{1}{9}$ of $27 = n$ 3

* Work each example. Then copy it using $=$ or $\neq$. Copy the example again using $>$ or $<$ or $=$.

10. (3×149) $\overset{\neq}{?}$ $(583 - 10)$ $\overset{<}{}$
11. $(88 \div 4)$ $\overset{=}{?}$ $(90 - 68)$
12. $(69 \div 3)$ $\overset{\neq}{?}$ $(19 + 13)$ $\overset{<}{}$
13. $(\frac{1}{5}$ of $35)$ $\overset{\neq}{?}$ $(\frac{1}{8}$ of $40)$ $\overset{>}{}$
14. (4×8) $\overset{=}{?}$ $(96 \div 3)$
15. (3×87) $\overset{\neq}{?}$ (6×43) $\overset{>}{}$
16. $(\frac{1}{4}$ of $36)$ $\overset{=}{?}$ $(\frac{1}{2}$ of $18)$
17. $(84 \div 2)$ $\overset{\neq}{?}$ $(93 \div 3)$ $\overset{>}{}$

* Dividing — 3-Place Numerals

Using the Distributive Property [O]

For our May Day party we want to put 128 people at 4 large tables with the same number at each table. How many people will be at each table? $128 \div 4 = n$

Study Ex. A. Are there enough hundreds to have as many as 1 hundred at each table? No If not, can 128 be renamed as 12 tens + 8, or as 120 + 8, as in Ex. B? Yes

Ex. B. Explain why 32 is the answer.
120 ÷ 4 = 30 8 ÷ 4 = 2 30 + 2 = 32

A	B
$128 = 100 + 20 + 8$ $4\overline{)100 + 20 + 8}$	$128 = 120 + 8$ $30 + 2$, or 32 $4\overline{)120 + 8}$

C

	Hundreds	Tens	Ones
		3	2
4)1	2	8	
1	2	0	
		8	
		8	
		0	

Ex. C shows another way to divide 128 by 4. How is the work really like that in Ex. B? 120 (12 tens) ÷ 4 = 30 (3 tens) 8 ÷ 4 = 2. Explain the work.
120 ÷ 4 = 30, 4 × 30 = 120, 128 − 120 = 8, 8 ÷ 4 = 2, 4 × 2 = 8, 8 − 8 = 0

Explain how the first digit was found for each answer of Ex. 1–10. Tell why it is written where it is. See below.

4?	8?	4?	4?	2?
1. $2\overline{)82}$	2. $3\overline{)249}$	3. $4\overline{)164}$	4. $7\overline{)287}$	5. $5\overline{)105}$
2?	4?	5?	9?	3?
6. $4\overline{)84}$	7. $6\overline{)246}$	8. $9\overline{)459}$	9. $4\overline{)368}$	10. $3\overline{)96}$

When you cannot divide hundreds, you think of the 3-place numeral as showing tens and ones. Ex. 1-10: Divided tens—Ex. 1,6,10. Renamed hundreds then divided tens—Ex. 2,3,4,5,7,8,9

* An extension in showing division—working with a product shown by a 3-place numeral, but with the unknown factor still shown by only a 2-place numeral!

Teaching Pages 338 and 339

Pupil's Objective

To learn to divide with a 3-place numeral and get a 2-place answer.

Background

This is an extension of the work on pages 334 and 335 where the distributive property was used in dividing with a 2-place numeral and getting a 2-place answer. Now, your pupils are introduced to dividing with a 3-place numeral and getting a 2-place answer. Later, they will encounter dividing with a 3-place numeral and getting a 3-place answer.

Providing adequate background in preparation for a new skill is an essential ingredient in the sequential development of a mathematics program. Basic to the division introduced on pages 338 and 339 is the idea of renaming hundreds, tens, and ones before dividing.

The renaming of numbers in many ways has been developed extensively throughout this program. Consider the number 128. Your pupils have learned that it may be named as 100 + 20 + 8. They have learned, also, that it may be renamed as 12 tens and 8 ones, 120 + 8. You are referred to the suggestions for Teaching Pages 166–172 for the many ways of renaming numbers.

The context in which the number is shown determines the renaming. If the hundreds cannot be divided to show a number of hundreds in the answer, the renaming should be in terms of tens and ones. In the example $4\overline{)128}$, the child should be able to see that a whole number of hundreds cannot be shown in the answer, so the number is renamed 12 tens (120) plus 8 ones. With the example thought of as $4\overline{)120 + 8}$, the method of dividing is very similar to what has been learned when dividing with a 2-place numeral.

The inverse relationship of multiplication and division by a number will receive continued emphasis as one is used as a check for the results of the other.

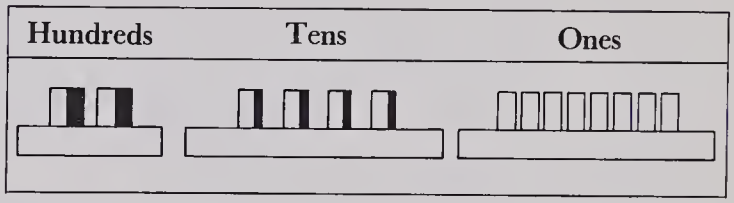
Pre-Book Lesson

• Engage in a review activity of renaming numbers shown with 3-place numerals. Write the numeral 248 on the chalkboard. Ask pupils to tell other ways of naming the number. You should encourage as many different names as possible:

a. 200 + 40 + 8
b. 200 + 48 (2 hundreds plus 48 ones)
c. 240 + 8 (24 tens plus 8 ones)
d. 230+ 18 (23 tens plus 18 ones)
e. 220 + 28 (22 tens plus 28 ones)

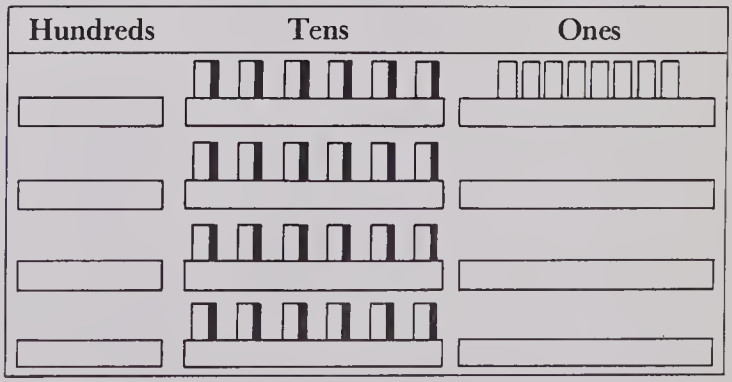
All of these examples are ways of renaming 248. Encourage creativity in thinking. Even though we may use only one of these ways of naming the number, the child should be praised for his independent thought.

• Point out that when we are dividing there will be times when we will use many of these ways of renaming. However, in this lesson, only two will be considered. First, work on the chalkboard or with a place-value teaching aid* to show 248 as 200 + 40 + 8.



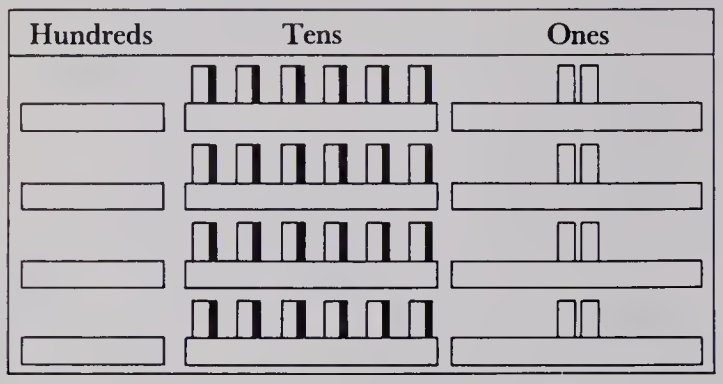
Ask a pupil to separate the 2 hundreds into 4 groups of the same size. Of course, since there are only 2 hundreds, it is not possible to separate them into 4 groups of hundreds. This should help pupils to *see* why it is necessary in the division example 248 ÷ 4, to rename 248 as 24 tens 8 ones. In so doing it becomes possible to proceed with division by 4.

Have pupils separate the 24 tens on the place-value teaching aid into 4 groups of ten.



* See 21, page xix.

Since each of the tens is placed in one of 4 groups, the pupils may proceed to separate the ones as shown below.



• It is now time to make the transition from work with the physical model to work with the division algorithm. Write the example $4\overline{)248}$ on the chalkboard and encourage the same thinking as was involved with the place-value demonstration.

Direct and guide the working of the example until children state in their own words something as follows: "I cannot divide the 2 hundreds by 4 and get a number of hundreds, so I rename 248 as 24 tens and 8 ones. 24 tens can be divided by 4 and give the result 6 tens. And, 8 ones can be divided by 4 and give the result 2 ones."

Ask children questions about the meaning of each of the digits in the answer. "What does the 6 tell us?" "What does the answer 62 mean?"

Using the Text Pages

• The oral work should easily follow from the Pre-Book Lesson. Refer to the place-value aid in order to insure that pupils understand the how and why of each step in finding the missing factor for these examples.

• Circulate among pupils as they are doing the written work to see that they are proceeding with the work correctly.

Individualizing Instruction

• You may find it necessary to modify the written assignment for *slower learners*. They often develop negative attitudes toward mathematics because of a lengthy assignment.

• Use Extra Examples Sets 81 and 82 as needed.

Reminder

Many ideas and skills have been developed in working with measurement, geometry, graphs, and charts. All pupils may profitably spend time doing additional work on one or more of these topics.

NOTES

Which of these answer digits are shown in the wrong places? Tell why each is wrong. 24 tens ÷ 4 = 6 tens

11. $6\overline{)306}$ (5) 12. $2\overline{)186}$ (9) 13. $4\overline{)248}$ (6 ∧) 14. $3\overline{)69}$ (2 ∨)

6 tens ÷ 3 = 2 tens

On the board, work Ex. 15–18. Work first as in Ex. B on page 338, then again as in Ex. D at the right.

15. $5\overline{)405}$ (81) 16. $3\overline{)279}$ (93) 17. $6\overline{)306}$ (51) 18. $7\overline{)147}$ (21)

When we divide numbers like \$2.13, as in Ex. E, we write the work just as when we divide other numbers. We put \$ before the answer and use two places for cents.

19. Explain the work in Ex. E. Why is 0 written at the left of the cent point in the answer? To show 0 dollars.

20. Ex. F shows the check for Ex. E. Is \$0.71 the correct answer for \$2.13 ÷ 3? Yes Explain the work. 3 × 1 one = 3 ones, 3 × 7 tens = 21 tens, or 2 hundreds and 1 ten. [W]

D

$$4\overline{)128}\quad 32,\ 120,\ 8,\ 8,\ 0$$

E

$$3\overline{)\$2.13}\quad \$0.71,\ 2\,10,\ 3,\ 3,\ 0$$

F

$$\$0.71 \times 3 = \$2.13$$

Copy each of Ex. 21–48. Write your work as in Ex. D or Ex. E. Check Ex. 21–34 by multiplying.

21. $4\overline{)248}$ (62) 22. $3\overline{)243}$ (81) 23. $8\overline{)248}$ (31) 24. $3\overline{)156}$ (52) 25. $6\overline{)246}$ (41) 26. $5\overline{)255}$ (51) 27. $7\overline{)357}$ (51)

28. $2\overline{)\$1.64}$ (\$0.82) 29. $3\overline{)\$1.83}$ (\$0.61) 30. $2\overline{)\$1.66}$ (\$0.83) 31. $3\overline{)\$1.26}$ (\$0.42) 32. $5\overline{)405}$ (81) 33. $3\overline{)276}$ (92) 34. $5\overline{)\$2.05}$ (\$0.41)

35. $4\overline{)\$0.48}$ (\$0.12) 36. $2\overline{)\$1.84}$ (\$0.92) 37. $7\overline{)287}$ (41) 38. $8\overline{)\$4.08}$ (\$0.51) 39. $7\overline{)217}$ (31) 40. $9\overline{)\$1.89}$ (\$0.21) 41. $6\overline{)306}$ (51)

42. $2\overline{)144}$ (72) 43. $9\overline{)369}$ (41) 44. $5\overline{)\$2.55}$ (\$0.51) 45. $4\overline{)\$1.24}$ (\$0.31) 46. $6\overline{)\$1.26}$ (\$0.21) 47. $9\overline{)\$4.59}$ (\$0.51) 48. $3\overline{)\$2.49}$ (\$0.83)

◆ **Extra Examples.** Work Sets 81 and 82.

Finishing and Solving Division Problems

[W]

For each of Ex. 1–8 write a question to make a problem worked by division. Then work the problem. Questions will vary.

1. Miss Wells gave Kay a box of 168 crayons and asked her to give 8 crayons to each child in the room. 21 children

2. On the last page of our book there are 48 examples. Miss Wells asked us to work $\frac{1}{2}$ of them. 24 examples

3. For our play we need 147 extra chairs. We will set them in rows of 7. 21 rows

4. A book we are reading in our class has 126 pages. Miss Wells said that we had finished reading $\frac{1}{3}$ of the book. 42 pages

5. Miss Wells said that 4 copies of the same book cost \$3.68 in all when they were new. \$0.92

6. In our whole school building there are 205 boys. It takes 5 boys to make a basketball team. 41 teams

7. Our room is 42 feet long. The board covers $\frac{1}{2}$ of the wall on that side. 21 feet

8. Miss Wells asked us to put 248 books on the shelves. Each of us can carry 8 books at a time. 31 trips

Teaching Page 340

Pupil's Objectives

(a) To have practice in writing questions to make division problems; and (b) to have practice in solving division problems.

Background

In the material on page 340, the children are given certain information from which they can write an appropriate division question. The mere act of finding an answer is not the whole of problem-solving. The child who can create problems shows an understanding of the problem situation. After understanding has taken place, he will be in a better position to find solutions to problems than the child who blindly searches for an answer. Thus, it is very important to place a great deal of emphasis on the writing of questions to make division problems.

Another activity you should use frequently is the writing of a mathematical sentence for a problem. This, too, encourages the child to understand the problem situation. If he is incapable of writing a mathematical sentence for relationships shown, then he will continue to have trouble in interpreting and solving problems.

Pre-Book Lesson

Review briefly division problems which involve partitive situations and those which involve measurement situations. Also, review problems which involve finding how many members are in part of a total set, such as $\frac{1}{3}$ of 12 books. This kind of problem may be solved by division.

Using the Text Page

- Make sure that all pupils understand that they are to write a question for each set of information to make a division problem. It would probably be a good idea to have pupils write all the problems and then go back to each one to find the solution.
- You may wish to have different children read the questions they have written. Have as many different questions as possible for each set of information. Impress upon the pupils that a question may be written in a variety of ways.

Individualizing Instruction

More capable children may refer to the chart on page 331 and show the M. and D. facts for the blank spaces in the lower right portion of the chart.

NOTES

Teaching Page 341

Pupil's Objective

To begin to learn some skills in estimating answers to multiplication and division examples.

Background

The ability to estimate includes the ability to determine whether or not an answer is reasonable. Many necessary relationships between numbers involved in an operation serve as a basis for accepting or rejecting a number found through computation. Of course, not all people give consideration to the reasonableness of answers derived from their computations. Learning mathematical skills without the concomitant attitudes and understandings which make one competent to determine the reasonableness of an answer leaves much to be desired.

The purpose of the material on page 341 is to extend understanding of various ways of estimating answers and to foster an attitude of "checking" one's work. By intuition, many pupils will have developed the notion that usually a product of two numbers is greater than either factor. It is this idea that is clarified in the formal treatment given to it in this lesson.

Notice that the generalizations on page 341 are stated in terms of working with counting numbers (frequently called natural numbers), since the generalizations would not be true for the number 0. This will become evident when you remember what was said about multiplying or dividing 0.

Pre-Book Lesson

• Show a M. fact on the chalkboard such as $6 \times 4 = 24$. Ask pupils if they can tell something about the relationship between the product and the number multiplied. It may be necessary to phrase the question in different ways, perhaps giving an indication that it is size that you wish to compare. This will be an opportunity to use the symbols for *is greater than* and *is less than*.

On the chalkboard, continue to show M. facts where the factor you multiply by is a counting number and then note relationships as shown below.

$6 \times 4 = 24$ Product $>$ the other factor
$2 \times 9 = 18$ Product $>$ the other factor
$8 \times 5 = 40$ Product $>$ the other factor
$1 \times 9 = 9$ Product $=$ the other factor

Analyze the facts and generalizations on the board and through guided discussion arrive at these two generalizations about multiplying by a counting number:

a. The product is never less than the other factor.

b. The product is always equal to or greater than the other factor.

c. When one factor is 1, the product is equal to the other factor.

• Write an example like $63 \times 15 = n$ on the chalkboard. Then show 58 and 945 on the board. Ask, "Which of these two numbers could not possibly be the answer to the multiplication sentence?" By inspection, pupils should be able to tell that the number 58 would not make sense. Have pupils analyze and discuss other examples of this type. It is not necessary that the children be able to find the answer, but they should be able to reject any answer that doesn't make sense.

• Conduct the same type of activity for division examples when dividing by a counting number. Make sure pupils arrive at the following generalizations:

a. A missing factor will never be greater than the product.

b. A missing factor will be equal to the product if the known factor is 1.

c. A missing factor will be less than the product if the known factor is a counting number greater than 1.

Using the Text Page

• The oral work on this page should follow easily from the Pre-Book Lesson. Have pupils state the generalizations in their own words as often as possible.

• In writing the answers to Ex. 9–16, pupils should show the number given that cannot possibly be correct.

Individualizing Instruction

Have pupils work each example in Ex. 9–16 to see if the correct answer is given.

*Does Your Answer Look Right?

M. and D. [O]

For boxes A and B, is each factor you are multiplying by or dividing by a counting number? Yes For box A is each product equal to or greater than the factor multiplied? Yes

If the factor you multiply by is a counting number, the product is equal to or greater than the other factor.

For box B is each answer equal to or less than the product you are dividing? Yes

If the factor you divide by is a counting number, the other factor is equal to or less than the product.

A		
8	6	7
$\times\ 4$	$\times\ 5$	$\times\ 1$
32	30	7
B		
$8\overline{)32}$	$6\overline{)18}$	$9\overline{)9}$

Ex. 1–8. Tell about the size of each answer. The answers agree with the generalizations about size.

1. 3×8
2. $6 \div 2$
3. $18 \div 9$
4. 8×1
5. 1×9
6. $7 \div 1$
7. 6×5
8. 1×1

Ex. 9–16. Just by looking, tell which answer shown after the problem or example must be wrong. Tell why.

9. May took 27 minutes to read a story. Joe took $\frac{1}{3}$ as long. How many minutes did Joe take? **9** **81**

81 is not equal to or less than the product.

10. May's necklace has 84 beads. To make 4 such necklaces how many beads would she need? **336** **21**

21 is not equal to or greater than the other factor.

11. 3×256 **86** **768**

86 is less than 256.

12. $106 \div 2$ **53** **318**

318 is greater than 106.

13. 8×104 **832** **13**

13 is less than 104.

14. $68 \div 2$ **34** **160**

160 is greater than 68.

15. 398×2 **212** **796**

212 is less than 398.

16. $408 \div 8$ **970** **51**

970 is greater than 408.

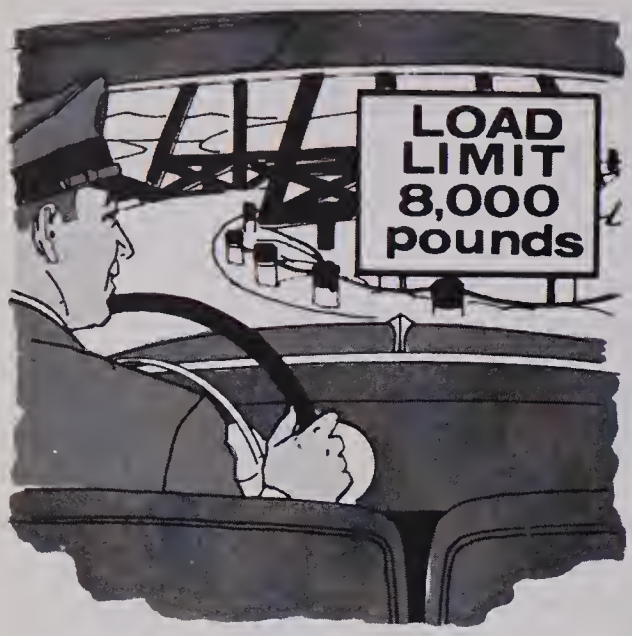
[W]

Now work Ex. 9–16.

* We often need to check an answer more quickly than by working another example. Here pupils learn how, by just looking at a product or factor shown, they can sometimes tell if an answer is incorrect.

Reading and Writing 4-Place Numerals

[o]



1. Why must the truck driver know how to read the numeral on the sign?

2. Can you read the numeral? 8 thousand Try.

3. We say that 8,000 is a 4-place numeral. Why? It has four digits.

4. What name do you think is given the place where you see 8 in 8,000? thousand

5. Count by 10's from 10 to 90.

6. Count by 100's from 100 to 900.

7. Count by 1,000's from 1,000 to 9,000.

Study Ex. 8. Then tell the missing words in Ex. 9–11.

8. 5,168 is read, "5 thousand 1 hundred 68."

9. 4,200 is read, "4 thousand 2 hundred."

10. 7,270 is read, "7 thousand 2 hundred 70."

11. 3,016 is read, "3 thousand 16."

12. Read these numerals:

a. 2,651	b. 1,005	c. 4,789	d. 8,018	e. 9,623
f. 6,205	g. 7,890	h. 1,080	i. 3,456	j. 5,700

On the board write the numeral for

13. two thousand one hundred twenty-nine. 2,129

14. five thousand five hundred. 5,500

15. seven thousand two. 7,002

16. four thousand two hundred eighty. 4,280

Teaching Pages 342 and 343

Pupil's Objectives

(a) To learn how to read and write 4-place numerals; and (b) to have mixed computational practice using 3-place numerals.

New Word

(page 342) *thousand*

Background

A common error in the reading of numerals with 3 or more digits is the use of the word "and." Because of the additive nature of our number system, it is not inaccurate to think of 462 as 4 hundreds and 6 tens and 2 ones. However, when we read the numeral, it is commonly accepted that it be read, "Four hundred sixty-two" and not, "Four hundred and sixty-two." The word "and" is used when reading a numeral containing a decimal fraction. For instance, the numeral 47.5 is read, "Forty-seven and five tenths." Third-grade children will have experience reading a numeral such as $5.75. It is read, "Five dollars and 75 cents."

In reading numerals, it is important to concentrate upon the names of the places of digits, as determined by the base of the system in which we are working. However, if we can begin to build accepted practices at the same time, it is worthy of our consideration.

Many children will already know how to read and write numerals of more than 3 places. This lesson may be considered as a review for them or you may wish to assign them one of the supplementary activities at the end of the chapter.

Pre-Book Lesson

• On the chalkboard, write some sentences which contain 4-place numerals.

a. There are 8,452 school children in our town.

b. Our school uses about 3,920 gallons of water each week.

Ask pupils to read the statements. If they are read correctly, call attention to the fact that each numeral contains four digits and that the fourth digit to the left is in thousand's place.

• Draw a place-value chart on the chalkboard (or work on a flannel board*) and have pupils fill in the words "Ones," "Tens," and so on, as shown below. Each pupil may make a chart of his own at his desk.

Thousands	Hundreds	Tens	Ones
2	3	6	0

List in the chart numerals like the following for pupils to read:

2,360	9,370	2,040
4,956	8,204	6,056

• If pupils do not ask about the comma in 4-place numerals, call it to their attention. Ask why they think it is used. Accept such answers as, "To make it easier to read" or "To break up the numeral for easier reading."

• Use the place-value chart to show 5,000. Then, show 400. Be sure to have pupils tell what each of the 0's means in 5,000 and in 400. Through this type of seeing and discussion, guide pupils to see that 5,472 may be renamed 5,000 + 400 + 70 + 2. Have pupils show several numbers in this expanded form. Emphasize the additive nature of the number system.

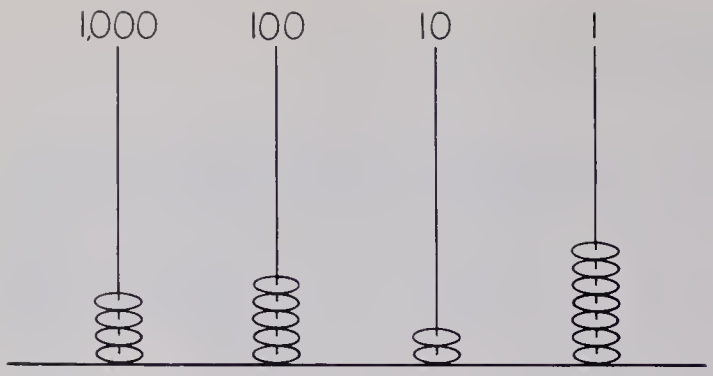
Using the Text Pages

• Use the place-value chart in connection with any of the oral material as may seem necessary.

• Some pupils may find the variety of other place-value teaching aids** more helpful than the one that has been used in this lesson. An abacus picture or some similar device may be helpful in connection with the oral exercises in the book. For instance, pupils can quickly draw pictures to demonstrate 4,527 (see Teacher's Page 343).

* See 2, page xix.

** See list on page xix.

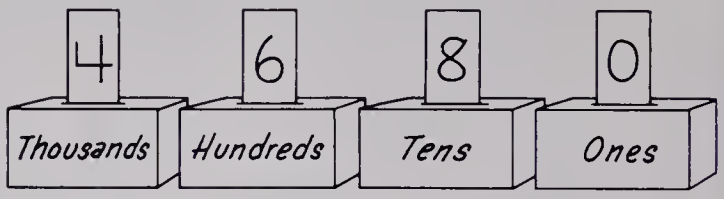


• Most pupils should be able to proceed independently with the mixed practice at the bottom of page 343.

Individualizing Instruction

• *All pupils* may participate in collecting and making a class list of 4-place numerals. These may be used for oral practice in reading numerals having four digits.

• *Slower learners* may need to use a more permanent place-value device than merely a chart drawn by hand to help in reading 4-place numerals. As shown below, insert a card on which is written a single digit into a box or can for each place value.



Let a pupil leader take charge as soon as the children are proceeding accurately. Eliminate this device as soon as possible and give further practice in the reading of 4-place numerals.

• *All pupils* may use either $>$ or $<$ to make each of the following true sentences.

8,000 ○ 7,000
3,550 ○ 3,505
7,001 ○ 7,002
5,483 ○ 5,485

NOTES

[W]

Study chart A. Then write the numerals that belong in each column

17. for row d. **18.** for row e.

A

	Thousands	Hundreds	Tens	Ones
a.	1,000	100	10	1
b.	2,000	200	20	2
c.	3,000	300	30	3
d.	4,000	? (400)	? (40)	? (4)
e.	5,000	? (500)	? (50)	? (5)

Copy and finish Ex. 19–21.

19. 1,000, 1,001, 1,002, ? (1,003)

20. 8,448, 8,44?, 8,450, ? (8,451)

21. 3,999, ?,000, ?,001, ? (4,002)

Show the numbers you would use in counting by 1's

22. from 4,577 to 4,589. **23.** from 7,996 to 8,008.

Other names for numbers

Study the renaming shown in chart B. Then copy and finish Ex. 24.

B

10 ones = 1 ten, or 10
10 tens = 1 hundred, or 100
10 hundreds = 1 thousand, or 1,000

24. 30 ones = ? (3) tens, or ? (30);
30 tens = ? (3) hundreds, or ? (300);
30 hundreds = ? (3) thousands, or ? (3,000)

Write the numeral for

25. 7 thousand eighteen. (7,018) **26.** 2 thousand one hundred. (2,100)

To Keep in Practice

A., S., M., D. [W]

Copy, work, and check.

1. $328 \div 8$ (41)
2. 149×5 (745)
3. $606 - 248$ (358)
4. $455 \div 5$ (91)
5. $597 - 406$ (191)
6. $75 + 869$ (944)
7. 6×154 (924)
8. $\frac{1}{4}$ of 368 (92)
9. $211 - 89$ (122)
10. $219 \div 3$ (73)
11. \$2.46 ÷ 6 (\$0.41)
12. \$4.38 − \$0.09 (\$4.29)
13. $8 \times \$0.45$ (\$3.60)
14. $\frac{1}{7}$ of \$3.57 (\$0.51)
15. \$0.55 + \$4.66 (\$5.21)
16. $\frac{1}{3}$ of \$2.79 (\$0.93)

Do You Make Mistakes?

Diagnostic Test 8

Write answers on folded paper. Work carefully.

	a	b	c	d	Study Pages	Practice Sets
1.	7×5^{35}	5×8^{40}	9×5^{45}	6×5^{30}	318–320	
2.	$5\overline{)40}^{8}$	$4\overline{)32}^{8}$	$45 \div 9^{5}$	$36 \div 9^{4}$	308, 326	79
3.	$\frac{1}{6}$ of 24^{4}	$\frac{1}{3}$ of 27^{9}	$\frac{1}{5}$ of 15^{3}	$\frac{1}{8}$ of 32^{4}	310–312	79
4.	$\begin{array}{r} 164 \\ \times 5 \\ \hline 820 \end{array}$	$\begin{array}{r} 172 \\ \times 4 \\ \hline 688 \end{array}$	$\begin{array}{r} 140 \\ \times 6 \\ \hline 840 \end{array}$	$\begin{array}{r} 123 \\ \times 8 \\ \hline 984 \end{array}$	302	78
5.	$8\overline{)80}^{10}$	$3\overline{)63}^{21}$	$4\overline{)284}^{71}$	$7\overline{)357}^{51}$	332–339	80–82

Do You Understand?

Test of Information and Meaning 8

Write an *n*-example for each of these on page 346:

1. Problem 1 $9 \times 34 = n$
2. Problem 4 $148 \div 2 = n$
3. Problem 8 $248 \div 4 = n$

4. Write the numeral for three thousand twenty-one. 3,021

Ex. 5–6. Copy an answer that cannot be correct.

5. 7×42 **6** **294** 6
6. $164 \div 4$ **41** 656 **656**

7. Write the answer for $(32 \div 8) + 12$. 16

8. $\frac{1}{4}$ of a pound is the same as $\underset{4}{?}$ ounces.

For each of Ex. 9–11, a whole is in parts of the same size. What is the fraction number for one of

9. 6 parts? $\frac{1}{6}$
10. 9 parts? $\frac{1}{9}$
11. 5 parts? $\frac{1}{5}$

Teaching Pages 344, 345, and 346

Pupil's Objective

To take the regular end-of-chapter tests for Chapter 8.

New Words

(page 346) *berry, flats, pine*

Background

You have probably established a regular routine for administering, scoring, discussing, and recording the end-of-chapter tests. These tests, like all previous tests, measure those ideas, skills, and concepts which have been developed or extended in Chapter 8. As has been the suggested practice, be sure to emphasize the *why* of an idea or an operation in discussing the various items on each test.

Using the Text Pages

• Diagnostic Test 8. You may wish to include the following items as a part of a diagnostic test in order to cover most of the major computational skills covered in the entire book.

a.	274	37	\$4.75	48¢
	83	518	0.29	7¢
	329	46	+2.65	11¢
	+58	+294		9¢
				3¢
b.	738	802	641	\$6.00
	−410	−683	−528	−3.49

• Use the tables below to find the per-cent scores for the individual record cards (see Teaching Pages 50 and 51).

Table of Per Cents for Chapter 8 Scores

Problem Test		Computation Test					
Score	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*	*Score*	*Per Cent*
1	13	1	4	9	36	17	68
2	25	2	8	10	40	18	72
3	38	3	12	11	44	19	76
4	50	4	16	12	48	20	80
5	63	5	20	13	52	21	84
6	75	6	24	14	56	22	88
7	88	7	28	15	60	23	92
8	100	8	32	16	64	24	96
						25	100

Suggestions for Material to Accompany End-of-Chapter 8 Tests

Pupils who encounter little difficulty in the tests just administered may be assigned any of the following activities to supplement and extend their mathematical skills and understandings:

Alternate Uses of Pages in Chapter 8

Page 309. Show the M. facts only for the division examples at the top of the page.

Page 319. Show a D. fact to go with each multiplication example at the bottom of the page.

Page 324. More capable children should extend their understanding of the factors-product relationship by engaging in the following activity.

Use particular examples in row 28 at the bottom of the page to help find answers for 48 ÷ 8; 90 ÷ 9; 30 ÷ 5.

Use particular examples in row 29 to help find answers for 80 ÷ 5; 48 ÷ 3; 54 ÷ 9; 50 ÷ 5.

Use examples in row 30 to help find answers for 48 ÷ 6; 54 ÷ 3; 56 ÷ 4.

Page 331. a. Show the factors only for the blank spaces in the lower right portion of the chart.

b. After the factors have been shown, see if you can find their product.

Page 337 (bottom). Change one number shown in each of Ex. 10–17 to use = and make a true sentence.

Page 342. Show each of the numbers in Ex. 12 in expanded form.

Supplementary Activities

• Make rows and columns of dots to show each example. Then copy the example and write the answer.

a. $(4 \times 5) + (3 \times 5) = n$
b. $(3 \times 6) + (3 \times 9) = n$
c. $(4 \times 8) + (4 \times 3) = n$
d. $(2 \times 6) + (2 \times 7) = n$
e. $(5 \times 3) + (5 \times 5) = n$
f. $(6 \times 4) + (6 \times 5) = n$

• *All pupils* have written tables of M. facts for 2, 3, 4, and 5. At your desks, complete the tables started below.

$4 \times 10 = 40$	$5 \times 10 = 50$
$4 \times 11 = ?$	$5 \times 11 = ?$
through	through
$4 \times 18 = 72$	$5 \times 18 = 90$

• Show the M. to go with each M. in the two tables above.

• Show the divisions to go with the multiplications shown in the tables.

• *More capable children* should be able to complete each of the following multiplication charts. After completing all three charts, the children should use them to review some of the generalizations about multiplying odd and even factors.

Factors Even

	0	2	4	6	8
0					
2					
4					
6					
8					

Factors Odd

	1	3	5	7	9
1					
3					
5					
7					
9					

One Factor Odd, One Factor Even

	0	2	4	6	8
1					
3					
5					
7					
9					

• Use the symbols > and < to make true sentences.

a. If $n = 9$,
then $(4 + n)$ ___ $(24 - n)$.
b. If $n = 14$,
then $(n - 5)$ ___ $(20 - n)$.
c. If $n = 47$,
then $(153 - 86)$ ___ $(n + 26)$.
d. If $n = 4$,
then $(n \times 6)$ ___ $(8 \times n) - (4 + 2)$.
e. If $n = 84$,
then $(384 - 214)$ ___ $(263 - n)$.

Coordinate Geometry

• Have pupils locate points on the coordinate system to

a. make different shapes or drawings.
b. make letters of the alphabet.
c. play the *Find the Treasure* game.

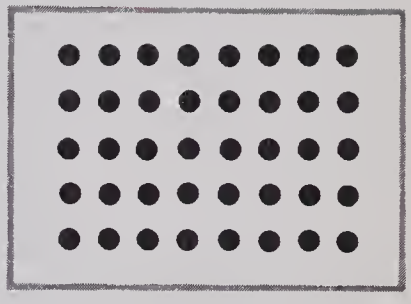
• The number line has been used for the most part throughout this program to illustrate the set of counting numbers or the set of whole numbers. However, points on a number line may be used to describe time, distance, speed, temperature, weight, and so on. Each of these concepts is directly related to another concept. This relation idea was mentioned earlier when pupils were pairing related

For each of Ex. 12–14, write the D. example two ways.

12. $\frac{1}{6}$ of 30 5 **13.** $\frac{1}{7}$ of 21 3 **14.** $\frac{1}{4}$ of 36 9

Write the work to check each of these divisions:

15. $284 \div 4 = 62$ $\begin{array}{r} 62 \\ \times 4 \\ \hline 248 \end{array}$ **16.** $168 \div 2 = 84$ $\begin{array}{r} 84 \\ \times 2 \\ \hline 168 \end{array}$



17. The picture shows 40 dots in rows of 8.

a. What product and factors can you find? product, 40 factors, 5 and 8

b. Show the set of M. and D. facts. ∨

$5 \times 8 = 40$ $8 \times 5 = 40$ $40 \div 5 = 8$ $40 \div 8 = 5$

Show the number for n in each of Ex. 18–20.

18. $n \div 7 = 4$ 28 **19.** $32 \div 8 = n$ 4 **20.** $5 \times n = 45$ 9

Copy Ex. 21–22, writing > or < or = for _?_. Copy each example again, writing = or ≠ for _?_.

21. $(35 - 7)$ _>_ $(28 - 8)$ **22.** (4×6) _>_ $(33 - 27)$

How Well Can You Compute?

Computation Test 8

Copy and work. Check each example.

1.	$168 \div 8$ 21	**8.**	$288 \div 4$ 72	**15.**	$6 \times \$1.45$ $8.70
2.	$457 + 209$ 666	**9.**	8×105 840	**16.**	$\frac{1}{7}$ of $3.57 $0.51
3.	5×89 445	**10.**	$88 + 99$ 187	**17.**	$\$6.81 - \$4.09$ $2.72
4.	$693 \div 3$ 231	**11.**	$\frac{1}{4}$ of 364 91	**18.**	$\$4.59 \div 9$ $0.51
5.	$916 - 309$ 607	**12.**	$812 - 764$ 48	**19.**	$\$1.45 + \$7.59$ $9.04
6.	3×230 690	**13.**	$307 + 568$ 875	**20.**	$\$7.78 - \$0.90$ $6.88
7.	$\frac{1}{6}$ of 246 41	**14.**	4×178 712	**21.**	$2 \times \$4.08$ $8.16

22. $30¢ + 6¢ + 8¢ + 9¢$ 53¢ **24.** $\$2.08 + \$5.30 + \$1.67$ $9.05

23. $3¢ + 9¢ + 8¢ + 7¢$ 27¢ **25.** $\$0.17 + \$3.88 + \$2.86$ $6.91

Can You Solve Problems?

Problem Test 8

1. Mr. Samson has trees, bushes, and plants to sell. He has 9 rows of pine trees, with 34 trees in each row. How many pine trees in all are there? 306 trees

2. He sells plants in boxes called flats. Each flat holds 12 plants. In 4 flats of tomato plants, how many tomato plants should there be? 48 plants

3. The town bought these numbers of trees of different kinds: 80, 26, 18, and 148. How many trees in all did the town buy from Mr. Samson? 272 trees

4. If $\frac{1}{2}$ of the 148 evergreen trees were pine trees, how many pine trees did the town buy? 74 trees

5. At a sale there were 600 cherry trees. All but 129 were sold. How many cherry trees were sold? 471 trees

6. One third of 129 cherry trees left were sweet cherry. How many sweet cherry trees were left? 43 trees

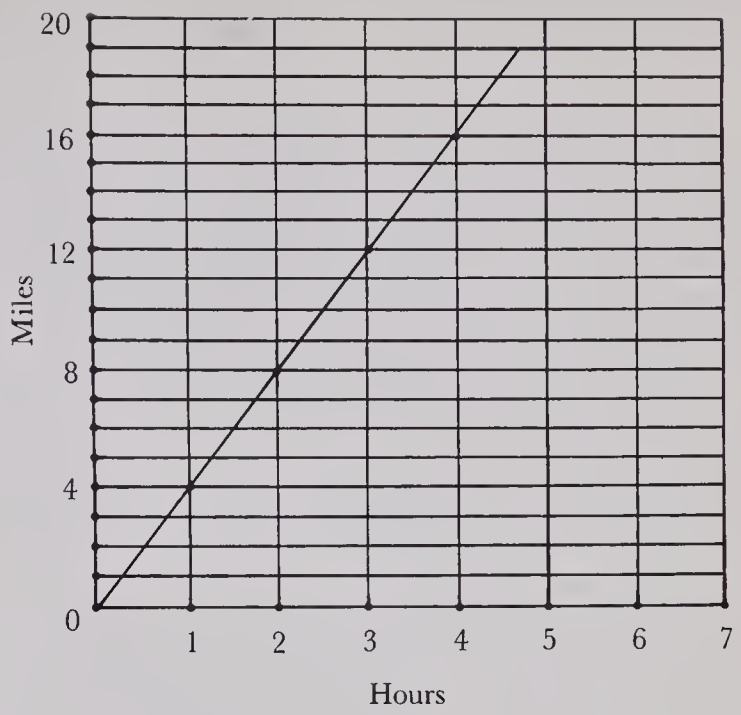
7. At the sale, 328 of 410 berry bushes were sold. How many berry bushes were not sold? 82 bushes

8. There are 248 pine trees to be set out. If they are planted in 4 rows, how many trees will be in each row? 62 trees

information in a chart in connection with problem-solving (see Teaching Page 81).

More capable children can now begin to pair information and show relationships by drawing two number-line pictures at right angles. You, as the teacher, will realize that a line graph is being suggested, but this need not be pointed out to pupils at this time.

The relationship between distance and time may be illustrated on number-line pictures. Discuss with pupils the idea that if you know that a man walks a distance of 4 miles in 1 hour, you can find how many miles he will travel in 3 hours; 5 hours; and so on. Then, have pupils locate points for each pair of data as shown on the number-line pictures at the right.



NOTES

Using the Reservoir for the Text

From time to time, through the pages of this Teachers' Edition, you have been referred to sets of Extra Problems, Extra Examples, and Extra Activities which appear at the back of the pupil's text. Reproductions of those pages with answers shown (pages 347–374) follow this Teacher's Page.

While each set of exercises is associated with a particular text page, you will recognize that you will be able to use a set of exercises at any time after the pupils have reached the text page for which that set was planned.

Immediately after text page 374, you will find a reproduction of pupil's page 375 on which appears a summary of symbols and terms used in this grade-3 program. This type of page was planned for use in the *Mathematics We Need* series because the authors felt that the ready-reference items with pictorial illustrations make a much more practical "dictionary" than the often used verbal glossary which is sometimes hard to interpret.

The index of the book follows on page 376–380. The authors work step by step as they build the many programs in the text pages but, in order to be most effective, you, the teacher, will probably want to have in mind the complete picture of the development of a topic. For instance, when *inequalities* is first introduced, it might be helpful for you to see what other pages in the text contribute to the development of that topic. By glancing at those pages, you should have an especially clear picture of the direction in which the learning is to proceed. That is, this index can become an aid in teaching, over and above the usual service of an index.

Set 1

Ice Fishing

For use after page 20

1. Dave and his father went ice fishing. Dave caught 4 little fish and 2 big fish. How many fish did he catch? 6 fish

2. His father caught 3 little fish and 5 big fish. How many fish did Dave's father catch? 8 fish

3. Dave's 2 big fish and his father's 5 big fish made how many big fish in all? 7 fish

4. Dave caught 4 little fish and his father caught 3 little fish. How many little fish did they catch together? 7 fish

5. Dave's father gave away 2 of his 8 fish. How many fish did he have left after that? 6 fish

6. For dinner the family ate some of the big fish Dave and his father caught. There were still 3 of the 7 big fish left. How many of the big fish did the family eat? 4 fish

Set 2

Setting the Dinner Table

For use after page 34

1. Five big people and 3 children are to be at the table. How many places must be set? 8 places

2. Mother washed 9 glasses. Molly must still dry 3 of them. How many glasses have been dried? 6 glasses

3. Molly has put 4 cups on the table. When she has put on 4 more cups, how many cups will there be? 8 cups

[continued on page 348]

4. Mother has 9 roses. She has put 6 in a bowl. How many roses are left for another bowl? 3 roses

5. Six of the 8 chairs are put in place at the table. How many more chairs must be set in place? 2 chairs

6. There were 9 pears in the kitchen. How many were there after Molly took away 8 of them? 1 pear

Set 3

A Day at Uncle Ned's Farm

For use after page 71

1. Jane counted 5 brown and 13 black horses in a field at Uncle Ned's farm. How many horses were in the field? 18 horses

2. Two of the 13 black horses went into the barn. How many black horses did not go into the barn? 11 horses

3. Three hens were drinking water and 7 were eating. How many hens were in the two groups together? 10 hens

4. There were 19 hens in the hen house. Six ran out. Then how many hens were in the hen house? 13 hens

5. Twenty-nine sheep were in a field. Four of them ran to Jane. How many sheep did not run to Jane? 25 sheep

Set 4

Rusty Helps His Mother and Father

For use after page 87

1. Rusty helped his mother 6 times and his father 8 times. In all, how many times did he help them? 14 times

2. Rusty helped his father plant potatoes. He put 8 of his 18 pieces of potato in one row. He then had how many pieces left to plant? 10 pieces

3. If Rusty had 12 tickets before he stopped at Mrs. Brown's, and 9 afterward, how many did he sell? 3 tickets

4. Rusty had 19 boxes to take to the basement. After the first trip there were 14 boxes to take to the basement. How many boxes did he take on the first trip? 5 boxes

5. Rusty cut 13 red roses, 2 yellow roses, and 4 pink roses for his mother. That was how many roses in all? 19 roses

6. For selling tickets, Rusty earned 35¢. He spent 5¢ for a pencil. How much money did Rusty have left? 30¢

7. Rusty gave 10¢ for ice cream and 7¢ for candy. How much more did he pay for ice cream than for candy? 3¢

Set 5

The School Circus

For use after page 95

1. Sam's group sold 34 tickets for the circus one day and 5 tickets the next, or how many in all? 39 tickets

2. In three days Nan sold 4, 5, and 9 tickets. All together she sold how many tickets? 18 tickets

3. In the first hour of the circus, Ida's group sold 28 bags of peanuts and Joe's group sold 15 bags. How many more bags did Ida's group sell than Joe's? 13 bags

4. There were 16 people at the doll stand. Six of them left. How many stayed a while longer? 10 people

5. Tom sold 17 bottles of orange pop and 12 bottles of other kinds. How many bottles of pop did Tom sell? 29 bottles

6. A clown sold all but 4 of his 19 balloons. How many balloons did he sell? 15 balloons

Set 6

Making Bird Houses

For use after page 135

1. Ricky counted 22 pieces of wood in a pile. After his father bought 14 of them, how many pieces were left? 8 pieces

2. Ricky gave a man 25¢ for some nails. The man gave Ricky 4¢ in change. How much did the nails cost? 21¢

3. Ricky and his father used 14 pieces of wood for one bird house and 9 pieces for another bird house. For the two bird houses they used how many pieces of wood? 23 pieces

4. For a third bird house they used 5 short boards, 2 long ones, and 6 little ones. How many boards was that? 13 boards

5. Ricky's father had 17 nails in his hand. He used all but 7 of them. How many nails did he use? 10 nails

6. One board was 37 inches long. Ricky's father cut an 8-inch piece off the board. Then how many inches long was the board? 29 inches

Set 7

Room Three

For use after page 162

1. In Room Three there are 15 boys and 16 girls. How many children are there in Room Three? 31 children

2. There are 6 children at the first table, 5 at the second, 7 at the third, and 9 at the fourth. How many children are at the tables? 27 children

3. Sue handed out 41 crayons and Molly handed out 35. How many more crayons did Sue hand out than Molly? 6 crayons

4. Room Three brought in 34 pounds of old papers one day, 28 pounds the next day, and 37 pounds the next. How many pounds of old papers did they bring in? 99 pounds

5. On Monday there were 40 pieces of red paper on the shelf. On Tuesday there were 24 pieces. How many pieces of red paper had been used? 16 pieces

6. Room Three had a party. The cookies cost 84¢. The drink cost 29¢ less. How much did the drink cost? 55¢

Set 8

Spring Clean-Up

For use after page 173

1. Mrs. Tucker told Mr. Tucker it took her 28 hours the first week and 24 hours the second week to clean the house. How many hours all together was that? 52 hours

2. In the second week Mrs. Tucker worked 15 of the 24 hours on Monday, Tuesday, and Wednesday. How many hours did she work on the other days of the week? 9 hours

3. Mike and June Tucker together washed 17 windows. June washed 5 windows. How many did Mike wash? 12 windows

4. When Mike worked in the garden, he counted 27 tomato plants and 18 cabbage plants. How many more tomato plants than cabbage plants were there? 9 tomato plants

5. Mike picked 15 roses. There were still 27 roses left. How many roses were there at first? 42 roses

6. June's mother washed 12 stair steps. June washed 9 stair steps. How many stair steps did June and her mother wash? 21 stair steps

Set 9

The Long Trip

For use after page 182

1. Thirty-one people went on a trip to a park. Four of them were teachers and the others were children. How many children were there? 27 children

2. First, they rode a train 29 miles from the school. Then they rode a bus 6 miles to the park. How many miles was it from the school to the park? 35 miles

3. For lunch there were 93 rolls. Mrs. Brown brought 48 rolls. How many rolls did others bring? 45 rolls

4. Mrs. Adams brought 4 cans of candy, 5 cans of peanuts, and 7 cans of other things. In all, how many cans did Mrs. Adams bring? 16 cans

5. Mrs. Wade had one box of things weighing 32 pounds and another box weighing 25 pounds. How many pounds heavier was the first box than the second box? 7 pounds

6. Eighteen of the 31 people who went on the trip left the park before the others. How many people stayed longer in the park? 13 people

Set 10

Up the Fish Ladder

For use after page 197

Hour	Number counted
First	213
Second	146
Third	84
Fourth	34
Fifth	18

The box at the left shows how many fish were counted going up a fish ladder in each of five different hours.

1. How many fish were counted (a) in the first hour? 213 (b) in the fifth hour? 18

2. How many fish were counted in the first hour and in the second hour together? 359

3. How many fish in all were counted in the third, fourth, and fifth hours? 136

4. How many more fish were counted in the first hour than in the second hour? 67

5. Of the 84 fish counted in the third hour, 49 went up the ladder in the first half hour. How many fish went up the ladder in the second half hour? 35

Set 11

The Milkman

For use after page 220

1. The milkman works 6 days each week. How many days does he work in 2 weeks? 12 days

2. One kind of milk costs 2¢ a quart more than another kind of milk. How much more would 8 quarts cost? 16¢

3. The milkman left 2 bottles of milk at each of 7 houses on Oak Street. How many bottles in all was that? 14 bottles

4. Don and Ed helped on the milk truck one day. The truck stopped at 9 houses on each of 2 streets. How many stops did the truck make on these streets? 18 stops

5. On Maple Street the boys left 6 quarts of milk at one house, 6 at another, and 5 at the last house. In all, how many quarts of milk did the boys leave there? 17 quarts

6. Don's mother gave the milkman $5.52 one week and $4.78 the next. How much more did she pay the first week than the second? $0.74

Set 12

The Picnic

For use after page 237

1. In each of the first 3 cars to get to the picnic grounds there were 5 people. How many came in these cars? 15 people

2. The boys were in pairs for a new kind of race. There were 8 pairs of boys in all. How many boys were there in the race? 16 boys

3. Seven groups of girls were in a potato race. There were 3 girls in each group. How many girls were there in this race? 21 girls

4. There were 7 boys, 6 girls, 8 men, and 5 women swimming. How many people was that in all? 26 people

5. At one table there were 2 men and 3 times as many boys as men. How many boys were there? 6 boys

6. Tickets for a glass of milk cost 5¢ each. How much did it cost to buy 3 tickets? 15¢

Set 13

Railroad Trains

For use after page 277

1. Mr. Tell's ticket to Atlanta cost $3.56. How much change did he get from $5.00? $1.44

2. At one town 93 tickets were sold last week. This week, 37 tickets have been sold. How many more tickets must be sold this week to make the two numbers equal? 56 tickets

3. Tickets costing $4.32, $1.76, and $2.47 were sold before nine o'clock in the morning. How much in all did these three tickets cost? $8.55

4. Mr. Bird bought 3 tickets to Montreal. How much did he pay if each ticket cost $3.09? $9.27

5. If a train runs 54 miles an hour for 3 hours, how many miles will it go? 162 miles

6. Twenty-eight tickets were collected in one car, 43 in the next car, and 36 in the third car. How many tickets were collected in the three cars? 107 tickets

Set 14

A Trip to the Shore

For use after page 295

1. On the way to the shore the 6 people in the Day family each ate 2 apples. How many apples did the Day family eat on the way to the shore? 12 apples

2. Only 2 people can sit at a table in the lunchroom at the shore. How many tables did the 6 people need? 3 tables

3. Joe found 12 little stones. If he gave 3 of them to one boy, 3 to another, and so on, how many boys would get stones? 4 boys

4. Peter Day saw 15 people get into boats, 3 people to a boat. How many boats did they use? 5 boats

5. Of the boats not used, 5 were red, 7 were white, and 5 were green. How many boats were not used? 17 boats

6. Mr. Day bought each of the 4 children a 59¢ toy. How much did he pay for all the toys? $2.36

7. Jane found 18 shells. She put them on a board, 3 in a row. How many rows did she have? 6 rows

Set 15

At Summer Camp

For use after page 314

1. It costs $7.00 to go to camp. Jan had $5.35. How much more money did she need? $1.65

2. The camp bus goes to town each day. The round trip is 34 miles. How many miles does the bus go in making 5 round trips to town? 170 miles

3. For the camp, Mr. Moore bought 35 pounds of meat, 43 pounds of potatoes, and 26 pounds of peas. How many pounds of food in all was that? 104 pounds

4. One day 36 children made up 4 groups of the same size for a game. How many were in each group? 9 children

5. Jan bought 4 books about camping. If they cost $1.49 each, how much did the 4 books cost? $5.96

Set 16

A New Store

For use after page 322

1. In a new store, 36 coats were in 4 rows with the same number in each. How many coats were in a row? 9 coats

2. One kind of toy was sold for $2.59 each. How much did 3 such toys cost? $7.77

3. There were 20 rings in a box, with 5 rings in each row. How many rows of rings were in the box? 4 rows

4. A dress cost $7.95. A hat cost $2.98. How much more did the dress cost than the hat? $4.97

5. How much did 4 pairs of 69¢ gloves cost? $2.76

Set 17

	a	b	c	d	e	f	g	h	i	j	k
1.	2	3	3	6	2	2	5	4	5	1¢	2¢
	+7 = 9	+6 = 9	+4 = 7	+4 = 10	+6 = 8	+4 = 6	+5 = 10	+3 = 7	+4 = 9	+3¢ = 4¢	+1¢ = 3¢
2.	7	4	6	4	7	3	8	5	2	6¢	1¢
	+3 = 10	+4 = 8	+3 = 9	+5 = 9	+2 = 9	+5 = 8	+2 = 10	+3 = 8	+5 = 7	+1¢ = 7¢	+4¢ = 5¢

Set 18

	a	b	c	d	e	f	g	h	i	j	k
1.	4	6	4	3	2	5	3	8	2	3¢	1¢
	+6 = 10¢	+2 = 8	+2 = 6	+7 = 10	+8 = 10	+2 = 7	+3 = 6	+1 = 9	+3 = 5	+1¢ = 4¢	+5¢ = 6¢
2.	1	5	1	7	3	1	9	2	4	1¢	1¢
	+9 = 10	+1 = 6	+6 = 7	+1 = 8	+2 = 5	+7 = 8	+1 = 10	+2 = 4	+1 = 5	+2¢ = 3¢	+8¢ = 9¢

Set 19

	a	b	c	d	e	f	g	h	i	j	k
1.	10	9	8	7	6	10	10	4	9	8¢	6¢
	−4 = 6	−6 = 3	−3 = 5	−5 = 2	−3 = 3	−5 = 5	−1 = 9	−2 = 2	−2 = 7	−1¢ = 7¢	−5¢ = 1¢
2.	5	7	10	6	8	9	7	4	9	5¢	4¢
	−2 = 3	−1 = 6	−9 = 1	−2 = 4	−7 = 1	−1 = 8	−6 = 1	−3 = 1	−8 = 1	−4¢ = 1¢	−1¢ = 3¢

Set 20

	a	b	c	d	e	f	g	h	i	j	k
1.	8	5	7	9	8	10	9	10	10	3¢	5¢
	−4 = 4	−3 = 2	−3 = 4	−7 = 2	−6 = 2	−3 = 7	−4 = 5	−2 = 8	−7 = 3	−2¢ = 1¢	−1¢ = 4¢
2.	9	7	8	9	6	8	7	10	10	6¢	3¢
	−5 = 4	−2 = 5	−2 = 6	−3 = 6	−4 = 2	−5 = 3	−4 = 3	−6 = 4	−8 = 2	−1¢ = 5¢	−1¢ = 2¢

Set 21

	a	b	c	d	e	f	g	h	i	j	k
1.	3	5	6	8	6	3	2	6	10	3¢	5¢
	+3 = 6	+2 = 7	−5 = 1	−5 = 3	−2 = 4	+7 = 10	+3 = 5	−3 = 3	−4 = 6	+6¢ = 9¢	+5¢ = 10¢
2.	2	7	9	4	9	10	4	10	6	9¢	10¢
	+8 = 10	−5 = 2	−6 = 3	+6 = 10	−2 = 7	−5 = 5	+2 = 6	−9 = 1	+2 = 8	−3¢ = 6¢	−8¢ = 2¢

Set 22

	a	b	c	d	e	f	g	h	i	j	k
1.	2	4	4	4	2	3	1	3	2	5¢	4¢
	6	1	3	2	2	4	5	5	1	3¢	1¢
	1 (9)	2 (7)	1 (8)	4 (10)	5 (9)	2 (9)	0 (6)	2 (10)	6 (9)	1¢ (9¢)	4¢ (9)
2.	2	3	2	6	1	1	6	4	3	3¢	1¢
	4	1	3	1	2	4	3	4	2	2¢	6¢
	2 (8)	6 (10)	5 (10)	3 (10)	3 (6)	4 (9)	1 (10)	1 (9)	3 (8)	5¢ (10¢)	2¢ (9¢)

Set 23

	a	b	c	d
1.	5 + 0 + 2 (7)	1 + 3 + 3 (7)	1 + 7 + 2 (10)	2¢ + 6¢ + 2¢ (10¢)
2.	2 + 5 + 2 (9)	3 + 0 + 2 (5)	3 + 3 + 2 (8)	4¢ + 2¢ + 4¢ (10¢)
3.	1 + 6 + 3 (10)	1 + 4 + 4 (9)	1 + 8 + 1 (10)	1¢ + 1¢ + 7¢ (9¢)
4.	2 + 7 + 0 (9)	2 + 3 + 3 (8)	3 + 4 + 1 (8)	3¢ + 1¢ + 4¢ (8¢)

Set 24

1.	5 + 1 + 3 (9)	5 + 2 + 3 (10)	5 + 4 + 1 (10)	2¢ + 4¢ + 4¢ (10¢)
2.	4 + 3 + 2 (9)	6 + 2 + 2 (10)	2 + 2 + 5 (9)	1¢ + 7¢ + 1¢ (9¢)
3.	1 + 7 + 0 (8)	4 + 2 + 2 (8)	2 + 3 + 2 (7)	5¢ + 2¢ + 2¢ (9¢)
4.	4 + 3 + 1 (8)	5 + 3 + 1 (9)	2 + 1 + 4 (7)	6¢ + 1¢ + 2¢ (9¢)

Set 25

	a	b	c	d	e	f	g	h	i
1.	11	3	72	13	66	21	6	47¢	74¢
	+5 (16)	+25 (28)	+5 (77)	+3 (16)	+2 (68)	+2 (23)	+13 (19)	+2¢ (49¢)	+3¢ (77¢)
2.	15	42	83	3	64	92	85	74¢	82¢
	+4 (19)	+7 (49)	+4 (87)	+46 (49)	+2 (66)	+4 (96)	+2 (87)	+4¢ (78¢)	+7¢ (89¢)

Set 26

	a	b	c	d	e	f	g	h	i
1.	11	52	3	98	73	64	54	21¢	7¢
	+7 (18)	+6 (58)	+42 (45)	+1 (99)	+6 (79)	+5 (69)	+4 (58)	+8¢ (29¢)	+52¢ (59¢)
2.	82	71	34	6	42	75	2	33¢	91¢
	+5 (87)	+4 (75)	+3 (37)	+41 (47)	+3 (45)	+4 (79)	+62 (64)	+5¢ (38¢)	+4¢ (95¢)

		a	b	c	d	e	f	g	h	i
Set 27	**1.**	43	35	21	52	91	12	71	11¢	82¢
		4	1	2	2	5	4	6	3¢	5¢
		2	2	5	5	1	3	1	4¢	1¢
		49	38	28	59	97	19	78	18¢	88¢
	2.	12	23	44	23	61	22	30	42¢	34¢
		3	2	1	1	1	1	3	6¢	3¢
		3	4	2	3	5	3	6	1¢	2¢
		18	29	47	27	67	26	39	49¢	39¢
Set 28	**1.**	31	23	63	50	51	81	22	40¢	62¢
		2	1	2	3	5	3	3	2¢	1¢
		5	4	2	3	3	5	4	7¢	5¢
		38	28	67	56	59	89	29	49¢	68¢
	2.	31	62	42	51	21	26	73	14¢	43¢
		1	1	2	7	4	1	4	2¢	2¢
		5	6	3	1	1	1	2	2¢	4¢
		37	69	47	59	26	28	79	18¢	49¢
Set 29	**1.**	25	56	20	24	30	61	73	20¢	25¢
		+30	+23	+14	+51	+10	+20	+10	+29¢	+63¢
		55	79	34	75	40	81	83	49¢	88¢
	2.	54	10	46	23	70	17	40	13¢	33¢
		+14	+15	+31	+72	+22	+40	+21	+73¢	+46¢
		68	25	77	95	92	57	61	86¢	79¢
Set 30	**1.**	66	52	21	54	67	18	20	23¢	72¢
		+30	+32	+44	+25	+22	+81	+78	+55¢	+26¢
		96	84	65	79	89	99	98	78¢	98¢
	2.	23	43	54	61	15	13	30	37¢	32¢
		+66	+34	+12	+11	+54	+21	+27	+31¢	+44¢
		89	77	66	72	69	34	57	68¢	76¢
Set 31	**1.**	18	19	29	38	27	45	76	59¢	36¢
		−3	−7	−4	−5	−3	−4	−3	−3¢	−5¢
		15	12	25	33	24	41	73	56¢	31¢
	2.	68	77	25	56	95	67	38	29¢	57¢
		−2	−5	−2	−4	−3	−4	−6	−2¢	−2¢
		66	72	23	52	92	63	32	27¢	55¢

		a	b	c	d	e	f	g	h	i
Set 32	1.	64	19	42	2	26	49¢	5¢	75	46¢
		+5 = 69	−5 = 14	+2 = 44	+16 = 18	−4 = 22	−3¢ = 46¢	+33¢ = 38¢	+4 = 79	+2¢ = 48¢
	2.	46	48	45	87	39	22¢	15¢	83	96¢
		+3 = 49	−6 = 42	+2 = 47	+2 = 89	−6 = 33	+5¢ = 27¢	−2¢ = 13¢	+4 = 87	−3¢ = 93¢
Set 33	1.	5	6	8	19	94	23	47	53¢	32¢
		−5 = 0	−0 = 6	−8 = 0	−0 = 19	−4 = 90	−0 = 23	−7 = 40	−3¢ = 50¢	−0 = 32¢
	2.	4	3	7	65	88	78	36	95¢	59¢
		−0 = 4	−3 = 0	−0 = 7	−5 = 60	−8 = 80	−0 = 78	−6 = 30	−0 = 95¢	−9¢ = 50¢
Set 34	1.	95	86	76	49	64	27	77	58¢	60¢
		−4 = 91	+3 = 89	−6 = 70	−0 = 49	+23 = 87	+42 = 69	−4 = 73	−8¢ = 50¢	+30¢ = 90¢
	2.	4	37	43	40	46	63	3	66¢	76¢
		+64 = 68	+31 = 68	−0 = 43	+50 = 90	+20 = 66	−3 = 60	+45 = 48	−3¢ = 63¢	−4¢ = 72¢
Set 35	1.	23	64	82	3	72	85	46	31¢	74¢
		+5 = 28	−4 = 60	−0 = 82	+36 = 39	+15 = 87	−3 = 82	−5 = 41	+28¢ = 59¢	−4¢ = 70¢
	2.	4	87	63	25	6	54	78	27¢	56¢
		+34 = 38	−3 = 84	+24 = 87	−5 = 20	+43 = 49	−0 = 54	−4 = 74	+32¢ = 59¢	−6¢ = 50¢
Set 36	1.	85	37	85	68	95	87	99	89¢	79¢
		−61 = 24	−25 = 12	−14 = 71	−18 = 50	−42 = 53	−76 = 11	−25 = 74	−53¢ = 36¢	−10¢ = 69¢
	2.	74	78	83	39	68	94	29	67¢	65¢
		−22 = 52	−14 = 64	−30 = 53	−11 = 28	−62 = 6	−84 = 10	−26 = 3	−40¢ = 27¢	−45¢ = 20¢
Set 37	1.	35	49	63	67	84	26	18	46¢	87¢
		−12 = 23	−24 = 25	−43 = 20	−21 = 46	−63 = 21	−15 = 11	−8 = 10	−16¢ = 30¢	−24¢ = 63¢
	2.	65	54	99	88	83	57	29	42¢	48¢
		−14 = 51	−10 = 44	−73 = 26	−23 = 65	−72 = 11	−36 = 21	−6 = 23	−2¢ = 40¢	−6¢ = 42¢

Set 38

	a	b	c	d	e	f	g	h	i
1.	42	65	18	54	68	76	33	95¢	24¢
	+50 = 92	−25 = 40	+50 = 68	+30 = 84	−37 = 31	−60 = 16	+45 = 78	−80¢ = 15¢	+43¢ = 67¢
2.	10	52	72	28	94	69	99	35¢	87¢
	+36 = 46	+43 = 95	−42 = 30	+21 = 49	−64 = 30	−17 = 52	−40 = 59	+2¢ = 37¢	−35¢ = 52¢

Set 39

	a	b	c	d	e	f	g	h	i
1.	2	5	5	6	3	7	61	53	34¢
	4	0	1	1	0	1	2	4	4¢
	1	1	2	5	6	0	2	0	1¢
	2 = 9	6 = 12	3 = 11	0 = 12	1 = 10	1 = 9	3 = 68	1 = 58	0 = 39¢
2.	22	4	0	2	44	50	81	42	20¢
	3	1	3	2	2	5	2	4	3¢
	3	5	1	3	3	2	4	2	6¢
	1 = 29	0 = 10	5 = 9	2 = 9	0 = 49	2 = 59	2 = 89	1 = 49	0 = 29¢

Set 40

	a	b	c	d	e	f	g	h	i
1.	9	15	13	4	7	17	15	9	11
	+8 = 17	−6 = 9	−8 = 5	+8 = 12	+6 = 13	−8 = 9	−7 = 8	+9 = 18	−6 = 5
2.	12	5	16	7	13	11	7	16	12
	−5 = 7	+9 = 14	−9 = 7	+7 = 14	−6 = 7	−8 = 3	+4 = 11	−8 = 8	−4 = 8

Set 41

	a	b	c	d	e	f	g	h	i
1.	3	2	5	3	2	53	84	5	41
	1	4	0	1	1	2	0	2	6
	4	0	2	2	4	0	3	1	0
	1 = 9	5 = 11	6 = 13	6 = 12	4 = 11	3 = 58	2 = 89	70 = 78	2 = 49
2.	4	3	2	6	5	60	91	4	22
	2	4	1	1	0	4	5	32	3
	0	2	0	2	2	3	0	3	2
	7 = 13	2 = 11	8 = 11	4 = 13	7 = 14	2 = 69	3 = 99	0 = 39	1 = 28

		a	b	c	d	e	f	g	h	i
Set 42	1.	64	36	8	58	49	53	67	24	68¢
		+4 = 68	+8 = 44	+47 = 55	+3 = 61	+7 = 56	+9 = 62	+4 = 71	+9 = 33	+8¢ = 76¢
	2.	61	5	7	8	37	15	87	16	54¢
		+8 = 69	+25 = 30	+35 = 42	+12 = 20	+8 = 45	+3 = 18	+3 = 90	+9 = 25	+8¢ = 62¢
Set 43	1.	3	8	9	4	6	6	7	1¢	7¢
		7	4	5	9	7	8	9	8¢	6¢
		9 = 19	9 = 21	8 = 22	8 = 21	9 = 22	7 = 21	6 = 22	9¢ = 18¢	8¢ = 21¢
	2.	2	6	2	5	8	13	7	5¢	3¢
		8	9	37	6	5	5	8	7¢	9¢
		45	4	6	9	6	8	9	6¢	6¢
		5 = 60	6 = 25	4 = 49	7 = 27	4 = 23	3 = 29	5 = 29	6¢ = 24¢	4¢ = 22¢
Set 44	1.	19	26	17	27	37	27	49¢	38¢	$0.39
		+65 = 84	+49 = 75	+46 = 63	+39 = 66	+53 = 90	+25 = 52	+37¢ = 86¢	+23¢ = 61¢	+0.29 = $0.68
	2.	28	45	79	58	39	18	49¢	76¢	$0.19
		+69 = 97	+47 = 92	+13 = 92	+16 = 74	+46 = 85	+74 = 92	+14¢ = 63¢	+17¢ = 93¢	+0.28 = $0.47
Set 45	1.	19	58	18	16	54	63	17¢	29¢	$0.41
		+52 = 71	+22 = 80	+0 = 18	+6 = 22	+3 = 57	+2 = 65	+50¢ = 67¢	+48¢ = 77¢	+0.20 = $0.61
	2.	30	37	48	67	31	46	28¢	53¢	$0.78
		+34 = 64	+14 = 51	+33 = 81	+27 = 94	+48 = 79	+28 = 74	+55¢ = 83¢	+29¢ = 82¢	+0.18 = $0.96
Set 46	1.	35	50	57	41	82	90	41¢	$0.27	86¢
		−8 = 27	−8 = 42	−9 = 48	−4 = 37	−8 = 74	−3 = 87	−7¢ = 34¢	−0.08 = $0.19	−9¢ = 77¢
	2.	63	32	31	55	22	21	60¢	$0.33	94¢
		−9 = 54	−6 = 26	−5 = 26	−6 = 49	−5 = 17	−6 = 15	−9¢ = 51¢	−0.08 = $0.25	−7¢ = 87¢

		a	b	c	d	e	f	g	h	i
Set 47	1.	29	35	42	86	39	64	31	84¢	$0.63
		−5 = 24	−7 = 28	−9 = 33	−3 = 83	−4 = 35	−6 = 58	−8 = 23	−5¢ = 79¢	−0.07 = $0.56
	2.	84	56	42	61	54	36	57	56¢	$0.89
		−9 = 75	−5 = 51	−5 = 37	−7 = 54	−4 = 50	−7 = 29	−4 = 53	−8¢ = 48¢	−0.07 = $0.82
Set 48	1.	39	57	83	65	33	54	92	$0.49	64¢
		+25 = 64	−24 = 33	−6 = 77	+8 = 73	−7 = 26	+29 = 83	−8 = 84	−0.22 = $0.27	+18¢ = 82¢
	2.	34	65	46	52	25	57	78	$0.29	95¢
		−6 = 28	+19 = 84	+30 = 76	−9 = 43	+16 = 41	−7 = 50	−36 = 42	−0.06 = $0.23	−5¢ = 90¢
	3.	8¢	15	3¢	5¢	4	9	0	$0.62	6¢
		6¢	1	5¢	3¢	10	2	2	0.04	14¢
		1¢	8	0	8¢	5	7	8	0.06	0
		2¢ = 17¢	4 = 28	7¢ = 15¢	1¢ = 17¢	8 = 27	4 = 22	5 = 15	0.03 = $0.75	8¢ = 28¢
Set 49	1.	83	65	91	24	73	64	48	51¢	$0.75
		−17 = 66	−49 = 16	−9 = 82	−11 = 13	−6 = 67	−27 = 37	−30 = 18	−26¢ = 25¢	−0.53 = $0.22
	2.	94	80	81	89	45	20	57	65¢	$0.90
		−15 = 79	−38 = 42	−48 = 33	−40 = 49	−7 = 38	−1 = 19	−28 = 29	−32¢ = 33¢	−0.84 = $0.06
	3.	86	62	75	42	23	24	42	32¢	$0.20
		−79 = 7	−57 = 5	−28 = 47	−18 = 24	−8 = 15	−6 = 18	−3 = 39	−9¢ = 23¢	−0.06 = $0.14
Set 50	1.	48	47	81	28	83	91	67	$0.98	88¢
		−19 = 29	−29 = 18	−6 = 75	−14 = 14	−9 = 74	−37 = 54	−40 = 27	−0.27 = $0.71	−29¢ = 59¢
	2.	86	50	34	72	32	40	36	$0.60	63¢
		−27 = 59	−27 = 23	−28 = 6	−60 = 12	−6 = 26	−2 = 38	−18 = 18	−0.05 = $0.55	−6¢ = 57¢
	3.	75	72	75	52	48	21	34	$0.30	86¢
		−66 = 9	−55 = 17	−29 = 46	−24 = 28	−8 = 40	−2 = 19	−9 = 25	−0.09 = $0.21	−30¢ = 56¢

Set 51

	a	b	c	d	e	f	g	h	i
1.	58	58	40	72	46	83	56	38¢	$0.49
	+36	−14	−20	−26	+29	−26	−24	−18¢	+0.34
	94	44	20	46	75	57	32	20¢	$0.83
2.	50	54	71	29	63	26	82	49¢	$0.41
	+48	−38	−39	+53	−37	+47	−39	+38¢	−0.26
	98	16	32	82	26	73	43	87¢	$0.15
3.	4¢	3	22	6	6¢	4¢	5	7¢	$0.15
	7¢	2	7	2	8¢	0	17	19¢	0.06
	3¢	1	2	0	3¢	8¢	4	2¢	0.03
	9¢	6	3	8	7¢	8¢	5	1¢	0.06
	23¢	12	34	16	24¢	20¢	31	29¢	$0.30

Set 52

	a	b	c	d	e	f	g	h	i
1.	12¢	46	12	26¢	35	23¢	44	16	21¢
	16¢	16	10	45¢	19	18¢	26	23	18¢
	17¢	12	32	10¢	22	21¢	16	13	25¢
	20¢	14	37	17¢	10	16¢	11	24	24¢
	65¢	88	91	98¢	86	78¢	97	76	88¢
2.	16¢	23	45	22¢	15	12¢	36	10	22¢
	27¢	18	16	19¢	48	11¢	14	24	15¢
	23¢	16	15	48¢	20	26¢	15	48	37¢
	13¢	40	23	10¢	14	30¢	24	17	20¢
	79¢	97	99	99¢	97	79¢	89	99	94¢

Set 53

	a	b	c	d	e	f	g	h
1.	$0.39	17	$0.38	25	$0.48	24	26	33
	0.09	25	0.16	39	0.05	17	15	19
	0.27	32	0.26	15	0.27	18	24	16
	0.10	14	0.04	14	0.15	28	28	24
	$0.85	88	$0.84	93	$0.95	87	93	92
2.	$0.07	32	$0.49	18	$0.28	$0.09	19	22
	0.28	29	0.17	19	0.02	0.12	28	18
	0.19	14	0.26	33	0.16	0.35	14	25
	0.27	19	0.03	21	0.29	0.18	32	19
	$0.81	94	$0.95	91	$0.75	$0.74	93	84

		a	b	c	d	e	f	g	h
Set 54	**1.**	245	$3.57	$3.28	469	205	652	562	$2.29
		+626	+5.04	+4.65	+426	+785	+239	+218	+3.67
		871	$8.61	$7.93	895	990	891	780	$5.96
	2.	$2.48	239	558	375	$4.39	536	215	$3.38
		+7.04	+533	+426	+409	+5.41	+325	+367	+2.29
		$9.52	772	984	784	$9.80	861	582	$5.67
	3.	285	$2.35	376	$2.26	358	366	647	$7.04
		+508	+3.36	+609	+4.44	+233	+306	+336	+2.59
		793	$5.71	985	$6.70	591	672	983	$9.63
Set 55	**1.**	474	586	229	563	468	$5.95	528	$8.67
		−231	−204	−126	−32	−326	−4.73	−217	−2.43
		243	382	103	531	142	$1.22	311	$6.24
	2.	469	897	609	787	889	$9.45	376	$7.78
		−55	−734	−104	−652	− 348	−2.41	−225	−2.35
		414	163	505	135	541	$7.04	151	$5.43
Set 56	**1.**	694	$9.85	$8.71	697	662	$8.91	582	$7.54
		−679	−3.47	−2.38	−452	−529	−8.03	−228	−3.27
		15	$6.38	$6.33	245	133	$0.88	354	$4.27
	2.	$8.58	581	465	373	$9.49	982	961	$4.53
		−3.39	−307	−450	−206	−6.37	−877	−429	−2.27
		$5.19	274	15	167	$3.12	105	532	$2.26
Set 57	**1.**	785	$8.75	$6.52	785	563	$9.23	545	$3.29
		−268	−3.46	−3.14	−224	−134	−1.09	−219	−2.18
		517	$5.29	$3.38	561	429	$8.14	326	$1.11
	2.	$6.52	470	567	484	$9.87	582	842	$7.74
		−3.25	−143	−420	−239	−2.59	−436	−219	−2.39
		$3.27	327	147	245	$7.28	146	623	$5.35
Set 58	**1.**	357	489	247	362	$0.79	$4.85	460	$4.66
		+424	−326	+535	−139	+5.16	+2.07	−248	+1.29
		781	163	782	223	$5.95	$6.92	212	$5.95
	2.	491	368	637	716	$6.41	$5.50	724	$7.45
		−258	−49	+136	+68	−2.27	−2.15	−208	−2.14
		233	319	773	784	$4.14	$3.35	516	$5.31

		a	b	c	d	e	f	g
Set 59	1.	454	367	726	873	$2.67	$5.81	$6.93
		−35	+303	−225	−726	+4.27	−2.49	−2.65
		419	670	501	147	$6.94	$3.32	$4.28
	2.	628	407	483	159	$7.41	$4.34	$8.62
		+366	+488	−38	+305	−5.24	+5.16	−4.48
		994	895	445	464	$2.17	$9.50	$4.14
Set 60	1.	713	456	632	365	544	221	$4.63
		+194	+443	+295	+463	+372	+588	+2.94
		907	899	927	828	916	809	$7.57
	2.	140	664	357	578	235	486	$3.42
		+699	+183	+562	+351	+362	+352	+2.85
		839	847	919	929	597	838	$6.27
Set 61	1.	$3.77	289	$1.85	568	$2.59	$3.75	$2.83
		+4.44	+482	+5.34	+257	+6.21	+3.67	+4.98
		$8.21	771	$7.19	825	$8.80	$7.42	$7.81
	2.	249	358	537	129	$2.27	$4.86	$5.76
		+289	+472	+165	+755	+3.98	+1.69	+2.48
		538	830	702	884	$6.25	$6.55	$8.24
Set 62	1.	449	$9.66	$5.39	719	$8.49	713	$6.49
		−155	−4.34	−1.84	−319	−6.78	−651	−2.83
		294	$5.32	$3.55	400	$1.71	62	$3.66
	2.	$7.30	917	826	419	$8.73	$4.27	$5.47
		−5.27	−483	−375	−277	−2.32	−3.92	−2.14
		$2.03	434	451	142	$6.41	$0.35	$3.33
Set 63	1.	583	$3.93	$4.72	879	$7.21	865	$4.56
		−289	−1.68	−2.89	−265	−3.84	−498	−0.57
		294	$2 25	$1.83	614	$3.37	367	$3.99
	2.	$6.48	521	762	438	$5.44	$2.82	$8.37
		−2.77	−235	−638	−238	−2.55	−1.56	−6.68
		$3.71	286	124	200	$2.89	$1.26	$1.69

Set 64

	a	b	c	d	e	f	g
1.	713	546	326	352	852	94	$4.32
	−192	+324	+259	−294	−76	+68	−0.84
	521	870	585	58	776	162	$3.48
2.	250	464	516	70	352	758	$4.36
	+689	+172	−282	−39	+364	+226	+2.86
	939	636	234	31	716	984	$7.22

Set 65

1.	502	$9.05	908	$8.29	$7.01	904	$5.09
	−124	−7.69	−617	−2.83	−4.49	−246	−2.63
	378	$1.36	291	$5.46	$2.52	658	$2.46
2.	902	905	$8.07	860	$6.12	809	$8.02
	−896	−557	−3.88	−645	−0.73	−537	−6.58
	6	348	$4.19	215	$5.39	272	$1.44

Set 66

1.	800	$7.06	$3.00	905	$6.00	$5.00	700
	−197	−0.48	−0.78	−239	−0.63	−1.89	−416
	603	$6.58	$2.22	666	$5.37	$3.11	284
2.	906	700	$4.00	600	$3.00	$6.00	807
	−667	−421	−0.85	−531	−2.54	−2.69	−298
	239	279	$3.15	69	$0.46	$3.31	509

Set 67

	a	b	c	d	e	f	g	h	i	j	k
1.	3	4	6	3	3	8	3	1	9	2	2
	×2	×3	×3	×3	×1	×3	×5	×3	×3	×2	×8
	6	12	18	9	3	24	15	3	27	4	16
2.	7	3	5	3	3	2	3	3	5	2	2
	×3	×6	×3	×8	×7	×3	×4	×9	×2	×7	×1
	21	18	15	24	21	6	12	27	10	14	2

Set 68

1.	4	6	4	4	8	4	3	4	3	7	2
	×2	×4	×4	×9	×4	×3	×4	×5	×6	×3	×9
	8	24	16	36	32	12	12	20	18	21	18
2.	5	9	4	1	4	2	7	4	4	3	8
	×4	×4	×7	×4	×8	×4	×4	×6	×1	×3	×3
	20	36	28	4	32	8	28	24	4	9	24

Set 69

	a	b	c	d	e	f	g
1.	681	163	445	136	$2.54	$3.72	$2.63
	172	455	363	684	5.46	2.91	1.85
	129	367	135	146	1.54	2.18	3.29
	982	985	943	966	$9.54	$8.81	$7.77
2.	255	146	324	217	365	138	$3.69
	177	328	246	135	159	364	2.38
	565	384	268	475	254	496	1.47
	997	858	838	827	778	998	$7.54

Set 70

	a	b	c	d	e	f	g	h	i	j
1.	84¢	43	32¢	73¢	62	42	61	53	82¢	91¢
	×2	×3	×4	×3	×2	×3	×4	×3	×3	×2
	$1.68	129	$1.28	$2.19	124	126	244	159	$2.46	$1.82
2.	93¢	92	31¢	93	83¢	52	82	41	63¢	32¢
	×2	×3	×5	×3	×3	×3	×4	×3	×2	×2
	$1.86	276	$1.55	279	$2.49	156	328	123	$1.26	64¢

Set 71

1.	51¢	72	53¢	81	82	92	71	23	81¢	31¢
	×3	×2	×2	×2	×3	×2	×3	×3	×4	×3
	$1.53	144	$1.06	162	246	184	213	69	$3.24	93¢
2.	94¢	82	61¢	63	72¢	62	42	31	52¢	82¢
	×2	×2	×2	×3	×3	×3	×3	×4	×4	×4
	$1.88	164	$1.22	189	$2.16	186	126	124	$2.08	$3.28

Set 72

1. 316 + 298 614
2. 3 × 63 189
3. 4 × 40 160
4. 159 − 63 96
5. 2 × 61¢ $1.22
6. 683 + 269 952
7. 800 − 263 537
8. 561 − 295 266
9. 4 × 52¢ $2.08
10. 2 × 62 124
11. 143 − 58 85
12. 83 × 3 249
13. 542 + 96 + 164 + 28 830
14. 77 + 408 + 45 + 357 887
15. 306 + 76 + 82 + 454 918
16. $0.58 + $1.67 + $4.87 $7.12
17. $0.93 + $6.46 + $1.15 $8.54
18. $4.18 + $0.62 + $1.68 $6.48

		a	b	c	d	e	f	g
Set 73	1.	\$0.57 × 2 = \$1.14	\$0.23 × 5 = \$1.15	48 × 3 = 144	85 × 4 = 340	\$0.23 × 9 = \$2.07	\$0.67 × 3 = \$2.01	\$0.44 × 3 = \$1.32
	2.	\$0.96 × 4 = \$3.84	32 × 7 = 224	32 × 8 = 256	\$0.75 × 2 = \$1.50	59 × 3 = 177	\$0.57 × 4 = \$2.28	\$0.79 × 3 = \$2.37
Set 74	1.	112 × 6 = 672	115 × 4 = 460	125 × 2 = 250	112 × 7 = 784	317 × 3 = 951	\$1.13 × 5 = \$5.65	\$4.17 × 2 = \$8.34
	2.	129 × 3 = 387	113 × 7 = 791	216 × 2 = 432	112 × 5 = 560	117 × 4 = 468	\$1.12 × 8 = \$8.96	\$1.19 × 3 = \$3.57
Set 75	1.	309 × 3 = 927	340 × 2 = 680	102 × 6 = 612	104 × 9 = 936	220 × 4 = 880	\$2.08 × 3 = \$6.24	\$3.07 × 3 = \$9.21
	2.	103 × 5 = 515	104 × 7 = 728	102 × 9 = 918	110 × 5 = 550	\$1.03 × 6 = \$6.18	103 × 8 = 824	\$1.08 × 2 = \$2.16
Set 76	1.	500 − 265 = 235	4 × 48 = 192	106 × 3 = 318	456 − 280 = 176	\$5.63	\$1.09	\$3.36
						2.17	0.66	1.18
						0.53	5.13	0.57
	2.	5 × 102 = 510	761 − 244 = 517	228 × 3 = 684	116 × 4 = 464	1.08	1.27	1.63
						0.54 = \$9.95	0.86 = \$9.01	0.79 = \$7.53
Set 77	1.	$8\overline{)24}^{3}$	$2\overline{)16}^{8}$	$4\overline{)8}^{2}$	$2\overline{)2}^{1}$	$3\overline{)27}^{9}$	$2\overline{)14}^{7}$	$3\overline{)18}^{6}$
	2.	$7\overline{)21}^{3}$	$6\overline{)18}^{3}$	$7\overline{)14}^{2}$	$5\overline{)15}^{3}$	$2\overline{)12}^{6}$	$9\overline{)18}^{2}$	$2\overline{)12}^{6}$
	3.	$3\overline{)6}^{2}$	$2\overline{)18}^{9}$	$6\overline{)12}^{2}$	$3\overline{)9}^{3}$	$4\overline{)12}^{3}$	$2\overline{)10}^{5}$	$3\overline{)3}^{1}$
	4.	$8\overline{)16}^{2}$	$3\overline{)21}^{7}$	$5\overline{)10}^{2}$	$9\overline{)27}^{3}$	$3\overline{)15}^{5}$	$3\overline{)24}^{8}$	$3\overline{)12}^{4}$

Set 78

	a	b	c	d	e	f	g
1.	168 × 3 = 504	123 × 8 = 984	309 × 3 = 927	3 × 239 = 717	123 × 7 = 861	284 × 3 = 852	256 × 3 = 768
2.	497 × 2 = 994	132 × 5 = 660	2 × 175 = 350	178 × 3 = 534	356 × 2 = 712	8 × 103 = 824	194 × 4 = 776
3.	385 × 2 = 770	103 × 9 = 927	295 × 3 = 885	468 × 2 = 936	247 × 4 = 988	132 × 6 = 792	208 × 4 = 832

Set 79

1.	$5\overline{)30}$ → 6	$9\overline{)36}$ → 4	$6\overline{)18}$ → 3	$7\overline{)35}$ → 5	$3\overline{)18}$ → 6	$5\overline{)15}$ → 3	$5\overline{)40}$ → 8
2.	$4\overline{)32}$ → 8	$8\overline{)24}$ → 3	$3\overline{)21}$ → 7	$9\overline{)27}$ → 3	$5\overline{)45}$ → 9	$3\overline{)27}$ → 9	$9\overline{)45}$ → 5
3.	$3\overline{)24}$ → 8	$3\overline{)3}$ → 1	$3\overline{)12}$ → 4	$7\overline{)21}$ → 3	$4\overline{)36}$ → 9	$9\overline{)18}$ → 2	$4\overline{)16}$ → 4

Set 80

1.	$2\overline{)48}$ → 24	$3\overline{)33}$ → 11	$2\overline{)64}$ → 32	$2\overline{)88}$ → 44	$3\overline{)63}$ → 21	$2\overline{)26}$ → 13	$4\overline{)48}$ → 12
2.	$3\overline{)93}$ → 31	$2\overline{)46}$ → 23	$3\overline{)66}$ → 22	$2\overline{)82}$ → 41	$3\overline{)96}$ → 32	$2\overline{)68}$ → 34	$2\overline{)66}$ → 33
3.	$3\overline{)36}$ → 12	$2\overline{)44}$ → 22	$4\overline{)84}$ → 21	$2\overline{)84}$ → 42	$2\overline{)22}$ → 11	$3\overline{)69}$ → 23	$3\overline{)39}$ → 13

Set 81

1.	$3\overline{)126}$ → 42	$6\overline{)186}$ → 31	$2\overline{)128}$ → 64	$9\overline{)279}$ → 31	$2\overline{)166}$ → 83	$3\overline{)\$2.19}$ → \$0.73	$8\overline{)168}$ → 21
2.	$2\overline{)188}$ → 94	$8\overline{)248}$ → 31	$3\overline{)246}$ → 82	$5\overline{)155}$ → 31	$7\overline{)217}$ → 31	$2\overline{)\$1.04}$ → \$0.52	$4\overline{)128}$ → 32
3.	$6\overline{)126}$ → 21	$2\overline{)142}$ → 71	$5\overline{)105}$ → 21	$3\overline{)159}$ → 53	$9\overline{)189}$ → 21	$4\overline{)\$2.88}$ → \$0.72	$5\overline{)255}$ → 51

Set 82

1.	$2\overline{)126}$ → 63	$3\overline{)186}$ → 62	$3\overline{)279}$ → 93	$2\overline{)186}$ → 93	$4\overline{)168}$ → 42	$6\overline{)\$1.86}$ → \$0.31	$9\overline{)369}$ → 41
2.	$9\overline{)189}$ → 21	$5\overline{)205}$ → 41	$2\overline{)148}$ → 74	$3\overline{)189}$ → 63	$5\overline{)155}$ → 31	$7\overline{)\$2.87}$ → \$0.41	$2\overline{)102}$ → 51
3.	$4\overline{)248}$ → 62	$4\overline{)124}$ → 31	$7\overline{)147}$ → 21	$7\overline{)357}$ → 51	$4\overline{)208}$ → 52	$4\overline{)\$3.24}$ → \$0.81	$4\overline{)204}$ → 51
4.	$3\overline{)243}$ → 81	$6\overline{)246}$ → 41	$8\overline{)328}$ → 41	$4\overline{)284}$ → 71	$3\overline{)123}$ → 41	$4\overline{)\$3.64}$ → \$0.91	$5\overline{)405}$ → 81

Set 83	Find out about other kinds of clocks—water clocks, candle clocks, and so on.	See any Children's Encyclopedia
Set 84	One kind of addition is done by using a clockface. Start at 12 and count to the right for the first addend. Then from that numeral count on, as the clock hand moves, for the second addend to find the sum. $4 + 5 = 9$ $5 + 8 = 1$ $4 + 9 = ?^{1}$ $11 + 8 = ?^{7}$ $8 + 3 = 11$ $7 + 9 = ?^{4}$ $10 + 5 = ?^{3}$ $6 + 9 = ?^{3}$ $6 + 6 = 12$ $8 + 6 = ?^{2}$ $7 + 7 = ?^{2}$ $9 + 7 = ?^{4}$	
Set 85	Try writing each of problems 1–6 on page 42 to make it an A. problem if it is now a S. problem and a S. problem if it is now an A. problem.	Answers will differ.
Set 86	For each of Ex. 13–21 on page 57, write the answer as a sentence using greater instead of less or less instead of greater. For Ex. 13, you would write, "20 is greater than (10 + 8)."	Number names will be transposed.
Set 87	Write each number sentence in rows 2–4 on page 77 so that, using =, what it says will be true. If what it says is now true, copy it as it is.	Answers will differ.
Set 88	See if you can make a set of 10 other problems using the table on page 81.	Answers will differ.
Set 89	Write Ex. 1–14 at the top of page 98 so that for each you can use the words is equal to.	Answers will differ.
Set 90	Write the *If* . . ., *Then* . . . examples on page 98 so that for each example you can write "T."	Answers will differ.

Set 91	Look up the word mile to find out about the difference between a mile on land and a mile on the sea or in the air.
Set 92	Write each of the n-examples on page 131, so that it shows n in three different places. For Ex. 1, write, "$n + 6 = 51$, $45 + n = 51$, $45 + 6 = n$."
Set 93	See if you can find out why dozen means twelve.
Set 94	Look up Weights or Measures and find some things to tell others about pounds and ounces.
Set 95	Write two different names for each number shown in rows 9–10 on page 167.
Set 96	Using A. and S., write four whole-number names for each of the following: 1. 134 4. 649 7. 295 2. 483 5. 312 8. 921 3. 758 6. 876 9. 567
Set 97	Try adding and subtracting with Roman numerals: a. X minus III VII d. X minus V V b. VI plus V XI e. IX minus VIII I c. VIII plus IV XII f. XII minus VII V
Set 98	Make up a set of five problems which have numbers you do not need to use in solving the problems.

Land mile 5,280 ft. Nautical mile (for sea and air), 6,076.1033 ft.

Work each example as in sample.

The French word *douze* means *twelve*.

See any encyclopedia.

Answers will differ.

Answers will differ.

Answers will differ.

Set 99

Try to make a magic square of your own by adding 21 to each of the numbers shown in the magic square for Ex. 1 on page 195. Check to see if it is a magic square. Make other magic squares.

Add 10 or 100, and so on to each number shown in Ex. 1.

Set 100

Find out about other uses for thermometers. See if you can find out how a thermometer is made. Also find when the freezing point is labeled 0 and the boiling point 100°. Centigrade thermometer

Thermometers:
candy
oven
clinical
electric

Set 101

Using A., S., and M., write three whole-number names for each of the following:

1. 123	**4.** 756	**7.** 435	**10.** 294
2. 314	**5.** 942	**8.** 378	**11.** 423
3. 862	**6.** 536	**9.** 522	**12.** 684

Answers will differ.

Set 102

Study the M. tables on page 242 to see what kind of number—even or odd—you get when you multiply two even numbers or an even number and an odd number. Then, without multiplying, write "even number" or "odd number" to tell what kind of number you think the product is for each of these:

1. 7×8 even	**5.** 28×32 even	**9.** 78×96 even
2. 9×6 even	**6.** 45×64 even	**10.** 285×172 even
3. 6×7 even	**7.** 35×84 even	**11.** 327×128 even
4. 16×25 even	**8.** 46×57 even	**12.** 462×534 even

Set 103

$$\begin{array}{r} 42 \\ ? \\ \hline 3?6 \end{array}$$

Make a new set of examples from rows 1–5 on page 260. In each example, show two ? marks in place of two digits. Ex. 1a might be written as at the left. Make sure that the example can be worked after you have written it.

Answers will differ.

Set 104

You have written the tables of D. facts for dividing by 2 through $18 \div 2 = 9$ and with answer 2 through $18 \div 9 = 2$. Now, beginning with $20 \div 2 = 10$, make the table for dividing by 2 through $36 \div 2 = 18$. Beside that table, start with $20 \div 10 = 2$ and make the table with answer 2 through $36 \div 18 = 2$.

$20 \div 2 = 10$ $20 \div 10 = 2$
$22 \div 2 = 11$ $22 \div 11 = 2$
$24 \div 2 = 12$ $24 \div 12 = 2$
$26 \div 2 = 13$ $26 \div 13 = 2$
$28 \div 2 = 14$ $28 \div 14 = 2$
$30 \div 2 = 15$ $30 \div 15 = 2$
$32 \div 2 = 16$ $32 \div 16 = 2$
$34 \div 2 = 17$ $34 \div 17 = 2$
$36 \div 2 = 18$ $36 \div 18 = 2$

Set 105

Study the D. tables at the top of page 293 and then copy and finish the following:

When an even number is divided by an even number, the answer is an (even or odd) number.

When an odd number is divided by an odd number, the answer is an (even or odd) number.

When an even number is divided by an odd number, the answer is an (even or odd) number.

Set 106

See if you can show that you cannot divide by zero. To help you, use a set of M. and D. facts with zero as a factor.

$0 \times 3 = 0$, so we would expect $0 \div 3 = 0$ and $0 \div 0 = 3$

Set 107

Study the tables of D. facts at the top of page 308 to see if you can find out why the product of an even number and an even number and also the product of an even number and an odd number are both even numbers.

Any number of even-numbered sets makes an even-numbered total.

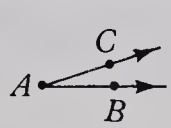
Symbols

=	is equal to	page 8*	ϕ	ten	page 4	>	is greater than page 98
$\neq$	is not equal to	page 27	|	one	page 4	<	is less than page 98
			()	parentheses	page 55		

Terms with Illustrations

Addend (pages 10, 15, 16)*
For $3 + 5 = 8$, 3 and 5 are addends.
For $8 - 3 = 5$, 3 and 5 are addends.

Angle (*See* page 257)



Array (page 206)

Associative Property (page 75)
For $5 + 2 + 8$, $(5 + 2) + 8 = 5 + (2 + 8)$

Binary operation (pages 10, 16, 204, 274)
A., S., M., and D. can be performed on only two numbers at a time, so we call them binary operations.

Closure (page 10)
The set of whole numbers has the property of closure for A. and M., but not for S. and D. For A., S., M., and D., only when *whole* numbers are added or multiplied is the answer always a *whole* number.

Commutative Property (pages 9, 206)
$2 + 3 = 3 + 2$ $\quad 2 \times 3 = 3 \times 2$

Digit (page 4) (base-10 system)
The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9

Distributive Property (pages 248, 334)
$2 \times (4 + 30) = (2 \times 4) + (2 \times 30)$
$(40 + 8) \div 2 = (40 \div 2) + (8 \div 2)$

Factor (page 207)
For $2 \times 3 = 6$, 2 and 3 are factors.
For $6 \div 2 = 3$, 2 and 3 are factors.

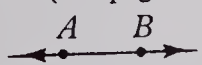
Fraction (pages 140, 283)
The symbols $\frac{1}{2}$, $\frac{2}{3}$, $\frac{11}{12}$, and so on

Graph (*See* page 317)

Identity element (pages 38, 227)
for A. is 0. $7 + 0 = 7$ and $0 + 7 = 7$
for M. is 1. $1 \times 6 = 6$ and $6 \times 1 = 6$

Inverses (pages 15, 283)
$7 + 4 = 11$; $11 - 4 = 7$
$3 \times 5 = 15$; $15 \div 3 = 5$

Line (*See* page 256)



This is read *line AB* or *line BA*.

Number (pages 2, 140)
A number is an idea.
Some counting numbers are 1, 2, and 3.
Some fraction numbers are $\frac{1}{2}$, $\frac{3}{8}$, and $\frac{11}{12}$.
Some whole numbers are 0, 1, 2, and 3.

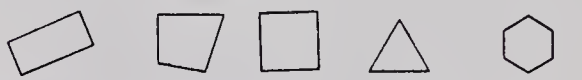
Number names (page 99)
$2 + 3$, 5, and $9 - 4$ each name the same number.

Number sentence (page 8, 77)
True sentences: $3 + 4 = 7$; $9 - 5 < 6$
False sentences: $3 + 4 \neq 7$; $10 + 2 > 15$
Open sentences: $n + 5 = 9$; $6 + 4 = ?$

Numeral (pages 2, 4, 166, 180, 342)
A numeral is a name for a number.

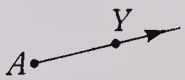
Path (*See* page 120)

Polygon (*See* page 121)



Product (pages 202, 283)
For $2 \times 3 = 6$, 6 is the product.
For $6 \div 2 = 3$, 6 is the product.

Ray (*See* page 256)



This is read, *ray AY*.

Renaming (pages 101, 123)
For addition:
2 tens and 12 ones = 3 tens and 2 ones
For subtraction:
3 tens and 5 ones = 2 tens and 15 ones

Segment (*See* page 256)

This is read *line segment AB* or *line segment BA*.

Set (pages 1, 3, 18, 202, 284)
A set is a well-defined collection of things such as objects or numbers or facts.

Simple closed curve (*See* pages 120–121)

A simple closed curve starts and ends at the same point and does not cross over itself.

Sum (pages 8, 15)
For $2 + 4 = 6$, 6 is the sum.
For $6 - 2 = 4$, 6 is the sum.

*Sampling of pages where idea used.

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